賽局理論與應用(Game Theory with Applications)
Homework 01 – 2016/09/29

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1. Give an example of a business situation that can be modeled as a game. Find an article from a reputable journalistic source detailing a business situation. Identify the players involved, the set of actions, and a description of the payoffs. If you can identify the classic game to which the situation belongs (e.g., Ice Cream stand example, Prisoner's Dilemma, etc.), then make this argument. Predict the outcome of this game; if applicable, describe how the outcome would be different if there were multiple players acting strategically. Make sure to detail your reference for this article explicitly.

範例來源與參考資料：

(1) The Economist, 1997 May 15th, Learning to play the game.
(2) BBC News, 2010 Mar 16th, US plans to give high-speed broadband to every American.
(3) Knowledge@Wharton, 2016 Mar 21st , The FCC's Spectrum Auction: Can the System Be Gamed?
(4) iThome, 2016 Jul 05th, FCC 擬以 864 億美元價碼拍賣電視頻譜

「隨著無線通信技術的快速發展與服務的多元演進，無線通信的供應及需求均日漸增加，對於無線頻譜的使用需求將益發強烈。然而，無線頻譜為稀有資源，且使用上具有排他性，因此，如何最大化頻譜的利用效率，已成為未來頻譜管理政策面上的重要目標。」[1]。1993 年美國國會在預算法案中，要求美國聯邦通訊委員會(Federal Communications Commission, FCC)須針對不同的出價方法提出最好的拍賣方針。

❖ **Players**

FCC, AT&T, Verizon, T-Mobile … etc

❖ **Actions**

FCC 如何將同樣數量的牌照賣出到盡可能高的價格？競拍電信商如何以盡可能低的價格購買到自己想要的牌照？自 1994 年啟用拍賣機制，時至今日每年皆有可能改變變動內容為「是否允許投標者進行組合報價（是否允許電信業者對於不同牌照組合報出總價，並不分別對每張牌照報出單價，然而然而其他電信業者依然可以針對每個牌照報出單獨的價格，在拍賣結束時則再比較總價與個別單價之組合）」，以下分析兩者情況。

❖ **Payoffs**

不允許組合報價	允許組合報價
當其中電信業者為贏得單一牌照，卻因無法有效得知其他電信業者出價之策略，致使超出預算範圍之錯誤報價。最終多數的電信業者都將損失，而 FCC 則為其中最大贏家獲得淨收入。即拍賣行為中所謂的風險暴露問題(Exposure problem)	在允許組合報價之情況下，電信業者得以直接對不同之牌照報出組合價格，若在拍賣中獲勝，則可以在預算範圍取得許可牌照（若在拍賣中被更高出價者擊敗，也不需要多付出一分錢），可以有效規避風險暴露問題(Exposure problem)。然而對於 FCC 則可能必須以不如預期地價格販售牌照，即所謂的門檻問題(Threshold Problem)

[1] 2007, 彭心儀、王郁琦、周韻采…等,《規劃頻率拍賣與回收制度之研究》

2. Consider the following game:

		Player 2	
		Z	W
Player 1	X	5, 5	-8, 8
	Y	-7, -8	0, 0

(1) What is the Nash equilibrium?

(2) Justify your answer in (1).

(3) Change only one of the eight entries in the table such that there is no equilibrium.

(1) A **Nash Equilibrium** in a game is a list of strategies, one for each player, such that no player can get a better payoff by switching to some other strategy that is available to her while all other players adhere to the strategies specified for them in the list.[2]

(2) 考慮 Player 1 選擇策略 X 時，Player 2 將選擇策略 W；
考慮 Player 1 選擇策略 Y 時，Player 2 亦將選擇策略 W；
因此考慮當 Player 2 選擇優勢策略 W 時，Player 1 將選擇 Y，
此時 (Y, W) 為 Nash Equilibrium。

(3) 只要使其策略組合中 Player 2 的報酬改為非零，即沒有均衡。
如：改變 Player 2 中 $(Y, W) = 0$ 使 $(Y, W) < -8$。

[2] Avinash Dixit, Games of Strategy 4th Edition, W. W. Norton & Company

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3. Read the file, “Review of fixed point theorem.pdf,” and answer the following questions.

(1) Find the fixed point of the function, $f(x) = x^2 - 3x + 4$.

(2) Does the function, $f(x) = x + 1$, have a fixed point? Why or why not?

(3) For each $x \in \mathbb{R}$ define $F(x) = (x, \infty) = \{y \in \mathbb{R} : y > x\}$. Then $F : \mathbb{R} \to \mathbb{R}$ is a correspondence. What is the correspondence, $F(2)$?

(4) Let C be a correspondence defined on the closed interval $[0,1]$ that maps a point x to the closed interval $[1 - x/2, 1 - x/4]$. Draw all fixed points on the graph.

(1) By **Brouwer Fixed Point Theorem**, Let

$$\begin{aligned} &f(x) = x^2 - 3x + 4 = x \\ \Longrightarrow\ &x^2 - 4x + 4 = (x-2)^2 = 0 \\ \Longrightarrow\ &x = 2 \end{aligned}$$

Then we can get the fixed point $(x, y) = (2, 2)$.

(2) It has no fixed point because that the line $y = x$ is parallel to the line $y = x + 1$.

(3) $\forall x \in \mathbb{R},\ F(x) = (x, \infty) = \{y \in \mathbb{R} : y > x\}$

Then $F(2) = (2, \infty)$.

(4) As the figure below, we can find all the fixed points on the thick segment (intersection by the line $y = x$ and the shadow area).