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1. Solve the following problems by the revised simplex method:

(a)

$$\begin{array}{lccccccc} \max & 3x_1 & + & 2x_2 & + & 4x_3 & & \\ \text{s.t.} & x_1 & + & x_2 & + & 2x_3 & \leq & 4 \\ & 2x_1 & & & + & 3x_3 & \leq & 5 \\ & 2x_1 & + & x_2 & + & 3x_3 & \leq & 7 \\ & & & x_1, x_2, x_3 & & & \geq & 0 \end{array}$$

(b)

$$\begin{array}{lcccccccccc} \max & 5x_1 & + & 6x_2 & + & 9x_3 & + & 8x_4 & & \\ \text{s.t.} & x_1 & + & 2x_2 & + & 3x_3 & + & x_4 & \leq & 5 \\ & x_1 & + & x_2 & + & 2x_3 & + & 3x_4 & \leq & 3 \\ & & & x_1, x_2, x_3, x_4 & & & & & \geq & 0 \end{array}$$

(c)

$$\begin{array}{lccccc} \max & 2x_1 & + & x_2 & & \\ \text{s.t.} & 2x_1 & + & 3x_2 & \leq & 3 \\ & x_1 & + & 5x_2 & \leq & 1 \\ & 2x_1 & + & x_2 & \leq & 4 \\ & 4x_1 & + & x_2 & \leq & 5 \\ & & x_1, x_2 & & \geq & 0 \end{array}$$

(a) Transfer the original problem into argument form and let

$$x_B = \begin{bmatrix} x_4 \\ x_5 \\ x_6 \end{bmatrix},\ x_N = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix},\ b = \begin{bmatrix} 4 \\ 5 \\ 7 \end{bmatrix},\ B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ N = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & 3 \\ 2 & 1 & 3 \end{bmatrix}$$

[Iteration 01] Choose x_3 the entering basic variable and x_5 the leaving variable

$$C_N - C_B B^{-1} N = \begin{bmatrix} 3 & 2 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 4 & 0 \end{bmatrix} \begin{bmatrix} 1 & -\frac{2}{3} & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 2 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 2 & -\frac{4}{3} \end{bmatrix}$$

$$a = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},\ d = B^{-1}a = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix},\ x_B = x_B^* - td = \begin{bmatrix} x_4 \\ x_3 \\ x_6 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} \\ \frac{5}{3} \\ 2 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} x_2$$

[Iteration 02] Choose x_2 the entering basic variable and x_4 the leaving variable

$$C_N - C_B B^{-1} N = \begin{bmatrix} 3 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 2 & 4 & 0 \end{bmatrix} \begin{bmatrix} 2 & -\frac{2}{3} & 0 \\ 0 & \frac{1}{3} & 0 \\ -1 & -\frac{1}{3} & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 2 & 0 & 1 \\ 2 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 0 \end{bmatrix}$$

$$a = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix},\ d = B^{-1}a = \begin{bmatrix} -\frac{1}{3} \\ \frac{2}{3} \\ \frac{1}{3} \end{bmatrix},\ x_B = x_B^* - td = \begin{bmatrix} x_2 \\ x_3 \\ x_6 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} \\ \frac{5}{3} \\ \frac{4}{3} \end{bmatrix} - \begin{bmatrix} -\frac{1}{3} \\ \frac{2}{3} \\ \frac{1}{3} \end{bmatrix} x_1$$

[Iteration 03] Choose x_1 the entering basic variable and x_3 the leaving variable

$$C_N - C_B B^{-1} N = \begin{bmatrix} 4 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 2 & 3 & 0 \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 \\ -1 & -\frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 3 & 0 & 1 \\ 3 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -\frac{3}{2} & -2 & -\frac{1}{2} \end{bmatrix}$$

$$x_B^* = B^{-1}b = \begin{bmatrix} x_2 \\ x_1 \\ x_6 \end{bmatrix} = \begin{bmatrix} \frac{3}{2} \\ \frac{5}{2} \\ \frac{1}{2} \end{bmatrix}$$

Since $C_N - C_B B^{-1} N$ shows that all the variable are negative, we find the best feasible solution.

$$z^* = \frac{21}{2} \text{ with } x_1 = \frac{5}{2},\ x_1 = \frac{3}{2},\ x_3 = x_4 = x_5 = 0,\ x_6 = \frac{1}{2}$$

(b) Transfer the original problem into argument form and let

$$x_B = \begin{bmatrix} x_5 \\ x_6 \end{bmatrix},\ x_N = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix},\ B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ N = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 1 & 2 & 3 \end{bmatrix},\ b = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

<u>[Iteration 01]</u> Choose x_3 the entering basic variable and x_6 the leaving variable

$$C_N - C_B B^{-1} N = \begin{bmatrix} 5 & 6 & 0 & 8 \end{bmatrix} - \begin{bmatrix} 0 & 9 \end{bmatrix} \begin{bmatrix} 1 & -\frac{2}{3} \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{3}{2} & -\frac{9}{2} & -\frac{11}{2} \end{bmatrix}$$

$$a = \begin{bmatrix} 2 \\ 1 \end{bmatrix},\ d = B^{-1}a = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix},\ x_B = x_B^* - td = \begin{bmatrix} x_5 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{3}{2} \end{bmatrix} - \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix} x_2$$

<u>[Iteration 02]</u> Choose x_2 the entering basic variable and x_5 the leaving variable

$$C_N - C_B B^{-1} N = \begin{bmatrix} 5 & 0 & 0 & 8 \end{bmatrix} - \begin{bmatrix} 6 & 9 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & -3 & 0 & 5 \end{bmatrix}$$

$$a = \begin{bmatrix} 1 \\ 3 \end{bmatrix},\ d = B^{-1}a = \begin{bmatrix} -7 \\ 5 \end{bmatrix},\ x_B = x_B^* - td = \begin{bmatrix} x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} -7 \\ 5 \end{bmatrix} x_4$$

<u>[Iteration 03]</u> Choose x_4 the entering basic variable and x_3 the leaving variable

$$C_N - C_B B^{-1} N = \begin{bmatrix} 5 & 0 & 0 & 9 \end{bmatrix} - \begin{bmatrix} 6 & 8 \end{bmatrix} \begin{bmatrix} \frac{3}{5} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{2}{5} \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 3 \\ 1 & 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -2 & -2 & -1 \end{bmatrix}$$

$$a = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\ d = B^{-1}a = \begin{bmatrix} \frac{2}{5} \\ \frac{1}{5} \end{bmatrix},\ x_B = x_B^* - td = \begin{bmatrix} x_2 \\ x_4 \end{bmatrix} = \begin{bmatrix} \frac{12}{5} \\ \frac{1}{5} \end{bmatrix} - \begin{bmatrix} \frac{2}{5} \\ \frac{1}{5} \end{bmatrix} x_1$$

<u>[Iteration 04]</u> Choose x_1 the entering basic variable and x_4 the leaving variable

$$C_N - C_B B^{-1} N = \begin{bmatrix} 8 & 0 & 0 & 9 \end{bmatrix} - \begin{bmatrix} 6 & 5 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 3 \\ 3 & 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} -5 & -2 & -4 & -2 \end{bmatrix}$$

$$x_B^* = B^{-1}b = \begin{bmatrix} x_2 \\ x_1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Since $C_N - C_B B^{-1} N$ shows that all the variables are negative, we find the best feasible solution

$$z^* = 17 \text{ with } x_1 = 1,\ x_2 = 2,\ x_3 = x_4 = x_5 = x_6 = 0$$

(c) Transfer the original problem into argument form and let

$$x_B = \begin{bmatrix} x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix},\ x_N = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix},\ B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},\ N = \begin{bmatrix} 2 & 3 \\ 1 & 5 \\ 2 & 1 \\ 4 & 1 \end{bmatrix},\ b = \begin{bmatrix} 3 \\ 1 \\ 4 \\ 5 \end{bmatrix}$$

<u>[Iteration 01]</u> Choose x_1 the entering basic variable and x_4 the leaving variable

$$C_N - C_B B^{-1} N = \begin{bmatrix} 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 2 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & -4 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 3 \\ 1 & 5 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & -9 \end{bmatrix}$$

$$x_B^* = B^{-1} b = \begin{bmatrix} x_3 \\ x_1 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}$$

Since $C_N - C_B B^{-1} N$ shows that all the variables are negative, we find the best feasible solution

$$z^* = 2 \text{ with } x_1 = 1,\ x_2 = x_4 = 0,\ x_3 = x_6 = 1,\ x_5 = 2$$

2. Solve the following problems:

(a)

$$\begin{array}{lcccccc} \max & 2x_1 & + & 5x_2 & & \\ \text{s.t.} & x_1 & + & 2x_2 & \le & 20 \\ & 2x_1 & + & x_2 & \le & 16 \\ & 2x_1 & & & \le & 2 \\ & & & x_2 & \le & 8 \\ & & x_1, x_2 & & \ge & 0 \end{array}$$

(b)

$$\begin{array}{lcccccccc} \max & 3x_1 & + & 5x_2 & + & 2x_3 & & \\ \text{s.t.} & x_1 & + & x_2 & + & 2x_3 & \le & 7 \\ & 2x_1 & + & 4x_2 & + & 3x_3 & \le & 15 \\ & x_1 & & & & & \le & 4 \\ & & & x_2 & & & \le & 3 \\ & & & & & x_3 & \le & 3 \\ & & & x_1, x_2, x_3 & & & \ge & 0 \end{array}$$

(a) Transfer the original problem into argument form.

$$\begin{array}{lcccccccc} \max & 2x_1 & + & 5x_2 & & & & \\ \text{s.t.} & x_1 & + & 2x_2 & + & x_3 & = & 20 \\ & 2x_1 & + & x_2 & + & x_4 & = & 16 \\ & 2x_1 & & & \le & 2 & & \\ & & & x_2 & \le & 8 & & \\ & & x_1, x_2, x_3, x_4 & & \ge & 0 & & \end{array}$$

[Iteration 01] Choose x_2 the basic entering variable ($x_2 = t$)

$$\begin{array}{lcccccccc} x_3 & = & 20 & - & 2t & ,t & \le & 10 \\ x_4 & = & 16 & - & t & ,t & \le & 16 \\ x_2 & = & & & t & & \le & 8 \end{array}$$

$$\text{Let } x_2 = t = 8 - x_2'$$

$$\begin{array}{lcccccc} x_3 & = & 4 & - & x_1 & + & 2x_2' \\ x_4 & = & 8 & - & 2x_1 & + & x_2' \\ z & = & 40 & + & 2x_1 & - & 5x_2' \end{array}$$

[Iteration 02] Choose x_1 the basic entering variable ($x_1 = t$)

$$\begin{array}{lcccccccc} x_3 & = & 4 & - & t & ,t & \le & 4 \\ x_4 & = & 8 & - & 2t & ,t & \le & 4 \\ x_1 & = & & & t & & \le & 1 \end{array}$$

$$\text{Let } x_1 = t = 1 - x_1'$$

$$\begin{array}{rcrcrcr} x_3 & = & 3 & + & x'_1 & + & 2x'_2 \\ x_4 & = & 6 & + & 2x'_1 & + & x'_2 \\ z & = & 42 & - & 2x'_1 & - & 5x'_2 \end{array}$$

Since the coefficients of x'_1 and x'_2 are all negative, we find the best solution

$$z^* = 42 \text{ with } x_1 = 1,\ x_2 = 8,\ x_3 = 3,\ x_4 = 6$$

(b) Transfer the original problem into argument form

$$\begin{array}{lrcrcrcrcr} \max & 3x_1 & + & 5x_2 & + & 2x_3 & & & & \\ \text{s.t.} & x_1 & + & x_2 & + & 2x_3 & + & x_4 & = & 7 \\ & 2x_1 & + & 4x_2 & + & 3x_3 & + & x_5 & = & 15 \\ & x_1 & & & & & & \le & & 4 \\ & & & x_2 & & & & \le & & 3 \\ & & & & & x_3 & & \le & & 3 \\ & & & x_1, x_2, x_3, x_4, x_5 & & & & \ge & & 0 \end{array}$$

[Iteration 01] Choose x_2 the basic entering variable ($x_2 = t$)

$$\begin{array}{rcrcrcl} x_4 & = & 7 & - & 2t & ,t & \le 7 \\ x_5 & = & 15 & - & t & ,t & \le \frac{15}{4} \\ x_2 & = & & & t & & \le 3 \end{array}$$

$$\text{Let } x_2 = t = 3 - x'_2$$

$$\begin{array}{rcrcrcrcr} x_4 & = & 4 & - & x_1 & + & 2x'_2 & - & 2x_3 \\ x_5 & = & 3 & - & 2x_1 & + & 4x'_2 & - & 3x_3 \\ z & = & 15 & + & 3x_1 & - & 5x'_2 & + & 2x_3 \end{array}$$

[Iteration 02] Choose x_1 the basic entering variable ($x_1 = t$)

$$\begin{array}{rcrcrcl} x_4 & = & 4 & - & t & ,t & \le 4 \\ x_5 & = & 3 & - & 2t & ,t & \le \frac{3}{2} \\ x_1 & = & & & t & & \le 4 \end{array}$$

$$\text{Let } x_1 = t = 4 - x'_1$$

$$\begin{array}{rcrcrcrcr} x'_2 & = & \frac{5}{4} & - & \frac{1}{2}x'_1 & + & \frac{7}{20}x_3 & + & \frac{1}{4}x_5 \\ x_4 & = & \frac{5}{4} & + & \frac{1}{2}x'_1 & - & \frac{5}{4}x_3 & + & \frac{1}{4}x_5 \\ z & = & \frac{83}{4} & - & \frac{1}{2}x'_1 & - & \frac{19}{4}x_3 & - & \frac{5}{4}x_5 \end{array}$$

Since the coefficient of x'_1, x_3 and x_5 are all negative, we find the best solution

$$z^* = \frac{83}{4} = 20.75 \text{ with } x_1 = 4,\ x_2 = \frac{7}{4} = 1.75,\ x_3 = 0,\ x_4 = \frac{5}{4} = 1.25,\ x_5 = 0$$

3. Solve the following problem by the dual simplex method.

$$\begin{array}{lrcrcrcr} \min & 3x_1 & + & 2x_2 & + & x_3 & & \\ \text{s.t.} & 3x_1 & + & x_2 & + & x_3 & \ge & 3 \\ & 3x_1 & - & 3x_2 & - & x_3 & \le & -6 \\ & x_1 & + & x_2 & + & x_3 & \le & 3 \\ & & & x_1, x_2, x_3 & & & \ge & 0 \end{array}$$

Transfer the original problem into standard argument form and the dual form

Primal

$$\begin{array}{lrcrcrcrcr} \max & -3x_1 & - & 2x_2 & - & x_3 & & & & \\ \text{s.t.} & -3x_1 & - & x_2 & - & x_3 & + & x_4 & = & -3 \\ & 3x_1 & - & 3x_2 & - & x_3 & + & x_5 & = & -6 \\ & x_1 & + & x_2 & + & x_3 & + & x_6 & = & 3 \\ & & & x_1, x_2, x_3, x_4, x_5, x_6 & & & & & \ge & 0 \end{array}$$

Dual

$$\begin{array}{lrcrcrcrcr} \min & -3y_1 & - & 6y_2 & + & 3y_3 & & & & \\ \text{s.t.} & 3y_1 & - & 3y_2 & - & y_3 & + & y_4 & = & 3 \\ & y_1 & + & 3y_2 & - & y_3 & + & y_5 & = & 2 \\ & y_1 & + & y_2 & - & y_3 & + & y_6 & = & 1 \\ & & & y_1, y_2, y_3, y_4, y_5, y_6 & & & & & \ge & 0 \end{array}$$

Since x_5 make the problem most infeasible, taking x_2, y_2 the basic entering variable and x_5, y_5 the leaving variable

$$\mathbf{D_p}\quad \begin{array}{rcrcrcrcr} x_4 &=& -1 &+& 4x_1 &+& \frac{2}{3}x_3 &+& \frac{1}{3}x_5 \\ x_2 &=& 2 &+& x_1 &-& \frac{1}{3}x_3 &+& \frac{1}{3}x_5 \\ x_6 &=& 1 &-& 2x_1 &-& \frac{2}{3}x_3 &-& \frac{1}{3}x_5 \\ -z &=& -4 &-& 5x_1 &-& \frac{1}{3}x_3 &-& \frac{2}{3}x_5 \end{array} \qquad \mathbf{D_d}\quad \begin{array}{rcrcrcrcr} y_4 &=& 5 &-& 4y_1 &+& y_3 &-& y_5 \\ y_2 &=& \frac{2}{3} &-& \frac{1}{3}y_1 &+& \frac{1}{3}y_3 &-& \frac{1}{3}y_5 \\ y_6 &=& \frac{1}{3} &-& \frac{2}{3}y_1 &+& \frac{2}{3}y_3 &+& \frac{1}{3}y_5 \end{array}$$

Take x_3, y_1 the basic entering variable and x_4, y_6 the leaving variable

$$\mathbf{D_p}\quad \begin{array}{rcrcrcrcr} x_3 &=& \frac{3}{2} &-& 6x_1 &-& \frac{2}{3}x_5 &+& \frac{3}{2}x_4 \\ x_2 &=& \frac{3}{2} &+& 3x_1 &+& \frac{1}{2}x_5 &-& \frac{1}{2}x_4 \\ x_6 &=& 2 && && &-& x_4 \\ -z &=& -\frac{9}{2} &-& 3x_1 &-& \frac{1}{2}x_5 &-& \frac{1}{2}x_4 \end{array} \qquad \mathbf{D_d}\quad \begin{array}{rcrcrcrcr} y_4 &=& 3 &-& 2y_3 &-& 3y_5 &+& 6y_6 \\ y_2 &=& \frac{1}{2} && &-& \frac{1}{2}y_5 &+& \frac{1}{2}y_6 \\ y_6 &=& \frac{1}{2} && y_3 &+& \frac{1}{2}y_5 &-& \frac{3}{2}y_6 \\ -w &=& \frac{9}{2} && &-& \frac{3}{2}y_5 &-& \frac{3}{2}y_6 \end{array}$$

All the coefficients are negative, we find the best feasible solution

$$z^* = -\frac{9}{2} \text{ with } x_1 = x_4 = x_5 = x_6 = 0,\ x_2 = x_3 = \frac{3}{2},\ y_1 = y_2 = \frac{1}{2},\ y_3 = y_5 = y_6 = 0$$