最佳化概論(Introduction to Optimization)
Homework 01 – 2017/09/20

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1. Consider the following linear programming problem:

$$\begin{array}{rrcrcrcl} \max & z & = & 4x_1 & + & 5x_2 & & \\ \text{s.t.} & 6x_1 & + & 4x_2 & \leq & 24 & & \\ & x_1 & + & 2x_2 & \leq & 6 & & \\ & -x_1 & + & x_2 & \leq & 1 & & \\ & & & x_2 & \leq & 2 & & \\ & & x_1, x_2 & & \geq & 0 & & \end{array}$$

(a) Solve the problem by the simplex method, where the entering variable is the nonbasic variable with the *most* positive z-row coefficient. (Please solve it in the algebra way introduced in class. Do not solve it by a tabular manner.)

(b) Resolve the problem by the simplex algorithm, always selecting the entering variable as the nonbasic variable with the *least* positive z-row coefficient. (Please solve it in the algebra way introduced in class. Do not solve it by a tabular manner.)

(c) Compare the number of iterations in (a) and (b). Does the selection of the entering variable as the nonbasic variable with the *most* positive z-row coefficient lead to a smaller number of iterations? What conclusion can be made regarding the optimality condition?

First, we need to turn this model to the standard form below and rewrite the constraints:

$$\begin{array}{rrcrcrcl} \max & z & = & 4x_1 & + & 5x_2 & & \\ \text{s.t.} & 6x_1 & + & 4x_2 & + & x_3 & = & 24 \\ & x_1 & + & 2x_2 & + & x_4 & = & 6 \\ & -x_1 & + & x_2 & + & x_5 & = & 1 \\ & & & x_2 & + & x_6 & = & 2 \\ & & & & & x_1, x_2, x_3, x_4, x_5, x_6 & \geq & 0 \end{array}$$

Then the constraints would be

$$\begin{array}{rcrcrcr} x_3 & = & 24 & - & 6x_1 & - & 4x_2 \\ x_4 & = & 6 & - & x_1 & - & 2x_2 \\ x_5 & = & 1 & + & x_1 & - & x_2 \\ x_6 & = & 2 & & & - & x_2 \end{array}$$

And obtain the initial solution:

$$x_1 = x_2 = 0,\ x_3 = 24,\ x_4 = 6,\ x_5 = 1,\ x_6 = z$$

(a) **[Iteration 01]** Choose x_2 as entering variable and x_5 will be the leaving basis.

$$\begin{array}{rrcrcrcr} \max & z & = & 5 & + & 9x_2 & - & 5x_5 \\ \text{s.t.} & x_2 & = & 1 & + & x_1 & - & x_5 \\ & x_3 & = & 20 & - & 10x_1 & + & 4x_5 \\ & x_4 & = & 4 & - & 3x_1 & + & 2x_5 \\ & x_6 & = & 1 & - & x_1 & + & x_5 \end{array}$$

[Iteration 02] Choose x_1 as entering variable and x_6 will be the leaving basis.

$$\begin{array}{rrcrcrcr} \max & z & = & 14 & + & 4x_5 & - & 9x_6 \\ \text{s.t.} & x_1 & = & 1 & + & x_5 & - & x_6 \\ & x_2 & = & 2 & - & x_6 & & \\ & x_3 & = & 10 & - & 6x_5 & + & 10x_6 \\ & x_4 & = & 1 & - & x_5 & + & 3x_6 \end{array}$$

[Iteration 03] Choose x_5 as entering variable and x_4 will be the leaving basis.

$$\begin{array}{rrcrcrcr} \max & z & = & 18 & - & 4x_4 & + & 3x_6 \\ \text{s.t.} & x_1 & = & 2 & - & x_4 & - & 2x_6 \\ & x_2 & = & 2 & - & x_6 & & \\ & x_3 & = & 4 & + & 6x_4 & + & 8x_6 \\ & x_5 & = & 1 & - & x_4 & + & 3x_6 \end{array}$$

[Iteration 04] Choose x_5 as entering variable and x_4 will be the leaving basis.

$$\begin{array}{rrcrcrcr}
\max & z & = & \frac{39}{2} & - & \frac{3}{8}x_3 & - & \frac{7}{4}x_4 \\
\text{s.t.} & x_1 & = & 3 & - & \frac{1}{4}x_3 & + & \frac{1}{2}x_4 \\
 & x_2 & = & \frac{3}{2} & + & \frac{1}{8}x_3 & - & \frac{3}{4}x_4 \\
 & x_5 & = & \frac{5}{2} & - & \frac{3}{8}x_3 & + & \frac{5}{8}x_4 \\
 & x_6 & = & \frac{1}{2} & - & \frac{1}{8}x_3 & + & \frac{3}{4}x_4
\end{array}$$

Finally we know that $z = \frac{39}{2}$ is the optimal solution where $x_1 = 3,\ x_2 = \frac{3}{2},\ x_3 = x_4 = 0,\ x_5 = \frac{5}{2}$ and $x_6 = \frac{1}{2}$.

(b) **[Iteration 01]** Choose x_1 as entering variable and x_3 will be the leaving basis.

$$\begin{array}{rrcrcrcr}
\max & z & = & 16 & + & \frac{7}{3}x_2 & - & \frac{2}{3}x_3 \\
\text{s.t.} & x_1 & = & 4 & - & \frac{2}{3}x_2 & - & \frac{1}{6}x_3 \\
 & x_4 & = & 2 & - & \frac{4}{5}x_2 & + & \frac{1}{6}x_3 \\
 & x_5 & = & 5 & - & \frac{5}{3}x_2 & - & \frac{1}{6}x_3 \\
 & x_6 & = & 2 & - & x_2 & &
\end{array}$$

[Iteration 02] Choose x_2 as entering variable and x_4 will be the leaving basis.

$$\begin{array}{rrcrcrcr}
\max & z & = & \frac{39}{2} & - & \frac{3}{8}x_3 & - & \frac{7}{4}x_4 \\
\text{s.t.} & x_1 & = & 3 & - & \frac{1}{4}x_3 & + & \frac{1}{2}x_4 \\
 & x_2 & = & \frac{3}{2} & + & \frac{1}{8}x_3 & - & \frac{3}{4}x_4 \\
 & x_5 & = & \frac{5}{2} & - & \frac{3}{8}x_3 & + & \frac{5}{8}x_4 \\
 & x_6 & = & \frac{1}{2} & - & \frac{1}{8}x_3 & + & \frac{3}{4}x_4
\end{array}$$

Finally we get the same result as (a), namely that $z = \frac{39}{2}$ is the optimal solution where $x_1 = 3,\ x_2 = \frac{3}{2},$ $x_3 = x_4 = 0,\ x_5 = \frac{5}{2}$ and $x_6 = \frac{1}{2}$

(c) As the result from (a) and (b), we can conclude that choosing the largest coefficient of z-row to enter may not always leads to the least iterations. Actually, it depends on the real graph about the objective function and constraints.