2. $X \sim N(\theta,1)$

$\theta \sim \frac{1}{\pi} \cdot \frac{1}{1+\theta^2}$

$$P(\theta|X) = \frac{P(X|\theta)\cdot P(\theta)}{P(X)}$$

claim: $E(\theta|X)$ and $Var(\theta|X)$

$P(\quad|\theta) \sim N(\theta,1)$

$$P(x) = \int_{-\infty}^{\infty} p(x|\theta)\, p(\theta)\, d\theta$$

①

$$E(\theta|X) = \int_{-\infty}^{\infty} \theta \frac{P(x|\theta)\cdot P(\theta)}{\int_{-\infty}^{\infty} P(x|\theta)\cdot P(\theta)\, d\theta} d\theta$$

$$= \int_{-\infty}^{\infty} \theta \frac{\frac{1}{\sqrt{2\pi}} e^{-\frac{(x-\theta)^2}{2}} \cdot \frac{1}{\pi} \cdot \frac{1}{1+\theta^2}}{\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-\theta)^2}{2}} \cdot \frac{1}{1+\theta^2}\, d\theta} d\theta$$

$$= \int_{-\infty}^{\infty} \theta \frac{e^{-\frac{(x-\theta)^2}{2}} \cdot \frac{1}{1+\theta^2}}{\int_{-\infty}^{\infty} e^{-\frac{(x-\theta)^2}{2}} \cdot \frac{1}{1+\theta^2}\, d\theta} d\theta$$

不会積 卡住

②

∵ $P(\theta|X)$ 裡 x 算是常數

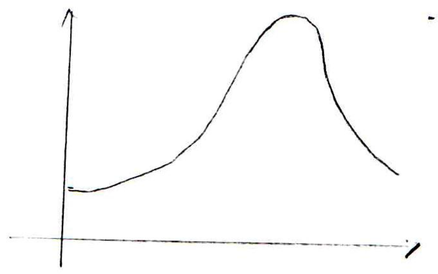
∴ 看 $P(X|\theta)\cdot P(\theta)$ 的樣子

$P(X|\theta)\cdot P(\theta) =$

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{(x-\theta)^2}{2}} \cdot \frac{1}{\pi} \cdot \frac{1}{1+\theta^2}$$

然後看不出來。

我有利用 R 模擬它的長相



看不出來像什麼分佈