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1. Solve the following question by using Land and Doig algorithm or Dakin algorithm.

$$\begin{array}{lrcrcrcrcl} \max & 12x_1 & + & 16x_2 & + & 22x_3 & + & 8x_4 & & \\ \text{s.t.} & 4x_1 & + & 5x_2 & + & 7x_3 & + & 3x_4 & \leq & 14 \end{array}$$
$$x_i \text{ are binary.}$$

Take $Z^* = -\infty$ and set all variables to 0 or 1.

$\boxed{\text{Node 01}}$ $x^0 = (x_1, x_2, x_3, x_4)$ and the LP Solution $Z = 44, x_1 = 0.5, x_2 = 1, x_3 = 1, x_4 = 1.$

$\boxed{\text{Node 02}}$ $x^1 = (1, x_2, x_3, x_4)$ and the LP Solution $Z = 43.71, x_1 = 1, x_2 = 1, x_3 = 0.714, x_4 = 0.$

$\boxed{\text{Node 03}}$ $x^1 = (0, x_2, x_3, x_4)$ and the LP Solution $Z = 43.33, x_1 = 0, x_2 = 1, x_3 = 1, x_4 = 0.667.$

$\Longrightarrow$ At Node 01, $Z^0 = \max\{43.71, 43.33\} = 43.71.$ $\Longrightarrow$ Choose Node 02 to next branching.

$\boxed{\text{Node 04}}$ $x^2 = (1, x_2, 0, x_4)$ and the Solution $Z = 36, x_1 = 1, x_2 = 1, x_3 = 0, x_4 = 1.$ This is a BIP Solution.

$\boxed{\text{Node 05}}$ $x^2 = (1, x_2, 1, x_4)$ and the LP Solution $Z = 43.6, x_1 = 1, x_2 = 0.6, x_3 = 1, x_4 = 0.$

$\Longrightarrow$ At Node 02, $Z^1 = \max\{43.33, 43.60\} = 43.60.$ $\Longrightarrow$ Choose Node 05 to next branching.

$\boxed{\text{Node 06}}$ $x^3 = (1, 1, 1, x_4)$ and there is no LP Solution.

$\boxed{\text{Node 07}}$ $x^3 = (1, 0, 1, x_4)$ and the Solution $Z = 42, x_1 = 1, x_2 = 0, x_3 = 1, x_4 = 1.$ This is a BIP Solution.

$\Longrightarrow$ Choose Node 03 to next branching

Node 08 $x^4 = (0, x_2, x_3, 0)$ and the Solution $Z = 38, x_1 = 0, x_2 = 1, x_3 = 1, x_4 = 0.$ This is a BIP Solution.

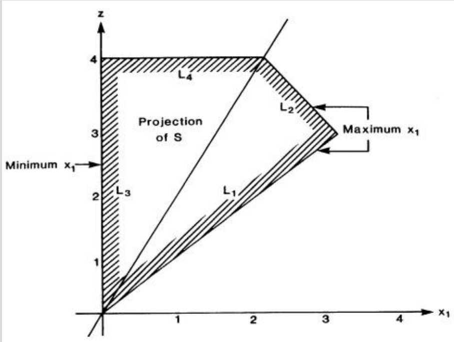
Node 09 $x^4 = (0, x_2, x_3, 1)$ and the LP Solution $Z = 42.85, x_1 = 0, x_2 = 1, x_3 = 0.857, x_4 = 1$

$\Longrightarrow$ Since the $Z^* = 42 = \lfloor 42.85 \rfloor.$

So we found the optimal solution: $Z = 42, x_1 = 1, x_2 = 0, x_3 = 1, x_4 = 1$

2. Consider the mixed integer program (Example 8.1)

$$\begin{array}{lrcrcrcrcrcl} \max & Z & = & x_1 & + & x_2 & + & y_1 \\ \text{s.t.} & x_1 & + & x_2 & + & y_1 & - & z & = & 0 \\ & 2x_1 & + & x_2 & + & y_1 & \le & 6 \\ & x_1 & + & 2x_2 & + & y_1 & \le & 6 \\ & x_1 & + & x_2 & + & 2y_1 & \le & 6 \\ & x_1 & + & x_2 & + & y_1 & \le & 4 \\ & y_1 & \ge & 0 \\ & & x_1, x_2 & & \ge & & 0 & , \text{integer.} \end{array}$$



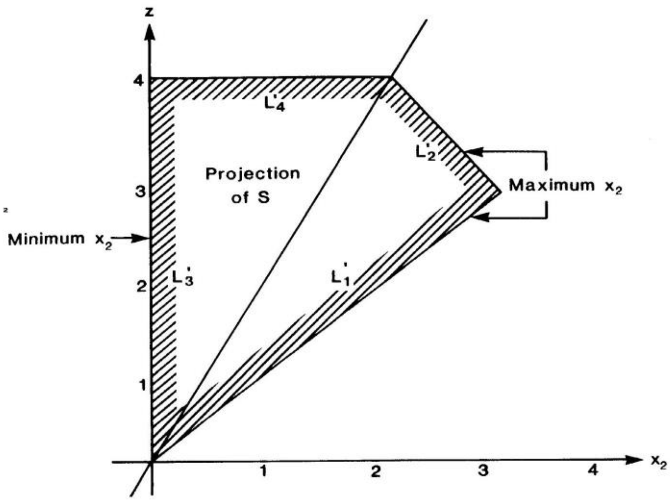
(a) Show that the maximum value of z is 4 for all x_1 in the closed interval $[0, 2]$.

(b) Find the projection of S onto the (x_2, z) plane.

(c) Is there a point (x_1, x_2, y_1, z) such that either x_1 or x_2, but not both, is integer and the coordinate (x_1, z) or (x_2, z) with x_1 or x_2 integer is in the projection of S onto the corresponding plane? Use a geometric argument to explain the question.

(a) From the constraints we know $z = x_1 + x_2 + y_1$. Note that the smallest possible value is 0 (when $x_1 = x_2 = y_1 = 0$) and the maximum value is 4 (Since the constraint $x_1 + x_2 + y_1 \le 4$). Moreover, the point $(x_1, x_2, y_1, z) = (2, 2, 0, 4)$ is in S.

(b) The projection of S onto the (x_2, z) plane shown below since all the constraints are symmetric with x_1 and x_2 (replace each other will not change the initial problem). Replace the x_1 axis to x_2 axis.



(c) Yes. As shown above, all the figure are the same!

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3. The following tableau corresponds to the optimal LP solution of ILP problem solved with the Dakin algorithm. For each following case, please derive a not-satisfied constraint for each variable at a non-integer value (x_1 and x_2), properly choose one of them for branching, and perform one dual simplex pivot iteration based on P_k^D and P_k^U.

		$(-x_3)$	$(-x_4)$	$(-x_5)$
x_0	$\frac{-71}{5}$	$\frac{3}{5}$	$\frac{2}{5}$	$\frac{11}{5}$
x_1	$\frac{2}{5}$	$\frac{-1}{5}$	$\frac{1}{5}$	$\frac{-2}{5}$
x_2	$\frac{19}{5}$	$\frac{8}{5}$	$\frac{-3}{5}$	$\frac{1}{5}$
x_3	0	-1	0	0
x_4	0	0	-1	0
x_5	0	0	0	-1

Observing the tableau. We can know that

(1) $\underline{P_1^D}$

$$P_1^D = \frac{\frac{2}{5} \cdot \frac{2}{5}}{\frac{1}{5}} = \frac{4}{5}$$

(2) $\underline{P_1^U}$

Calculate that $\min \left\{ \frac{\frac{3}{5}}{\frac{1}{5}}, \frac{\frac{11}{5}}{\frac{2}{5}} \right\} = \frac{\frac{3}{5}}{\frac{1}{5}}$.

So that

$$P_1^U = \frac{(\frac{2}{5} - 1) \cdot \frac{3}{5}}{-\frac{1}{5}} = \frac{9}{5}$$

Then the ILP Gomory Cut:

$$x_1^G = (-\frac{2}{5}) + (-\frac{4}{5})(-x_3) + (-\frac{1}{5})(-x_4) + (-\frac{3}{5})(-x_5)$$

4. Solve the following $0-1$ ILP problem by the search enumeration method with a node algorithm that includes the zero completion and the infeasibility test.

$$\begin{array}{lrcrcrcrcrcrcr} \max & z & = & 4x_1 & + & 5x_2 & + & 6x_3 & + & 2x_4 & + & 3x_5 & & \\ \text{s.t.} & -4x_1 & - & 2x_2 & + & 3x_3 & - & 2x_4 & + & x_5 & \leq & -1 \\ & -x_1 & - & 5x_2 & - & 2x_3 & + & 2x_4 & - & 2x_5 & \leq & -5 \\ \multicolumn{12}{c}{x_j \in \{0,1\}, j = 1, \cdots, 5} \end{array}$$

$\boxed{\text{Problem 01}}$

Zero Completion Infeasible: $b_1 = -1, b_2 = -5 \implies Z_I = 0$

Infeasible Test: $P_1 = (-1+4+2+2) = 7 > 0$ and $P_2 = (-5+1+5+2+2) = 5 > 0$

$\boxed{\text{Problem 02}}$ $x = (1, \#, \#, \#, \#)$

Zero Completion Infeasible: $b_2 = -4 \implies Z_I = 4$

Infeasible Test: $P_1 = 7 > 0$ and $P_2 = 5 > 0$

$\boxed{\text{Problem 03}}$ $x = (1, 1, \#, \#, \#)$

Zero Completion feasible: $b_1 = 5, b_2 = 3 \implies$ A **feasible** Solution $(1,1,0,0,0)$ and $Z_U = 9$

Infeasible Test: $P_1 = 5 > 0$ and $P_2 = 0 \geq 0$

$\boxed{\text{Problem 04}}$ $x = (1, 0, 1, \#, \#)$

Zero Completion Infeasible: $b_2 = -2 \implies Z_I = 10 > Z_U$. **Backfoward!**

Infeasible Test: NULL.

$\boxed{\text{Problem 05}}$ $x = (1, 0, 0, \#, \#)$

Zero Completion Infeasible: $b_2 = -4 \implies Z_I = 4$

Infeasible Test: $P_1 = 5 > 0$ and $P_2 = -2 < 0 \implies$ **Infeasible!**

$\boxed{\text{Problem 06}}$ $x = (0, \#, \#, \#, \#)$

Zero Completion Infeasible: $b_1 = -1, b_2 = -5 \implies Z_I = 0$

Infeasible Test: $P_1 = 3 > 0$ and $P_2 = 4 > 0$

$\boxed{\text{Problem 07}}$ $x = (0, 1, \#, \#, \#)$

Zero Completion feasible: $b_1 = 1, b_2 = 0 \implies$ A **feasible** Solution $(0,1,0,0,0)$ and $Z_U = 5$

$\boxed{\text{Problem 08}}$ $x = (0, 0, \#, \#, \#)$

Zero Completion Infeasible: $b_1 = -1, b_2 = -5 \implies Z_I = 0$

Infeasible Test: $P_1 = 1 > 0$ and $P_2 = -1 < 0 \implies$ **Infeasible!**

Thus, the optimal solution is $x = (0,1,0,0,0)$ and the objective value is 5.

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5. Suppose, at a node x^l, c^1 and c^2 are the first and second smallest costs corresponding to the free variables. Under what circumstances is it valid to say that to produce an improved solution from x^l we must have

$$z^l + c^1 + c^2 + \sum_{j \in \bar{F}} c_j < z^*$$

where $\bar{F} = \{j \in F | c_j < 0\}$. What implicit enumerations tests can be deduced from the inequality? What calculations are necessary for their implementation?

Assume that c^1 and c^2 are positive. Both of the corresponding variables of c^1 and c^2 are set to one so that all constraints are satisfied (feasible). In the case, the variables associated with $c^3, \cdots$ are set to zero.

Note that the problem is infeasible only either the variable of c^1 or the variable of c^2 set to 1