Integer and Combinatorial Optimization

Spring 2017

Homework 1

(Due at the beginning of the class on March 16)

1. (20%) Consider the following LP:

$$\begin{aligned} &\max \ 4x_1 + 3x_2 \\ &\text{S.t.} \\ &\ 2x_1 + x_2 \le 12 \\ &-x_1 + 2x_2 \le 4 \\ &x_1, x_2 \ge 0 \end{aligned}$$

(a) Solve the problem graphically and indicate each basic feasible solution.
(b) Solve the problem using the primal simplex method.
(c) Associate each tableau with basic feasible solution in (a).
(d) The dual model of the problem and draw the dual problem graphically.

2. (20%) Given the following LP:

$$\begin{aligned} &\max \ 4x_1 + 2x_2 \\ &\text{S.t.} \\ &\ 2x_1 \le 16 \\ &\ x_1 + 3x_2 \le 17 \\ &\ x_2 \le 5 \\ &\ x_1, x_2 \ge 0 \end{aligned}$$

(a) Solve the problem graphically.
(b) Determine how many additional units of resource 1 (constraint 1) would be needed to increase the optimal value by 15. Justify your answer.

3. (20%) Show the feasible region of a LP model, $\{A\boldsymbol{X} \le 0, \boldsymbol{X} \ge 0\}$ is convex.

4. (20%) A caterer to "The Ritz" motel collects the dirty napkins and sends them to laundry every day. Due to different room occupation levels during a week, the number of napkins needed on day i is d_i. The caterer can wash and dry at most u napkins every day and the cleaned napkins will be ready for use next day. If the dirty napkin is not cleaned, a new one is purchased at the price of p. If the laundry room is used on day i, a fixed cost of f_i is incurred. Assume that at the beginning of a week, there are n clean napkins and no dirty napkins left. Find the best laundry plan for the caterer so that the entire week's cost is minimized.

5. (20%) Assume that you have spare saving $10,000 each year. There are three investment tools you can choose from: (1) deposit, (2) mutual funds and (3) bonds. The annual interest rate is 2% if you deposit your money in a bank. If you buy mutual funds, the investment length is two years and the return is estimated to be 7% after two years. If you invest in bonds, you can get 4% of interest payment every year but the investment length is 4 years. At the end of year, you will re-invest all your available money and renew your portfolio. In addition, you are advised to deposit at least 30% of your available money in a bank and the amount of money invested in mutual funds not greater than twice of the amount invested in bonds throughout the entire investment period. Please formulate a mathematical model to maximize your money at the end of the fifth year. (At the beginning you already have $10,000 and the investment length is five years.)