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1. Ford has four automobile plants. Each plant is capable of producing the Taurus, Lincoln, or Escort, but it can only produce one of these cars. The fixed cost of operating each plant for a year and the variable cost of producing a car of each type at each plant are given as follows:

Plant	**Fixed Cost**	**Taurus**	**Lincoln**	**Escort**
1	7 Billions	12000	16000	9000
2	6 Billions	15000	18000	11000
3	4 Billions	17000	19000	12000
4	2 Billions	19000	22000	14000

Each year, Ford must produce 500,000 of each type of car. Formulate an IP which minimize the annual cost of producing cars based on the following constrains:

(a) Each plant can produce only one type of car.

(b) The total production of each type of car must be at a single plant.

(c) If plant 2 is used, plant 3 cannot be used.

(d) If plant 3 and plant 4 are used, plant 1 must also be used.

- ✓ The factory i $(i = 1, 2, 3, 4)$ produce j $(j = 1, 2, 3)$ car type (1 for Taurus, 2 for Lincoln and 3 for Escort).
- ✓ Whether i produce j or not, $y_{ij} = 0$ or 1. If $y_{ij} = 1$ how much x_{ij}.
- ✓ Fixed Cost: $(f_{1j}, f_{2j}, f_{3j}, f_{4j}) = (7, 6, 4, 2)$ billion $\forall j$.
- ✓ Variable Cost:

$$\mathbf{C} = [c_{ij}] = \begin{bmatrix} 12000 & 16000 & 9000 \\ 15000 & 18000 & 11000 \\ 17000 & 19000 & 12000 \\ 19000 & 22000 & 14000 \end{bmatrix}$$

Then

$$\begin{aligned}
\min \quad & \sum_{i=1}^{4}\sum_{j=1}^{3}(c_{ij}x_{ij} + f_{ij}y_{ij}) \\
\text{s.t.} \quad & \sum_{j=1}^{3} y_{ij} \le 1,\ \forall i = 1, 2, 3, 4 \\
& \sum_{i=1}^{4} y_{ij} = 1,\ \forall j = 1, 2, 3 \\
& \sum_{j=1}^{3} (y_{2j} + y_{3j}) \le 1,\ \forall j = 1, 2, 3 \\
& \sum_{j=1}^{3} y_{1j} \ge \sum_{j=1}^{3} y_{3j} + \sum_{j=1}^{4} y_{4j} - 1 \\
& \sum_{i=1}^{4} x_{ij} \ge 500000 \\
& My_{ij} \ge x_{ij},\ \forall i = 1, 2, 3, 4;\ \forall j = 1, 2, 3 \\
& x_{ij}, y_{ij} \in \{0, 1\},\ \forall i = 1, 2, 3, 4;\ \forall j = 1, 2, 3
\end{aligned}$$

2. Consider the following problem:

$$\begin{array}{lrcrcrcr} \max & 4x_1 & - & {x_1}^2 & + & 10{x_2}^2 & - & {x_2}^3 \\ \text{s.t.} & x_1 & + & x_2 & \leq & 3 & & \\ & x_1, x_2 & \geq & 0 & & & & \\ & x_1, x_2 & \in & \mathbb{Z} & & & & \end{array}$$

This problem can be reformulated as an equivalent pure Binary Integer Programming (BIP) problem, depending on the definitions of the binary variables. Assume that the binary variables are interpreted as: $y_{ij} = 1$ if $x_i \geq j$ $(i = 1, 2$ and $j = 1, 2, 3)$, and $y_{ij} = 0$ otherwise.

$$\begin{array}{lll} \max & (3y_{11} + y_{12} - y_{13}) + (9y_{21} + 23y_{22} + 31y_{23}) & \\ \text{s.t.} & \displaystyle\sum_{i=1}^{2}\sum_{j=1}^{3} y_{ij} = 3 & y_{i1} \geq y_{i2}, y_{i2} \geq y_{i3} \quad \forall i = 1, 2 \\ & x_{ij}, y_{ij} \in \{0, 1\} & \forall i = 1, 2;\ \forall j = 1, 2, 3 \end{array}$$

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3. John is buying stocks. His broker suggests six different stocks, namely, $1, 2, \cdots, 6$. Let c_j denote the return of purchasing stock j. Formulate the stock selection problem subject to the following constraints, using $0-1$ variables as needed:
   (a) To lower the risk of losing money, John should buy at least two stocks.
   (b) Due to John's budget limit, he cannot buy more than four stocks.
   (c) Since stocks 3 and 5 belong to the same company, the broker recommends purchase of at most of one of these.
   (d) The broker suggests the following two combinations: **either** choose two from stocks $1, 2, 3$ and 4, **or** at least two from stocks $3, 4, 5$ and 6.
   (e) Stock 4 can only be purchase if stock 1 is bought.

✓ Take y_j represent buy the stock j $(j = 1, 2, \cdots, 6)$ or not. ($y_j = 1$ for 'YES' and $y_j = 0$ 'NO')

✓ Let $k_i \in \{0, 1\}$, $i = 1$

$$
\begin{aligned}
\min \quad & \sum_{j=1}^{6} (c_j \times y_j) \\
\text{s.t.} \quad & \sum_{j=1}^{6} y_j \geq 2 \\
& \sum_{j=1}^{6} y_j \leq 4 \\
& y_3 + y_5 \leq 1 \\
& \sum_{j=1}^{4} y_j \leq (2 + k_1 \times M), \text{ if } k_1 = 0 \\
& \sum_{j=1}^{4} y_j \geq (2 - k_1 \times M), \text{ if } k_1 = 0 \\
& \sum_{j=3}^{6} y_j \geq [2 - M(1 - k_1)], \text{ if } k_1 = 1 \\
& y_1 - y_4 \geq 0
\end{aligned}
$$

4. Four jobs need to be done on a certain machine. However, the setup time for each job depends on which job immediately preceded it, as shown in the following table. Formulate an IP or MIP model to determine the sequence of jobs that minimizes the total setup times.

		Job			
		1	**2**	**3**	**4**
	None	4	5	8	9
Immediately	**1**	-	7	12	10
Preceding	**2**	6	-	10	14
Job	**3**	10	11	-	12
	4	7	8	15	-

(For example, if job 1 is sequenced as the first one, it requires 4 units of setup time. Assume that job 3 is right after job 1, it will take 12 units of time to setup the machine.)

- ✓ Take y_{ij} represent whether job i $(i = 0, 1, 2, 3, 4)$ is done before job j $(j = 1, 2, 3, 4)$ or not. ($y_{ij} = 1$ for 'YES' and $y_{ij} = 0$ 'NO')
- ✓ Note that $i = 0$ means there is no work been done.
- ✓ Apparently, $y_{ij} = 0\ \ \forall i = j$.

Consider the Model below.

$$
\begin{aligned}
\min \quad & (4y_{01} + 5y_{02} + 8y_{03} + 9y_{04}) + (7y_{12} + 12y_{13} + 10y_{14}) \\
& +(6y_{21} + 10y_{23} + 14y_{24}) + (10y_{31} + 11y_{32} + 12y_{34}) \\
& +(7y_{41} + 8y_{42} + 15y_{43})
\end{aligned}
$$

$$
\begin{aligned}
\text{s.t.} \quad & y_{01} + y_{11} + y_{12} + y_{13} + y_{14} = 1 \\
& y_{02} + y_{21} + y_{22} + y_{23} + y_{24} = 1 \\
& y_{03} + y_{31} + y_{32} + y_{33} + y_{34} = 1 \\
& y_{04} + y_{41} + y_{42} + y_{43} + y_{44} = 1
\end{aligned}
$$

$$
\begin{aligned}
& y_{01} + y_{02} + y_{03} + y_{04} = 1 \\
& y_{11} + y_{21} + y_{31} + y_{41} = 1 \\
& y_{12} + y_{22} + y_{32} + y_{42} = 1 \\
& y_{13} + y_{23} + y_{33} + y_{43} = 1 \\
& y_{14} + y_{24} + y_{34} + y_{44} = 1
\end{aligned}
$$

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5. Using the classical (Gomory) cut and the Basic Approach to solve the following Knapsack problem.

$$\begin{array}{rl} \max & 3x_1 + 4x_2 + 8x_3 + 2x_4 + 7x_5 = x_0 \\ \text{s.t.} & 2x_1 + x_2 + 3x_3 + 5x_4 + 6x_5 \leq 14 \\ & x_i = 0 \text{ or } 1 \quad \forall \ i \end{array}$$

[Dual]

$$\begin{array}{rrl} & w_0 = & 14w_1 \\ \text{s.t.} & -2w_1 \geq & 3 \\ & -w_2 \geq & 4 \\ & -3w_3 \geq & 8 \\ & -5w_4 \geq & 2 \\ & -6w_5 \geq & 7 \\ & w_i \geq & 0 \end{array}$$

$$\forall i = 1,2,3,4,5$$

$\Longrightarrow$

[Primal]

$$\begin{array}{rl} \max & 3x_1 + 4x_2 + 8x_3 + 2x_4 + 7x_5 = x_0 \\ \text{s.t.} & 2x_1 + x_2 + 3x_3 + 5x_4 + 6x_5 \leq 14 \\ & x_i = 0 \text{ or } 1 \quad \forall \ i \end{array}$$

$$x_1 + x_7 = x_2 + x_8 = x_3 + x_9 = x_4 + x_{10} = x_5 + x_{11} = 1$$

# 01		$-x_1$	$-x_2$	$-x_3$	$-x_4$	$-x_5$
x_0	0	-3	-4	-8	-2	-7
x_1	0	-1	0	0	0	0
x_2	0	0	-1	0	0	0
x_3	0	0	0	-1	0	0
x_4	0	0	0	0	-1	0
x_5	0	0	0	0	0	-1
x_6	14	2	1	3	5	6
x_7	1	1	0	0	0	0
x_8	1	0	0	0	0	0
x_9	1	0	0	1	0	0
x_{10}	1	0	0	0	1	0
x_{11}	1	0	0	0	0	1
S	5	1	1	1	1	1

$\Longrightarrow$

# 02		$-x_1$	$-x_2$	$-x_3$	$-x_4$	$-x_5$
x_0	40	5	4	8	6	1
x_1	0	-1	0	0	0	0
x_2	0	0	-1	0	0	0
x_3	5	1	1	1	1	1
x_4	0	0	0	0	-1	0
x_5	0	0	0	0	0	-1
x_6	-1	-1	-2	-3	2	3
x_7	1	1	0	0	0	0
x_8	1	0	1	0	0	0
x_9	-4	-1	-1	-1	-1	-1
x_{10}	1	0	0	0	0	0
x_{11}	1	0	0	0	0	1
S	0	0	0	-1	0	0

and so on …

As the Final Approaching Result show at the right.

We can know that:

$$x_0 = 22$$
$$x_1 = 1$$
$$x_2 = 1$$
$$x_3 = 1$$
$$x_4 = 0$$
$$x_5 = 1$$

$\Longrightarrow$

Final		$-x_6$	$-x_7$	$-x_8$	$-x_{11}$	$-x_9$
x_0	22	1	2	6	2	3
x_1	1	1	0	0	0	0
x_2	1	0	1	0	0	0
x_3	1	0	0	1	0	0
x_4	0	1	-1	-1	1	-2
x_5	0	0	0	0	0	0
x_6	2	3	4	2	-5	4
x_7	0	-1	0	0	0	0
x_8	0	0	-1	0	0	0
x_9	0	0	0	-1	0	0
x_{10}	1	1	1	1	-5	2
x_{11}	0	0	0	0	0	-1
x_{12}	0	0	0	0	-1	0

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