Integer and Combinatorial Optimization

Spring 2017

Homework 2

(Due on 9:00AM March 30)

1. (20%) Ford has four automobile plants. Each plant is capable of producing the Taurus, Lincoln, or Escort, but it can only produce one of these cars. The fixed cost of operating each plant for a year and the variable cost of producing a car of each type at each plant are given as follows:

Plant	Fixed cost	Taurus	Lincoln	Escort
1	7 billion	12000	16000	9000
2	6	15000	18000	11000
3	4	17000	19000	12000
4	2	19000	22000	14000

Each year, Ford must produce 500,000 of each type of car. Formulate an IP which minimize the annual cost of producing cars based on the following constrains:

(a) Each plant can produce only one type of car.

(b) The total production of each type of car must be at a single plant.

(c) If plant 2 is used, plant 3 cannot be used.

(d) If plant 3 and plant 4 are used, plant 1 must also be used.

2. (20%) Consider the following problem:

$$\text{Max } 4x_1 - x_1^2 + 10x_2^2 - x_2^3$$

subject to

$$x_1 + x_2 \le 3$$

x_1 and x_2 are ≥ 0 and integers.

This problem can be reformulated as an equivalent pure Binary Integer Programming (BIP) problem, depending on the definitions of the binary variables. Assume that the binary variables are interpreted as: $y_{ij} = 1$ if $x_i \ge j$ (i = 1, 2, and j = 1, 2, 3), and $y_{ij} = 0$ otherwise.

3. (20%) John is buying stocks. His broker suggests six different stocks, namely, 1, … , 6. Let c_j denote the return of purchasing stock j. Formulate the stock selection problem subject to the following constraints, using 0-1 variables as needed:
   (a) To lower the risk of losing money, John should buy at least two stocks.
   (b) Due to John's budget limit, he cannot buy more than four stocks.
   (c) Since stocks 3 and 5 belong to the same company, the broker recommends purchase of at most of one of these.
   (d) The broker suggests the following two combinations: **either** choose two from stocks 1, 2, 3, and 4, **or** at least two from stocks 3, 4, 5, and 6.
   (e) Stock 4 can only be purchase if stock 1 is bought.

4. (20%) Four jobs need to be done on a certain machine. However, the setup time for each job depends on which job immediately preceded it, as shown in the following table. Formulate an IP or MIP model to determine the sequence of jobs that minimizes the total setup times.

		Job			
		1	**2**	**3**	**4**
Immediately preceding job	None	4	5	8	9
	1	-	7	12	10
	2	6	-	10	14
	3	10	11	-	12
	4	7	8	15	-

(For example, if job 1 is sequenced as the first one, it requires 4 units of setup time. Assume that job 3 is right after job 1, it will take 12 units of time to setup the machine.)

5. (20%) Using the classical (Gomory) cut and the Basic Approach to solve the following Knapsack problem.

$$\text{Max} \quad 3x_1 + 4x_2 + 8x_3 + 2x_4 + 7x_5 = x_0$$
$$\text{subject to} \quad 2x_1 + x_2 + 3x_3 + 5x_4 + 6x_5 \le 14$$
$$x_i = 0 \text{ or } 1 \quad \forall i$$