高等數位訊號處理(Advanced Digital Signal Processing)
Homework 05 – 2017/06/30

R05546030 **彭新翔**

1. Write a Matlab program that can generate the forward and inverse N-point number theoretic transform matrices (modulus M).

$$[A, B] = \text{NTTm}(N, M) \qquad \text{\% A: forward, B: inverse}$$

The outputs A and B are $N \times N$ matrices. Choose the smallest positive α.

The program should be able to run for large N (avoid calculating α^k directly).

The MATLAB program should be mailed to me.

期末太忙來不及寫惹 QQ

R05546030 **彭新翔**

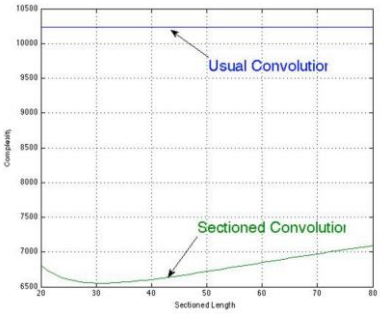
2. In addition to the linear complexity, what is the other important advantage of the sectioned DFT convolution?

使用離散傅立葉轉換(DFT, Discrete Fourier Transform)進行運算，由於其計算複雜度僅為 $O(N\log_2 N)$ 屬於較為快速的演算法，因此適合做頻譜分析，但其輸出結果較為複雜且指數函數並非二元形式(binary form)。因此在進行摺積計算時，可以被下述方法取代：

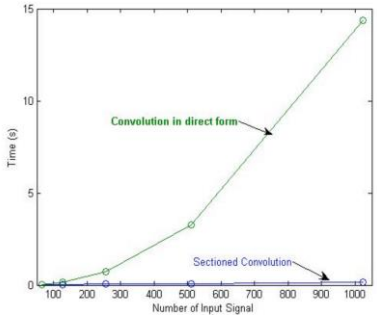
(1) DCT
(2) DST
(3) DHT
(4) Directly Computing
(5) Sectioned DFT Convolution
(6) Wingograd Algorithm
(7) Number Theoretic Transform (NTT)
(8) Z-Transform Based Recursive Method

當採用分段 DFT 摺積運算時，最佳化的分段長度與 N 值無關，其中又若當 M 為固定常數時可以使得計算複雜度與 N 值呈線性關係。除此之外還具備節省能源與穩定硬體架構的優點，如下所示：

× **Saving Energy**



× **Saving Time**



× **Fixed Hardware Architecture**

The Complexity of Sectioned Convolution is $C = 2 \times \frac{N}{L-M+1} \cdot \frac{L}{2}\log_2 L.$

Optimal Sectioned Length is $M = \frac{L+1+\log L}{1+\log L} = 1 + \frac{L}{1+\log_2 L}$

R05546030 **彭新翔**

3. What are the most important applications of (a) the Walsh transform, (b) the Haar transform, and (c) the NTT nowadays?

關於 Walsh Transform、Harr Transform 和 NTT 於目前發展的重要應用如下：

(a) **Walsh Transform**

生物醫學影像辨識(Medical and Biological Image Processing)、語音辨識(Speech Recognition)、數位全息投影(Digital Holography)…等。

(b) **Haar Transform**

影像壓縮處理(Image Compression)

(c) **Number Theoretic Transform (NTT)**

快速摺積運算(Fast Digital Convolution)，如：應用於數位編碼加密解密(Digital Coding for Encryption and Decryption)

R05546030 **彭新翔**

4. How many entries of (a) the N-point Walsh transform and (b) the N-point Haar transform that are equal to 0, 1, and -1?

R05546030 **彭新翔**

5. What are the advantages and limitation when using the NTT to calculate the convolution?.

× 優點

(1) 僅只需要對整數進行加法與乘法運算。

(2) 若適當地使用 LUT，則無須任何的加法與乘法運算

(3) 根據其摺積性質，當計算環狀摺積(Circular Convolution)時，可以使用 NTT 取代 DFT。

× 限制

(1) 由於不易觀察較高頻率項，因此並不適合作頻譜分析。

(2) 對於摺積運算而言，不適合處理存在有非整數的數列或其輸出不在 $[0, M-1]$ 者。

R05546030 **彭新翔**

6. Why the orthogonal transform plays an important role in signal processing?

由於訊號處理中常出現有遺失的項需要進行重建(Reconstruction)，對於正交形式(Orthogonal Case)而言：

(1) Perfect Reconstruction：

$$x[n] = \sum_{m=0}^{N-1} C_m^{-1} y[m] \phi_m[n]$$

(2) Partial Reconstruction：

$$x_k[n] = \sum_{m=0}^{K-1} C_m^{-1} y[m] \phi_m[n], K < N$$

(3) Reconstruction Error of Partial Reconstruction：

$$||x[n] - x_k[n]||^2 = \sum_{m=k}^{N-1} \sum_{m_1=k}^{N-1} C_m^{-1} y[n] C_{m_1}^{-1} y^*[m_1] C_m \delta[m - m_1] = \sum_{m=K}^{N-1} C_m^{-1} |y[m]|^2$$

其中由於 $C_m^{-1}|y[m]|^2$ 必為正值，因此當 K 越大時，Reconstruction Error 必定越小；而為非正交形式(Non-Orthogonal Case)而言，則無法保證當 K 越大時，Reconstruction Error 會越小。由此可知 The Orthogonal Transform 在訊號處理中扮演了極為重要的角色。

R05546030 **彭新翔**

7. (a) What is the results of CMDA if there are three data [1 0 0], [1 0 1], [0 1 1] and these three data are modulated by the 1^{st}, 5^{th}, and 10^{th} rows of the 16-point Walsh transform?
   (b) Is it better to use the Haar transform and the number theoretic transform for CMDA? Why?

(a) QQ 不會寫

(b) 選擇 Haar Transform 較佳，雖然 Haar Wavelet 因為不具連續性而不可微分為其缺點，但對於分析具有 Sudden Transitions 的 CDMA 來說則是一個益處。此外 Number Theoretic Transform (NTT)矩陣中由於存在多個值為零，因而不適合用以處理 CMDA。

R05546030 **彭新翔**