Factor analysis

National Central University

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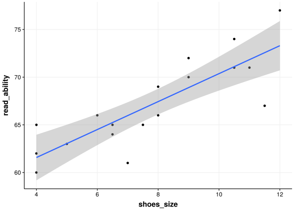
Motivation example (Using a Regression example)

For children in elementrays school
Observed variable:Shoe size and reading ability
latent variable: age



Purpose of factor analysis

- Dimension reduction (it is not the most important in factor analysis)
- Describe the covariance relationship among many observable vaiables in terms of a few underlying, but unobservable (latent) variable.

The Orthogonal Factor Model

The observable random vector $\mathbf{X}$, with components has mean μ and covariance matrix Σ. $\mathbf{X}$ is linearly dependent upon a few unobservable random variables $F_1, F_2, \ldots, F_m$ called common factor

$$\begin{aligned} X_1 - \mu_1 &= \ell_{11}F_1 + \ell_{12}F_2 + \cdots + \ell_{1m}F_m + \varepsilon_1 \\ X_2 - \mu_2 &= \ell_{21}F_1 + \ell_{22}F_2 + \cdots + \ell_{2m}F_m + \varepsilon_2 \\ &\vdots \\ X_p - \mu_p &= \ell_{p1}F_1 + \ell_{p2}F_2 + \cdots + \ell_{pm}F_m + \varepsilon_p \end{aligned}$$

in matrix notation

$$\mathbf{X} - \boldsymbol{\mu} = \mathbf{LF} + \boldsymbol{\varepsilon}$$

Assume that

$$\mathrm{E}(\mathbf{F}) = \mathbf{0}_{(m\times 1)}, \mathrm{Cov}(\mathbf{F}) = \mathrm{E}(\mathbf{FF}') = \mathrm{I}_{(m\times m)}$$
$$\mathrm{E}(\boldsymbol{\varepsilon}) = \mathbf{0}_{(p\times 1)}, \mathrm{Cov}(\boldsymbol{\varepsilon}) = \mathrm{E}(\boldsymbol{\varepsilon\varepsilon}') = \Psi_{(p\times p)}$$

where Ψ is a diagonal matrix.

So the covariance structure for **X** From this model

$$
\begin{aligned}
\Sigma = \mathsf{Cov}(\mathbf{X}) &= \mathrm{E}\left((\mathbf{X}-\boldsymbol{\mu})(\mathbf{X}-\boldsymbol{\mu})'\right) \\
&= \mathbf{L}\mathrm{E}(\mathbf{F}\mathbf{F}')\mathbf{L}' + \cancelto{0}{\mathrm{E}(\varepsilon\mathbf{F}')\mathbf{L}'} + \cancelto{0}{\mathbf{L}\mathrm{E}(\mathbf{F}\varepsilon')} + \mathrm{E}(\varepsilon\varepsilon') \\
&= \mathbf{L}\mathbf{L}' + \Psi
\end{aligned}
$$