(3rd Assignment is due on March 22. The paper work must be submitted on the class.)

To estimate $I = \int_a^b f(x)dx$, assume that $X_1, X_2, ..., X_n$ are independently sampled from some distribution wiht the p.d.f $p(x) : [a, b] \to R^+$, and monte carlo integration is defined by $\hat{I}_M = \frac{1}{n}\sum_{i=1}^n \frac{f(X_i)}{p(X_i)}$. Please show

(1) $E(\hat{I}_M) = I$;

(2) Please show that $\hat{I}_M \to^p I$ when $E(\frac{f(X)}{p(X)})^2 < \infty$, where X is a random variable with p.d.f $p(x)$. (Hint: Chebyshev's inequality);

(3) Please apply monte carlo integration for $\int_0^1 \exp(-x^2)dx$ with the uniform samples (Use: "runif: in R to generate data from U(a,b))

(4) Suppose $f(x) = x^{-1/3} + \frac{x}{10}, 0 < x \leq 1$. When $p_1(x) = 1, 0 < x < 1$, and $p_2(x) = \frac{2}{3}x^{-1/3}, 0 < x \leq 1$, please compare their variances of monte carlo integrations, and interpret what we earn.