

XV. Applications of Wavelet Transforms

Wavelet 所適用的 applications，通常有以下兩大特點：

- (1) 信號的頻率分佈，會隨著不同的時間(或地點)有較大變異
- (2) Multiscale 的分析扮演重要的角色

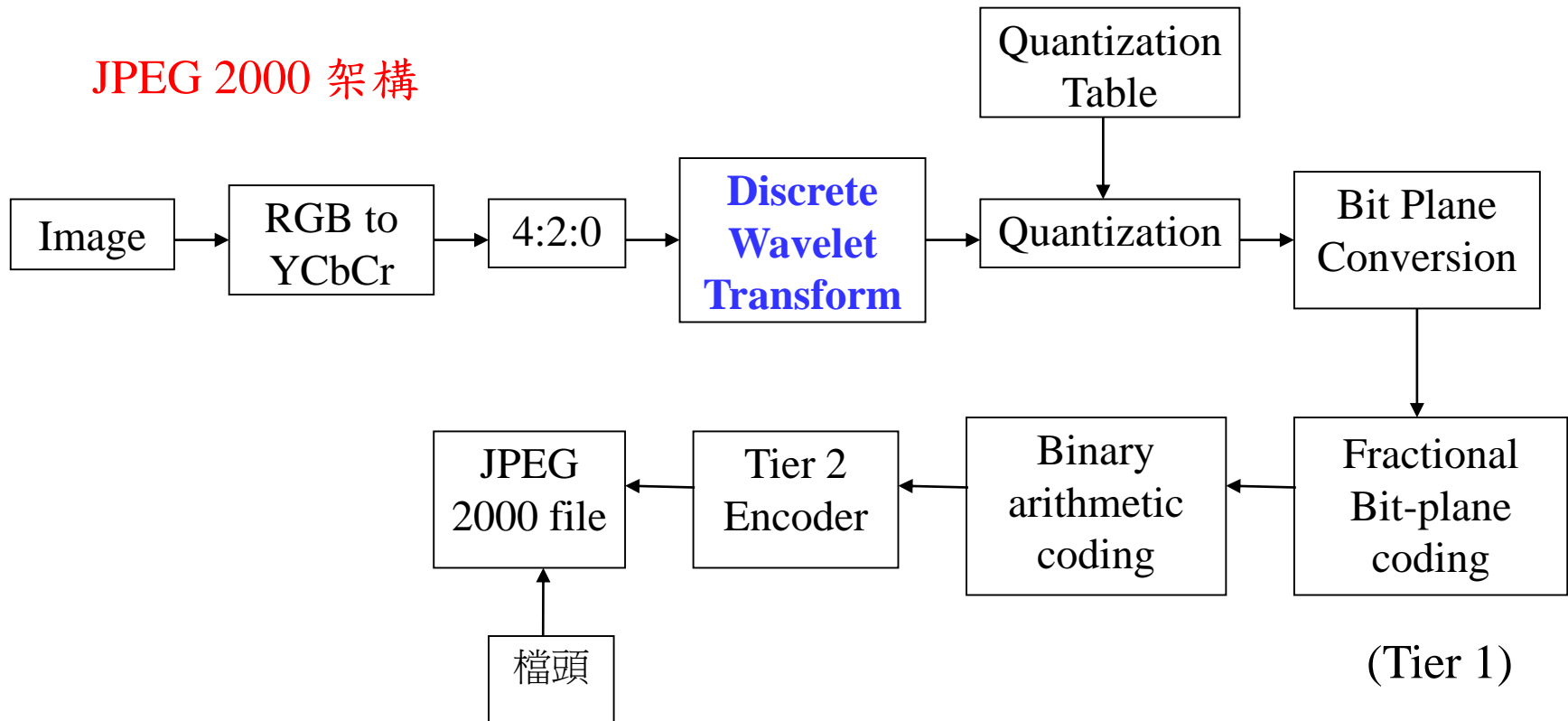
Larger sampling interval → ignoring the detail

Smaller sampling interval → requiring a lot of data

Wavelet transforms compromise them.

目前，文獻上，80% 以上的應用和 image processing 有關

JPEG 2000 架構

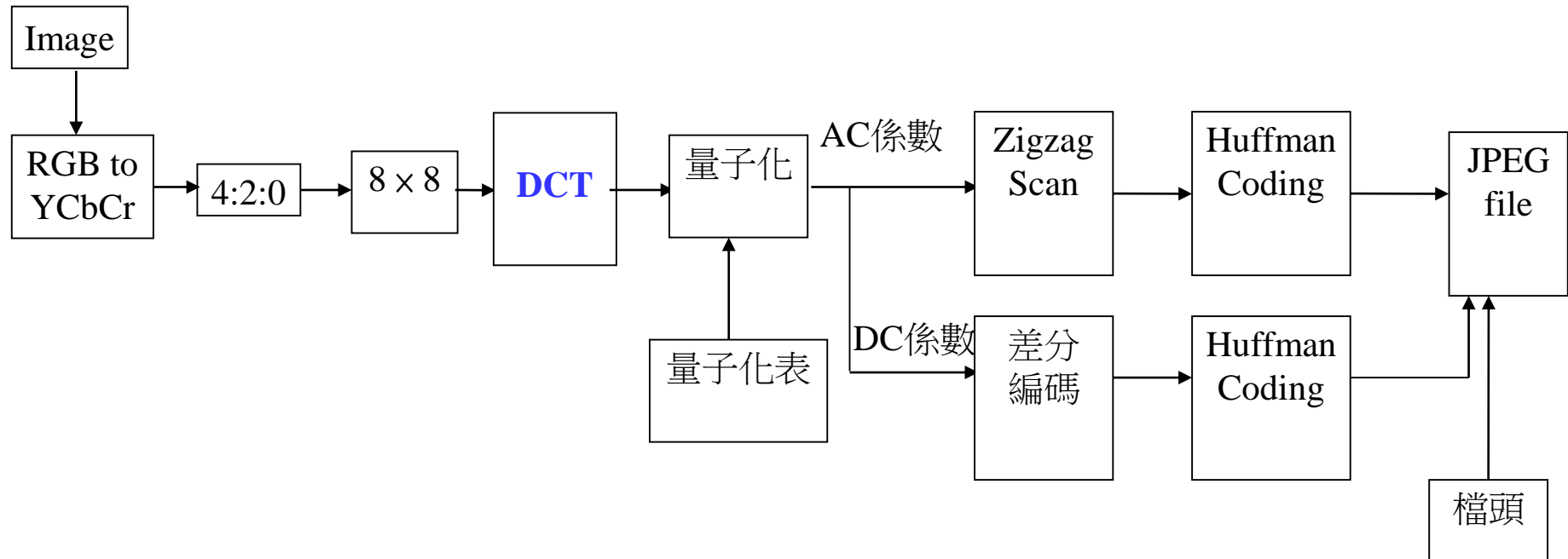


Tier 1: zero coding, sign coding, magnitude refinement coding, run length coding

Tier 2: 用以控制檔案大小 (例如只取比較重要的地方編碼)

註：感謝 2010年修課的潘冠臣同學幫忙整理

比較：傳統 JPEG 架構



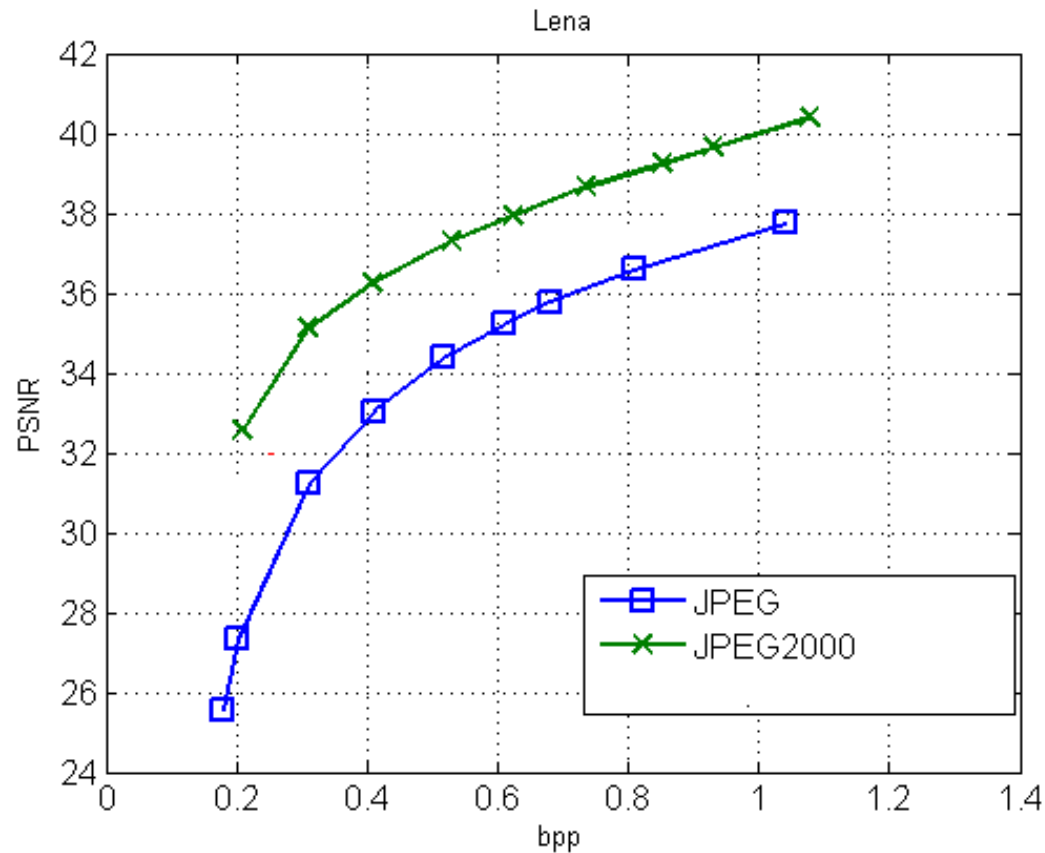
問題：由於 8×8 的切割，在高壓縮率時會造成 blocking effect

Original image

DCT-based
image compression $CR = 53.4333$ Wavelet-based
image compression $CR = 51.3806$

CR: compression ratio

註：感謝 2006年修課的黃俊德同學



bpp: bit per pixel (每一點平均需要多少個 bits)

PSNR: peak signal to noise ratio (PSNR), see page 477

使用 JPEG 2000 做影像壓縮的優點：

(1)

(2)

(3)

所以，在高壓縮率之下，重建的影像仍有不錯的品質

Question:

Why JPEG 2000 has not replaced the status of JPEG now?

參考資料

C. Christopoulos, A. Skodras, and T. Ebrahimi, “The JPEG2000 still image coding system: An overview,” *IEEE Trans. Consumer Electronics*, vol. 46, no. 4, pp.1103-1127, Nov. 2000.

Another Compression Algorithm: **SPIHT**

Using the correlation among high frequency parts in different layers

B.J. Kim, Z. Xiong, and W.A. Pearlman. “Low bit-rate scalable video coding with 3-D set partitioning in hierarchical trees (3-D SPIHT),” *IEEE Trans. Circuits Syst. Video Technol.*, vol. 10, pp. 1374-1387, 2000.

(2) Edge and Corner Detection

(3) Pattern recognition

(a) Feature extraction

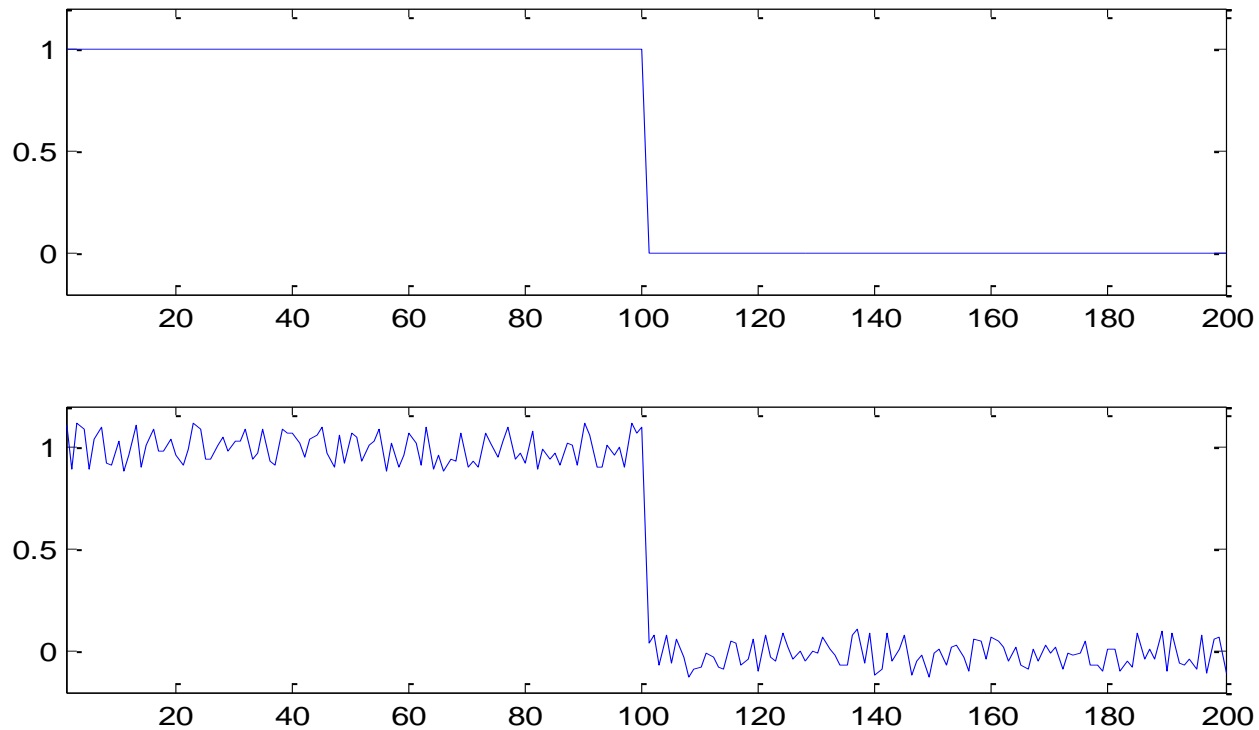
(Using the wavelet features)

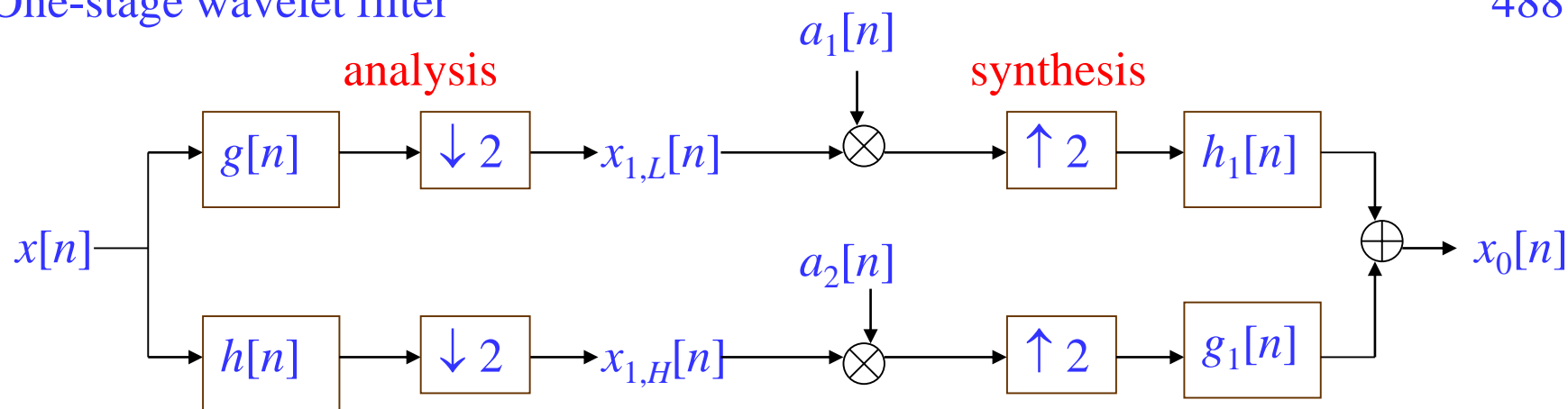
(b) Computation Time 和縮小的 pattern 互相比較 (節省運算)

(4) 強調前景，壓縮背景

(5) Filter Design

如何不傷到 edge，又能夠將 noise 去除掉？





做 filter design 時，可以令

$$a_1[n] = 1,$$

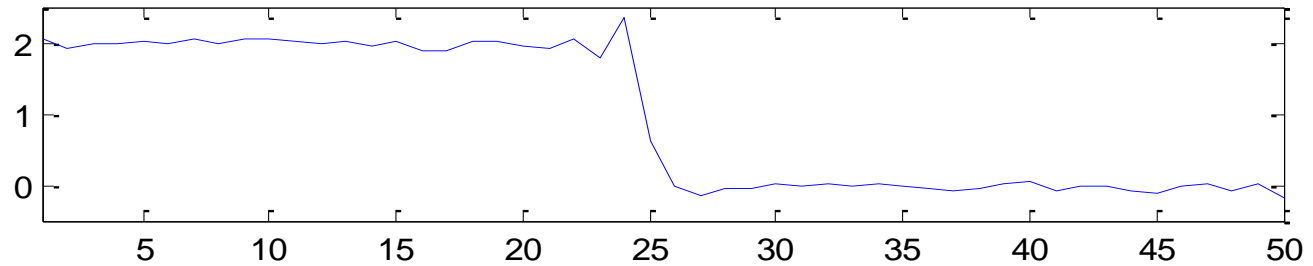
$$a_2[n] = 0 \text{ for non-edge region}$$

$$a_2[n] = 1 \text{ for edge region}$$

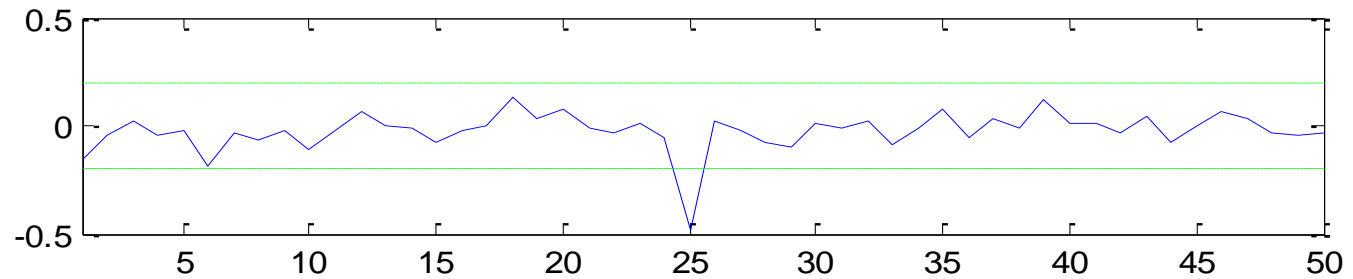
以 $x_{1,H}[n]$ 的 amplitude
來區分

必要時可使用 two-stage 以上的 wavelet filter

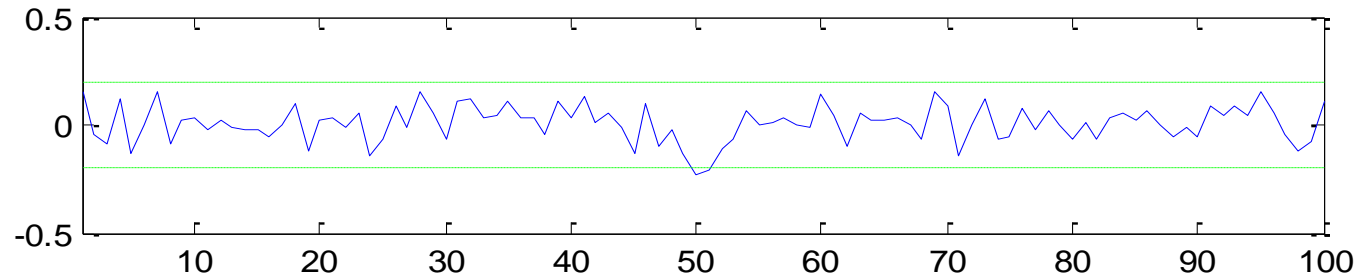
$x_{2,L}[n]$
(2nd stage,
lowpass)



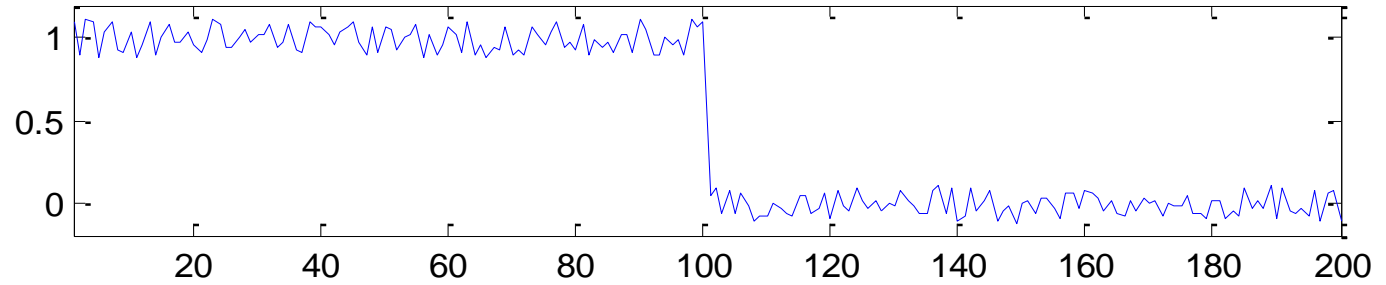
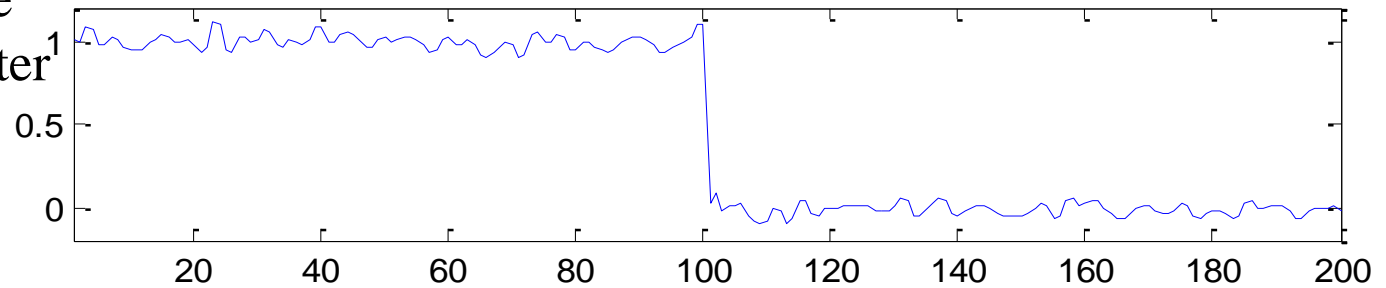
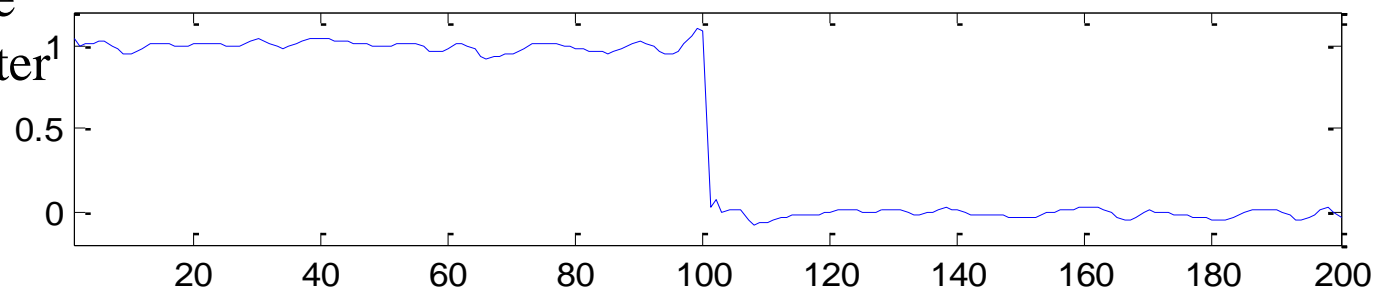
$x_{2,H}[n]$
(2nd stage,
highpass)



$x_{1,H}[n]$
(1st stage,
highpass)



原信號

使用 one-stage
的 wavelet filter¹使用 two-stage
的 wavelet filter¹

(6) Music

音樂當中，音每高一個音階，頻率就增為二倍

音樂 每一音階有12個半音，增加一個半音，頻率增加 $2^{1/12}$ 倍
(等比級數)

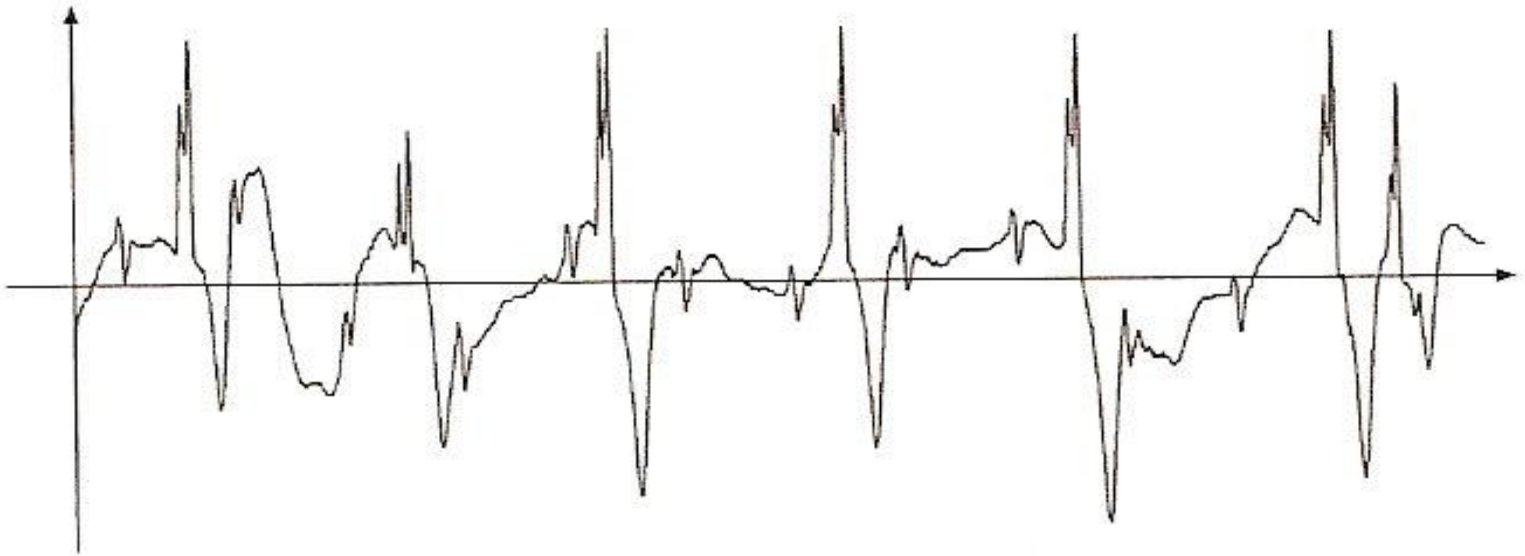
	Do	升Do	Re	升Re	Me	Fa	升Fa	So	升So	La	升La	Si
Hz	270	286	303	321	340	360	382	405	429	454	481	510
Hz	540	572	606	642	680	721	764	810	857	908	962	1019

(7) Acoustics

(8) Analyzing the Electrocardiogram (ECG)

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- Is the rhythm of the cardiac valve in synchronization with that of the heart muscle?
- Does the heart muscle relax between beats?



From: A. K. Louis, P. Maab, and A. Rieder, “*Wavelets Theory and Applications*”, John Wiley & Sons, Chichester, 1997.

(9) 「短期因素」和「長期因素」的分析

population

economical data

temperature

(10) 其他奇奇怪怪的應用

指紋的辨識

羊毛質料的辨識

Time-frequency Analysis 和 Wavelet 在應用上的異同處

相同：都能夠處理一個信號的頻率分佈會隨時間而改變的情形

不同：Time frequency analysis 對於瞬間頻率的分析比較精確

Wavelet 可作「巨觀」和「微觀」的分析

附錄十五 希臘字母大小寫與發音一覽表

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大寫	A	B	Γ	Δ	E	Z	H	Θ
小寫	α	β	γ	δ	ε	ζ	η	θ
英文拚法	alpha	beta	gamma	delta	epsilon	zeta	eta	theta
KK 音標	`ælfə	`betə	`gæmə	`deltə	`epsələn	`zetə	`itə	`θitə

大寫	I	K	Λ	M	N	Ξ	O	Π
小寫	ι	κ	λ	μ	ν	ξ	ο	π
英文拚法	iota	kappa	lambda	mu	nu	xi	omicron	pi
KK 音標	aɪ`otə	`kæpə	`læmdə	mju	nu	sai	`amɪkrən	pai

大寫	P	Σ	T	Υ	Φ	X	Ψ	Ω
小寫	ρ	σ	τ	υ	φ, ϕ	χ	ψ	ω, ω
英文拚法	rho	sigma	tau	upsilon	phi	chi	psi	omega
KK 音標	ro	`sɪgmə	taʊ	`jʊpsələn	fai	kai	sai	`omɪgə

1. Directional Form 2-D Wavelet Transforms

一般的 2-D wavelet transform，其實可分解成沿著 x -axis 以及沿著 y -axis 的 1-D wavelet transforms 的組合

其實，2-D wavelet transform 不一定要沿著 x -axis， y -axis 來做

Directional 2-D wavelet transforms:

- curvelet
- contourlet
- bandlet
- shearlet
- Fresnelet
- wedgelet
- brushlet

- **Curvelet** (ridgelet)

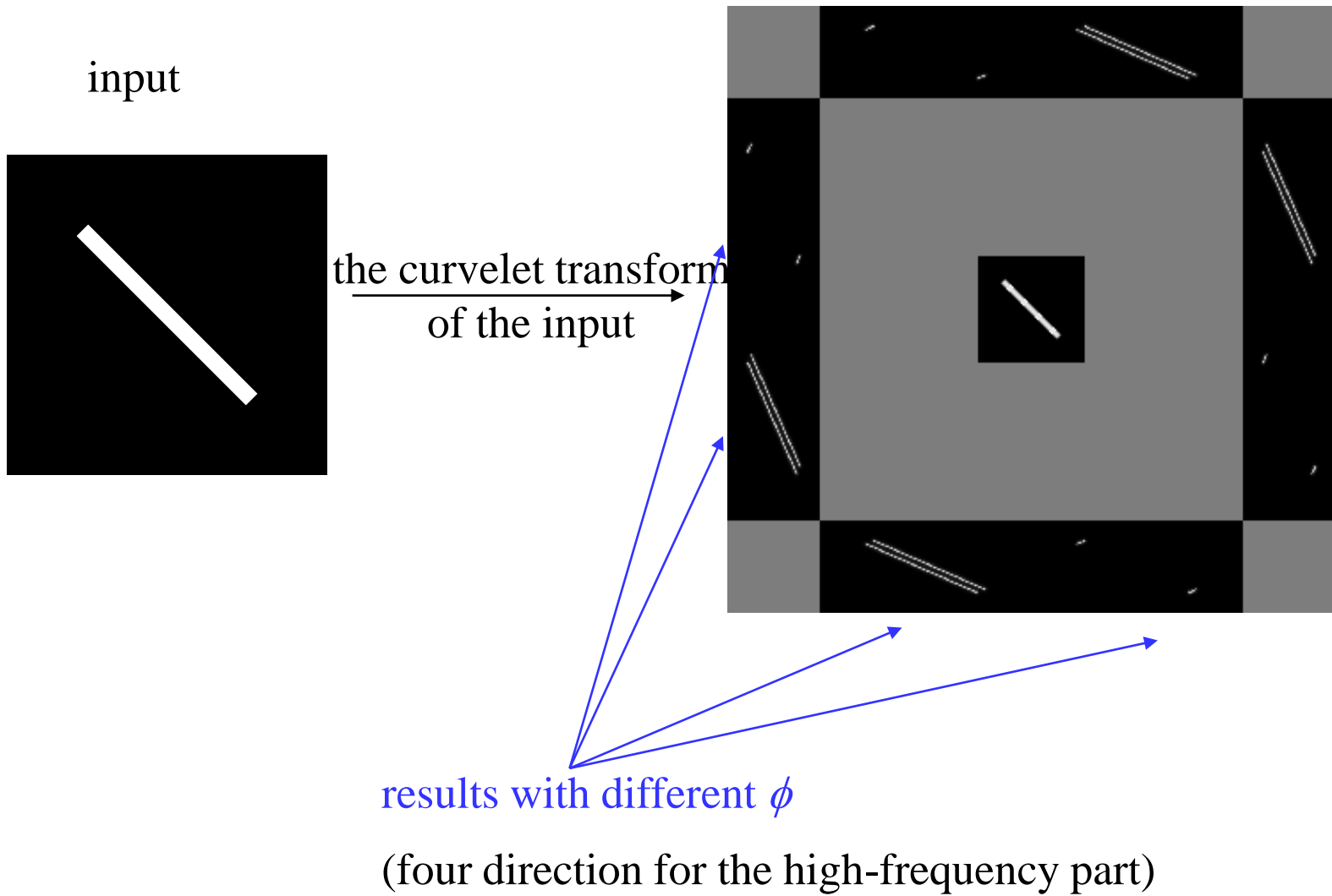
$$F_w(a, b, \phi) = \frac{1}{\sqrt{b}} \int_{-\infty}^{\infty} f(r \cos \phi, r \sin \phi) \psi\left(\frac{r-a}{b}\right) dr$$

rotation

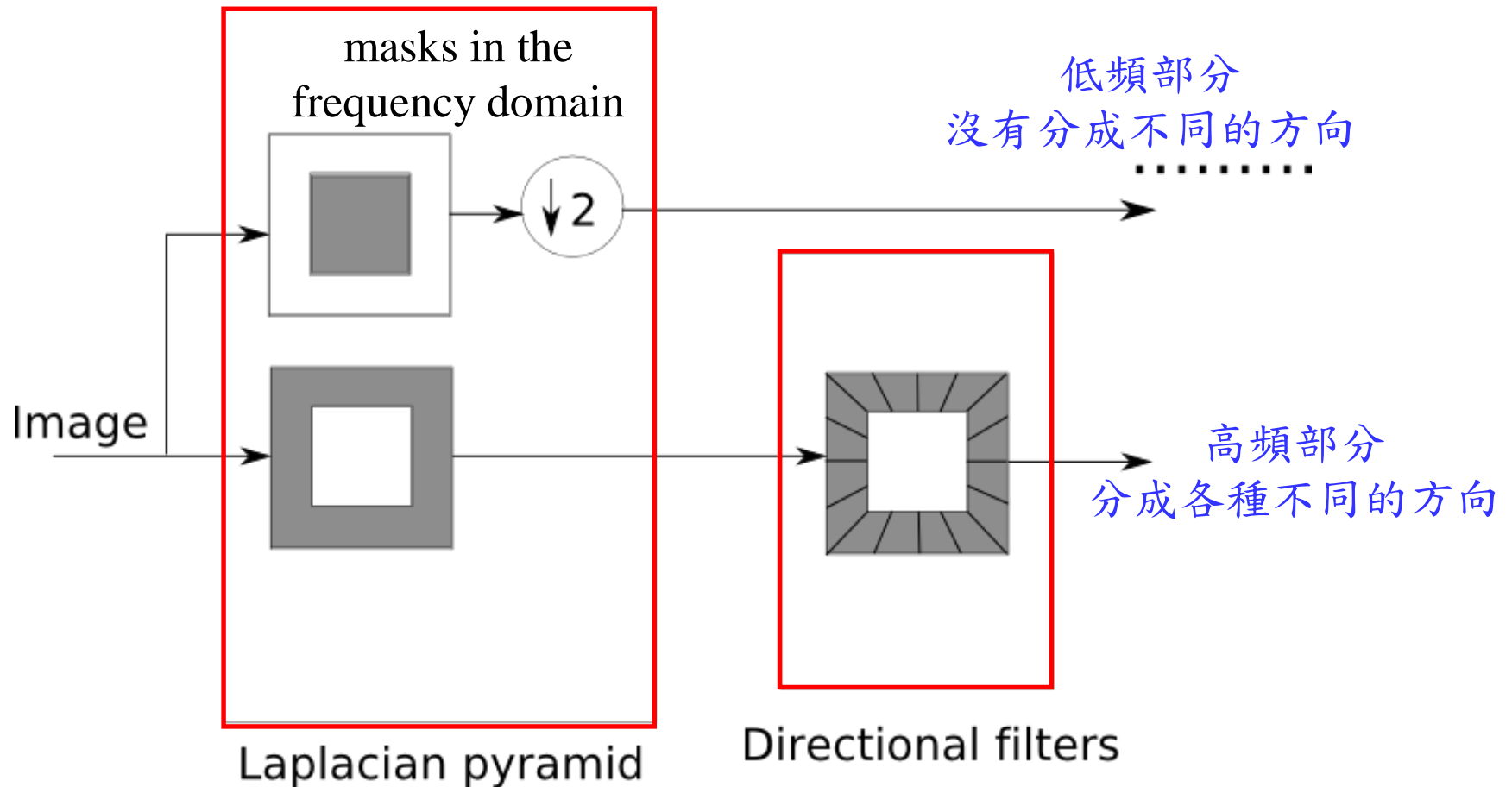
比較：原本的 1-D wavelet

$$F_w(a, b) = \frac{1}{\sqrt{b}} \int_{-\infty}^{\infty} f(x) \psi\left(\frac{x-a}{b}\right) dx$$

E. Candès and D. Donoho, "Curvelets – a surprisingly effective nonadaptive representation for objects with edges." In: A. Cohen, C. Rabut and L. Schumaker, Editors, *Curves and Surface Fitting*: Saint-Malo 1999, Vanderbilt University Press, Nashville (2000), pp. 105–120.

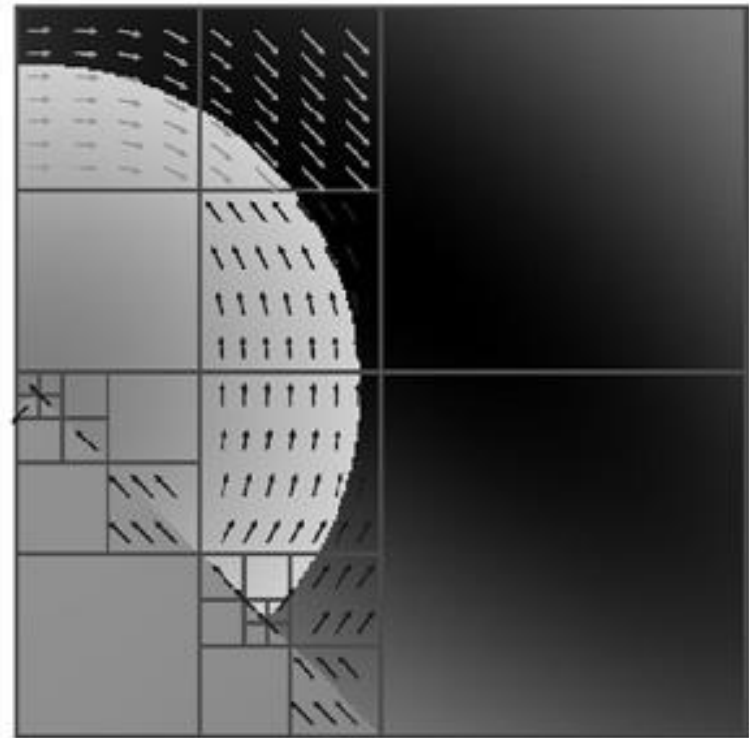
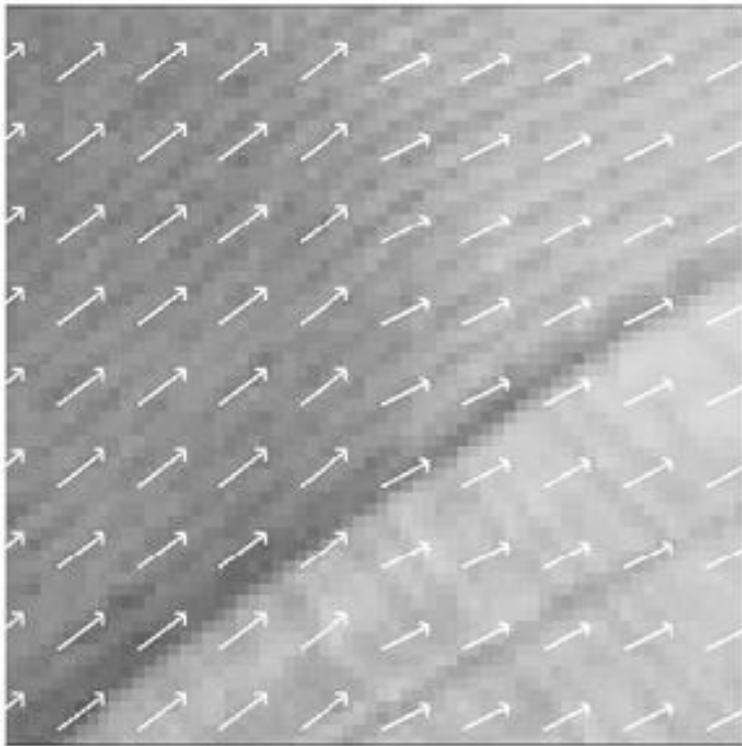


• Contourlet



M. Do and M. Vetterli, "The contourlet transform: An efficient directional multiresolution image representation," *IEEE Trans. Image Processing*, vol.14, no.12, pp.2091–2106, Dec. 2005.

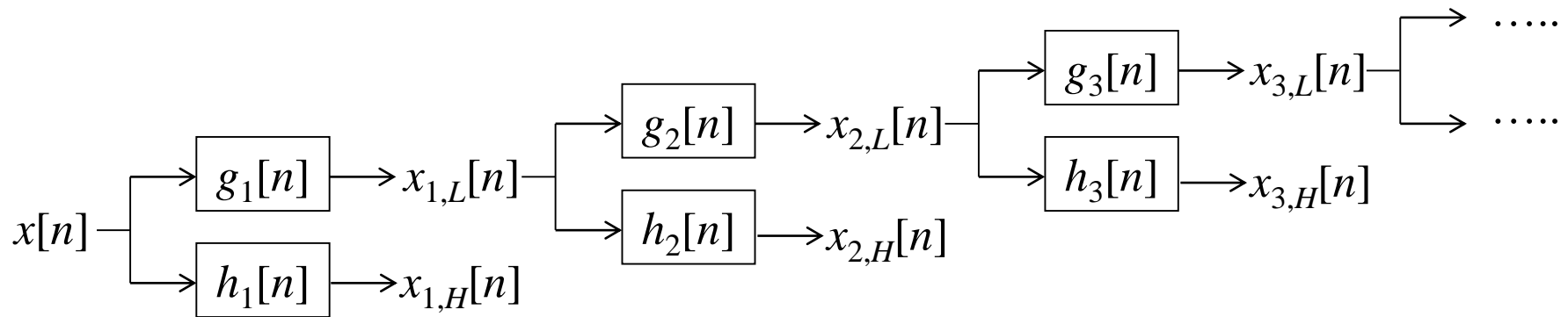
根據物體的紋理或邊界，來調整 wavelet transforms 的方向



Stephane Mallet and Gabriel Peyre, "A review of Bandlet methods for geometrical image representation," *Numerical Algorithms*, Apr. 2002.

2. Stationary Wavelet Transforms

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其中 $g_j[n] \rightarrow \uparrow 2 \rightarrow g_{j+1}[n]$ $h_j[n] \rightarrow \uparrow 2 \rightarrow h_{j+1}[n]$

Q: 和原本 discrete wavelet transform 不一樣的地方在哪裡？

G. P. Nason and B. W. Silverman, “The stationary wavelet transform and some statistical applications,” *Lecture Notes in Statistics*, available in <http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.49.2662&rep=rep1&type=pdf>

3. Bandwidth Form Wavelet Transforms

A little modification for $g[n]$ and $h[n]$

4. Multi-Band Wavelet Transforms

Instead of only two outputs

Happy New Year!

祝各位期末考順利，寒假愉快！