The final exam. The paper work must be submitted between 10:00 am - 11:59 am of July 3rd, and the problem of the oral test will be given during the mentioned shedule. Grade: paper work (50%)+the oral test (50%).

(1) Assume thay $(Y_1, Y_2, Y_3, Y_4) \sim$ multinomial with $(0.5-0.5\theta, 0.25\theta, 0.25\theta, 0.5)$. When the incomplete data $(y_1, y_2, y_3+y_4) = (38, 34, 125)$ are observed, please use EM algorithm to estimate θ with the initial value =0.5.

Assume that E-Step is replaced by MC step (Monte Carlo integration). Please compare the convergence property with the previous result, and whether the monotonical property for log-likelihood function still holds.

(2) Assume that $X_1, X_2, ..., X_n \sim^{i.i.d} \pi(x|\theta)$, where $\pi(x|\theta) = \sum_{j=1}^{3} p_j \phi_{\sigma_j}(x-\mu_j)$, $\phi_\sigma(x-\mu) = \frac{1}{\sigma}\phi(\frac{x-\mu}{\sigma})$, ϕ is the p.d.f of the standard normal distribution and $\sum_{j=1}^{3} p_j = 1$.

(a) Please estimate the unknown parameter $(\mu_i, \sigma_j, p_j)_{j=1}^{3}$ by EM algorithm. (Hint: introduce the latent variable Z_i to present the label information. For example, $Z_i = j$ if the ith observation (X_i) comes from the jth component.)

(b) Please provide (i) a simulation scheme for the mentioned mixture model. (ii) Simulate data by the proposed simulation scheme, and check your EM performanace based on the simulated data.

(3) Apply Metropolis-Hasting algorithm with $g(\cdot|x) \sim U(x-\epsilon, x+\epsilon)$ to simulate data from N(0,1).

(i) Please check the MC chain performance under different ϵ and the initial value x_0.

(ii) How to estimate $E(\exp(Z^{16}))$ by (i), where Z is the standard normal.

(Hint of (i): You could discuss the MC convergence and burn-in performance. Please refer to "ConceptMarkov chain Monte Carlo.pdf" and "Convergence and error estimation.pdf" sent by 6/20 email.)