Representation Theorem

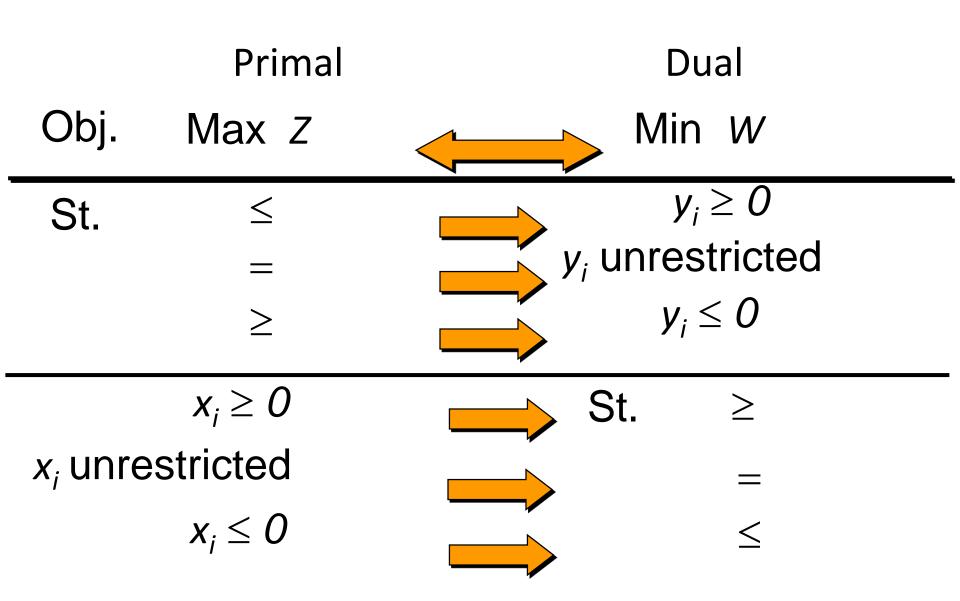
$$S=\{Ax \leq b, x \geq 0\}$$

Let x^1 , x^2 ,..., x^k be the extreme points of S and r^1 ,..., r^j be the extreme directions of S.

Assume $x \in S$, then x can be represented as

$$x=\sum_{i=1}^k \lambda_i x^i + \sum_{i=1}^j \mu_i r^i$$
 S.T.
$$\sum_{k=0}^n \lambda_i = 1 \ and \ \lambda_i \geq 0, \ \mu_i \geq 0$$

Primal-Dual Forms



Dual Problem

Dual

$$\begin{split} \text{maximize} \quad c^T \Bigg(\sum_{k \in K} \alpha^k x^k + \sum_{j \in J} \beta^j r^j \Bigg), \\ \text{subject to} \quad & \sum_{k \in K} \alpha^k \\ & A^l \Bigg(\sum_{k \in K} \alpha^k x^k + \sum_{j \in J} \beta^j r^j \Bigg) \leq b^l, \\ & \alpha^k, \beta^j \geq 0, \qquad k \in K, \ j \in J. \\ z_D = \text{maximize} \quad & c^T x, \\ \text{subject to} \quad & A^l x \leq b^l, \\ & x \in \text{conv}(Q), \\ \text{where } Q = \Big\{ x \colon A^2 x \leq b^2 \ , \ x \geq 0, \ \text{integer} \Big\}. \end{split}$$

Primal

minimize
$$\eta$$

subject to $\eta + \lambda^T (A^1 x^k - b^1) \ge c^T x^k$, $k \in K$: α^k
 $\lambda^T A^1 r^j \ge c^T r^j$, $j \in J$: β^j
 $\lambda \ge 0$.

η is unrestricted => "=" $λ \ge 0$ => " \le "

First set of constraints: $\alpha \ge 0$

Second set of constraints: $\beta \ge 0$

Result 7

$$z_{IP} = 28 < z_{LD} = 28 \frac{8}{9} < z_{LP} = 30 \frac{2}{11}$$

$$x^{6} = (3,3) \in S.$$
For $\lambda = \frac{1}{15}$, $z_{LR}(\lambda) = 28 \frac{14}{15}$

$$\lambda(b^{1} - A^{1}x^{6}) = \frac{1}{15}(4 - 3) = \frac{1}{15} = \delta_{1}$$

$$z_{LR}(\lambda) - z(\lambda, x^{6}) = (29 - \lambda) - (27 + \lambda) = 1 \frac{13}{15} = \delta_{2}$$

$$cx^{6} \ge z_{IP} - 1 \frac{14}{15} \Rightarrow 27 > 28 - 1 \frac{14}{15}$$