Integer and Combinatorial Optimization

Spring 2017

Homework 3 (Due on 12:00 (noon) April 17)

1. Home Grocery is a new company that makes same-day deliveries of groceries to people's homes. The company is launching its business in Metropolis, a large urban area. The marketing department has identified eight neighborhoods in Metropolis where the company should concentrate its business. The logistics manager has identified six locations where the company may locate grocery depots. Table 1 shows the average time (in minutes) required to travel from each of the six potential depot locations to the center of each of the eight neighborhoods. It also shows the target population (in thousands) for the company's service in each neighborhood. The company wishes to locate **two** depots so that they maximize the population served within 12 min of average travel time. Formulate the problem as an IP model.

Table 1 Travel Times from Depots to Neighborhoods

	Depots						
Neighborhoods	1	2	3	4	5	6	population
1	15	17	27	5	25	22	22
2	10	12	24	4	22	20	8
3	5	6	17	9	21	17	11
4	7	6	8	15	13	10	14
5	14	12	6	23	6	8	22
6	18	17	10	28	9	5	18
7	11	10	5	21	10	9	16
8	24	22	22	33	6	16	20

2. Consider a tableau which is neither primal nor dual feasible. Suppose we add the following redundant constraint

$$s = M + \sum_{j=1}^{n} 1(-x_{J(j)}) \geq 0$$

to the bottom of the tableau (s is a nonnegative slack variable and M is a large positive number). Show that after a pivot on the 1 element in the new row which is in the lexicographically smallest column, the tableau exhibits dual feasibility. (See this example on page 44 in slides of Dual Fractional IP.)

3. Under what condition will the Gomory slack variable in a mixed integer cut be an integer constrained variable? Please justify your answer. (There are four cases when generating a cut. The condition is derived from these four cases.)

4. Solving the mixed integer program below. Relate the computations to (x_1, x_2) space graphically. Also show that it is possible for the tableau to exhibit infeasibility after all the integer constrained variables have reached their lower bounds.

$$\begin{aligned} &\text{Max} \quad 8x_1 + 2x_2 = x_0 \\ &\text{S.T.} \quad 3x_1 + 2x_2 \le 1 \\ &\qquad\quad 7x_1 + \ \ x_2 \ge 2 \\ &x_1, x_2 \ge 0; x_0, x_1 \text{ integer.} \end{aligned}$$

5. Solving the following problem by the all-integer algorithm.

$$\begin{aligned} &\text{Max} \quad -2x_1 - 5x_2 = x_0 \\ &\text{S.T.} \quad -2x_1 - 2x_2 \le -9 \\ &\qquad\quad -2x_1 - 6x_2 \le -22 \\ &\qquad\quad x_1, x_2 \text{ integer.} \end{aligned}$$