Game **Theory** with Applications

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Ice Cream Carts on the Beach

- If you were the owner of the ice cream carts, where to locate your ice cream carts?

Ice Cream Cart Example (only one cart)

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  - Consider a beach that can be represented by unit interval. There is a continuum of several tourists who are located uniformly at the beach.
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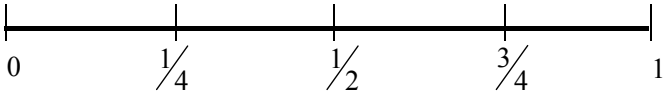

Ice Cream Cart Example (one owner runs two carts)

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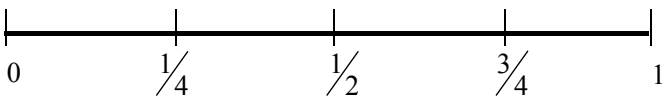

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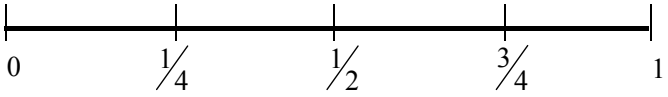
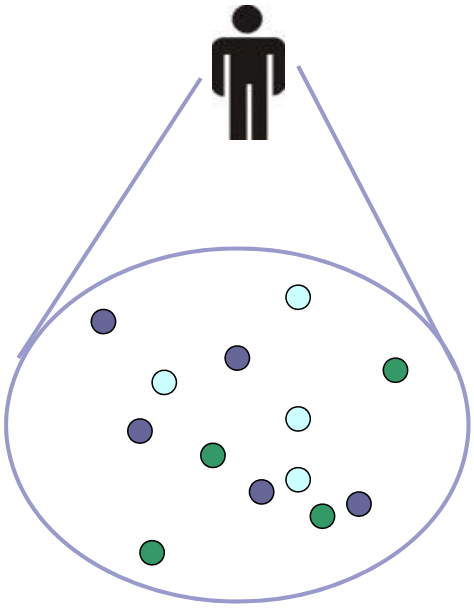


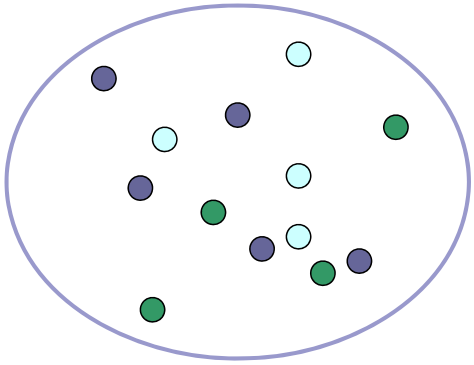
Overview of Game Theory

- Game theory is the study of multi-person decision problems.
- Single decision maker (DM)
- Multiple decision makers (DMs)



Centralized

- Each entity acts in a self-interested manner with respect to its own objective and firm constraints.
- No, or limited, information sharing is allowed.



Decentralized

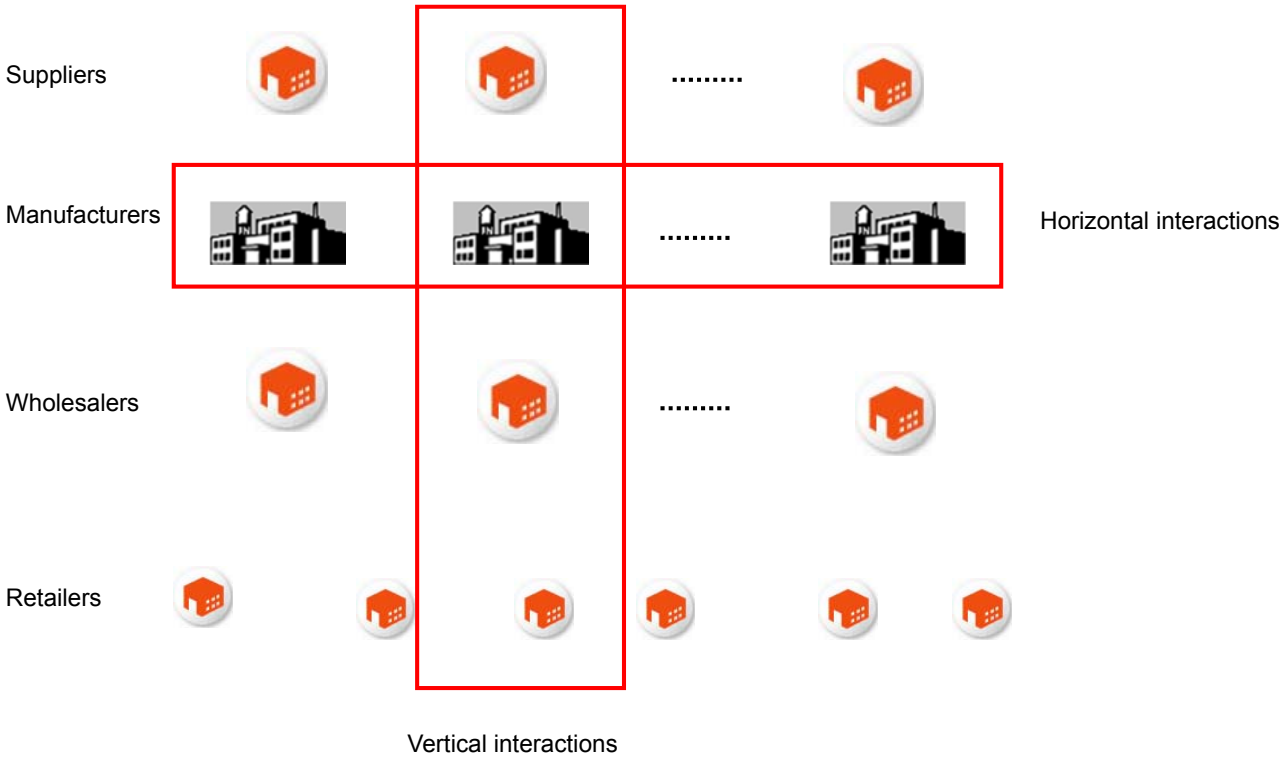
Overview of Game Theory

	One DM	Many DMs
Static	Mathematical programming	**Static game theory**
Dynamic	Optimal control theory	**Dynamic games**

Source: S. C. Chang and Y. N. Yang's Lecture notes

Main tool for analysis: Game Theory

Applications of Game Theory – players in supply chains



Classification of Games

	Complete Information	Incomplete Information
Static	Static games of complete information	Static games of incomplete information
Dynamic	Dynamic games of complete information	Dynamic games of incomplete information

What is a Game?

- A zero-sum game is one in which the players' interests are in direct conflict, e.g. in baseball game, one team wins and the other loses. (+1)+(-1)=0
- A game is non-zero sum, if players interests are not always in direct conflict, so that there are opportunities for both to gain.
- Properties of a game
  - There are 2 or more players.
  - There is some choice of action chosen by players.
  - The game has one or more outcomes, e.g., someone wins, someone loses.
  - The outcome depends on the actions chosen by all players.

Key Elements of a Game

- Players: Who is interacting?
- Strategies: What are their actions?
- Payoffs: What are their incentives?
- Information: What do they know?
- Rationality: How do they think?

Elements of a Game

- The players
  - how many players are there?
- Actions (Strategies)
  - A complete description of what the players can do
    - a set of possible actions
- Payoffs
  - A description of the payoff consequences for each player for every possible combination of actions chosen by all players playing the game.

A Well-known Example: Prisoners' Dilemma

- Two people are arrested for a crime. The police lack sufficient evidence to convict each suspect and consequently need them to give testimony against each other. The policy put each suspect in a different cell to prevent the two suspects form communicating with each other.
- The police tell each suspect that if he testifies against (doesn't cooperate with) the other, he will be released and will receive a reward (2 units) for testifying if the other suspect does not testify against him.
- If neither suspect testifies, both will be released on account of insufficient evidence, and no rewards will be paid.
- If one testifies, the other will go to prison for 10 yrs; if both testify, both will go to prison for 5 yrs.

Overview of Game Theory

- Players: Prisoner 1 and prisoner 2
- Set of all possible actions: C (cooperate) and D (defect)
- Payoffs: The rewards or years in the prison
- Information: Rewards or years in prison
- Rationality: Maximization of her/his own "utility"

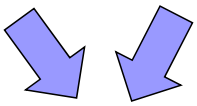
How to represent a game?

- Payoff of Prisoner 1

		P2	
		C (cooperate)	D (defect)
P1	C		
	D		

- Payoff of Prisoner 2

		P2	
		C (cooperate)	D (defect)
P1	C		
	D		



		Prisoner 2	
		C (cooperate)	D (defect)
Prisoner 1	C		
	D		

- Optimal strategy: (C, C)?

Overview of Game Theory

		Prisoner 2	
		C (cooperate)	D (defect)
Prisoner 1	C	0, 0	-10, 2
	D	2, -10	-5, -5

- Optimal strategy: (C, C)?
- If Prisoner 1 chose C, will Prisoner 2 still choose C?
- If Prisoner 2 chose C, will Prisoner 1 still choose C?
- Any “equilibrium” (stable) combination?

Assuming….

- Payoffs are known and fixed.
- All players behave rationally. They understand and seek to maximize their own payoffs.

Solution Approach

		Prisoner 2	
		C (cooperate)	D (defect)
Prisoner 1	C	0, 0	-10, 2
	D	2, -10	-5, -5

Equilibrium Solution (Stable solution)

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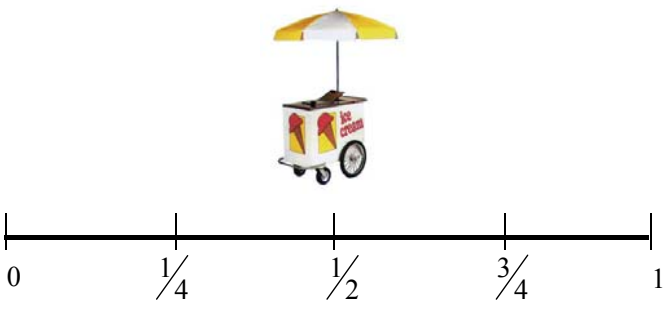

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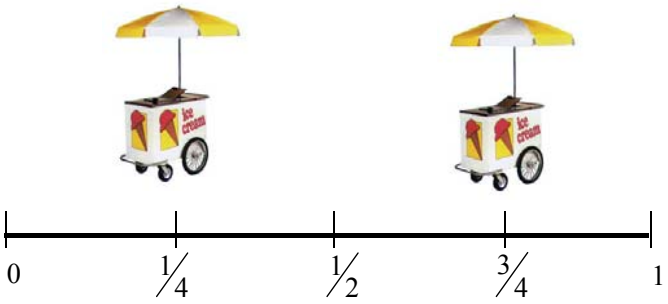

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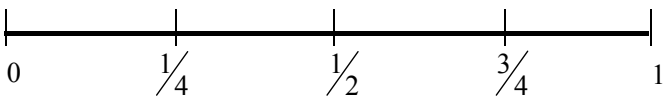

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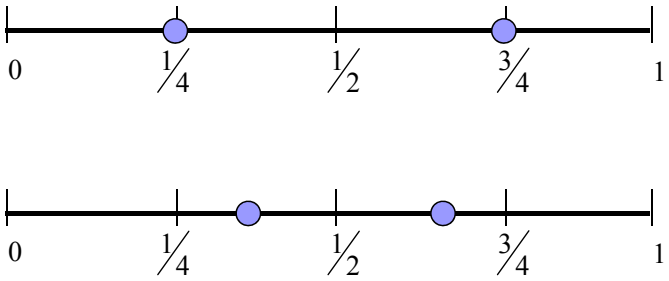

Concept of Equilibrium

- No one can be better-off by a unilateral change in its solution.
- No player has anything to gain by changing only his or her own strategy.