Game Theory with Applications

Homework #3 – Due Thursday, November 10

1. Let

$$x_n = \frac{1}{n},\ n \geq 1$$

   By the definition of sequence convergence, prove the sequence x_n converges to 0.

2. Let X and Y be sequences in $\mathbf{R}^p$ that converge to $\underline{x}$ and $\underline{y}$ respectively. Prove that $X + Y$ converges to $\underline{x} + \underline{y}$.

3. Closed set

   (a) True or False? $\{x \in \mathbf{R} : 0 \leq x \leq 1\}$ is closed in $\mathbf{R}$.

   (b) True or False? $\{x \in \mathbf{R} : x \geq 0\}$ is closed in $\mathbf{R}$.

   (c) True or False? $\{(x, y) \in \mathbf{R}^2 : x^2 + y^2 \leq 1\}$ is closed in $\mathbf{R}^2$.

   (d) True or False? $\{x \in \mathbf{R} : 0 \leq x < 1\}$ is closed in $\mathbf{R}$.

4. Please give an example of the sequence and closed set to demonstrate that a set S is closed if and only if for any convergent sequence of points $\{x_k\}$ contained in S with limit point $\bar{x}$, we also have that $\bar{x} \in S$.

5. Disprove that $\left\{x \in \mathbf{R}^n : \sum_{i=1}^n x_i^2 = 1\right\}$ is convex.