AM 609 HOMEWORK ASSIGNMENT 2 Spring 2018

1.

Homework Problem 2.

✓ **No. 4**. A truncation of a distribution function F can be defined for $a < b$ as

$$\tilde{F}(x) = \begin{cases} 0, & \text{if } x < a \\ F(x), & \text{if } a \le x < b \\ 1, & \text{if } b \le x \end{cases}$$

$x \in [a, b)$

Find a method for generating from the distribution function $\tilde{F}$, assuming that we already have a method for generating from F. Demonstrate the validate of your algorithm.

No. 8. In each of the following cases, give an algorithm that uses exactly "*one*" random number for generating a random variate with the same distribution as X.
(a) $X = \min\{U_1, U_2\}$, where U_i's are i.i.d. $\sim U(0, 1)$.
(b) $X = \max\{U_1, U_2\}$, where U_i's are i.i.d. $\sim U(0, 1)$.
(c) $X = \min\{Y_1, Y_2\}$, where Y_i's are i.i.d. $\sim \exp(1/\beta)$.

No. 10. For the acceptance-rejection method, find the distribution of the number of (Y, U) pairs that are rejected before acceptance occurs. What is the expected number of rejections?

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Lecture Note 4

(a) Draw flowcharts for the main programs of Example 1 and 2.

(b) A system has 10 components, each of which may be either up, down or on standby. At time t=0, 8 components are up and 2 are standby. The system is operational as long as there are 8 components that are up. When a component ① failure occurs, a unit on standby is activated (provided that a spare unit is available), to bring the number of operational components up to 8. When a unit fails, it enters a single-server repair shop to be repaired one at a time. Furthermore, when a system failure occurs, the system shuts down (i.e. components are either under repair or sit idle) until 8 operational units are again available. Note that when the system is down, the remaining life times of those up components are frozen until the system is brought up again. Suppose that we are interested in the long-run fraction of time that the system is operational.

機器 shut down 元件剩7個 損壞時間要暫停.

Lecture Note 2. p.5. Composition

Assume that the repair time distribution is hyper-exponential, i.e., exponential with mean 12 hours with probability 0.2 or exponential with mean 4 hours with probability 0.8. Simulate this system for 10 years to estimate the desired fraction efficiently (to make efficient use of both computer time and memory) for if

→ $24 \times 365 \times 10$

(i) the failure occurs according to an exponential distribution with mean 48 hours.

$\exp(48)$

$\frac{k}{\lambda}$

(ii) the failure occurs according to a 2-Erlang distribution with mean 48 hours.

$48 = \lambda$

see Ex6. Lecture

Gamma(2) $E(G) = 48$.

$E(X) = 48$ $k = 2$

$48 = \frac{k}{\lambda} = \frac{2}{\lambda} \Rightarrow \lambda = \frac{1}{24}$

① Flow chart

② State variable

③ Statistics counters

④ Events

state n = n down comp.

$S = \{0, 1, 2, 3\}$

$Z = \sum_{i=1}^{2} X_i$

$X_i \sim \exp(k\lambda)$

AM 609 ASSIGNMENT 5 Spring 2018

An airplane lands with N passengers aboard and reaches the arrival gate at time 0. The time for the first arriving passenger to reach the baggage claim area has an exponential distribution with mean 1/N and the interarrival time between the (k-1)st and kth passenger. passenger to reach the baggage claim area is exponentially distributed with mean $1/(N-k+1)$, k=1,2,...,N. Baggage reaches the claim area according to the following stochastic process: an Baggage exponentially distributed time Z (mean $1/u$) → 去到門口 接一堆 elapses following airplane landing. Thereafter, at regular time intervals of duration c (>0), → 放到輸送帶 suitcases are deposited one at a time in the claim area. Thus, the kth suitcase arrives in the claim area at time $Z + kc$ → 行李到達 passenger的地方 for k = 1,2,...,N.

行李做洗牌

Assume that each passenger has one suitcase and that the arrival order of suitcases in the claim area is random in the sense that all permutations are equally likely. Also assume that once a passenger's suitcase arrives and the passenger is present, the suitcase is immediately removed. Denote by $\boxed{T}$ the total amount of time until all passengers have obtained their baggage and let $\boxed{D_k}$ by the delay experienced by the kth passenger to arrive in the claim area, k=1,2,...,N.

state → 要包含哪些資訊

(a) What continuous time stochastic process $\{X(t): t > 0\}$ would you simulate to estimate E(T) and E(max$\{D_1, D_2, ..., D_N\}$)? Specify the state space $\boxed{S}$ of the process.

(b) Specify the events associated with simulation of the process, the state transition mechanism, and the event scheduling mechanism.

Ex 5. Ex 6. 為例，參考.

下週三

AM 609 ASSIGNMENT 6 Spring 2018

Continuation of Assignment 5: Consider N = 150, u = N/2 and c = 1/N.

(a) Estimate E(T) and E(max{D_1,D_2,...,D_N}) with 10,000 replications, construct the estimator with 90% and 95 % confidence intervals.

(b) Redo Part (a) with 1,000,000 replications.

$|\alpha_n - \alpha| \le 0.01$

(c) If the acceptable relative error is 1% under 95% confidence, construct a simulation run to yield the estimator and the confidence interval. How large is the sample size to achieve that?

$$n \ge \frac{V_n z_\delta^2}{\varepsilon^2 \alpha_n^2}$$

~~δ = 0.95~~

delta 95 = 0.05

$\varepsilon = 0.01$

AM 609 ASSIGNMENT 7 Spring 2018

Consider a telephone system with N (>1) telephones connected to a switchboard by *lines* numbered 1,2,...,N. The switchboard has M (<N) *links* numbered 1,2,...,M, each of which can be used to connect any two lines, subject to the restriction that only one connection can be made to each line. If more than one link is available, a placed call is connected (instantaneously) on the lowest numbered available link. Assume that the system is a "lost-call" system: any call that can not be connected when it is placed is immediately abandoned. A call is lost if there is at least one link available but the called line is in use (a *busy call*) and a call is lost if no link is available (a *blocked call*).

有電話進來 找空閒且 編號最小 的 link m

Assume that the duration of a call placed at line i is a positive random variable L_i having finite mean, i = 1,2,...,N. Also assume that calls are placed randomly in the sense that the time from the end of a call (placed, losed or received) at line i to the next call placed at line i is a positive random variable A_i; with (independent) probability P_{ij} the called line is line j, where $j \neq i$. After a lost call placed at line i the time to the next call placed at line i is distributed as A_i.

有通電話 佔線時間

電話掛掉之後隔多久 撥電話 or 電話來

(a) Define a continuous time stochastic process {X(t): t>0} that you would simulate to estimate the fraction of calls that are successfully completed, busy or blocked. Specify the process {X(t):t>0} as a generalized semi-Markov process.

(b) Write a program to simulate this model for M = 4 links and N = 10 lines. Suppose that (for all i) A_i is exponentially ($\lambda=2$) distributed with E(A_i) = 0.5 and that L_i has a gamma distribution with shape parameter k = 0.25 and scale parameter l = 1.0 (λ). Also suppose that P_{ij} = 1/(N-1) for all $j \neq i$ (P_{ii} = 0).

(c) Obtain point estimates for the limiting probability that (i) a call is successfully completed; (ii) a call is busy and (iii) a call is blocked. Also obtain a point estimate and a 95% confidence interval for the limiting probability that no links are available.

Ans: 是 should

Check (a) 做出的是否為 regenerative process

P.11 Thm 2

(1) serving

0

customer [S_1 . . . . S_4]

(i-1) waiting

AM 609 ASSIGNMENT 8 Spring 2018

Exp(4), mean $= \frac{1}{4}$

理論值 E(W) P(N=2) 已知 Exp(5), mean $= \frac{1}{5}$

$\frac{1}{100000}\sum_{i=10000}^{110000} W_i$

Consider a single-server system with Poisson arrivals at rate 4/min and exponential service with rate 5/min, an M/M/1 queue. 用不同方式看 Var 的變化

原: Customer average

Let W_i be the waiting time in the system of the ith customer and $N(t)$ be the number of customers in the system at time t. Let the system state at t=0 with "one" arrival finding the system empty.

(a) Form a non-terminating simulation to estimate E(W_∞) with a warm-up period of 10,000 customers and a production period for 100,000 customers. not iid 不能做 C.I

前面10000個不要

(b) Use batch means with 5/batch for 20,000 batches and 10/batch with 10,000 batches to estimate E(W_∞) with 95% confident intervals 不用重新生成

平均 busy cycle times $E(K) = \frac{1}{1-\rho} = \frac{1}{1-\frac{4}{5}} = \frac{1}{0.2} = 5$

(c) Use ratio estimator by 20,000 "regenerative cycles" with 95% confidence interval. In the case, you will need to identify regeneration cycles of this system. 存起來 (d)用 ①講清楚 M/M/1 是 regenerative cycles

(d) Use the data collected in (c) to form a Jack-Knife estimator with 95% confidence interval. $\bar{r}_n = \mu + \frac{c_1}{n} + \frac{c_2}{n^2} + \cdots$ [0.9707448, 1.0783291]

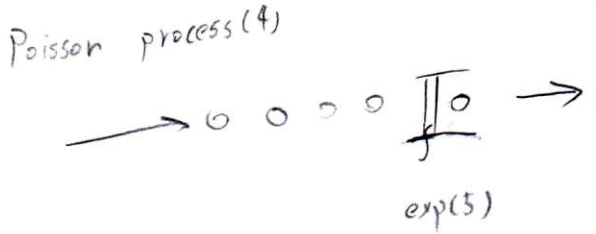
(e) Do (a) ~ (d) for P($N(\infty)$=2) with (a) warm-up = 2,000 min, production = 20, 000 min, (b) 1min/batch for 20,000 batches and 2min/batch for 10,000 batches, (c) and (d) are the same.

改用 time average

$\lim_{t\to\infty} P(N(t)=2)$ limiting prob

M/M/1 Queue

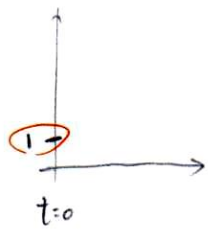
$\lim_{t\to\infty}\frac{t(2)}{t} = P(N(\infty)=2)$



$\frac{1}{t}\int_0^t I[N(s)=2]\,ds$

$W_i \to W_\infty$

$N(t)$



Hint: e_1: arrival

e_2: service completion

event list

$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$

有人在被服務才有

AM 609 ASSIGNMENT 9 SOLUTION Spring 2018

1. First of all, we need to show that $exp(U^2)(1 + exp(1\text{-}2U))/2$ is an unbiased estimator, which can be done by taking its expectation and realizing that $1\text{-}U$ has the same distribution as U. Secondly, since $Cov(exp(U^2), exp((1\text{-}U)^2))$ is negative, we have

$$Var(\ exp(U^2)(1 + exp(1\text{-}2U))/2\ ) \leq Var(\ [exp(U_1^2) + exp(U_2^2)]/2).$$

2. (a) We can simply generate 5 uniform random numbers, U_1, U_2, U_3, U_4, U_5, then set

$$I_1 = 1 \text{ if } -(\ln U_1 + 2\ln U_2 + 3\ln U_3 + 4\ln U_4 + 5\ln U_5) \geq 21.6;\ 0 \text{ otherwise.}$$

We then repeat this procedure for n times and estimate the probability by $\sum I_i/n$.
(b) For each set of $(U_1, U_2, U_3, U_4, U_5)$, we also compute

$$I'_1 = 1 \text{ if } -(\ln(1\text{-}U_1) + 2\ln(1\text{-}U_2) + 3\ln(1\text{-}U_3) + 4\ln(1\text{-}U_4) + 5\ln(1\text{-}U_5) \geq 21.6;$$
$$0 \text{ otherwise.}$$

We then repeat (a) and (b) for $n/2$ times and estimate the probability by $\sum(I_i + I'_i)/n$.
(c) Not only having reduced variance as shown in class, we also generate $n/2$ less uniform random numbers. Therefore, the use of antithetic variable is efficient here.

3. (a) We need to compute several things before to calculate the variance reduction. Those are

$$Cov(X, I) = E(XI)\text{-}E(X)E(I) = (a^2\text{-}a)/2,$$
$$Var(I) = a\text{-}a^2 \quad and \quad Var(X) = 1/12.$$

Thus, the variance reduction $Cov^2(X,I)/(Var(X)Var(I))$ can be calculated accordingly.
(b) Just follow the same procedure as in (a).
(c) It can be seen that, for fixed constant a, large X implies small I, namely, 0. On the other hand, if we decrease X, I will be more likely to be large, i.e., $I = 1$. So, they are negatively correlated.

4. First we note that

$$Z(\lambda) = h(Y) + \lambda[X - h(Y)].$$

We want to minimize the variance of $Z(\lambda)$, or equivalently, its second moment:

$\because E(X) = E(h(Y)) = \alpha$

$\parallel$

$E(E(X|Y))$

AM 609 ASSIGNMENT 10 Spring 2018

Consider 10 elements that are initially arranged in the order of increasing indexes. At each time, a request is made for one of these elements--the ith element being requested, independently of the past, with probability P(i). After being requested the element is then moved according to some policy. Assume that P(1)=0.06, P(2)=0.01, P(3)=0.12, P(4)=0.14, P(5)=0.08, P(6)=0.18, P(7)=0.04, P(8)=0.10, P(9)=0.22, P(10)=0.05.

There are three policies:

Policy A: After being requested, the element is then moved to the front of the list. 叫出來後放最前面

Policy B: After being requested, the element is move one position closer to the front. 叫出來後往前一個

Policy C: A policy of your own. $\frac{1}{n}\sum_{j=1}^{n} x_j$ ← 第j次叫到第幾個順位的書

以前n次判斷要移幾格

Please use simulation to estimate the expected positions of the element requested after this process has been in operation under these policies for a long time. In simulation, we can use the first 10,000 requests to wash out the initial effect, then start collecting statistics from then on."

Lecture Note 6 p.2 去比較，用 two stage Procedure

(a) Use confidence intervals of the estimators to compare these policies. 沒有 constraint.

(b) With a constraint on the number of requests, use the "two-stage procedure" to select the best policy, where $p^* = 0.95$, $d^* = 0.1$ and n_0=40. 用 P.3

$h=2.786$. By Text Book.

下下禮拜三交 X 在香港 $\boxed{\text{下禮拜五交}}$ 6/15.

AM 609 ASSIGNMENT 11 Spring 2018

Coefficient Test

找到 $\boxed{F_e}$

(a) For a given (ordered) set of numbers of size 10,

66.2, 72.4, 81.0, 94.8, 112.0, 116.4, 124.3, 140.1, 145.2, 155.8, r.n.g.

please conduct the Kolmogorov-Smirnov test for the null hypothesis that the values are generated from an exponential distribution with mean 100.

(b) Use your exponential random number generator to generate 10 numbers, then do the Kolmogorov-Smirnov test to find the p-value. 再檢查 random number generator

$\lambda = \frac{1}{100}$ c.d.f. $= 1 - e^{-\lambda x}$

$D = \underline{66.1} (= d)$

H_0 : $D \equiv \max_x |F_e(x) - F(x)|$

期末考!!

* Variance Reduction
* Output Analysis

$I_i \{D > d\}$

$\frac{\sum_{i=1}^{r} I_i}{r} \sim$ pvalue