Game Theory with Applications

Homework #4 – Due Thursday, November 17

Recall the 2-player game mentioned in the class. The normal-form is shown as follows:

		Player 2	
		(H) Head (q)	(T) Tail (1-q)
Player 1	(H) Head (p)	-1, $\underline{1}$	$\underline{1}$, -1
	(T) Tail (1-p)	$\underline{1}$, -1	-1, $\underline{1}$

Let us follow the notations introduced in class to verify several important conditions mentioned in the Kakutani fixed-point theorem.

Notations:

- The action (or strategy) profile: $\sigma = (p, q)$
- The action (or strategy) profile except player *i*'s action: σ_{-i}
- The space of action (or strategy) profile: Σ
- Player *i*'s payoff function: $u_i(\sigma)$
- Player *i*'s *best response correspondence*, r_i, maps each strategy profile σ to the set of mixed strategies that maximize player *i*'s payoff when his opponents play σ_{-i}.
- The correspondence $r : \Sigma \to \Sigma$ to be the Cartesian product of the r_i.
- The graph G_R of the correspondence *r*: $G_R = \{(p, q, \hat{p}, \hat{q}) : (\hat{p}, \hat{q}) \in r(p, q)\}$

1. Please explicitly write the following terms in this example.
   (a) The space of action (or strategy) profile Σ
   (b) Player 1's expected payoff function $u_1(\sigma)$

2. Suppose $\sigma' = (p', q') \in r(p, q)$ and $\sigma'' = (p'', q'') \in r(p, q)$ where $\sigma = (p, q)$. Show that $\lambda p' + (1-\lambda) p''$ is player 1's best response to q for $\lambda \in (0,1)$.

3. Show that correspondence $r(p, q)$ in this example is convex for all $(p, q) \in \Sigma$.

4. Let sequence $(p_n) = \left(\frac{1}{4^n}\right)$ and $(q_n) = \left(\frac{1}{4^n}\right)$. What are the best response sequence $(\hat{p}_n)$ and $(\hat{q}_n)$?

5. What is the limit point of the sequence $(p_n, q_n, \hat{p}_n, \hat{q}_n)$, where $p_n = \frac{1}{4^n}$ and $q_n = \frac{1}{4^n}$ for all *n*?

6. Suppose that a sequence $(p_n, q_n, \hat{p}_n, \hat{q}_n)$ converges to $(p, q, \hat{p}, \hat{q})$ for all sequence index *n*, but $\hat{p}$ is not player 1's best response to *q*. Show that $\hat{p}_n$ cannot be player 1's best response to q_n.

7. Show that $r(\cdot)$ has a closed graph.