高等數位訊號處理(Advanced Digital Signal Processing)
Homework 01 – 2017/04/07

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1. Design a Mini-max **highpass** FIR filter such that

   ① Filter length = 21 ② Sampling frequency $f_s = 5000$ Hz ③ Pass Band $1100 \sim 2500$ Hz

   ④ Transition band: $900 \sim 1100$ Hz ⑤ Weighting function: $W(F) = 1$ for passband, $W(F) = 0.5$ for stop band.

   ⑥ Set $\Delta = 0.0001$ in Step 5.

   紙本上要有：

   (a) the Matlab program

   (b) the frequency response

   (c) the impulse response $h[n]$

   (d) the maximal error for each iteration

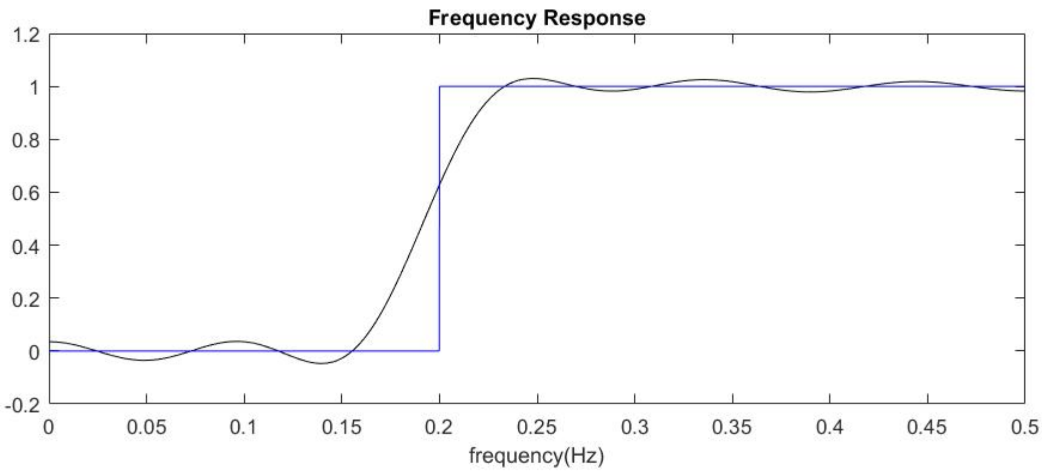
(a) **MATLAB Code**

```
function hw01()
N = 21;              % N: Filter Length
k = (N - 1)/2;       % k = (n-1)/2
delta = 0.0001;      % Delta = 0.0001

% Initial
A = zeros(12,12);    % Matrix A
S = zeros(12,1);     % Column Vector S
H = zeros(12,1);     % Column Vector H

% STEP 01
F = [0; 0.05; 0.10; 0.13; 0.16; 0.23; 0.26; 0.29; 0.35; 0.4; 0.45; 0.5];
H = F >= 0.22;
f = 0:delta:0.5;
Hd = f >= 0.2;
W1 = 1.0*(f>=0.22); % Define Weight Function
W2 = 0.5*(f<=0.18);
W = W1+W2;
n = 0;

% Do While Loop
E1 = 99;
E0 = 5;
while((E1-E0 > delta) | (E1-E0 < 0))
    % STEP 02
    for i = 1:1:12
        for j = 1:1:11
            A(i,j) = cos(2*pi*(j-1)*F(i));
```

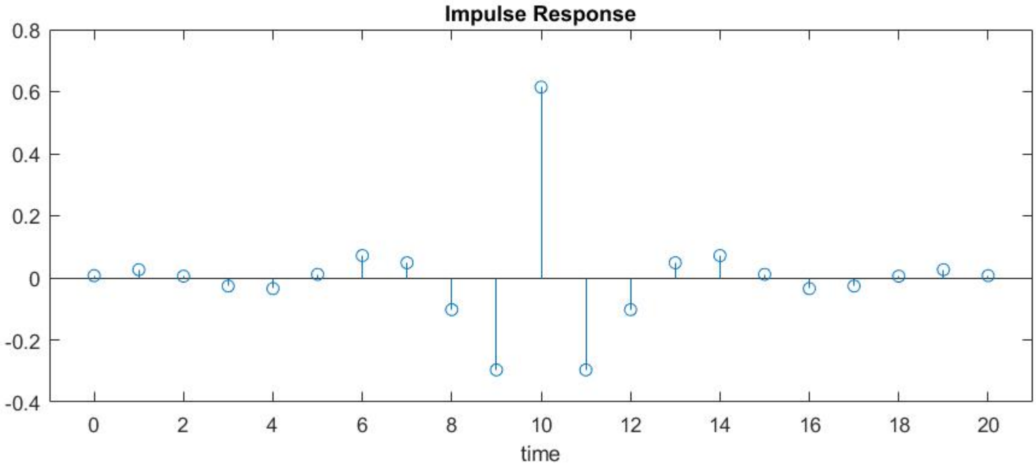
```
        end
        A(i,12)=(-1)^(i-1)*1/Wf(F(i));
    end
    S = A^(-1)*H;   % Another Method: S = A\H;
    % STEP 03
    RF = 0;
    for i = 1:11
        RF = RF + S(i)*cos(2*(i-1)*pi*f);
    end
    err = (RF-Hd).*W;
    % STEP 04
    q = 2;
    for i = 2:length(f)-1
        if(err(i) > err(i-1) && err(i) > err(i+1))
            F(q) = delta*i;
            q = q+1;
        end
        if(err(i) < err(i-1) && err(i) < err(i+1))
            F(q) = delta*i;
            q = q+1;
        end
    end
    % STEP 05
    E1 = E0;
    n = n+1;
    [max_value, max_locs] = findpeaks(err);
    [min_value, min_locs] = findpeaks(-err);
    E0 = max(max(abs(err)))
    F = sort([0 (max_locs-1)*delta (min_locs-1)*delta 0.5]);
    F = F';
end

% STEP 06
h(k+1) = S(0+1);
for i = 1:k
    h(k+i+1) = S(i+1)/2;
    h(k-i+1) = S(i+1)/2;
end

% Plot Frequency Response
subplot(211)
plot(f, RF,'k',f ,Hd,'b')
```

```
title('Frequency Response');
xlabel('frequency(Hz)');
x = 0:1:20;

% Plot Impulse Response
subplot(212)
stem(x,h)
title('Impulse Response');
xlabel('time');
xlim([-1 21])

function w = Wf(F)
if F >= 0.22
    w = 1;
else
    w = 0;
end
if F <= 0.18
    w = 0.5;
end
```

(b) **Frequency Response**



(c) **Impulse Response**



(d) **Maximal Error for Each Iteration**

	Iteration 01	Iteration 02	Iteration 03	Iteration 04
Maximal Error	0.1433	0.0843	0.0658	0.0651

2. (a) What are the two most important applications of the Fourier transform?
(b) From the view point of implementation, what are the disadvantages of the discrete Fourier transform?

(a) 實際上傅立葉轉換在各個學科中皆有許多相對應的重要應用，此處若要談及訊號處理中的重要應用，則應為：

① 可以將時域與空間域的訊號，轉換產生頻域訊號進行頻譜分析

② 計算線性非時變(LTI)系統的卷積(convolution)

(b) DFT：$X[m] = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi}{N}mn}$

IDFT：$x[n] = \frac{1}{N}\sum_{m=0}^{N-1} X[m]\, e^{j\frac{2\pi}{N}mn}$

如上所示，離散傅立葉轉換及其逆轉換的計算複雜度皆為 $O(N^2)$，最為人所詬病之缺點即為計算量龐大。此外在實際操作中仍有許多限制，如：

[1] 取樣速率(Sampling Rate)必須為受測訊號最高成分頻率的兩倍或兩倍以上，亦即要符合 Nyquist Sampling Theorem。否則將使低頻部分反射高頻部分而造成混疊效應(aliasing effect)。

[2] 要求受測訊號必須具備週期性。

3. Suppose that $x[n] = y(0.0002n)$ and the length of $x[n]$ is 25000 and $X[m]$ is the FFT of $x[n]$. Find m_1 and m_2 such that $X[m_1]$ and $X[m_2]$ correspond to the 300 Hz and -100 Hz components of $y(t)$, respectively.

Hence $x[n] = y(0.0002n)$, we can know that $f_s = \frac{1}{\Delta_t} = \frac{1}{0.0002} = 5000$ Hz and $N = 25000$.

As we know that $f = m\frac{f_s}{N}$:

$$300 = m_1 \frac{5000}{25000}$$

$$\Longrightarrow m_1 = 1500$$

$$-100 + 5000 = m_2 \frac{5000}{25000}$$

$$\Longrightarrow m_2 = 24500$$

4. Why ① the <u>transition band</u> and ② the <u>weighting function</u> are important in Minimax FIR digital filter design?

(a) 已知：

$$N = \frac{2}{3}\frac{1}{\Delta F}\log_{10}\left(\frac{1}{10\delta_1\delta_2}\right)$$

由上式可知當濾波器長度 N 為定值時 ΔF 和 $\delta_1\delta_2$ 呈負相關，因此可以藉由降低過渡頻帶(transition band)的頻率響應(frequency response)來換取較高的通帶(passband)和截止區(stopband)的精確度。

(b) 頻率響應(frequency response)在通帶(passband)和截止區(stopband)的誤差可以根據加權函數(Weighting function)進行調整，用以避免較大振盪的漣波(ripple)，由此達到較佳的結果。

5. Estimate <u>the length of the digital filter</u> if both the passband ripple and the stopband ripple are smaller than 0.02, the sampling interval $\Delta_t = 0.0001$, and the transition band is from 2000 Hz to 2200 Hz.

Take $\delta_1 = \delta_2 = \delta = 0.02$ and $\Delta_t = 0.0001$.

$$\Longrightarrow f_s = \frac{1}{\Delta_t} = \frac{1}{0.0001} = 10000,\ \Delta F = \tfrac{\Delta f}{f_s} = \tfrac{2200-2000}{10000} = 0.02$$

Then $N = \dfrac{2}{3}\dfrac{1}{\Delta F}\log_{10}\left(\dfrac{1}{10\delta_1\delta_2}\right) = \dfrac{2}{3} \times \dfrac{1}{0.02} \times \log_{10}\left(\dfrac{1}{10 \times 0.02 \times 0.02}\right) = 79.9313 \cong 80$

6. Make a comparison among the methods of MSE, Minimax, and frequency sampling for FIR filter design and show their advantages and disadvantages.

(a) **MSE**

優點：三種方法中平均誤差最小者，以設計方法論亦處於三者之中。

缺點：設計方法較為彈性，並沒有過多缺點。

(b) **Minimax**

優點：三種方法中最大誤差最小者，因此可以保證系統的計算結果在一定的誤差以內，保持相當的精度限制。

缺點：要求濾波器的頻率響應必須具備偶對稱性質或奇對稱性質，且必須給定過渡頻帶(transition band)，此外其中使用了遞迴的方式以致於設計相對較為複雜。

(c) **Frequency sampling**

優點：為三種方法中最為簡單者，並沒有使用到加權函數(weighting function)，僅只使用到了傅立葉反轉換(inverse Fourier transform) 配合 FIR 濾波器係數和頻率響應間的線性關係來將問題表為一個線性方程式的解。

缺點：其最顯著的缺點在於無法使得過渡頻帶(transition band)得到最佳化的結果，並且容易產生疊頻效應(Aliasing Effect)和 Gibb’s 現象…等。