Introduction to Optimization

Homework #2 – Due Wednesday, October 11

1. Answer the following short questions:

   (a) Classify each of the following sets as open, closed, neither, or both.

   (i) $\{x : |x-5| \le \frac{1}{2}\}$

   (ii) $\{x : x^2 > 0\}$

   (b) Find the interior of $[0,3] \cup (3,5)$.

   (c) Find the boundary points of $[0,3] \cup (3,5)$.

   (d) Find the closure of $\{x : x^2 > 0\}$.

   (e) Find all cluster points of $A = \{(x, y) \in \mathbf{R}^2 : 0 \le x \le 1 \text{ or } x = 2\}$.

   (f) Find all cluster points of $S = \{(x, y) \in \mathbf{R}^2 : y < x^2 + 1\}$.

2. Let $0 < b < 1$ and $x_n = b^n$, $n \ge 1$. Show that the sequence (x_n) converges to 0. (Hint: we may write for some for some $a > 0$,

$$b = \frac{1}{1+a}$$

   and use the Bernoulli's inequality if $a > -1$, $a \in \mathbf{R}$ $(1+a)^n \ge 1 + na$, $n \ge 1$.)

3. Let $X = (x_n)$ be a sequence in $\boldsymbol{R}^p$ which is convergent to x, and let $c \in \boldsymbol{R}$. Show that $\lim(cx_n) = cx$.

4. If $X = (x_n)$ and $Y = (y_n)$ are sequences of real numbers which both converge to c and if $Z = (z_n)$ is a sequence such that $x_n \le z_n \le y_n$ for $n \in \boldsymbol{N}$, then Z also converges to c.