Integer and Combinatorial Optimization

Spring 2017

Homework 4

(Due at the beginning of the class on May 18)

1. Solve the following question by using Land and Doig algorithm or Dakin algorithm

$$\begin{aligned}&\text{Max } 12x_1 + 16x_2 + 22x_3 + 8x_4\\ &\text{S.t.} \quad 4x_1 + 5x_2 + 7x_3 + 3x_4 \le 14,\\ &\qquad x_i \text{ are binary.}\end{aligned}$$

2. Consider the mixed integer program (Example 8.1)

$$\begin{aligned}&\text{Max } z = x_1 + x_2 + y_1\\ &\text{S.t.} \quad x_1 + x_2 + y_1 - z = 0\\ &\quad 2x_1 + x_2 + y_1 \le 6,\\ &\quad x_1 + 2x_2 + y_1 \le 6,\\ &\quad x_1 + x_2 + 2y_1 \le 6,\\ &\quad x_1 + x_2 + y_1 \le 4,\\ &\quad y_1 \ge 0,\\ &\quad x_1, x_2 \ge 0, \text{integer.}\end{aligned}$$

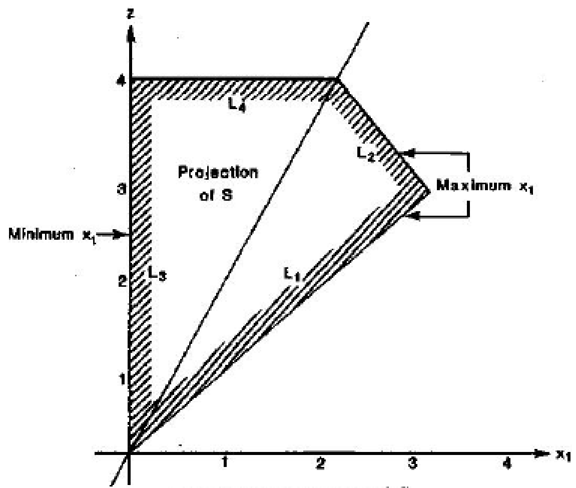

Figure 8.3: Projection of S

(a) Show that the maximum value of z is 4 for all x_1 in the closed interval [0, 2].

(b) Find the projection of S onto the (x_2, z) plane.

(c) Is there a point (x_1, x_2, y_1, z) such that either x_1 or x_2, but not both, is integer and the coordinate (x_1, z) or (x_2, z) with x_1 or x_2 integer is in the projection of S onto the corresponding plane? Use a geometric argument to explain the question.

3. The following tableau corresponds to the optimail LP solution of ILP problem solved with the Dakin algorithm. For each following case (a and b), please derive a not-satisfied constraint for each variable at a noninteger value (x_1 and x_2), properly choose one of them for branching, and perform one dual simplex pivot iteration based on P_k^D *and* P_k^U.

		$(-x_3)$	$(-x_4)$	$(-x_5)$
x_0	$-\frac{71}{5}$	$\frac{3}{5}$	$\frac{2}{5}$	$\frac{11}{5}$
x_1	$\frac{2}{5}$	$-\frac{1}{5}$	$\frac{1}{5}$	$-\frac{2}{5}$
x_2	$\frac{19}{5}$	$\frac{8}{5}$	$-\frac{3}{5}$	$\frac{1}{5}$
x_3	0	-1	0	0
x_4	0	0	-1	0
x_5	0	0	0	-1

4. Solve the following 0-1 ILP problem by the search enumeration method with a node algorithm that includes the zero completion and the infeasibility test.

$$\begin{aligned} \text{minimize} \quad & z = 4x_1 + 5x_2 + 6x_3 + 2x_4 + 3x_5, \\ \text{subject to} \quad & -4x_1 - 2x_2 + 3x_3 - 2x_4 + x_5 \leq -1, \\ & -x_1 - 5x_2 - 2x_3 + 2x_4 - 2x_5 \leq -5, \\ & x_j \in \{0,1\}, \quad j=1,\ldots,5, \end{aligned}$$

5. Suppose, at a node x^l, c^1 and c^2 are the first and second smallest costs corresponding to the free variables. Under what circumstances is it valid to say that to produce an improved solution from x^l we must have

$$z^l + c^1 + c^2 + \sum_{j \in \bar{F}} c_j < z^*, \qquad (1)$$

where $\bar{F} = \{ j \in F \mid c_j < 0\}$. What implicit enumerations tests can be deduced from inequality (1)? What calculations are necessary for their implementation?