Representation Theorem

S={Ax$\leq$ b, x $\geq$ 0}

Let x^1, x^2,…, x^k be the extreme points of S and r^1,…, r^j be the extreme directions of S.

Assume $x \in$S, then x can be represented as

$$x = \sum_{i=1}^{k} \lambda_i x^i + \sum_{i=1}^{j} \mu_i r^i$$

S.T. $\sum_{k=0}^{n} \lambda_i = 1 \; and \; \lambda_i \geq 0, \mu_i \geq 0$

Primal-Dual Forms

	Primal		Dual	
Obj.	Max Z	$\Longleftrightarrow$	Min W	
St.	$\leq$	$\Rightarrow$	$y_i \geq 0$	
	$=$	$\Rightarrow$	y_i unrestricted	
	$\geq$	$\Rightarrow$	$y_i \leq 0$	
	$x_i \geq 0$	$\Rightarrow$	St.	$\geq$
	x_i unrestricted	$\Rightarrow$		$=$
	$x_i \leq 0$	$\Rightarrow$		$\leq$

Dual Problem

Dual

$$
\begin{aligned}
\text{maximize} \quad & c^T\left(\sum_{k\in K}\alpha^k x^k + \sum_{j\in J}\beta^j r^j\right), \\
\text{subject to} \quad & \sum_{k\in K}\alpha^k = 1, \\
& A^1\left(\sum_{k\in K}\alpha^k x^k + \sum_{j\in J}\beta^j r^j\right) \le b^1, \\
& \alpha^k, \beta^j \ge 0, \qquad k\in K,\ j\in J.
\end{aligned} \qquad (**)
$$

$$
\begin{aligned}
z_D = \text{maximize} \quad & c^T x, \\
\text{subject to} \quad & A^1 x \le b^1, \\
& x \in \text{conv}(Q),
\end{aligned}
$$

where $Q=\left\{x\colon A^2x \le b^2\ , x\ge 0, \text{integer}\right\}$.

Primal

$$
\begin{aligned}
\text{minimize} \quad & \eta \\
\textit{subject to} \quad & \eta + \lambda^T\left(A^1x^k - b^1\right) \ge c^T x^k, \quad k\in K \qquad : \alpha^k \\
& \lambda^T A^1 r^j \ge c^T r^j, \qquad j \in J \qquad : \beta^j \\
& \lambda \ge 0.
\end{aligned}
$$

η is unrestricted => "="

$\lambda\ge 0$ => "$\le$"

First set of constraints: $\alpha\ge 0$

Second set of constraints: $\beta\ge 0$

Result 7

$$z_{IP} = 28 < z_{LD} = 28\frac{8}{9} < z_{LP} = 30\frac{2}{11}$$

$$x^6 = (3,3) \in S.$$

$$\text{For } \lambda = \frac{1}{15}, z_{LR}(\lambda) = 28\frac{14}{15}$$

$$\lambda(b^1 - A^1 x^6) = \frac{1}{15}(4-3) = \frac{1}{15} = \delta_1$$

$$z_{LR}(\lambda) - z(\lambda, x^6) = (29-\lambda) - (27+\lambda) = 1\frac{13}{15} = \delta_2$$

$$cx^6 \geq z_{IP} - 1\frac{14}{15} \Rightarrow 27 > 28 - 1\frac{14}{15}$$