Introduction to Optimization

Homework #5 – Due Wednesday, January 3

1. Solve the following problems by the revised simplex method:

   a. $\quad$ *maximize* $\quad 3x_1 + 2x_2 + 4x_3$

   $\quad\quad$ *subject to* $\quad x_1 + x_2 + 2x_3 \leq 4$

   $$2x_1 \quad\quad + 3x_3 \leq 5$$
   $$2x_1 + x_2 + 3x_3 \leq 7$$
   $$x_1, x_2, x_3 \geq 0$$

   b. $\quad$ *maximize* $\quad 5x_1 + 6x_2 + 9x_3 + 8x_4$

   $\quad\quad$ *subject to* $\quad x_1 + 2x_2 + 3x_3 + x_4 \leq 5$

   $$x_1 + x_2 + 2x_3 + 3x_4 \leq 3$$
   $$x_1, x_2, x_3, x_4 \geq 0$$

   c. $\quad$ *maximize* $\quad 2x_1 + x_2$

   $\quad\quad$ *subject to* $\quad 2x_1 + 3x_2 \leq 3$

   $$x_1 + 5x_2 \leq 1$$
   $$2x_1 + x_2 \leq 4$$
   $$4x_1 + x_2 \leq 5$$
   $$x_1, x_2 \geq 0.$$

2. Solve the following problems.

   a. $\quad$ *maximize* $\quad 2x_1 + 5x_2$

   $\quad\quad$ *subject to* $\quad x_1 + 2x_2 \leq 20$

   $$2x_1 + x_2 \leq 16$$
   $$2x_1 \leq 2$$
   $$x_2 \leq 8$$
   $$x_1, x_2 \geq 0.$$

   b. $\quad$ *maximize* $\quad 3x_1 + 5x_2 + 2x_3$

   $\quad\quad$ *subject to* $\quad x_1 + x_2 + 2x_3 \leq 7$

   $$2x_1 + 4x_2 + 3x_3 \leq 15$$
   $$x_1 \leq 4, x_2 \leq 3, x_3 \leq 3$$
   $$x_1, x_2, x_3 \geq 0$$

3. Solve the following problem by the dual simplex method.

   $\quad\quad$ *minimize* $\quad 3x_1 + 2x_2 + x_3$

   $\quad\quad$ *subject to* $\quad 3x_1 + x_2 + x_3 \geq 3$

   $$3x_1 - 3x_2 - x_3 \leq -6$$
   $$x_1 + x_2 + x_3 \leq 3$$
   $$x_1, x_2, x_3 \geq 0$$