

Game Theory with Applications

Homework #2 – Due Thursday, October 13

1. Consider the following bargaining game. Players 1 and 2 are bargaining over how to split one dollar. Both players simultaneously name shares they would like to have, s_1 and s_2 , where $0 \leq s_1, s_2 \leq 1$, if $s_1 + s_2 \leq 1$, then the players receive the shares they named; if $s_1 + s_2 > 1$, then both players receive zero. What are the pure strategy Nash equilibria of this game?

2. Consider the Cournot model we discussed in class:

- Two competing firms, selling a homogeneous good.
- The marginal cost of producing each unit of the good is c .
- The market price, P is determined by (inverse) market demand: $P = a - Q$ if $a > Q$, $P = 0$ otherwise.
- Each firm decides on the quantity to sell (market share): q_1 and q_2 .
- $Q = q_1 + q_2$ is the total market demand.
- Both firms seek to maximize profits.

(a) Solve for the equilibrium quantity q_1^* and q_2^* .

(b) Please verify your solution in (a) by showing that the statement “In the equilibrium, no one can be better-off by a unilateral change in its solution” is satisfied.

3. Following Question 2, suppose that each firm produces the half of monopoly quantity q_m , i.e., $q_1 = q_2 = \frac{1}{2}q_m$.

(a) Solve for the monopoly quantity q_m .

(b) Please compare each firm's profit in Question 3 with the solution you obtained in Question 2.

(c) Show that $q_1 = q_2 = \frac{1}{2}q_m$ is not an equilibrium solution.

4. In an industry there are N firms producing a homogeneous product. Let q_i denote the output level of firm i , $i = 1, 2, \dots, N$, and let Q denote the aggregate industry production level. That is, $Q = \sum_{i=1}^N q_i$.

Assume that the demand curve facing the industry is $p = 100 - Q$. Suppose that the cost function of each firm i is given by

$$TC_i(q_i) = \begin{cases} F + q_i^2 & \text{if } q_i > 0 \\ 0 & \text{if } q_i = 0 \end{cases}$$

Suppose that the number of firms in the industry N is sufficiently small so that all the N firms make above-normal profits. Calculate the output and profit levels of each firm in a Cournot equilibrium. (Hint: you can assume that all firms have identical cost functions.)