R05546030 **彭新翔**

1. Home Grocery is a new company that makes same-day deliveries of groceries to people's homes. The company is launching its business in Metropolis, a large urban area. The marketing department has identified eight neighborhoods in Metropolis where the company should concentrate its business. The logistics manager has identified six locations where the company may locate grocery depots. Table 1 shows the average time (in minutes) required to travel from each of the six potential depot locations to the center of each of the eight neighborhoods. It also shows the target population (in thousands) for the company's service in each neighborhood. The company wishes to locate **two** depots so that they maximize the population served within 12 min of average travel time. Formulate the problem as an IP model.

Table 1 Travel Times from Depots to Neighborhoods.

	Depots						
Neighbor	1	2	3	4	5	6	**Population**
1	15	17	27	5	25	22	22
2	10	12	24	4	22	20	8
3	5	6	17	9	21	17	11
4	7	6	8	15	13	10	14
5	14	12	6	23	6	8	22
6	18	17	10	28	9	5	18
7	11	10	5	21	10	9	16
8	24	22	22	33	6	16	20

- ✓ Take x_i represent the Depot i $(i = 1, 2, \cdots, 6)$ be selected or not.
- ✓ Vector $P = [22, 8, 11, 14, 22, 18, 16, 20]$ represent the population of the neighbor
- ✓ Take y_j represent the neighborhood j $(j = 1, 2, \cdots, 8)$ can be served.
- ✓ If t_{ij} (travel times from i to j) is less than 12 minutes, $A_{ij} = 1$, else $A_{ij} = 0$.

$$\begin{aligned} \max \quad & \sum_{j=1}^{8} P_i \cdot y_j \\ \text{s.t.} \quad & \sum_{i=1}^{6} x_j = 2 \\ & \sum_{i=1}^{6} A_{ij} \cdot x_i \leq M y_j \end{aligned}$$

R05546030 彭新翔

2. Consider a tableau which is neither primal nor dual feasible. Suppose we add the following redundant constraint

$$s = M + \sum_{j=1}^{n} 1(1 - x_{J(j)}) \geq 0$$

to the bottom of the tableau (s is a nonnegative slack variable and M is a large positive number). Show that after a pivot on the 1 element in the new row which is in the lexicographically smallest column, the tableau exhibits dual feasibility. (See this example on page 44 in slides of Dual Fractional IP.)

Neither dual nor primal are feasible, and thus it means at least one $a_{0j} < 0$ in the tableau. Assume α_u has the smallest lexicographically order for all $a_{0j} < 0$, then we use α_k to pivot, the new $a'_{0k} = a'_{0k} > 0$; and all other $a'_{0j} = a_{0j} - a_{0k} \geq 0$, then the tableau become dual feasible.

For Example:

$$\begin{array}{lrcrcr}
\max & -2x_1 & + & 3x_2 & & \\
\text{s.t.} & x_1 & + & x_2 & \leq & 3 \\
 & 2x_1 & + & x_2 & \leq & -4 \\
 & x_1, x_2 & & & \geq & 0
\end{array}$$

3. Under what condition will the Gomory slack variable in a mixed integer cut be an integer constrained variable? Please justify your answer. (There are four cases when generating a cut. The condition is derived from these four cases.)

Combining all four cases, we obtain Gomory mixed integer cut in the form

$$\sum_j f_j^* x - j \geq f_0$$

Where

$$
\begin{aligned}
f_j^* &= \bar{a_j} && \text{if } \bar{a_j} \geq 0 \text{ and } x_j \text{ noninteger} \\
f_j^* &= \frac{f_0}{f_0 - 1}\bar{a_j} && \text{if } \bar{a_j} < 0 \text{ and } x_j \text{ noninteger} \\
f_j^* &= f_q && \text{if } f_q \leq f_0 \text{ and } x_j \text{ integer} \\
f_j^* &= \frac{f_0}{1 - f_0}(1 - f_q) && \text{if } f_q > f_0 \text{ and } x_j \text{ integer}
\end{aligned}
$$

4. Solving the mixed integer program below. Relate the computations to (x_1, x_2) space graphically. Also show that it is possible for the tableau to exhibit infeasibility after all the integer constrained variables have reached their lower bounds.

$$\begin{array}{rrcrcr} \max & 8x_1 & + & 2x_2 & = & x_0 \\ \text{s.t.} & 3x_1 & + & 2x_2 & \leq & 1 \\ & 7x_1 & + & x_2 & \geq & 2 \\ & & x_1, x_2 & & \geq & 0 \\ & & x_0, x_1 & & \in & \mathbb{Z} \end{array}$$

	z	x_1	x_2	x_3	x_4	RHS	r
z	1	-8	-2	0	0	*	
x_3	0	3	2	1	0	1	$\frac{1}{3}$
x_4	0	-7	-1	0	1	-2	$\frac{2}{7}$
z	1	0	$\frac{10}{3}$	$\frac{8}{3}$	0	*	
x_1	0	1	$\frac{2}{3}$	$\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{1}{2}$
x_4	0	0	$\frac{11}{3}$	$\frac{7}{3}$	1	$\frac{1}{3}$	$\frac{1}{11}$

$\Longrightarrow$

# 01	1	$(-x_2)$	$(-x_3)$
x_0	$\frac{8}{3}$	$\frac{1}{3}$	$\frac{8}{3}$
x_1	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$
x_2	0	-1	0
x_3	0	0	-1
x_4	$\frac{1}{3}$	$\frac{11}{3}$	$\frac{7}{3}$
x_5	$\frac{-1}{3}$	$\frac{-2}{3}$	$\frac{-1}{3}$

# 02	1	$(-x_2)$	$(-x_3)$
x_0	1	5	1
x_1	0	1	0
x_2	$\frac{1}{2}$	$\frac{-3}{2}$	$\frac{1}{2}$
x_3	0	0	-1
x_4	$\frac{-3}{2}$	$\frac{11}{2}$	$\frac{1}{2}$
x_5	0	-1	0

In this case, there is no feasible solution.

R05546030 **彭新翔**

5. Solving the following problem by the all-integer algorithm.

$$\begin{array}{rrcrcr} \max & -2x_1 & - & 5x_2 & = & x_0 \\ \text{s.t.} & -2x_1 & - & -2x_2 & \leq & -9 \\ & -2x_1 & - & 6x_2 & \leq & -22 \\ & \multicolumn{5}{c}{x_1, x_2 \in \mathbb{Z}} \end{array}$$

# 01	1	$(-x_1)$	$(-x_2)$
x_0	0	2	5
x_1	0	-1	0
x_2	0	0	-1
x_3	-9	-2	-2
x_4	-22	-2	-6
x_5	-8	-1	-2

 # 01

$(x_1 = x_2 = 0)$

$p = 1$

$u_1 = 1,\ u_2 = 1$

$\lambda_1 = 2,\ \lambda_2 = 3$

$\lambda = \max(\lambda_1, \lambda_2) = 3$

# 02	1	$(-x_5)$	$(-x_2)$
x_0	-16	2	1
x_1	8	-1	2
x_2	0	0	-1
x_3	7	-2	2
x_4	-6	-2	-2
x_5	0	-1	0
x_6	-3	-1	-1

 # 02

$p = 2$

$u_1 = 1,\ u_2 = 1$

$\lambda_1 = 2,\ \lambda_2 = 2$

$\lambda = \max(\lambda_1, \lambda_2) = 2$

# 03	1	$(-x_5)$	$(-x_6)$
x_0	-19	1	1
x_1	2	1	2
x_2	3	1	-1
x_3	1	0	2
x_4	0	0	-2
x_5	0	-1	0
x_6	0	0	-1

 # 03

$x_0 = -19$

$x_1 = 2$

$x_2 = 3$