V. Homomorphic Signal Processing

◉ 5-A Homomorphism

Homomorphism is a way of "carrying over" operations from one algebra system into another.

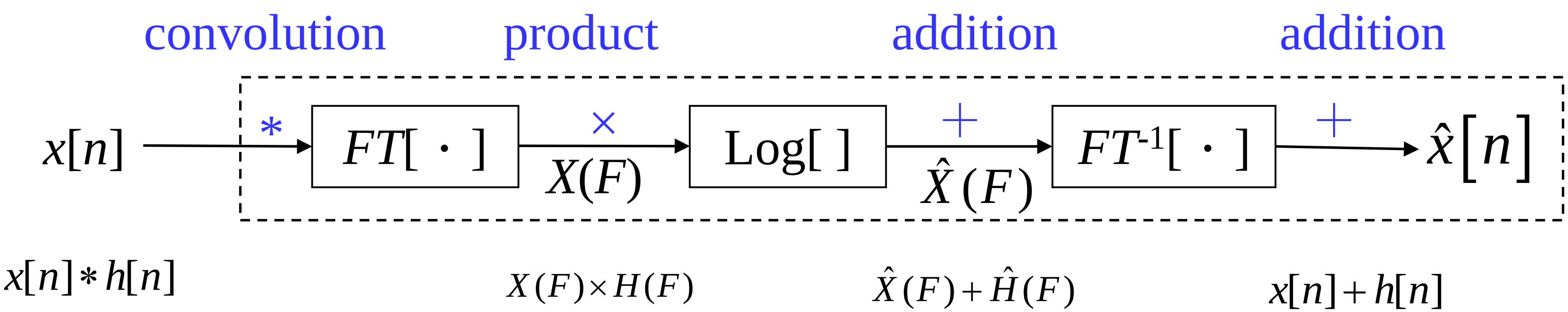
Ex. convulution $\xrightarrow[\textit{Fourier}]{}$ multiplication $\xrightarrow[\log]{}$ addition

把複雜的運算，變成效能相同但較簡單的運算

◉ 5-B Cepstrum

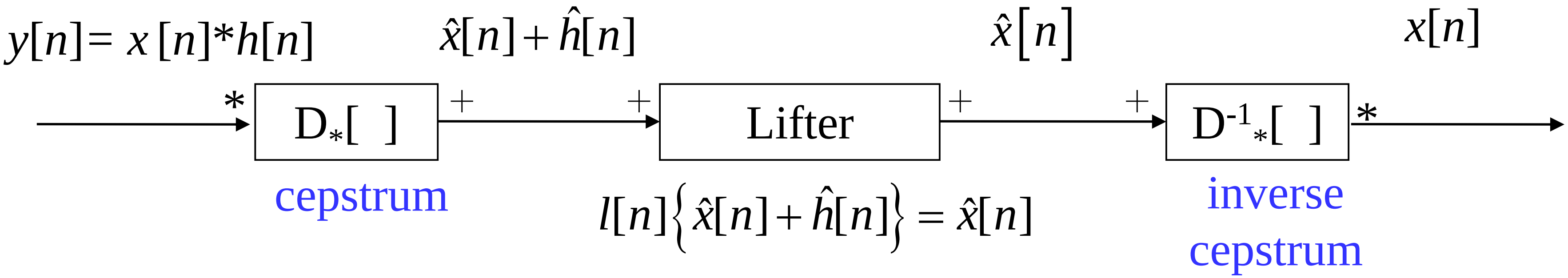
$$\hat{X}(Z)\Big|_{z=e^{i2\pi F}} = \log X(Z)\Big|_{z=e^{i2\pi F}} = \log\left|X(Z)\right|_{z=e^{i2\pi F}} + j\arg[X(e^{i2\pi F})]$$

For the process of cepstrum (denoted by $D_*[\,\cdot\,]$)

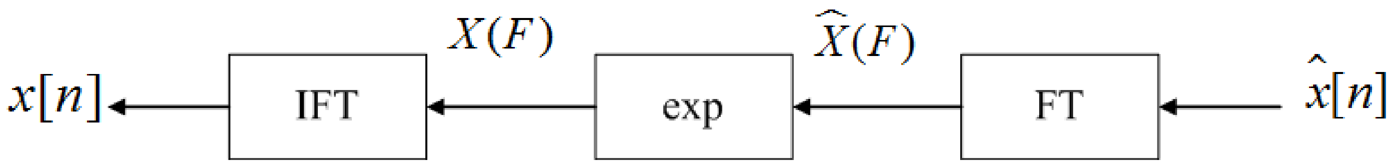


FT: discrete-time Fourier transform

- 由 $y[n]=x[n]*h[n]$ 重建 $x[n]$



For the inverse cepstrum $D^{-1}_*[\cdot]$

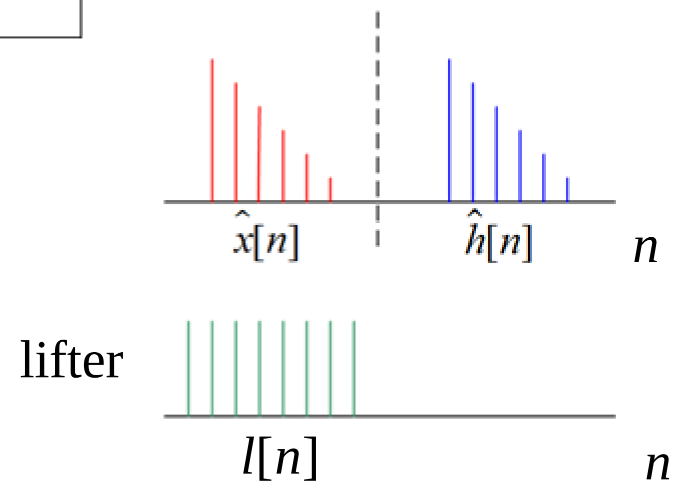


- **有趣的名詞**

$\hat{x}[n]$ cepstrum

n quefrency

$l[n]$ lifter



$$l[n]\left\{\hat{x}[n]+\hat{h}[n]\right\}=\hat{x}[n]$$

◉ 5-C Methods for Computing the Cepstrum

- **Method 1**: Compute the inverse discrete time Fourier transform:

$$\hat{x}[n] = \int_{-1/2}^{1/2} \hat{X}(F) e^{i2\pi nF} dF \qquad : \text{inverse F.T}$$

where $\hat{X}(F) = \log|X(F)| + j\arg[X(F)]$

ambiguity for phase

Problems: (1)

(2)

Actually, the COMPLEX Cepstrum is REAL for real input

- **Method 2 (From Poles and Zeros of the Z Transform)**

實際上計算 cepstrum的方法

time delay

$$X(Z)=\frac{A\cancel{Z^{r}}\prod_{k=1}^{m_i}(1-a_kZ^{-1})\prod_{k=1}^{m_0}(1-b_kZ)}{\prod_{k=1}^{P_i}(1-c_kZ^{-1})\prod_{k=1}^{P_0}(1-d_kZ)}$$

where

$$|a_k|\ ,\ |b_k|\ ,\ |c_k|\ ,\ |d_k| \le 1$$

Poles & zeros inside unit circle

Poles & zeros outside unit circle

$$\therefore \hat{X}(Z)=\log X(Z)=\log A+r\cdot\log Z+\sum_{k=1}^{m_i}\log(1-a_kZ^{-1})+\sum_{k=1}^{m_0}\log(1-b_kZ)$$

$$-\sum_{k=1}^{P_i}\log(1-c_kZ^{-1})-\sum_{k=1}^{P_0}\log(1-d_kZ)$$

$$\therefore \hat{X}(Z) = \log X(Z) = \log A + r \cdot \log Z + \sum_{k=1}^{m_i} \log(1 - a_k Z^{-1}) + \sum_{k=1}^{m_0} \log(1 - b_k Z)$$

$$- \sum_{k=1}^{P_i} \log(1 - c_k Z^{-1}) - \sum_{k=1}^{P_0} \log(1 - d_k Z)$$

Z^{-1}

(inverse Z transform)

?

Taylor series

$$f(t) = f(t_0) + \sum_{n=1}^{\infty} \frac{f^{(n)}(t_0)}{n!} (t - t_0)^n$$

Taylor series expansion

$\Big\downarrow Z^{-1}$

(Suppose that $r = 0$)

$$\hat{x}[n] = \begin{cases} \log(A) & , n = 0 \\ -\sum_{k=1}^{m_i} \frac{a_k^{\ n}}{n} + \sum_{k=1}^{P_i} \frac{c_k^{\ n}}{n} & , n > 0 \\ \sum_{k=1}^{m_0} \frac{b_k^{\ -n}}{n} - \sum_{k=1}^{P_0} \frac{d_k^{\ -n}}{n} & , n < 0 \end{cases}$$

Poles & zeros inside unit circle, right-sided sequence

Poles & zeros outside unit circle, left-sided sequence

Note:

(1) $\hat{x}[n]$ always decays with $|n|$.

(2) 在 complex cepstrum domain

Minimum phase 及 maximum phase 之貢獻以 $n = 0$ 為分界切開

(3) For FIR case, $c_k = 0$, $d_k = 0$

(4) The complex cepstrum is unique and of infinite duration for both positive & negative n, even though $x[n]$ is causal & of finite durations

$\hat{x}[n]$ is always IIR

- **Method 3**

$$Z \cdot \hat{X}'(Z) = Z \cdot \frac{X'(Z)}{X(Z)}$$

$$\therefore ZX'(Z) = Z\hat{X}'(Z) \cdot X(Z)$$

$$\Big\downarrow Z^{-1}$$

$$n\,x[n] = \sum_{k=-\infty}^{\infty} k\,\hat{x}[k]\,x[n-k]$$

$$\therefore x[n] = \sum_{k=-\infty}^{\infty} \frac{k}{n}\hat{x}[k]\,x[n-k] \qquad for\ n \neq 0$$

Suppose that $x[n]$ is causal and has minimum phase, i.e. $x[n] = \hat{x}[n] = 0$, $n < 0$

$$x[n] = \sum_{k=-\infty}^{\infty} \frac{k}{n} \hat{x}[k] x[n-k] \qquad \textit{for } n \neq 0$$

$$\Rightarrow x[n] = \sum_{k=0}^{n} \frac{k}{n} \hat{x}[k] x[n-k] \qquad \textit{for } n > 0 \qquad (\textit{causal sequence})$$

$$x[n] = \hat{x}[n] x[0] + \sum_{k=0}^{n-1} \frac{k}{n} \hat{x}[k] x[n-k]$$

For a minimum phase sequence $x[n]$

$$\hat{x}[n] = \begin{cases} 0 & , n < 0 \\ \dfrac{x[n]}{x[0]} - \displaystyle\sum_{k=0}^{n-1} \left(\frac{k}{n}\right) \hat{x}[k] \frac{x[n-k]}{x[0]} & , n > 0 \\ \log A & , n = 0 \end{cases} \qquad \text{recursive method}$$

Determining $\hat{x}[n]$ from $\hat{x}[0], \hat{x}[1], \cdots, \hat{x}[n-1]$

For anti-causal and maximum phase sequence, $x[n] = \hat{x}[n] = 0$, $n > 0$

$$x[n] = \sum_{k=n}^{0} \frac{k}{n} \hat{x}[k] x[n-k] \qquad , n < 0$$

$$= \hat{x}[n] x[0] + \sum_{k=n+1}^{0} \frac{k}{n} \hat{x}[k] x[n-k]$$

For maximum phase sequence,

$$\hat{x}[n] = \begin{cases} 0 & , n > 0 \\ \log A & , n = 0 \\ \dfrac{x[n]}{x[0]} - \displaystyle\sum_{k=n+1}^{0} \left(\frac{k}{n}\right) \hat{x}[k] \frac{x[n-k]}{x[0]} & , n < 0 \end{cases}$$

◉ 5-D Properties

P.1) The complex cepstrum decays at least as fast as $\dfrac{1}{n}$

$$\left|\hat{x}[n]\right| < c\left|\frac{\alpha^{n}}{n}\right| \qquad -\infty < n < \infty$$

$$\alpha = \max\left(a_k, b_k, c_k, d_k\right)$$

P.2) If $X(Z)$ has no poles and zeros outside the unit circle, i.e. $x[n]$ is minimum phase, then

$$\hat{x}[n] = 0 \qquad \textit{for all } n < 0$$

because of no b_k, d_k

P.3) If $X(Z)$ has no poles and zeros inside the unit circle, i.e. $x[n]$ is maximum phase, then

$$\hat{x}[n] = 0 \qquad \textit{for all } n > 0$$

because of no a_k, c_k

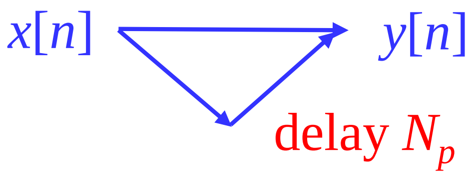
P.4) If $x[n]$ is of finite duration, then

$\hat{x}[n]$ has infinite duration

◉ 5-E Application of Homomorphic Deconvolution

(1) Equalization for Echo

$$y[n] = x[n] + \alpha\ x[n - N_p]$$



Let $p[n]$ be $p[n] = \delta[n] + \alpha\delta[n - N_p]$

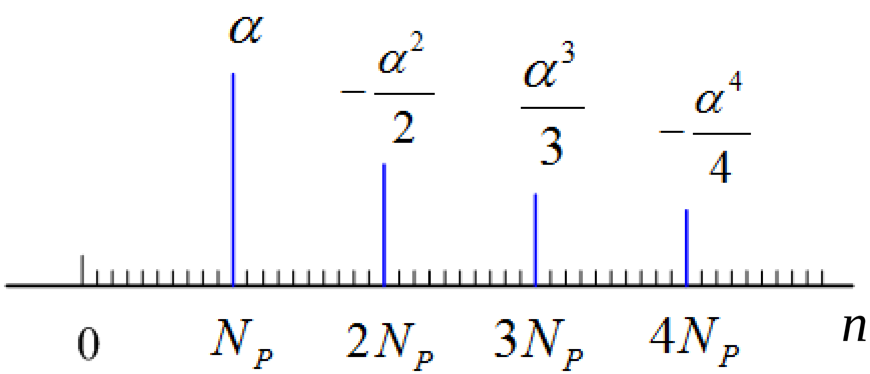
$$y[n] = x[n] + \alpha\, x[n - N_p] = x[n] * p[n]$$

$$P(Z) = 1 + \alpha Z^{-N_p}$$

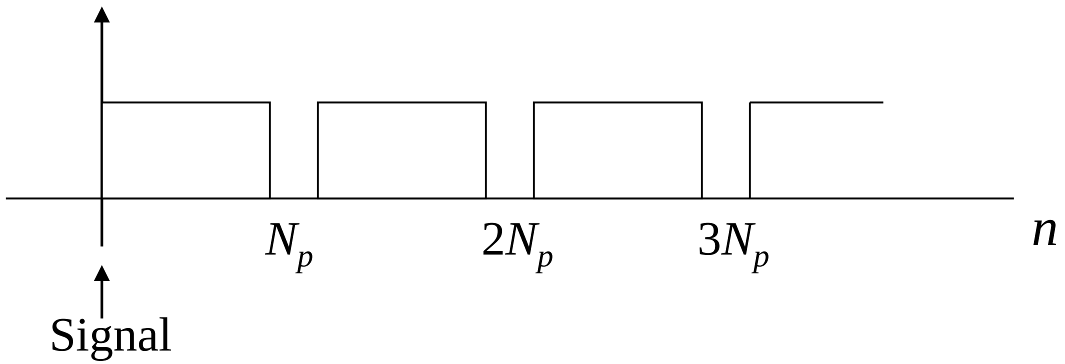
$$\hat{P}(Z) = \log\,(1 + \alpha Z^{-N_p}) = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{\alpha^k}{k} Z^{-kN_p}$$

$$\downarrow\ Z^{-1}$$

$$\hat{p}[n] = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{\alpha^k}{k} \delta(n - k \cdot N_p)$$



Filtering out the echo by the following "lifter":



(2) Representation of acoustic engineering

$$y[n] \quad = \quad x[n] \quad * \quad h[n]$$

Synthesized music — music — building effect：e.g. 羅馬大教堂的 impulse response

(3) Speech analysis

$$s[n] \;=\; g[n] \;*\; v[n] \;*\; p[n]$$

Speech wave | Global wave shape | Vocal tract impulse | Pitch

They can be separated by filtering in the complex cepstrum domain

(4) Seismic Signals

(5) Multiple-path analysis for any wave-propagation problem

用 cepstrum 將 multipath的影響去除

From 王小川，“語音訊號處理”，全華出版，台北，民國94年。

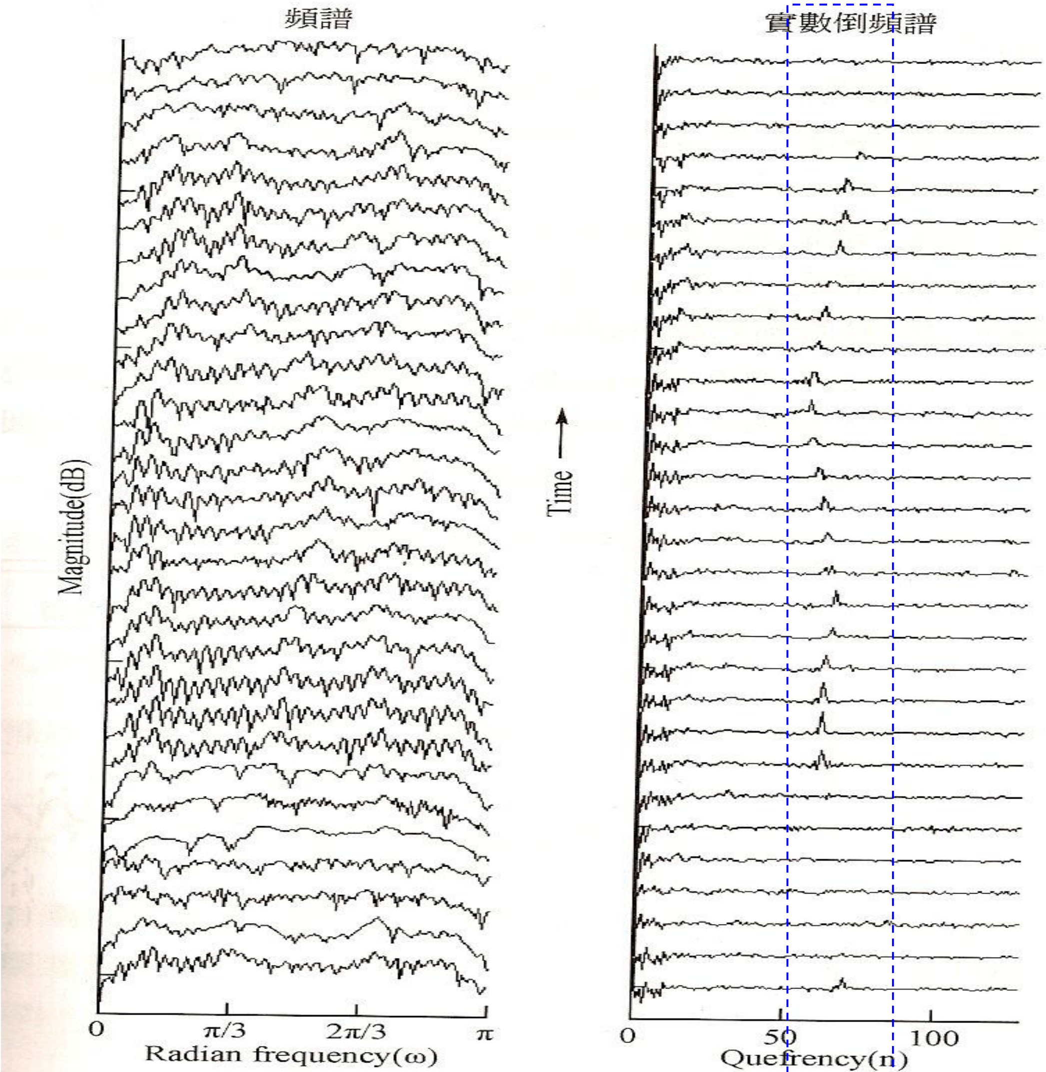


圖 7-5　元音頻譜與倒頻譜

From 王小川，“語音訊號處理”，全華出版，台北，民國94年。

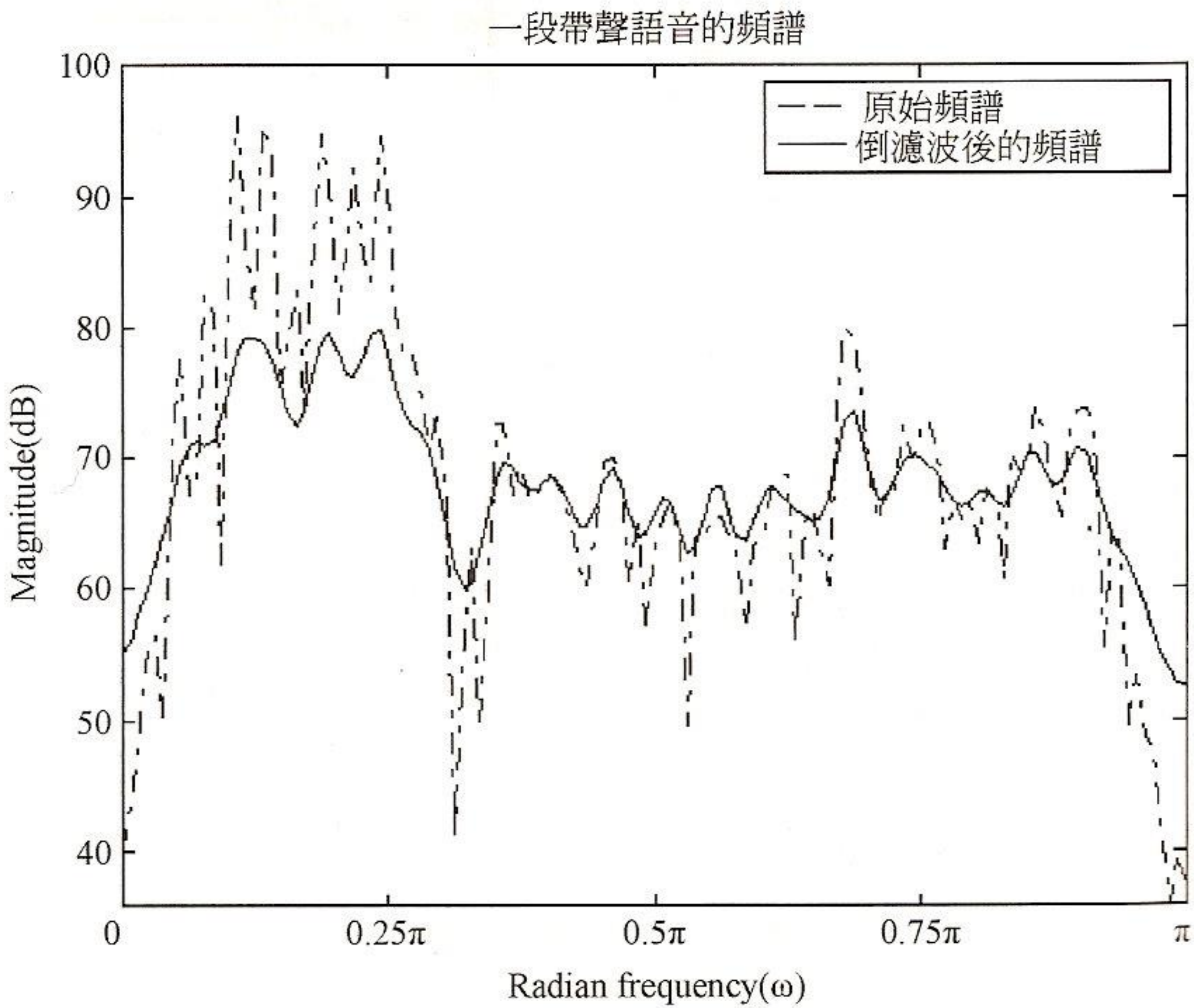


圖 7-6　經過倒濾波器作平滑處理的頻譜

⦿ 5-F Problems of Cepstrum

(1) $|\log(X(Z))|$

(2) Phase

(3) Delay Z^{-k}

(4) Only suitable for the multiple-path-like problem

◉ 5-G Differential Cepstrum

$$\hat{x}_d(n) = Z^{-1}\left[\frac{X'(Z)}{X(Z)}\right] \quad \text{或} \quad \hat{x}_d[n] = \int_{-1/2}^{1/2} \frac{X'(F)}{X(F)} e^{i2\pi F} dF$$

inverse Z transform

Note: $$\frac{d}{dZ}\hat{X}(Z) = \frac{d}{dZ}\log(X(Z)) = \frac{X'(Z)}{X(Z)}$$

If $x(n) = x_1(n) * x_2(n)$

$$X(Z) = X_1(Z)\cdot X_2(Z)$$

$$X'(Z) = X_1'(Z)\cdot X_2(Z) + X_1(Z)\cdot X_2'(Z)$$

$$\frac{X'(Z)}{X(Z)} = \frac{X_1'(Z)}{X_1(Z)} + \frac{X_2'(Z)}{X_2(Z)} \qquad \therefore \quad \hat{x}_d(n) = \hat{x}_{1d}(n) + \hat{x}_{2d}(n)$$

Advantages: no phase ambiguity

able to deal with the delay problem

• Properties of Differential Cepstrum

(1) The differential Cepstrum is shift & scaling invariant

不只適用於 multi-path-like problem

也適用於 pattern recognition

If $y[n] = A\,X[n - r]$

$$\Rightarrow \quad \hat{y}_d(n) = \begin{cases} \hat{x}_d(n) & , \quad n \neq 1 \\ -r + \hat{x}_d(1) & , \quad n = 1 \end{cases}$$

(Proof): $Y(z) = Az^{-r} X(z)$

$$Y'(z) = Az^{-r} X'(z) - rAz^{-r-1} X(z)$$

$$\frac{Y'(z)}{Y(z)} = \frac{X'(z)}{X(z)} - rz^{-1}$$

(2) The complex cepstrum $\hat{C}[n]$ is closely related to its differential cepstrum $\hat{x}_d[n]$ and the signal original sequence $x[n]$

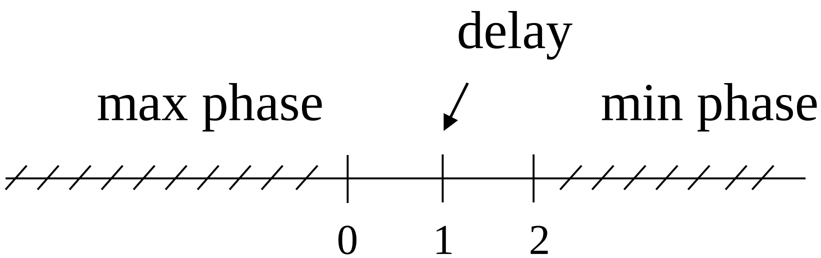
$$\hat{C}(n) = \frac{-\hat{x}_d(n+1)}{n} \qquad n \neq 0 \qquad \textit{diff cepstrum}$$

$$\textit{and} \quad -(n-1)\,x(n-1) = \sum_{k=-\infty}^{\infty} \hat{x}_d(n)\,x(n-k) \qquad \textit{recursive formula}$$

Complex cepstrum 做得到的事情, differential cepstrum 也做得到！

(3) If $x[n]$ is minimum phase (no poles & zeros outside the unit circle), then $\hat{x}_d[n] = 0$ for $n \leq 0$

(4) If $x[n]$ is maximum phase (no poles & zeros inside the unit circle) , then $\hat{x}_d[n] = 0$ for $n \geq 2$



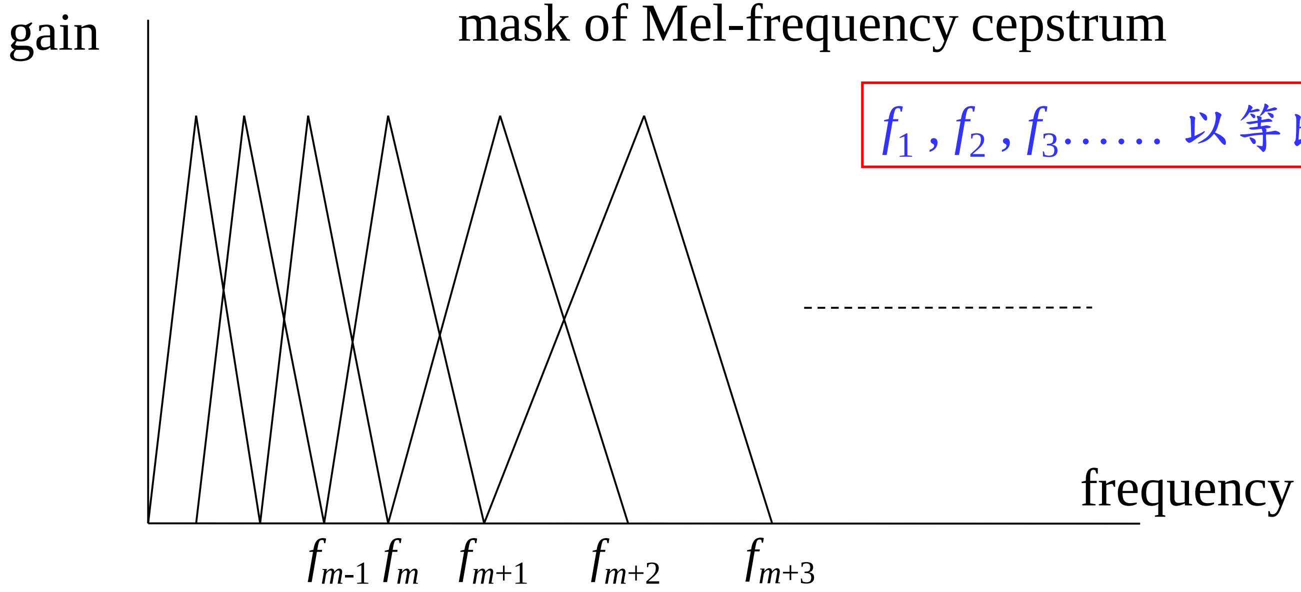
(5) If $x(n)$ is of finite duration, $\hat{x}_d[n]$ has infinite duration

Complex cepstrum decay rate $\propto \dfrac{1}{n}$

Differential Cepstrum decay rate 變慢了, ∵ $\hat{x}_d(n+1) = n \cdot \hat{c}(n) \propto n \cdot \dfrac{1}{n} = 1$

⦿ 5-H Mel-Frequency Cepstrum (梅爾頻率倒頻譜)

Take log in the frequency mask



f_1 , f_2 , f_3...... 以等比級數增加

$B_m[k] = 0$ for $f < f_{m-1}$ and $f > f_{m+1}$

$B_m[k] = (k - f_{m-1})/(f_m - f_{m-1})$ for $f_{m-1} \le f \le f_m$

$B_m[k] = (f_{m+1} - k)/(f_{m+1} - f_m)$ for $f_m \le f \le f_{m+1}$

$f = k\, f_s / N$

Process of the Mel-Cepstrum

summation of the effect inside the m^{th} mask

(1) $x[n] \xrightarrow{FT} X[k]$

(2) $$Y[m] = \log\left\{ \sum_{k=f_{m-1}}^{f_{m+1}} \left|X[k]\right|^2 B_m[k] \right\}$$

(3) $$c_x[n] = \frac{1}{M}\sum_{m=1}^{M} Y[m]\cos\left(\frac{\pi n(m-1/2)}{M}\right)$$

Q: What are the difference between the Mel-cepstrum and the original cepstrum?

Advantages :

Mel-frequency cepstrum 更接近人耳對語音的區別性

用 $c_x[1]$, $c_x[2]$, $c_x[3]$, ……., $c_x[13]$ 即足以描述語音特徵

◉ 5-I References

- R. B. Randall and J. Hee, “Cepstrum analysis,” *Wireless World*, vol. 88, pp. 77-80. Feb. 1982
- 王小川，“語音訊號處理”，全華出版，台北，民國94年。
- A. V. Oppenheim and R. W. Schafer, *Discrete-Time Signal Processing*, London: Prentice-Hall, 3rd ed., 2010.
- S. C. Pei and S. T. Lu, “Design of minimum phase and FIR digital filters by differential cepstrum,” *IEEE Trans. Circuits Syst. I*, vol. 33, no. 5, pp. 570-576, May 1986.
- S. Imai, “Cepstrum analysis synthesis on the Mel-frequency scale,” *ICASSP*, vol. 8, pp. 93-96, Apr. 1983.

附錄六：聲音檔和影像檔的處理 (by Matlab)

A. 讀取聲音檔

- 電腦中，沒有經過壓縮的聲音檔都是 *.wav 的型態

- 讀取： **wavread** 或 audioread

  註：2015版本以後的 Matlab，**wavread** 將改為 **audioread**

- 例： [**x**, **fs**] = wavread('C:\WINDOWS\Media\ringin.wav');
  可以將 ringin.wav 以數字向量 **x** 來呈現。 **fs**: sampling frequency
  這個例子當中 size(**x**) = 9981 1 fs = 11025

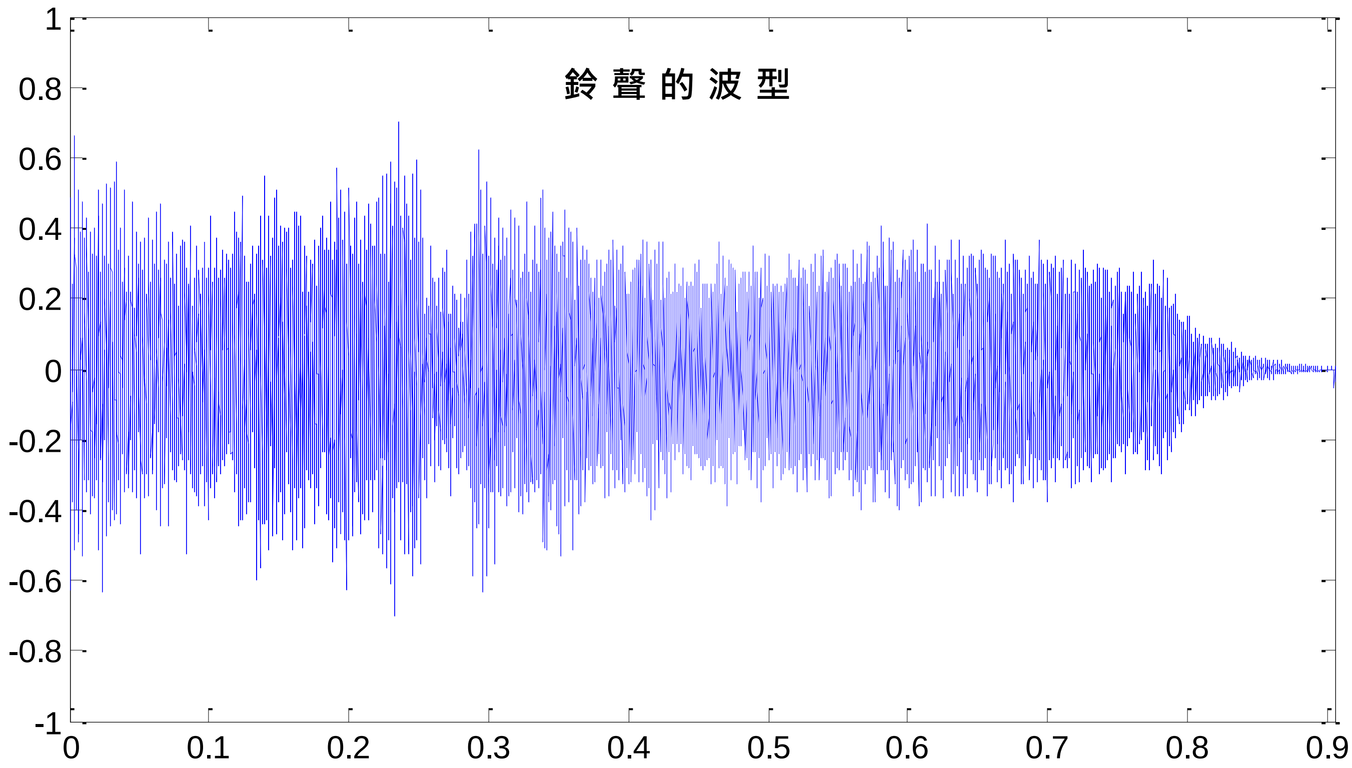
- 思考: 所以，取樣間隔多大?

- 這個聲音檔有多少秒？

畫出聲音的波型

```
time = [0:length(x)-1]/fs;    % x 是前頁用 wavread 所讀出的向量
```

plot(time, x)



注意：*.wav 檔中所讀取的資料，值都在 –1 和 +1 之間

一個聲音檔如果太大，我們也可以只讀取它部分的點

[x, fs]=wavread('C:\WINDOWS\Media\ringin.wav', [4001 5000]);

% 讀取第4001至5000點

[x, fs, **nbits**] = wavread('C:\WINDOWS\Media\ringin.wav');

nbits: x(n) 的bit 數

第一個bit : 正負號，第二個bit : 2^{-1}，第三個bit : 2^{-2}， .....，

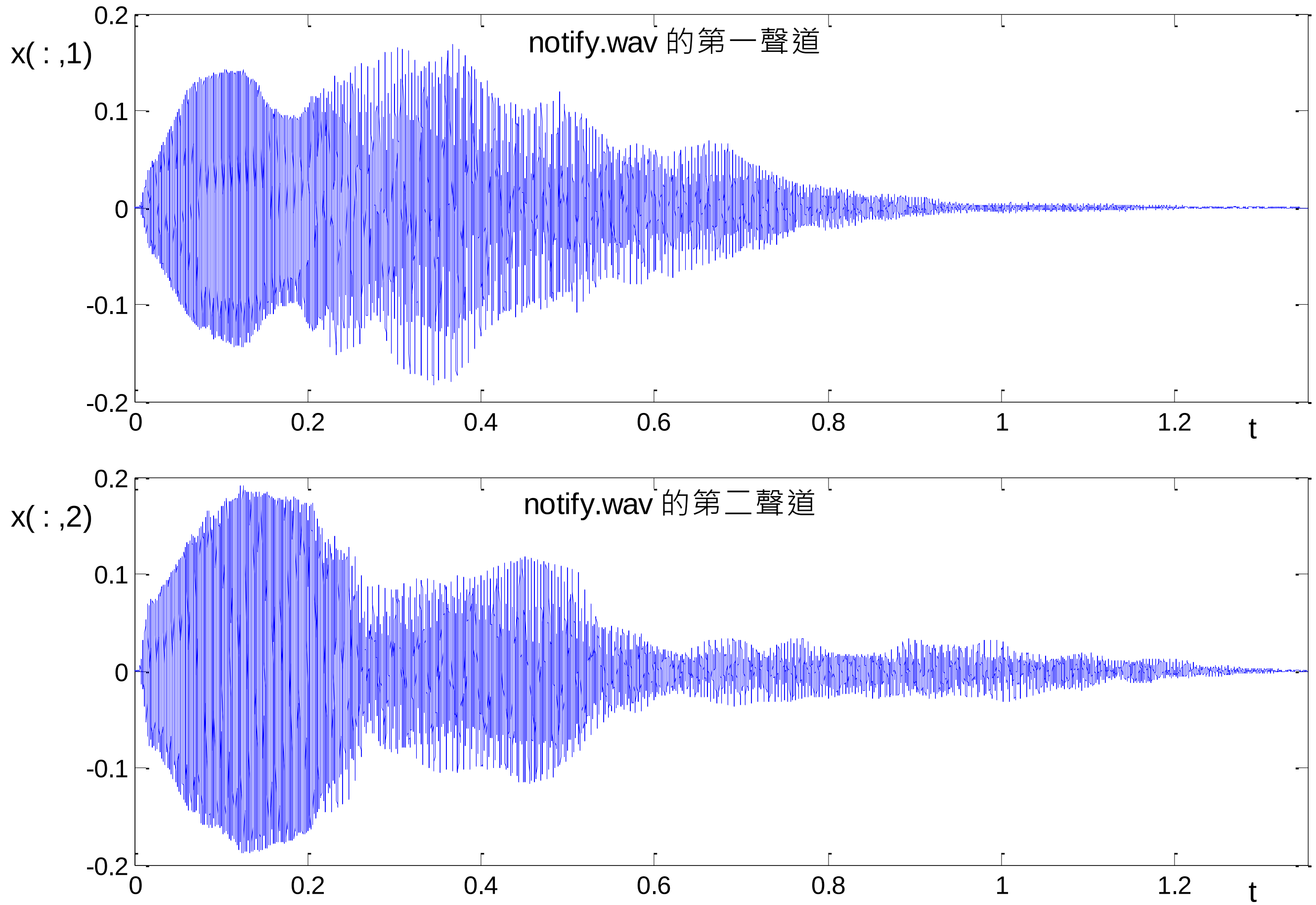
第 n 個bit : $2^{-nbits+1}$， 所以 **x** 乘上$2^{nbits-1}$ 是一個整數

以鈴聲的例子， nbits = 8，所以 **x** 乘上 128是個整數

•有些聲音檔是 **雙聲道**（**Stereo**）的型態 (俗稱**立體聲**)

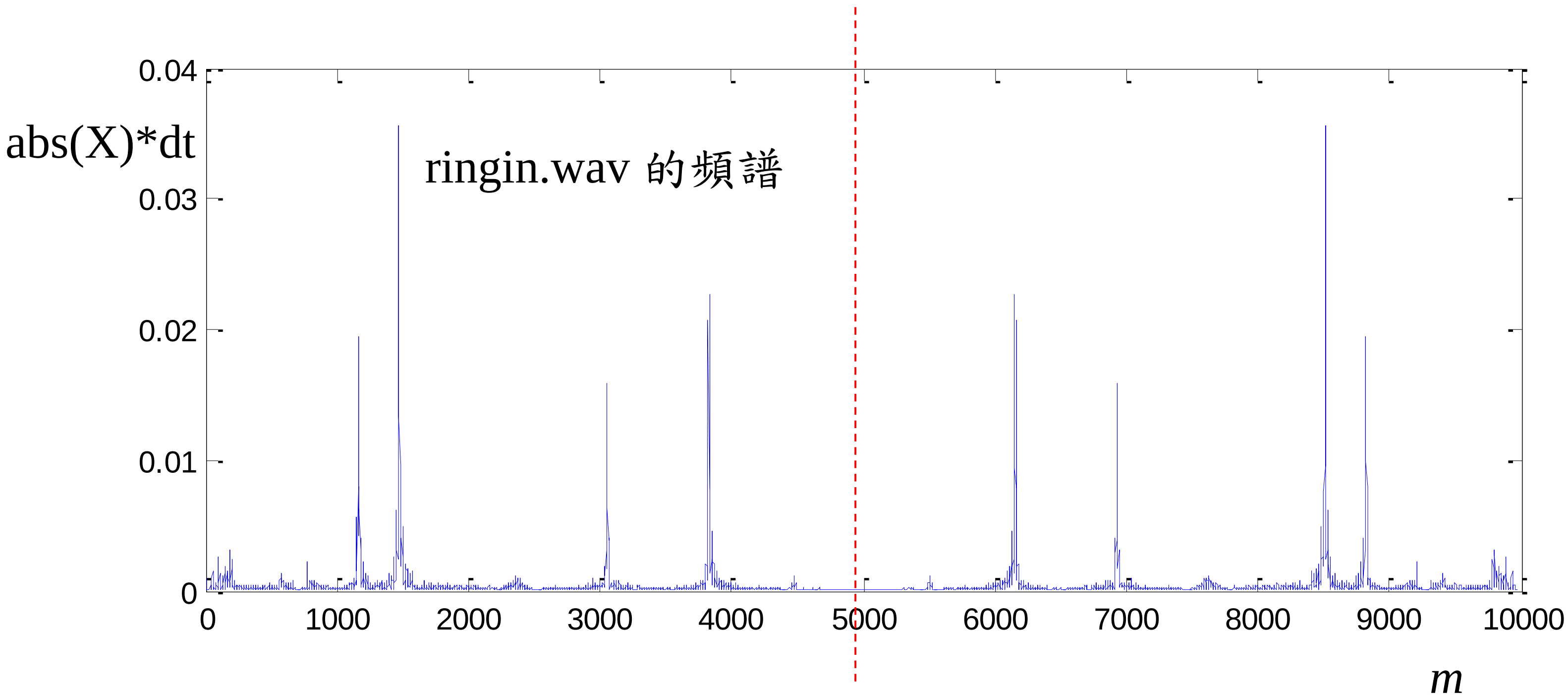
例： [x, fs]=wavread('C:\WINDOWS\Media\notify.wav');

size(x) = 29823   2   fs = 22050



B. 繪出頻譜 (請參考附錄二)

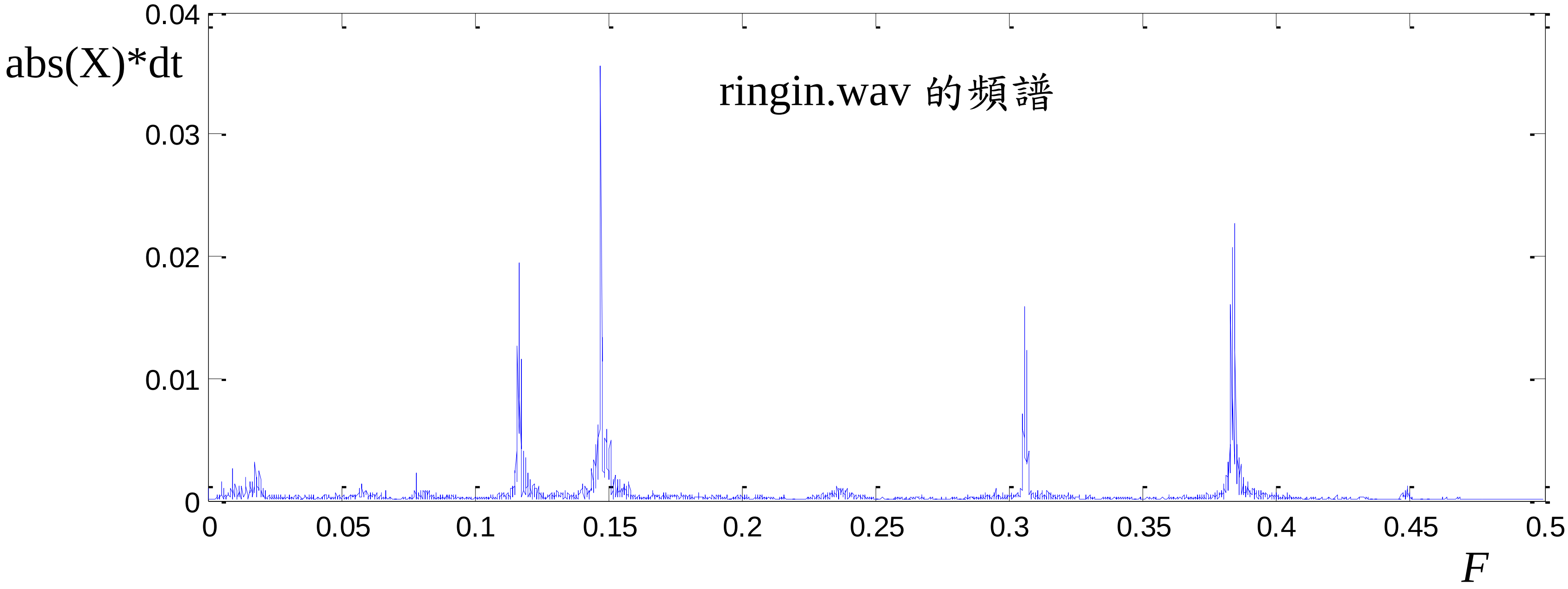
X = fft(x); plot(abs(X)*dt); % dt = 1/fs

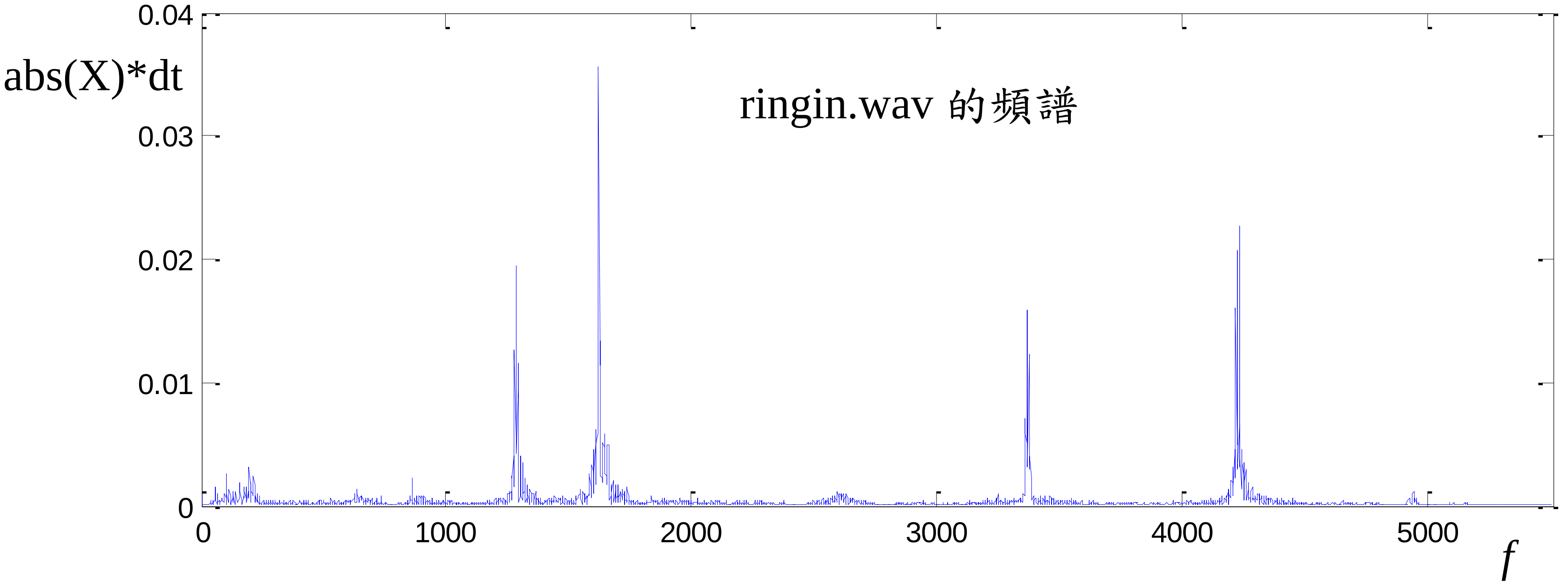


fft 橫軸 轉換的方法

(1) Using normalized frequency F: $F = m / N$.

(2) Using frequency f, $f = F \times f_s = m \times (f_s / N)$.


abs(X)*dt
ringin.wav 的頻譜
0.04
0.03
0.02
0.01
0
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
F


abs(X)*dt
ringin.wav 的頻譜
0.04
0.03
0.02
0.01
0
0
1000
2000
3000
4000
5000
f

C. 聲音的播放

(1) wavplay(**x**): 將 **x** 以 11025Hz 的頻率播放

(時間間隔 = 1/11025 = 9.07×10^{-5} 秒)

(2) sound(**x**): 將 **x** 以 8192Hz 的頻率播放

(3) wavplay(**x, fs**) 或 sound(**x, fs**): 將 **x** 以 **fs** Hz 的頻率播放

Note: (1)~(3) 中 **x** 必需是1 個column (或2個 columns)，且 **x** 的值應該 介於 –1 和 +1 之間

(4) soundsc(**x, fs**): 自動把 **x** 的值調到 –1 和 +1 之間 再播放

D. 用 Matlab 製作 *.wav 檔： wavwrite

wavwrite(x, fs, waveFile)

將數據 **x** 變成一個 *.wav 檔，取樣速率為 **fs** Hz

① **x** 必需是1 個column (或2個 columns)　② **x** 值應該 介於 –1 和 +1 之間

③ 若沒有設定fs，則預設的fs 為 8000Hz

E. 用 Matlab 錄音的方法

錄音之前，要先將電腦接上麥克風，且確定電腦有音效卡

(部分的 notebooks 不需裝麥克風即可錄音)

範例程式：

```
Sec = 3;
Fs = 8000;
recorder = audiorecorder(Fs, 16, 1);
recordblocking(recorder, Sec);
audioarray = getaudiodata(recorder);
```

執行以上的程式，即可錄音。

錄音的時間為三秒，sampling frequency 為 8000 Hz

錄音結果為 audioarray，是一個 column vector (如果是雙聲道，則是兩個 column vectors)

範例程式 (續)：

```
wavplay(audioarray, Fs);                % 播放錄音的結果
t = [0:length(audioarray)-1]./Fs;
plot (t, audioarray‘);                  % 將錄音的結果用圖畫出來
xlabel('sec','FontSize',16);
wavwrite(audioarray, Fs, ‘test.wav’)    % 將錄音的結果存成 *.wav 檔
```

指令說明：

recorder = audiorecorder(Fs, nb, nch);　　(提供錄音相關的參數)

Fs: sampling frequency,

nb: using nb bits to record each data

nch: number of channels (1 or 2)

recordblocking(recorder, Sec);　　(錄音的指令)

recorder: the parameters obtained by the command “audiorecorder”

Sec: the time length for recording

audioarray = getaudiodata(recorder);

(將錄音的結果，變成 audioarray 這個 column vector，如果是雙聲道，則 audioarray 是兩個 column vectors)

以上這三個指令，要並用，才可以錄音

F、MP3 檔的讀和寫

要先去這個網站下載 mp3read.m, mp3write.m 的程式

http://www.mathworks.com/matlabcentral/fileexchange/13852-mp3read-and-mp3write
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mp3read.m ： 讀取 mp3 的檔案
mp3write.m ： 製作 mp3 的檔案

不同於 *.wav 檔 (未壓縮過的聲音檔)，*.mp3 是經過 MPEG-2 Audio Layer III的技術壓縮過的聲音檔

範例：

```
%% Write an MP3 file by Matlab

fs=8000;            % sampling frequency
t = [1:fs*3]/3;
filename = ‘test’;
Nbit=32;            % number of bits per sample
x= 0.2*cos(2*pi*(500*t+300*(t-1.5).^3));
mp3write(x, fs, Nbit, filename);   % make an MP3 file test.mp3

%% Read an MP3 file by Matlab

[x1, fs1]=mp3read('phase33.mp3');
x2=x1(577:end);    % delete the head
sound(x2, fs1)
```

F：影像檔的處理

Image 檔讀取:　imread

Image 檔顯示:　imshow,　image,　imagesc

Image 檔製作:　imwrite

基本概念：灰階影像在 Matlab 當中是一個矩陣

彩色影像在 Matlab 當中是三個矩陣，分別代表 Red, Green, Blue

*.bmp:　沒有經過任何壓縮處理的圖檔

*.jpg:　有經過 JPEG 壓縮的圖檔

Video 檔讀取:　aviread

範例一：(黑白影像)

```
im=imread('C:\Program Files\MATLAB\pic\Pepper.bmp');
```

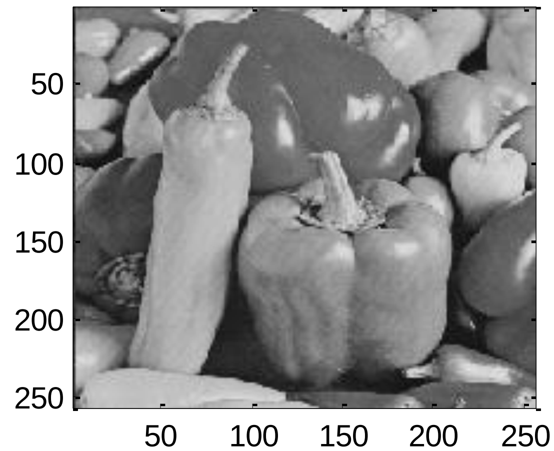
(注意，如果 Pepper.bmp 是個灰階圖，im 將是一個矩陣)

```
size(im)
```

(用 size 這個指令來看 im 這個矩陣的大小)

```
ans =
  256   256
```

```
image(im);
colormap(gray(256))
```



範例二：(彩色影像)

```
im2=imread('C:\Program Files\MATLAB\pic\Pepper512c.bmp');
```

```
size(im2)
```

(注意，由於這個圖檔是個彩色的，所以 im2 將由三個矩陣複合而成)

```
ans =
  512   512   3
```

```
imshow(im);
```

or

```
image(im/255);
```

注意：要對影像做運算時，<u>要先變成 double 的格式</u>

否則電腦會預設影像為 integer 的格式，在做浮點運算時會產生誤差

例如，若要對影像做 2D Discrete Fourier transform

```
im=imread('C:\Program Files\MATLAB\pic\Pepper.bmp');
im=double(im);
Imf=fft2(im);
```