R05546030 **彭新翔**

1. Write the Matlab program to compute the FFT of two N-point real signals x and y using only one N-point FFT.

$$[Fx, Fy] = \text{fftreal}(x, y)$$

<u>The Matlab file should be mailed to displab531@gmail.com</u>.

Code 已寄至信箱。

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2. How do we use <u>three real multiplications</u> to implement a complex multiplication?

$$
\begin{aligned}
(a+bj)(c+dj) &= (ac-bd)+(ad+bc)j \\
&= e+fj
\end{aligned}
$$

$$
\begin{bmatrix} e \\ f \end{bmatrix} = \begin{bmatrix} c & -d \\ d & c \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} c & c \\ c & c \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} 0 & -c-d \\ d-c & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}
$$

$$
\begin{gathered}
e_1 = c(a+b) \\
f_1 = e_2 \\
e_2 = b(-c-d) \\
f_2 = b(d-c)
\end{gathered}
$$

$$
\Longrightarrow \; e = e_1 + e_2, \;\; f = f_1 + f_2
$$

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3. How do we implement the following matrix operations with the lest number of multiplications?

(a)

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} b & a & b \\ a & b & a \\ b & a & b \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

(b)

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} a & b & c & d \\ b & -d & -a & -c \\ c & -a & d & b \\ d & -c & b & -a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

(a)

整理可得

$$\begin{aligned} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} &= \begin{bmatrix} b & a & b \\ a & b & a \\ b & a & b \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \\ &= \frac{1}{2}\left(\begin{bmatrix} (a+b) & (a+b) & (a+b) \\ (a+b) & (a+b) & (a+b) \\ (a+b) & (a+b) & (a+b) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} -(a-b) & (a-b) & -(a-b) \\ (a-b) & -(a-b) & (a-b) \\ -(a-b) & (a-b) & -(a-b) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \right) \end{aligned}$$

考慮

$$\begin{bmatrix} y_4 \\ y_5 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} (a+b) & (a+b) & (a+b) \\ (a+b) & (a+b) & (a+b) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \text{ 和 } \begin{bmatrix} y_6 \\ y_7 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -(a-b) & (a-b) & -(a-b) \\ (a-b) & -(a-b) & (a-b) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

其中

$$y_4 = y_5 = \frac{(a+b)(x_1+x_2+x_3)}{2},\ y_6 = -y_7 = \frac{(b-a)(x_1-x_2+x_3)}{2}$$

則有

$$y_1 = y_4 + y_6,\ y_2 = y_3 = y_5 + y_7$$

(b)

整理可得

$$\begin{aligned} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} &= \begin{bmatrix} a & b & c & d \\ b & -d & -a & -c \\ c & -a & d & b \\ d & -c & b & -a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \\ &= \frac{1}{3}\left(\begin{bmatrix} (a+b+c+d) & (a+b+c+d) & (a+b+c+d) & (a+b+c+d) \\ (a+b+c+d) & (a+b+c+d) & (a+b+c+d) & (a+b+c+d) \\ (a+b+c+d) & (a+b+c+d) & (a+b+c+d) & (a+b+c+d) \\ (a+b+c+d) & (a+b+c+d) & (a+b+c+d) & (a+b+c+d) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \right) \end{aligned}$$

4. Determining the numbers of real multiplications for the
   (a) $100-\text{point DFT}$
   (b) $176-\text{point DFT}$
   (c) $338-\text{point DFT}$

- 假設 $N-\text{point DFT}$, 其中 $N = P_1 \times P_2 \times \cdots \times P_k$ 且 $P_1, P_2, \cdots, P_k$ 彼此互質，則其總乘法量為

$$\frac{N}{P_1}B_1 + \frac{N}{P_2}B_2 + \cdots + \frac{N}{P_k}B_k$$

- 假設 $N-\text{point DFT}$, 其中 $N = P^c$ 且 P 為質數，則其總乘法量為

$$N_2B_1 + N_1B_2 + 3D_1 + 2D_2$$

(a) $N = 100 = 10 \times 10$

$\Longrightarrow$ The numbers of real multiplications: $10 \times 10 + 10 \times 20 + 3 \times 80 = 640$

(b) $N = 176 = 11 \times 16$

$\Longrightarrow$ The numbers of real multiplications: $11 \times 20 + 16 \times 46 = 956$

(c) $N = 338 = 2 \times 13^2$

$\Longrightarrow$ The numbers of real multiplications: $2(13 \times 52 + 13 \times 52 + 3 \times 144) = 3568$

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5. Suppose that a $1-D$ ridge detection filter is:

$x_s[n] = x[n] * h[n]$ $\qquad h[1] = h[-1] = -0.3$ $\qquad h[2] = h[-2] = -0.125$

$h[3] = h[-3] = -0.075$ $\qquad h[0] = 1$ $\qquad h[n] = 0$ otherwise

Design <u>an efficient way</u> to implement the above filter operation.

上述一維濾波器由於具備偶函數對稱，採用 Directly Computing Method 最有效率。

6. Suppose that $\text{length}(x[n]) = 1200$. What is the best way to implement the convolution of $x[n]$ and $y[n]$ if

(a) $\text{length}(y[n]) = 600$ (b) $\text{length}(y[n]) = 50$

(c) $\text{length}(y[n]) = 9$ (d) $\text{length}(y[n]) = 3$

(a) $\underline{\text{length}(y[n]) = 600}$

採用 $1799-\text{point DFT/IDFT}$ 實現折積計算。

(b) $\underline{\text{length}(y[n]) = 50}$

先將 $x[n]$ 拆分等分（每一等分大小取 324），採用 $373-\text{point DFT/IDFT}$ 實現折積計算。

(c) $\underline{\text{length}(y[n]) = 9}$

直接計算。

(d) $\underline{\text{length}(y[n]) = 3}$

直接計算。