

## ◎ 2-H Relations among Filter Length $N$ , Transition Band, and Accuracy

◆ Suppose that we want:

- ① passband ripple  $\leq \delta_1$ ,
- ② stopband ripple  $\leq \delta_2$ ,
- ③ width of transition band  $\leq \Delta F$  (expressed by **normalized frequency**)

$$\Delta F = (f_1 - f_2)/f_s = (f_1 - f_2)T \quad (f_s: \text{sampling frequency}, T: \text{sampling interval})$$

Then, the estimated length  $N$  of the digital filter is:

$$N = \frac{2}{3} \frac{1}{\Delta F} \log_{10} \left( \frac{1}{10\delta_1\delta_2} \right)$$

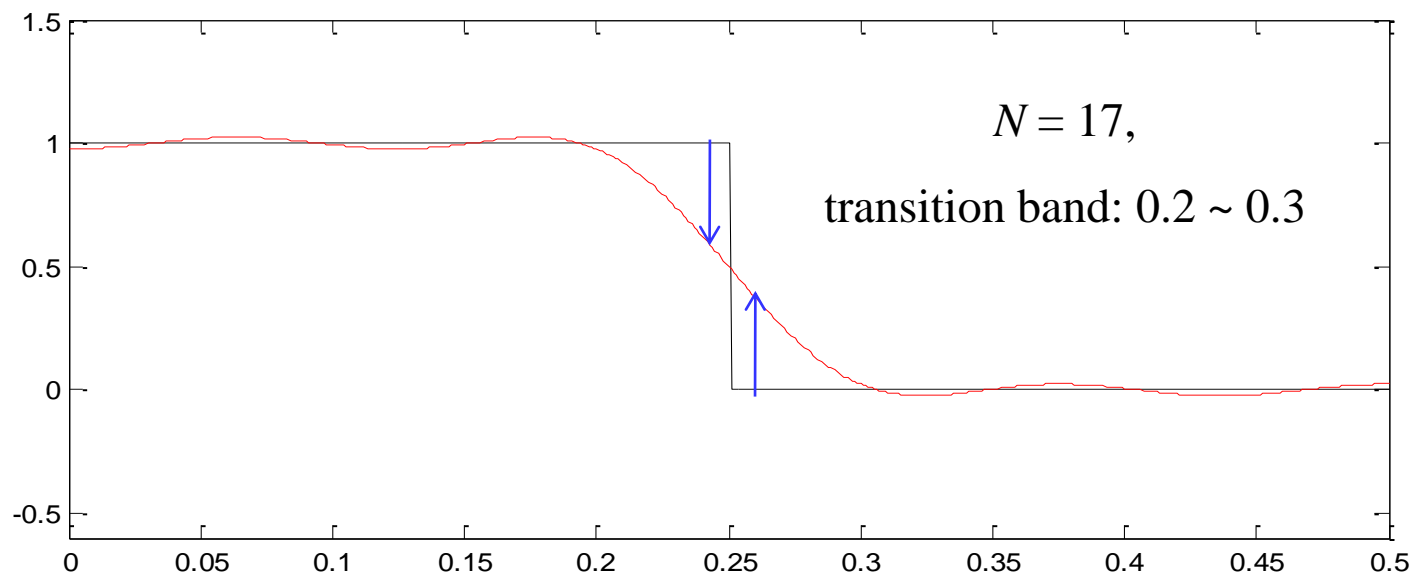
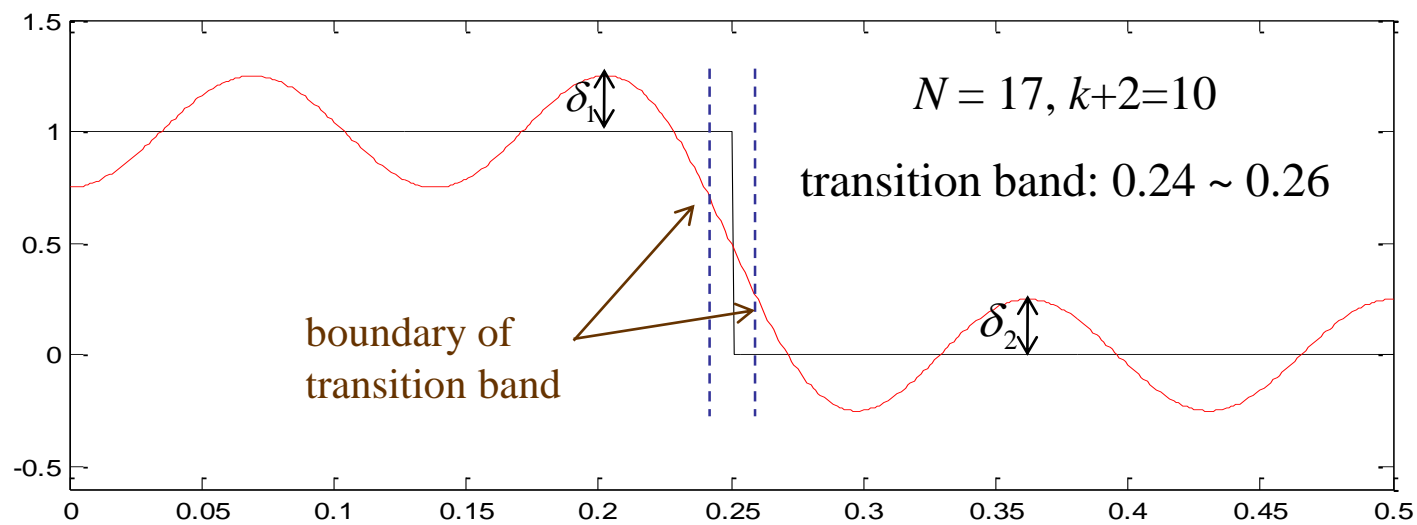
- When there are two transition bands,  $\Delta F = \min(\Delta F_1, \Delta F_2)$
- 犧牲 transition band 的 frequency response, 換取較高的 passband and stopband accuracies

$$N = \frac{2}{3} \frac{1}{\Delta F} \log_{10} \left( \frac{1}{10\delta_1\delta_2} \right) \quad \frac{3}{2} N \Delta F = \log_{10} \left( \frac{1}{10\delta_1\delta_2} \right)$$

[Ref] F. Mintzer and L. Bede, “Practical design rules for optimum FIR bandpass digital filter”, *IEEE Trans. ASSP*, vol. 27, no. 2, pp. 204-206, Apr. 1979.

問題：假設  $\delta_1 = \delta_2 = \delta$ ， $N$  為固定，

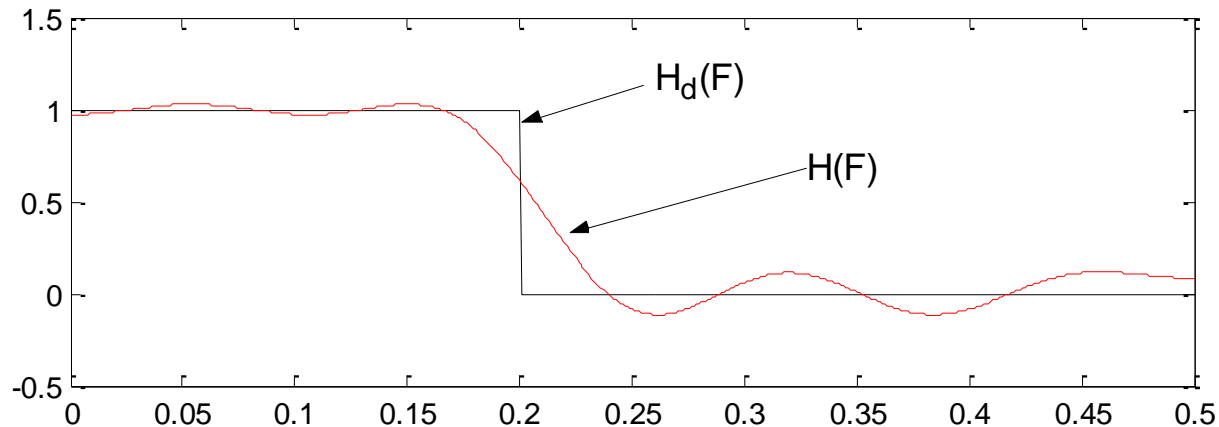
當  $\Delta F$  變為  $A$  倍時， $\delta$  變為多少？



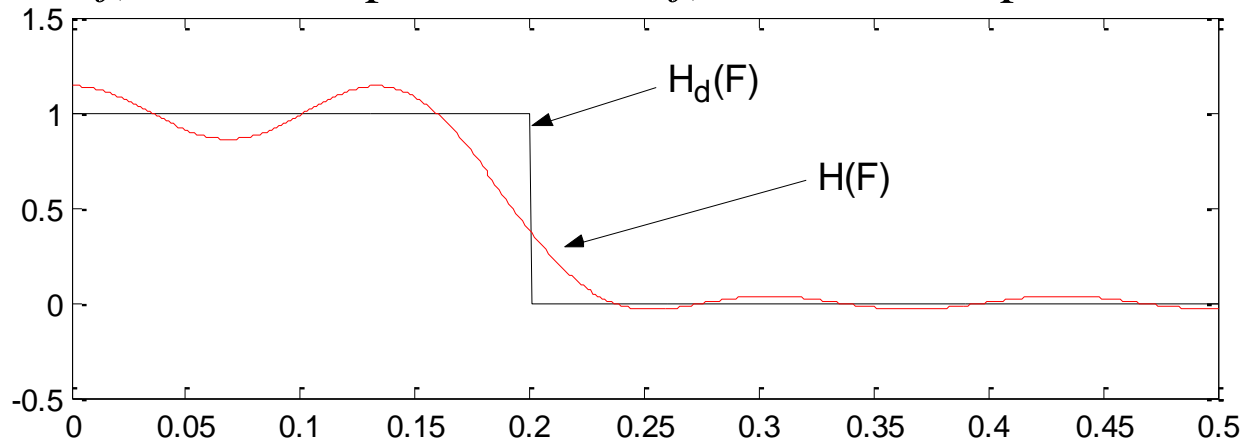
## © 2-I Relations between Weight Functions and Accuracy

79

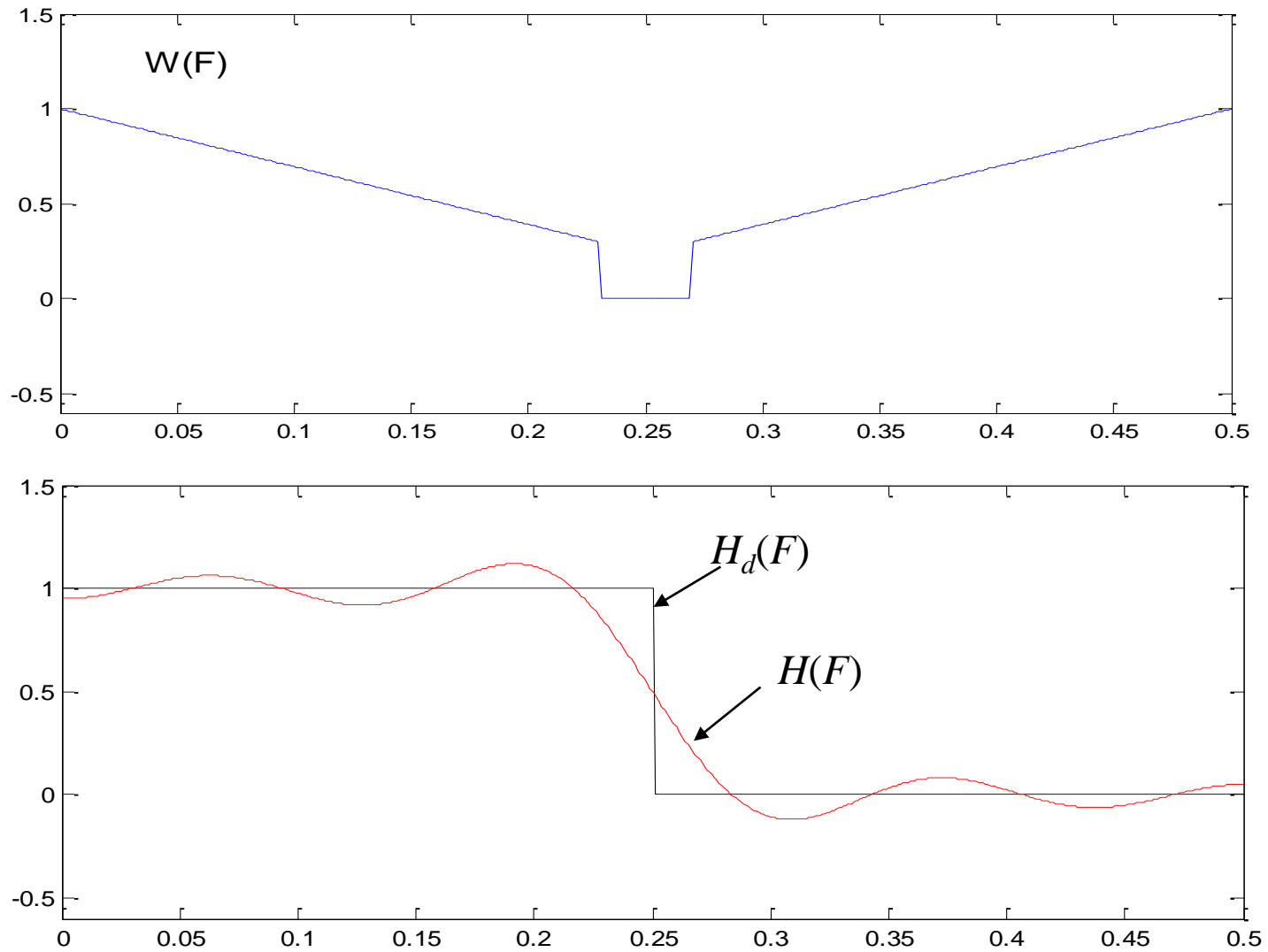
If we treat the passband more important than the stop band  
 $W(f) = 1$  in the passband,  $0 < W(f) < 1$  in the stopband



If we treat the stop band more important than the pass band  
 $0 < W(f) < 1$  in the passband,  $W(f) = 1$  in the stopband



# Larger error near the transition band



## ◎ 2-J FIR Filter in MSE Sense with Weight Functions

$$R(F) = \sum_{n=0}^k s[n] \cos(2\pi n F)$$

可對照 pages 47~49

$$MSE = \int_{-1/2}^{1/2} W(F) |R(F) - H_d(F)|^2 dF \quad \textcolor{red}{F = f / f_s}$$

$$= \int_{-1/2}^{1/2} W(F) \left( \sum_{\tau=0}^k s[\tau] \cos(2\pi \tau F) - H_d(F) \right)^2 dF$$

$$\frac{\partial MSE}{\partial s[n]} = 2 \int_{-1/2}^{1/2} W(F) \cos(2\pi n F) \left( \sum_{\tau=0}^k s[\tau] \cos(2\pi \tau F) - H_d(F) \right) dF = 0$$

$$\sum_{\tau=0}^k s[\tau] \int_{-1/2}^{1/2} W(F) \cos(2\pi n F) \cos(2\pi \tau F) dF - \int_{-1/2}^{1/2} W(F) H_d(F) \cos(2\pi n F) dF = 0$$

$$n = 0 \sim k$$

問題：  $\int_{-1/2}^{1/2} W(F) \cos(2\pi n F) \cos(2\pi \tau F) dF \neq 0$  when  $n \neq \tau$   
(not orthogonal)

$$\sum_{\tau=0}^k s[\tau] \int_{-1/2}^{1/2} W(F) \cos(2\pi n F) \cos(2\pi \tau F) dF - \int_{-1/2}^{1/2} W(F) H_d(F) \cos(2\pi n F) dF = 0$$

$\tau = 0 \sim k, n = 0 \sim k$

可以表示成  $(k+1) \times (k+1)$  matrix operation

$$\begin{matrix} & \tau=0 & \tau=1 & \tau=2 & & \tau=k \\ \begin{matrix} n=0 \\ n=1 \\ n=2 \\ \vdots \\ n=k \end{matrix} & \begin{bmatrix} B[0,0] & B[0,1] & B[0,2] & \cdots & B[0,k] \\ B[1,0] & B[1,1] & B[1,2] & \cdots & B[1,k] \\ B[2,0] & B[2,1] & B[2,2] & \cdots & B[2,k] \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ B[k,0] & B[k,1] & B[k,2] & \cdots & B[k,k] \end{bmatrix} & \begin{bmatrix} s[0] \\ s[1] \\ s[2] \\ \vdots \\ s[k] \end{bmatrix} & = & \begin{bmatrix} C[0] \\ C[1] \\ C[2] \\ \vdots \\ C[k] \end{bmatrix} \end{matrix}$$

**B**

**S**

**= C**

**S = B<sup>-1</sup>C**

$$B[n, \tau] = \int_{-1/2}^{1/2} W(F) \cos(2\pi n F) \cos(2\pi \tau F) dF$$

$$C[n] = \int_{-1/2}^{1/2} W(F) H_d(F) \cos(2\pi n F) dF$$

Think : Is it possible to apply the **transition band** to the FIR filter in the **MSE sense**?



## ◎ 2-K Four Types of FIR Filter

$$h[n] = 0 \text{ for } n < 0 \text{ and } n \geq N$$

點數為  $N$

$$H(F) = \sum_{n=0}^{N-1} h[n] \exp(-j2\pi n F)$$

- Type 1  $R(F) = \sum_{n=0}^k s[n] \cos(2\pi n F)$  ← 之前的方法只討論到 Type 1

$$h[n_1] = h[n_2 - n] \quad \text{and} \quad N \text{ is odd.}$$

(even symmetric)

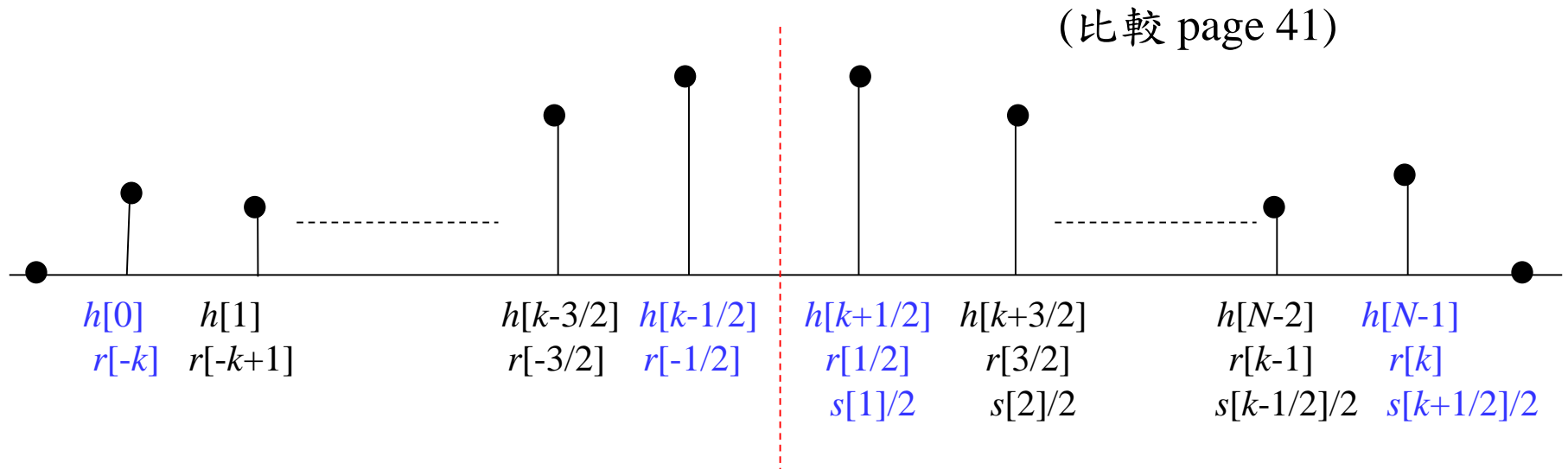
$$k = (N-1)/2$$

- Type 1:  $R(F) = \sum_{n=0}^k s[n] \cos(2\pi n F)$   
 $\underline{h[n] = h[N-1-n]}$  (even symmetric) and  $N$  is odd.
- Type 2:  $R(F) = \sum_{n=1}^{k+1/2} s[n] \cos(2\pi (n-1/2) F)$   
 $\underline{h[n] = h[N-1-n]}$  (even symmetric) and  $N$  is even.
- Type 3:  $R(F) = \sum_{n=1}^k s[n] \sin(2\pi n F)$   
 $\underline{h[n] = -h[N-1-n]}$  (odd symmetric) and  $N$  is odd.
- Type 4:  $R(F) = \sum_{n=1}^{k+1/2} s[n] \sin(2\pi (n-1/2) F)$   
 $\underline{h[n] = -h[N-1-n]}$  (odd symmetric) and  $N$  is even.

$$k = (N-1)/2$$

- Type 2: When  $h[n] = h[N-1-n]$  and  $N$  is even:  
(even symmetric)

令  $r[n] = h[n + k]$ , where  $k = (N-1)/2$  (注意此時  $k$  不為整數)



當  $R(F) = \sum_{n=-k}^k r[n] \exp(-j2\pi n F)$

$$R(F) = e^{j2\pi F k} H(F)$$

$$\begin{aligned} R(F) &= \sum_{n=1/2}^k \{r[n] \exp(-j2\pi n F) + r[-n] \exp(j2\pi n F)\} \\ &= \sum_{n=1/2}^k r[n] \{\exp(-j2\pi n F) + \exp(j2\pi n F)\} = \sum_{n=1/2}^k 2r[n] \cos(2\pi n F) \end{aligned}$$

$$R(F) = \sum_{n=1}^{k+1/2} s[n] \cos(2\pi (n-1/2) F)$$

$$n_{(new)} = n_{(old)} + \frac{1}{2}$$

$$n_{(old)} = n_{(new)} - \frac{1}{2}$$

$$s[n] = 2r[n-1/2] \quad n = 1, 2, \dots, k+1/2$$

設計出  $s[n]$  之後

$$r[n] = s[n+1/2]/2, \quad h[n] = r[n-k],$$

$$R(F) = \sum_{n=1}^{k+1/2} s[n] \cos(2\pi(n-1/2)F)$$

由於  $n$  和  $n+1$  兩項相加可得

$$\cos(2\pi(n-1/2)F) + \cos(2\pi(n+1/2)F) = 2\cos(\pi F)\cos(2\pi nF)$$

所以可以「判斷」 $R(F)$  能被改寫成

$$R(F) = \cos(\pi F) \sum_{n=0}^{k_1} s_1[n] \cos(2\pi nF)$$

求  $s_1[n]$  和  $s[n]$  之間的關係

$$\begin{aligned} R(F) &= \sum_{n=0}^{k_1} s_1[n] \cos(\pi F) \cos(2\pi nF) \\ &= \sum_{n=0}^{k_1} \frac{1}{2} s_1[n] \cos(2\pi(n-1/2)F) + \sum_{n=0}^{k_1} \frac{1}{2} s_1[n] \cos(2\pi(n+1/2)F) \\ &= \sum_{n=0}^{k_1} \frac{1}{2} s_1[n] \cos(2\pi(n-1/2)F) + \sum_{n=1}^{k_1+1} \frac{1}{2} s_1[n-1] \cos(2\pi(n-1/2)F) \end{aligned}$$

$$R(F) = \sum_{n=0}^{k_1} \frac{1}{2} s_1[n] \cos(2\pi(n-1/2)F) + \sum_{n=1}^{k_1+1} \frac{1}{2} s_1[n-1] \cos(2\pi(n-1/2)F)$$

$$R(F) = \frac{1}{2} s_1[0] \cos(\pi F) + \sum_{n=1}^{k_1} \frac{1}{2} (s_1[n] + s_1[n-1]) \cos(2\pi(n-1/2)F) \\ + \frac{1}{2} s_1[k_1] \cos(2\pi(k_1+1/2)F)$$

$$R(F) = \left( s_1[0] + \frac{1}{2} s_1[1] \right) \cos(\pi F) + \sum_{n=2}^{k-1/2} \frac{1}{2} (s_1[n] + s_1[n-1]) \cos(2\pi(n-1/2)F) \\ + \frac{1}{2} s_1[k-1/2] \cos(2\pi(k)F) \quad (\text{令 } k_1 + 1/2 = k)$$

比較係數可得  $s[1] = s_1[0] + \frac{1}{2} s_1[1]$

$$s[n] = \frac{1}{2} (s_1[n] + s_1[n-1]) \quad \text{for } n = 2, 3, \dots, k-1/2$$

$$s[k+1/2] = \frac{1}{2} s_1[k-1/2]$$

$$\begin{aligned}
err(F) &= [H_d(F) - R(F)]W(F) \\
&= \left[ H_d(F) - \cos(\pi F) \sum_{n=0}^{k-1/2} s_1[n] \cos(2\pi nF) \right] W(F) \\
&= \left[ \sec(\pi F) H_d(F) - \sum_{n=0}^{k-1/2} s_1[n] \cos(2\pi nF) \right] \cos(\pi F) W(F)
\end{aligned}$$

只需將 pages 53-56 的方法當中， $H_d(F)$  換成  $\sec(\pi F) H_d(F)$

$W(F)$  換成  $\cos(\pi F) W(F)$

$k$  換成  $k - 1/2 = N/2 - 1$

注意  $s_1[n]$  和  $s[n]$  之間的關係即可

## Design Method for Type 3

91

$$R(F) = \sum_{n=1}^k s[n] \sin(2\pi n F)$$

由於  $n-1$  和  $n+1$  兩項相減可得

$$\sin(2\pi(n+1)F) - \sin(2\pi(n-1)F) = 2\sin(2\pi F)\cos(2\pi n F)$$

所以「判斷」可將  $R(F)$  改寫為

$$R(F) = \sin(2\pi F) \sum_{n=0}^{k_1} s_1[n] \cos(2\pi n F)$$

$$\begin{aligned} R(F) &= \frac{1}{2} \sum_{n=0}^{k_1} s_1[n] \sin(2\pi(n+1)F) - \frac{1}{2} \sum_{n=0}^{k_1} s_1[n] \sin(2\pi(n-1)F) \\ &= \frac{1}{2} \sum_{n=0}^{k_1} s_1[n] \sin(2\pi(n+1)F) + \frac{s_1[0]}{2} \sin(2\pi F) - \frac{1}{2} \sum_{n=2}^{k_1} s_1[n] \sin(2\pi(n-1)F) \\ &= \frac{1}{2} \sum_{n=1}^{k_1+1} s_1[n-1] \sin(2\pi n F) + \frac{s_1[0]}{2} \sin(2\pi F) - \frac{1}{2} \sum_{n=1}^{k_1-1} s_1[n+1] \sin(2\pi n F) \end{aligned}$$



$$\begin{aligned}
R(F) = & \frac{s_1[0]}{2} \sin(2\pi F) + \frac{1}{2} (s_1[0] - s_1[2]) \sin(2\pi F) \\
& + \frac{1}{2} \sum_{n=2}^{k_1-1} (s_1[n-1] - s_1[n+1]) \sin(2\pi n F) \\
& + \frac{1}{2} s_1[k_1-1] \sin(2\pi k_1 F) + \frac{1}{2} s_1[k_1] \sin(2\pi(k_1+1) F)
\end{aligned}$$

令  $k_1 = k - 1$ , 比較係數可得

$$s[1] = s_1[0] - \frac{1}{2} s_1[2]$$

$$s[n] = \frac{1}{2} s_1[n-1] - \frac{1}{2} s_1[n+1] \quad \text{for } n = 2, 3, \dots, k-2$$

$$s[k-1] = \frac{1}{2} s_1[k-2]$$

$$s[k] = \frac{1}{2} s_1[k-1]$$

$$\begin{aligned}
err(F) &= [H_d(F) - R(F)]W(F) \\
&= \left[ H_d(F) - \sin(2\pi F) \sum_{n=0}^{k-1} s_1[n] \cos(2\pi nF) \right] W(F) \\
&= \left[ \csc(2\pi F) H_d(F) - \sum_{n=0}^{k-1} s_1[n] \cos(2\pi nF) \right] \sin(2\pi F) W(F)
\end{aligned}$$

將 pages 53-56 的方法當中， $H_d(F)$  換成  $\csc(2\pi F) H_d(F)$

$W(F)$  換成  $\sin(2\pi F) W(F)$

$k$  換成  $k-1$

注意  $s_1[n]$  和  $s[n]$  之間的關係即可

Think : Design Method for Type 4

### 一、各種程式語言寫程式共通的原則

- (1) 能夠不在迴圈內做的運算，則移到迴圈外，以節省運算時間
- (2) 寫一部分即測試，不要全部寫完再測試 (縮小範圍比較容易 debug)
- (3) 先測試簡單的例子，成功後再測試複雜的例子

### 二、Matlab 寫程式特有的技巧

- (1) 迴圈能避免就儘量避免
- (2) 儘可能使用 Matrix 及 Vector operation

Example: 由 1 加到 100，用 Matlab 一行就可以了

```
sum([1:100])
```

完全不需迴圈

### 三、一些重要的 Matlab 指令

(1) **function**: 放在第一行，可以將整個程式函式化

(2) **tic, toc**: 計算時間

tic 為開始計時，toc 為顯示時間

(3) **find**: 找尋一個 vector 當中不等於 0 的 entry 的位置

範例：find([1 0 0 1]) = [1, 4]

find(abs([-5:5])<=2) = [4, 5, 6, 7, 8]

(因為 abs([-5:5])<=2 = [0 0 0 1 1 1 1 1 0 0 0])

(4) **'**: Hermitian (transpose + conjugation), **.'**: transpose

(5) **imread**: 讀圖，**image, imshow, imagesc**: 將圖顯示出來，

(註：較老的 Matlab 版本 imread 要和 double 並用

A=double(imread('Lena.bmp'));

(6) **imwrite**: 製做圖檔

- (7) `xlsread`: 由 Excel 檔讀取資料
- (8) `xlswrite`: 將資料寫成 Excel 檔
- (9) `aviread`: 讀取 video 檔
- (10) `dlmread`: 讀取 \*.txt 或其他類型檔案的資料
- (11) `dlmwrite`: 將資料寫成 \*.txt 或其他類型檔案