R05546030 彭新翔

Two players are playing an infinitely-repeated prisoner's dilemma game of the following form

Player 2

		C	D
Player 1	C	(2,2)	(0,3)
	D	(3,0)	(1,1)

The players simultaneously choose action at regular intervals.

Consider the following "grim trigger strategy" in which player i, $i=1,2$ chooses C in the first stage. In the t^{th} stage, if the outcome of all $t-1$ preceding stages has been (C, C) then player i chooses C; otherwise, player i chooses D.

R05546030 彭新翔

1. Explain how you show that the "grim trigger strategy" is a Nash equilibrium strategy of the infinitely repeated game of this game.

We can just discuss that whether the "grim trigger strategy" are the Best Response to each other (Player 1 and Player 2). If there is any action that no single player wants to deviate from his or her predicted action, we can conclude that the "grim trigger strategy" is a Nash Equilibrium.

R05546030 彭新翔

2. Suppose Player 1 adopts the grim trigger strategy
   (a) What is Player 2's best response in stage t if the outcomes of stage $1,\cdots,t-1$ are other than (C, C)?
   (b) What is Player 2's best response in stage t if the outcomes of stage $1,\cdots,t-1$ are (C,C)?

(a) If the outcomes of stage $1,\cdots,t-1$ are other than (C, C)

Player 1 will choose D at stage t and forever after (Player 1 adapts the trigger strategy).

Player 2 will choose D at stage t and forever after (Player 2's best response to Player 1's choices)

(b) If the outcomes of stage $1,\cdots,t-1$ are (C, C)

Player 1 will choose C at t (Because that Player 1 adopts the trigger strategy). There are 2 possible choices for Player 2:

(Discussion each conditions next page)

- Player 2 choose D at t.
- Player 2 choose C at t.

R05546030 彭新翔

3. Find the condition on the discount factor δ under which the strategy pair in which each player uses this strategy is a Nash equilibrium of the infinitely repeated game of the Prisoner's Dilemma in the above table.

(1) If Player 2 choose D at t, Player 2 receives "3" at t and 1 forever after.

The total payoff V:

$$V = 3 + \delta \cdot 1 + \delta^3 \cdot 1 + \cdots = 3 + \frac{\delta}{1-\delta}$$

(2) If Player 2 choose C at t, Player 2 receives "2" at t and face the same game again.

The total payoff V:

$$V = 2 + V\delta$$

$$\implies V = \frac{2}{1-\delta}$$

(3) According to the statement (1) and (2)

$$\begin{aligned} \frac{2}{1-\delta} &= 3 + \frac{\delta}{1-\delta}, \text{ if } \delta \geq \frac{1}{2} \text{ choose C.} \\ \delta &= \frac{1}{2}, \text{ if } \delta \leq \frac{1}{2} \text{ choose D.} \end{aligned}$$