

# Session 4

## Recurrent Neural Network

# Acknowledgements

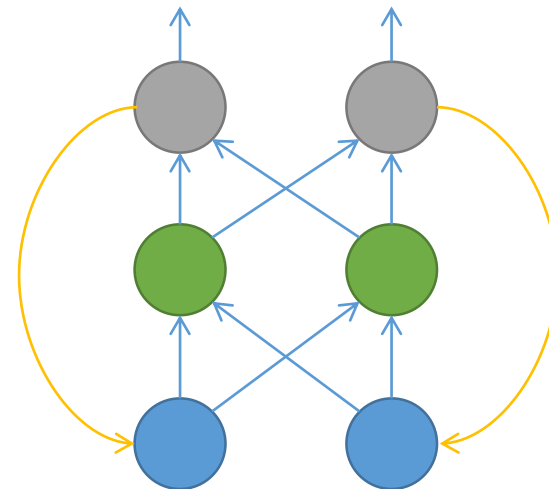
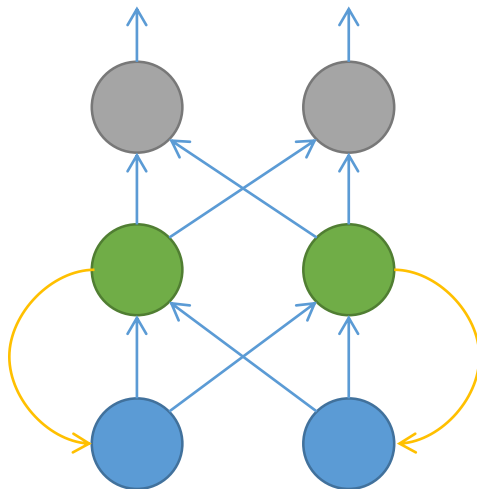
- The materials majorly derived from Prof. Fabien Moutarde. Some slides come from the online classes.
- **Fei-Fei Li + J.Johnson + S.Yeung: slides on “Recurrent Neural Networks”** from the “*Convolutional Neural Networks for Visual Recognition*” course at Stanford  
[http://cs231n.stanford.edu/slides/2019/cs231n\\_2019\\_lecture10.pdf](http://cs231n.stanford.edu/slides/2019/cs231n_2019_lecture10.pdf)
- **Yingyu Liang: slides on “Recurrent Neural Networks”** from the “*Deep Learning Basics*” course at Princeton  
[https://www.cs.princeton.edu/courses/archive/spring16/cos495/slides/DL\\_lecture9\\_RNN.pdf](https://www.cs.princeton.edu/courses/archive/spring16/cos495/slides/DL_lecture9_RNN.pdf)
- **Arun Mallya: slides “Introduction to RNNs”** from the “*Trends in Deep Learning and Recognition*” course of Svetlana LAZEBNIK at University of Illinois at Urbana-Champaign  
[http://slazebni.cs.illinois.edu/spring17/lec02\\_rnn.pdf](http://slazebni.cs.illinois.edu/spring17/lec02_rnn.pdf)
- **Tingwu Wang: slides on “Recurrent Neural Network”** for a course at University of Toronto  
[https://www.cs.toronto.edu/%7Etingwuwang/rnn\\_tutorial.pdf](https://www.cs.toronto.edu/%7Etingwuwang/rnn_tutorial.pdf)
- **Christopher Olah: online tutorial “Understanding LSTM Networks”**  
<https://colah.github.io/posts/2015-08-Understanding-LSTMs/>

# Introduction

- Recurrent neural networks have been an important focus of research and development during the 1990's. → It is much older than ConvNet!
- They are designed to learn sequential or time varying patterns.
- A recurrent net is a neural network with feedback (closed loop) connections [Fausett, 1994]. Examples include BAM, Hopfield, Boltzmann machine, and recurrent backpropagation nets [Hecht-Nielsen, 1990].
- A dynamic neural network can be defined as a neural networks that consists of **inter-layer feedback loops** (i.e., from output layer to input layer) and **intra-layer feedback loops** (i.e., between different neurons within the same layer) or self-feedback loops.
- From the computational perspective, a dynamic neural network that contains the feedback loop that may provide more computational advantages than a static neural network, which contains only feed-forward architecture
- Applications: natural language processing (NLP), forecasting, signal processing and control require the treatment of dynamics associated with the unknown model.

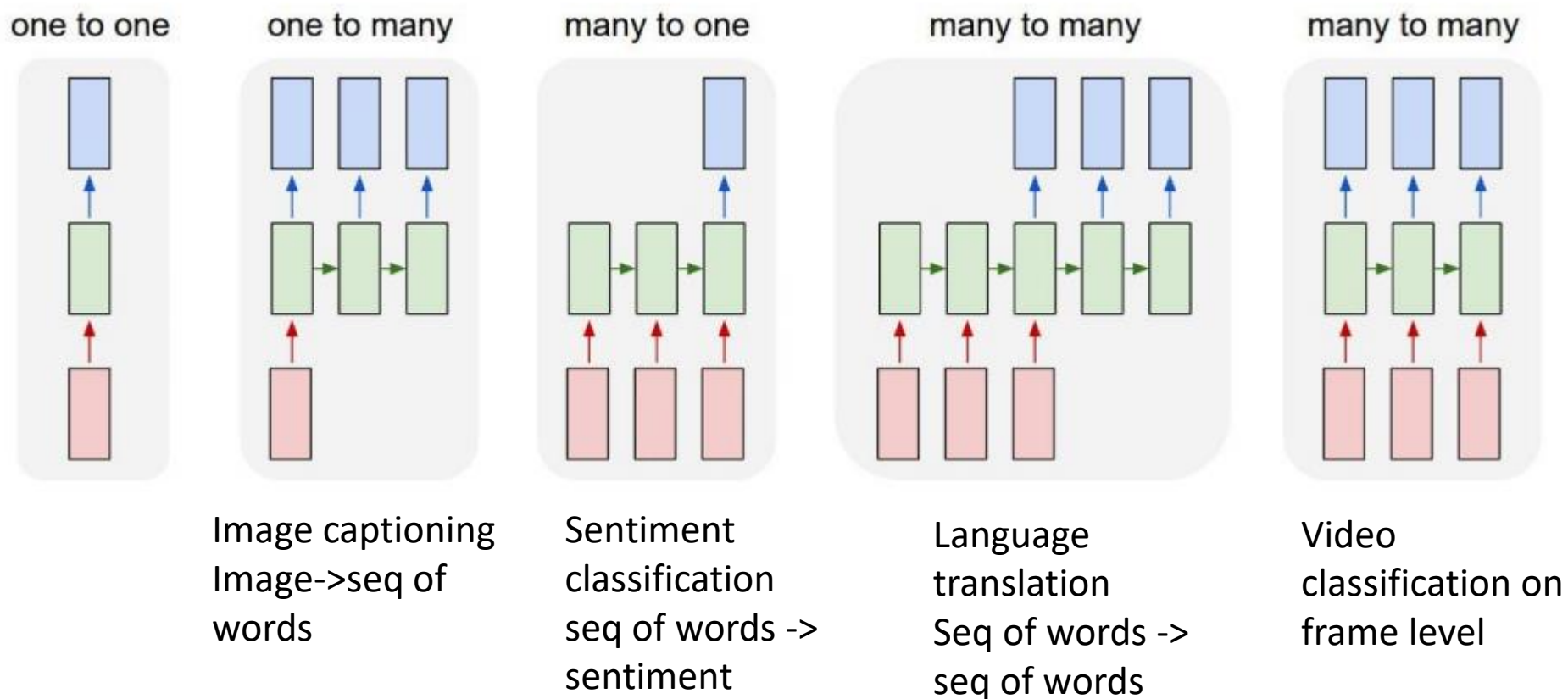
# Old style of RNN

- Elman introduced feedback from the hidden layer to the context portion of the input layer.
  - This approach pays more attention to the sequence of input values.
- Jordan recurrent neural networks, Jordan use feedback from the output layer to the context nodes of the input layer and give more emphasis to the sequence of output values.



However, these methods did not succeed in bigger data set due to the design of gradient flow

- In some context of machine learning, we want to have flexibility of input and output



# Gradient flow

- Gradient flow is very important in network
- We already saw a lots in the last section
- Risk to have feed-back connection:
  - stability,
  - Controllability
  - Observability

# Advantages of RNN

- The hidden state  $s$  of the RNN builds a kind of lossy summary of the past
- RNN totally adapted to processing SEQUENTIAL data (same computation formula applied at each time step, but modulated by the evolving “memory” contained in state  $s$ )
- Universality of RNNs: any function computable by a Turing Machine can be computed by a finite-size RNN (Siegelmann and Sontag, 1995)

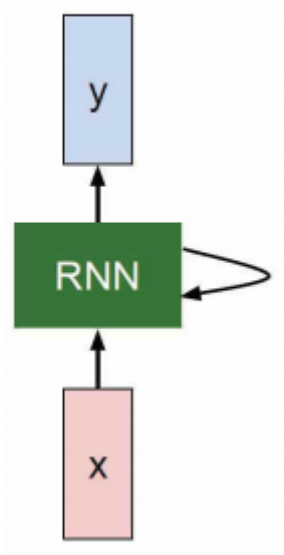
# Simply RNN: Vanilla

State vector  $s \leftrightarrow$  vector  $h$  of hidden neurons

$$h_t = f_W(h_{t-1}, x_t)$$

Diagram illustrating the equation  $h_t = f_W(h_{t-1}, x_t)$  with labels:

- New hidden state (points to  $h_t$ )
- Input vector at time  $t$  (points to  $x_t$ )
- Old hidden state (points to  $h_{t-1}$ )
- Some function with parameters  $W$  (points to  $f_W$ )



$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t)$$

$$y_t = W_{hy}h_t$$

$$\text{Or } y_t = \text{softmax}(W_{hy}h_t)$$

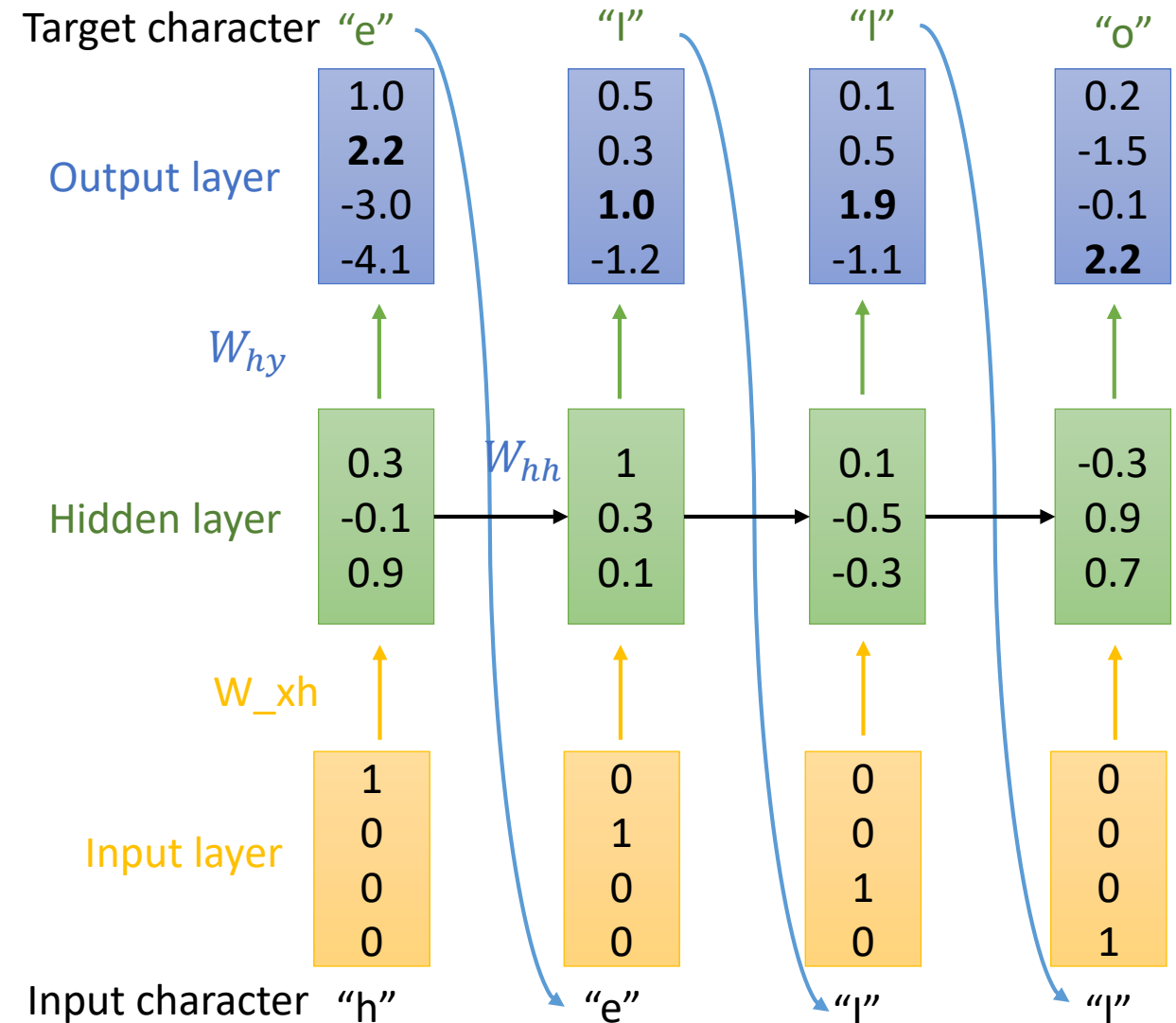


# Character-level Language Model

- Given four letters [h,e,l,o] as input vector
- First we transfer this into one-hot vector
- Then we randomize weights  $W_{xh}$ ,  $W_{hy}$
- Then we can train a sequence to predict “hello”

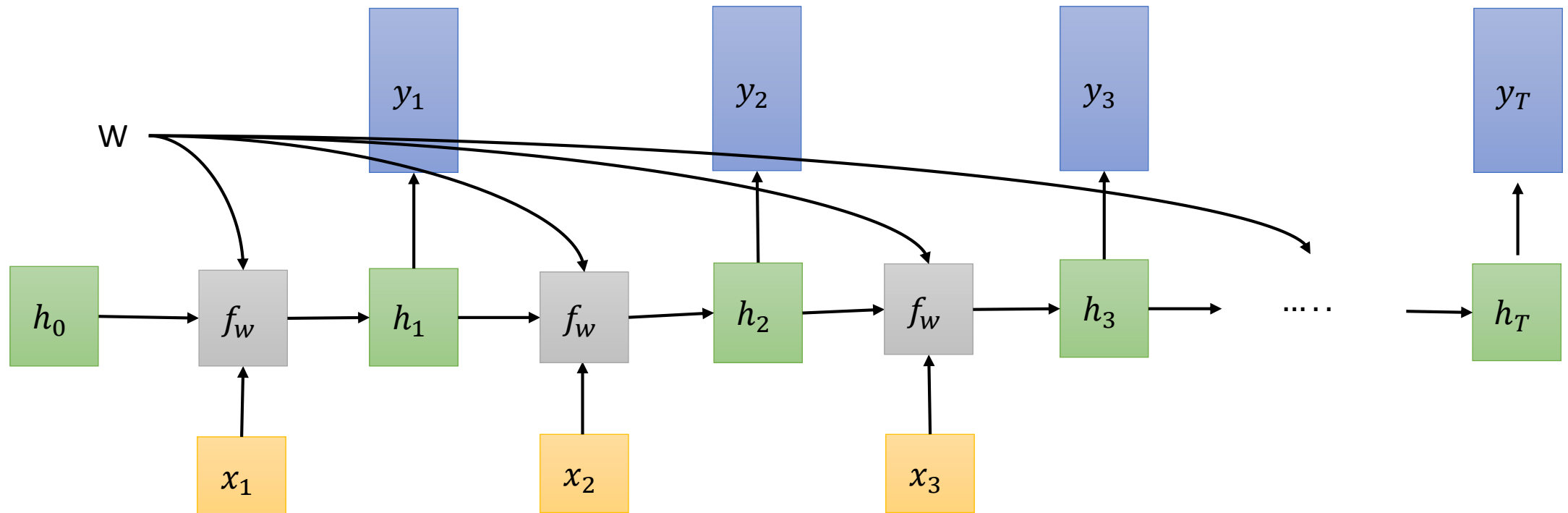
$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t)$$

Be aware that weights are shared in the sequence processing.

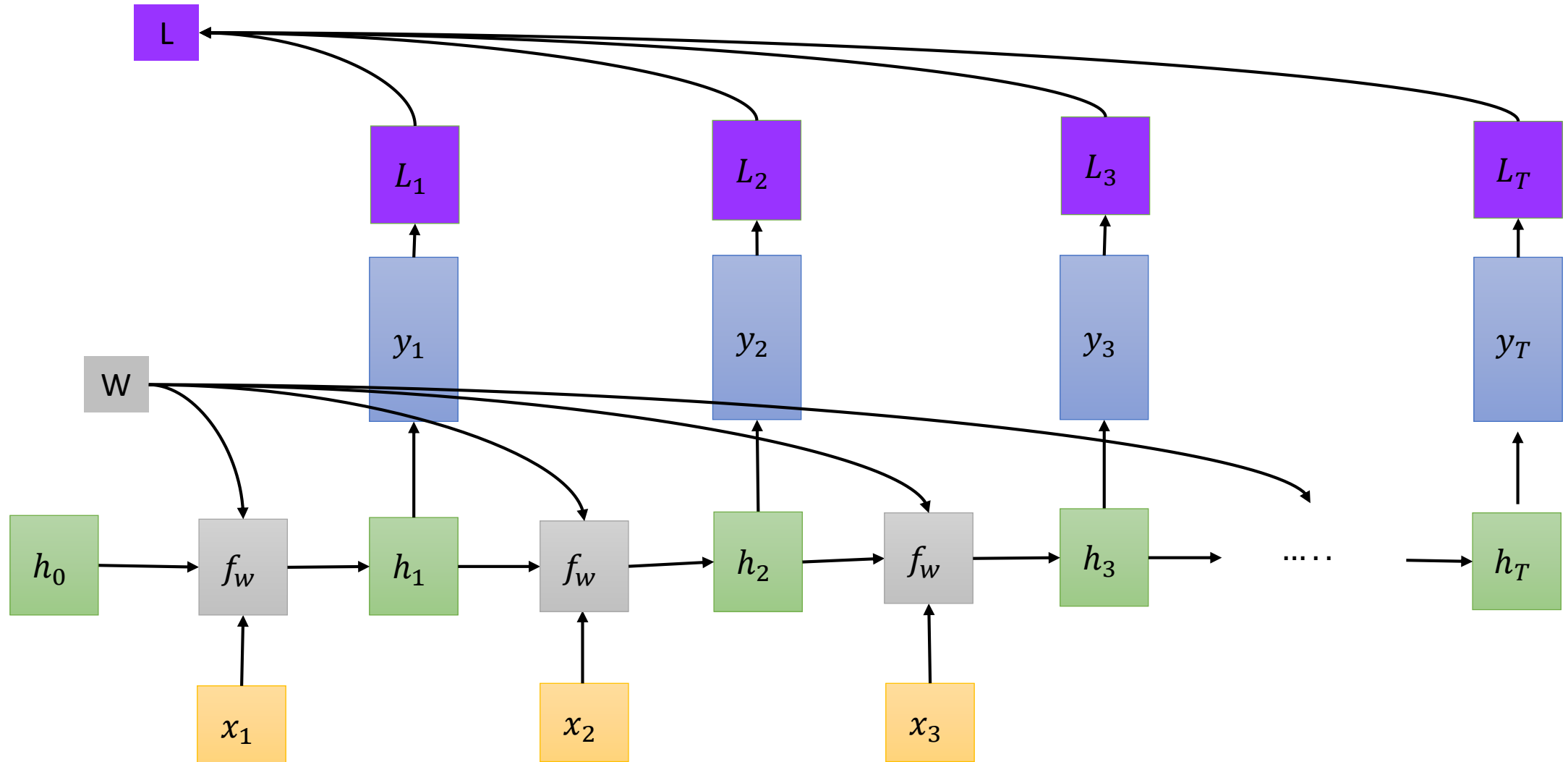


# RNN: computation graph M2M

- RNN forward pass: Many to many: (language translation)

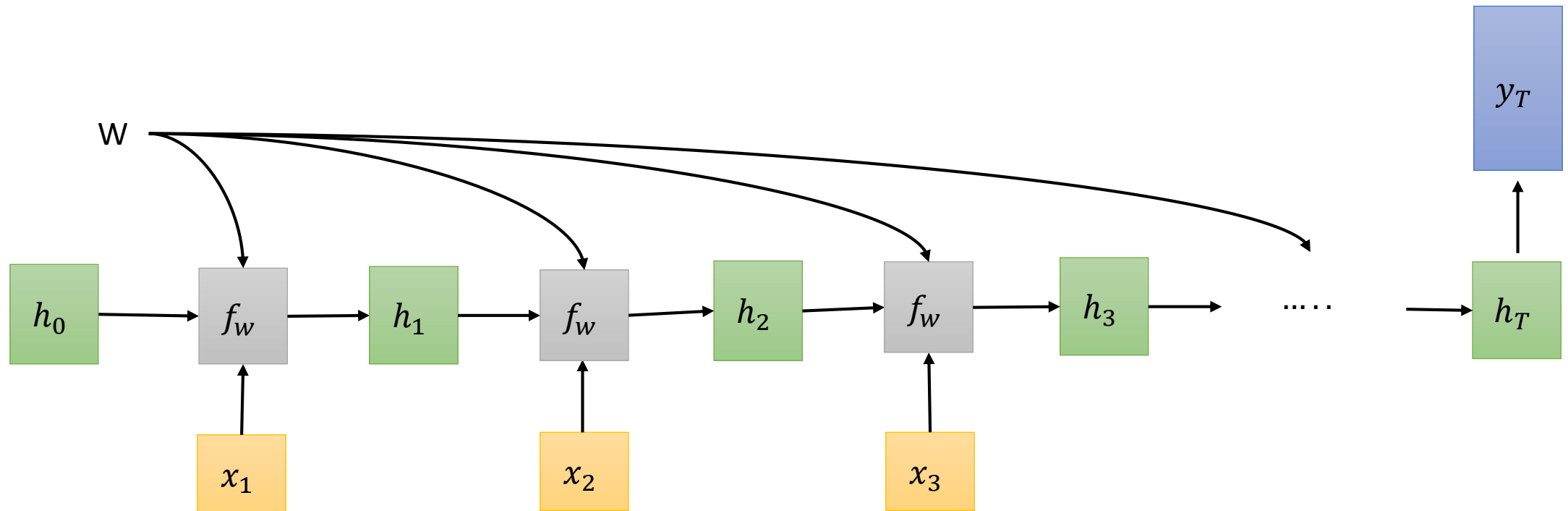


# RNN: computation graph M2M training



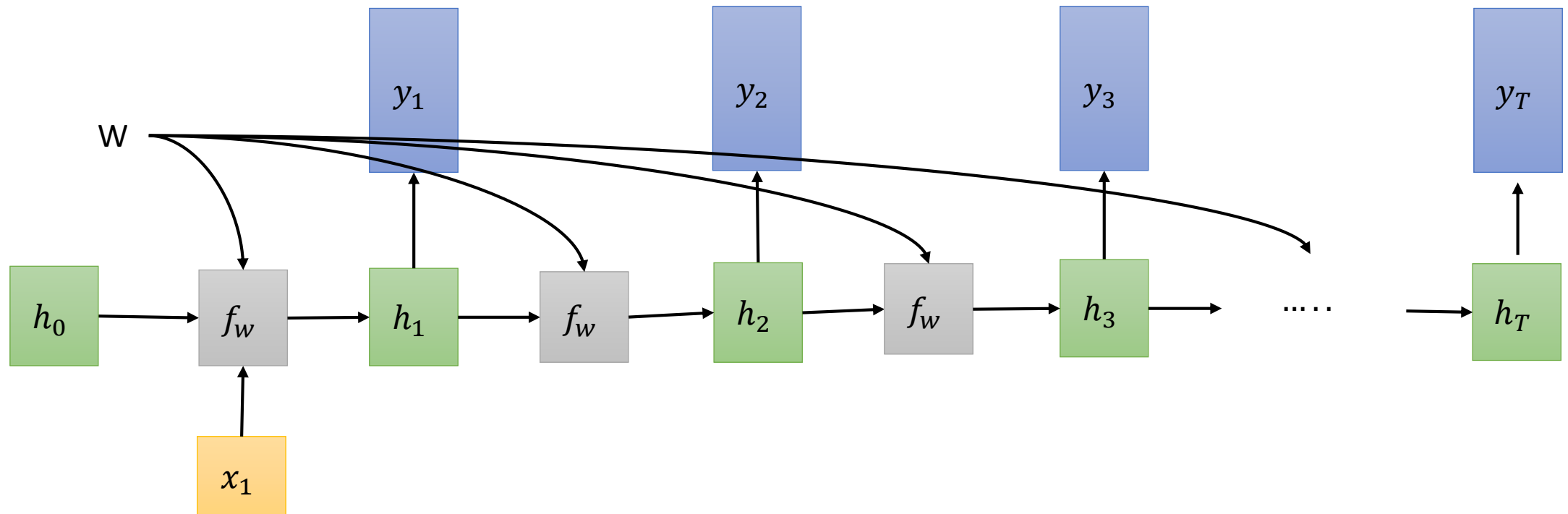
# RNN: computation graph M2O

- RNN forward pass: Many to one: (semantic judge)



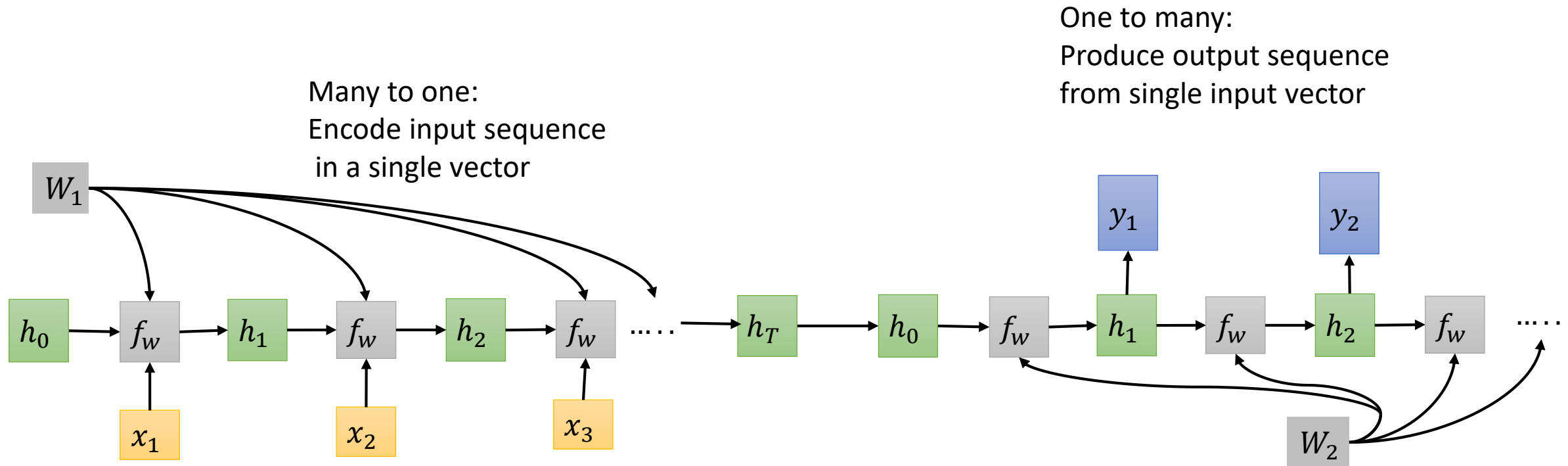
# RNN: computation graph O2M

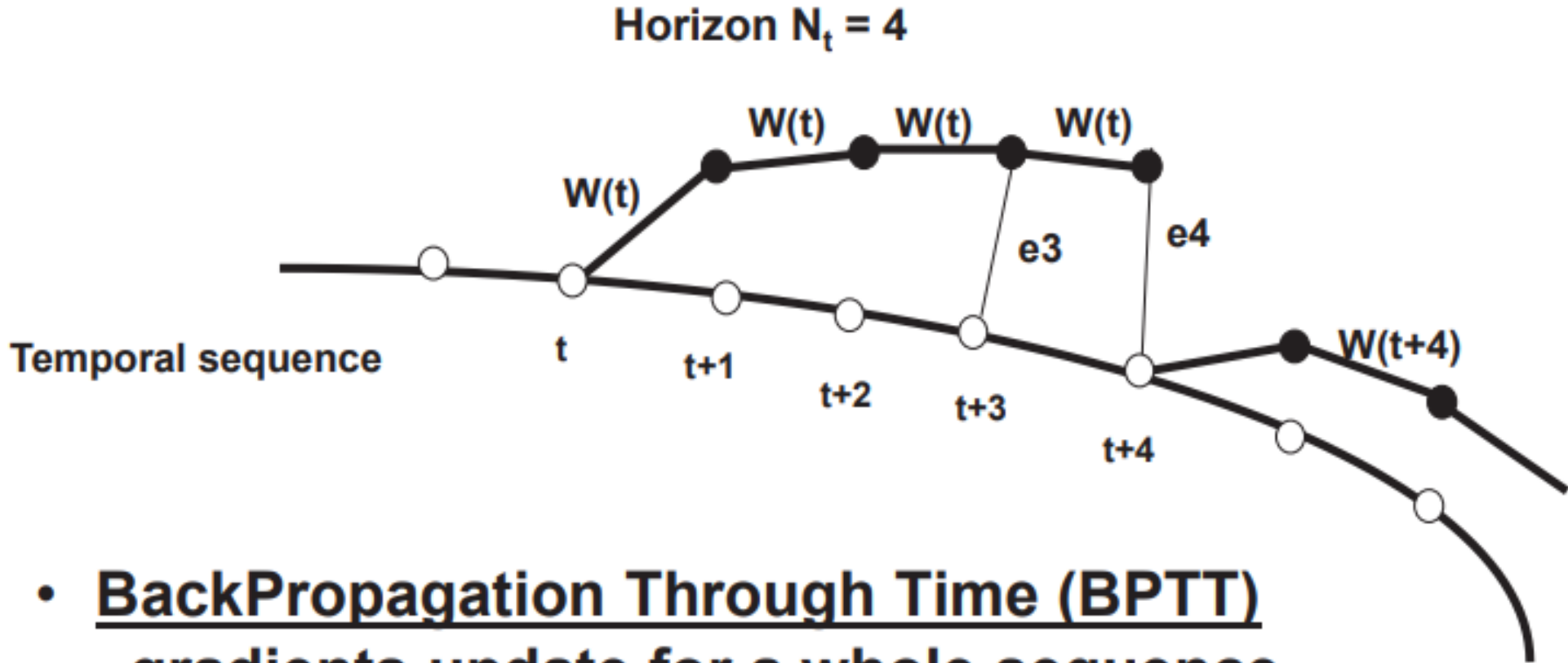
- RNN forward pass: One to many: (captioning)



# Sequence to Sequence: O2M+M2O

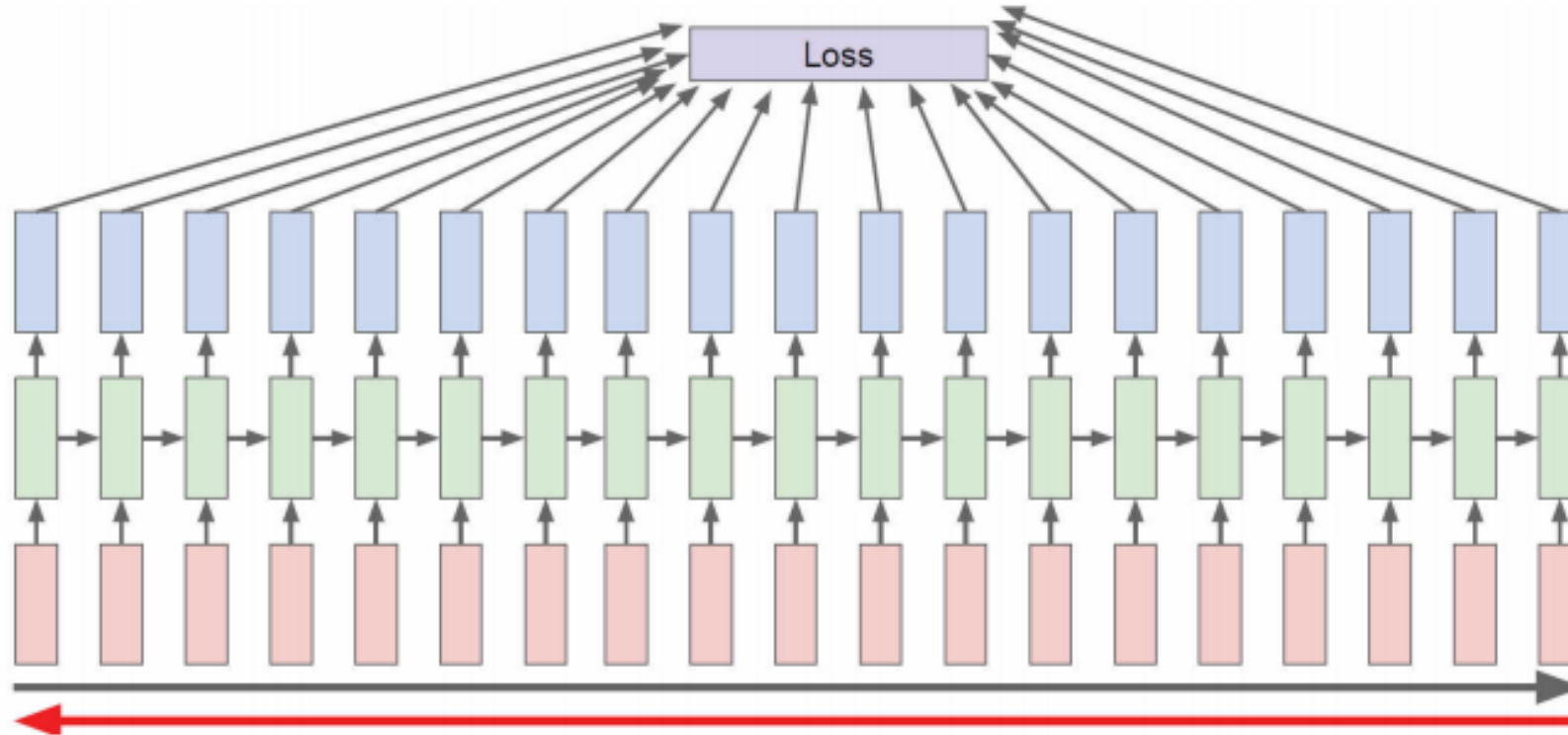
- O2M+M2O





- **BackPropagation Through Time (BPTT)**  
gradients update for a whole sequence
- **or Real Time Recurrent Learning (RTRL)**  
gradients update for each frame in a sequence

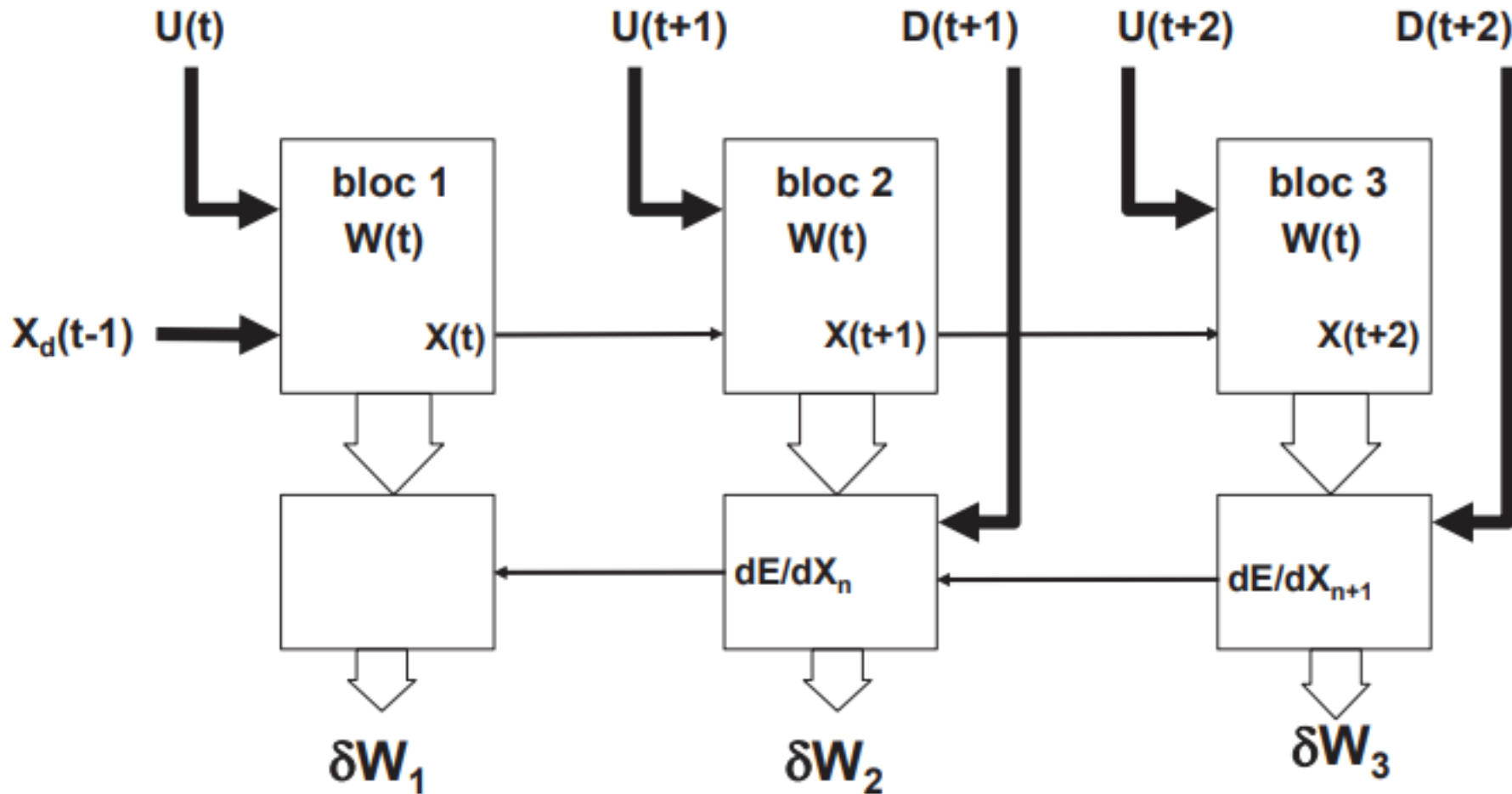
# BackPropagation THROUGH TIME (BPTT)



- **Forward through entire sequence to compute SUM of losses at ALL (or part of) time steps**
- **Then backprop through ENTIRE sequence to compute gradients**

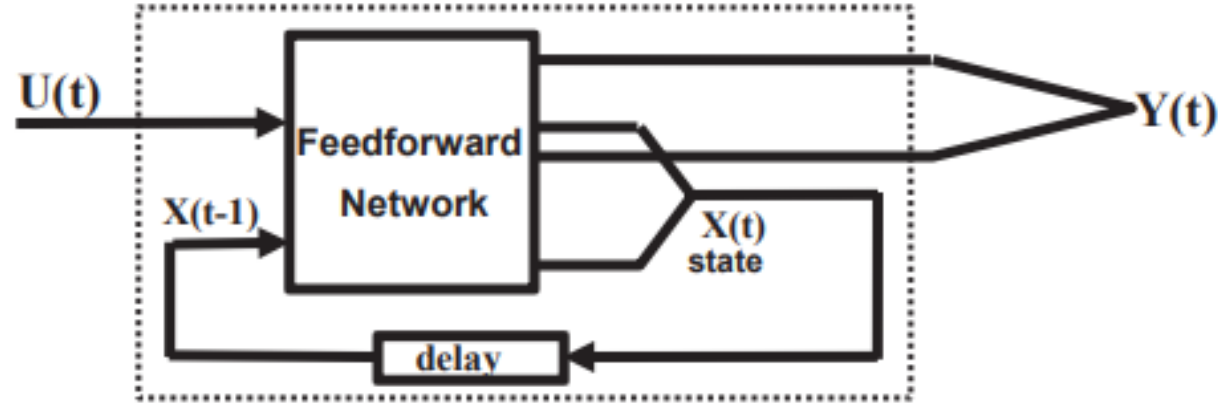


# BPTT computation principle



$$\delta W = \delta W_1 + \delta W_2 + \delta W_3$$

# BPTT algorithm



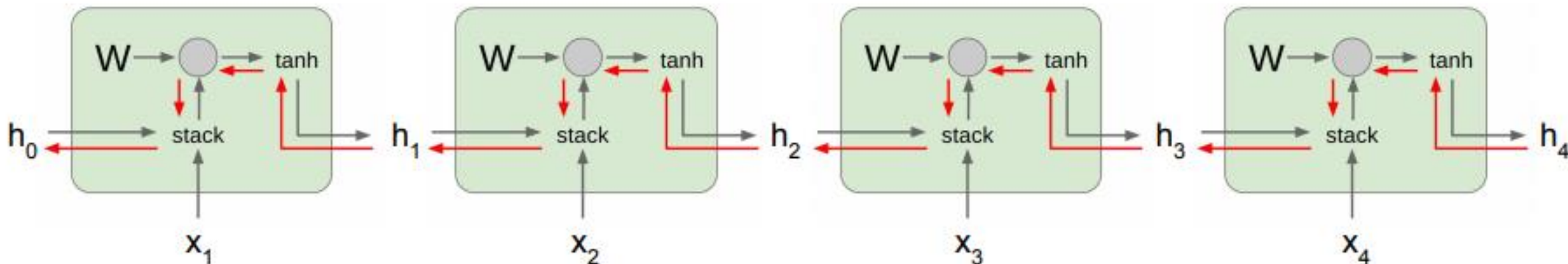
$$W(t+N_t) = W(t) - \lambda \text{grad}_W(E) \text{ avec } E = \sum_{\tau} (Y_{\tau} - D_{\tau})^2$$

$$\frac{\partial E}{\partial W} = \sum_{t=1}^T \frac{\partial E_t}{\partial W} \quad \text{and} \quad \forall t, \frac{\partial E_t}{\partial W} = \frac{\partial E_t}{\partial Y_t} \frac{\partial Y_t}{\partial X_{t-1}} \frac{\partial X_{t-1}}{\partial W} \quad (\text{chain rule})$$

$$\frac{\partial X_t}{\partial W} = \sum_{k=1}^{t-1} \frac{\partial X_t}{\partial X_{t-k}} \frac{\partial X_{t-k}}{\partial W} \quad \frac{\partial X_t}{\partial X_{t-k}} = \prod_{j=1}^k \frac{\partial X_j}{\partial X_{j-1}} \quad \text{Jacobian matrix of the Feedforward net}$$

# Vanilla RNN Gradient Flow

Bengio et al, "Learning long-term dependencies with gradient descent is difficult", IEEE Transactions on Neural Networks, 1994  
Pascanu et al, "On the difficulty of training recurrent neural networks", ICML 2013



Computing gradient of  $h_0$  involves many factors of  $W$  (and repeated tanh)

Largest singular value  $> 1$ :  
**Exploding gradients**

Largest singular value  $< 1$ :  
**Vanishing gradients**

**Gradient clipping:** Scale gradient if its norm is too big

```
grad_norm = np.sum(grad * grad)
if grad_norm > threshold:
    grad *= (threshold / grad_norm)
```

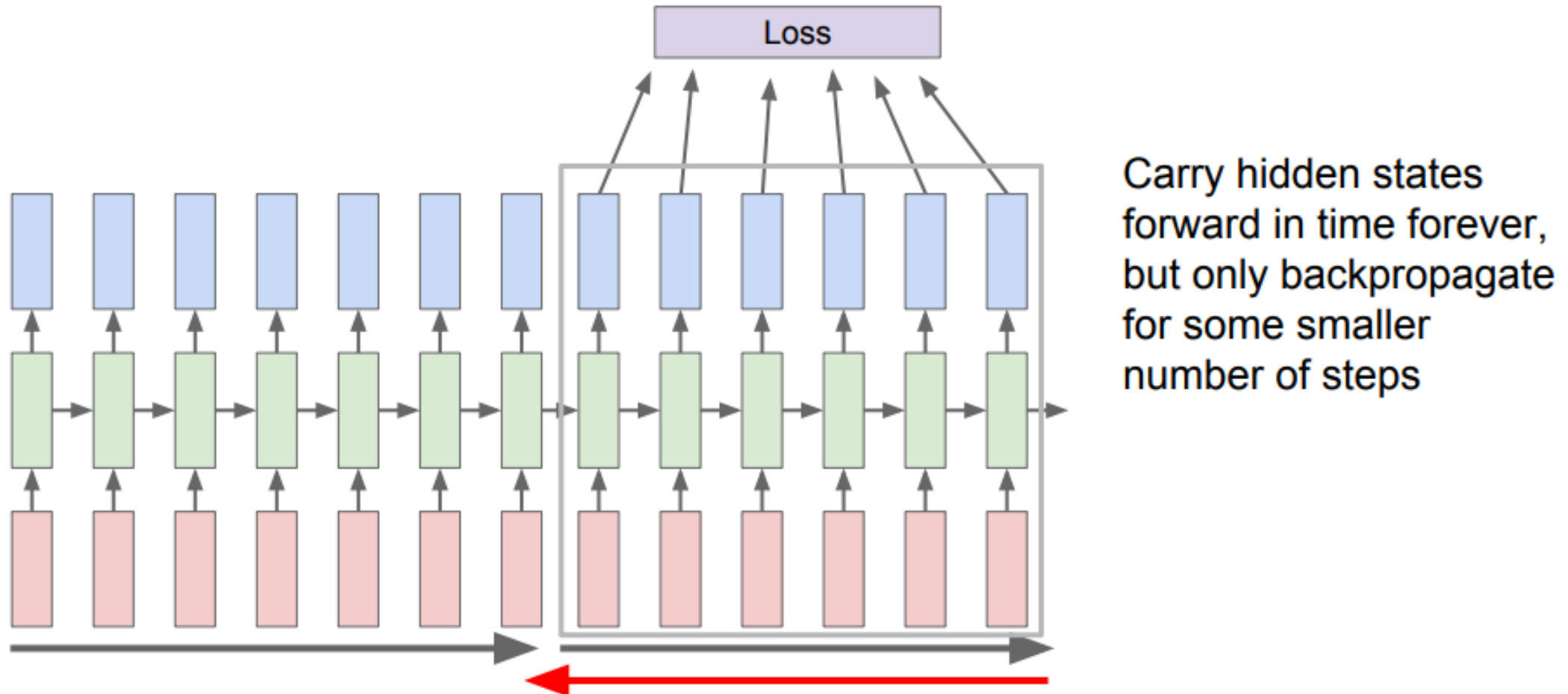
Change architecture!

# Vanishing/exploding gradient problem

- If eigenvalues of Jacobian matrix  $>1$ , then gradients tend to EXPLODE  
→ Learning will never converge.
- Conversely, if eigenvalues of Jacobian matrix then gradients tend to VANISH  
→ Error signals can only affect small time lags  
→ short-term memory.
- Possible solutions for exploding gradient: **CLIPPING** trick (limited values in an array, see `numpy.clip`), truncated.
- Possible solutions for vanishing gradient:
  - use ReLU instead of tanh
  - **change what is inside the RNN!**

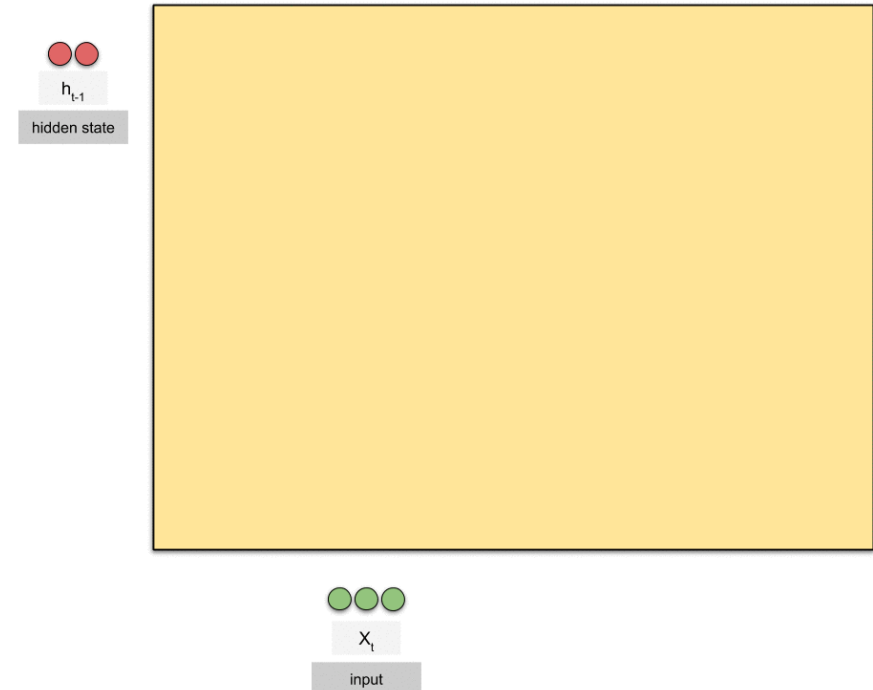
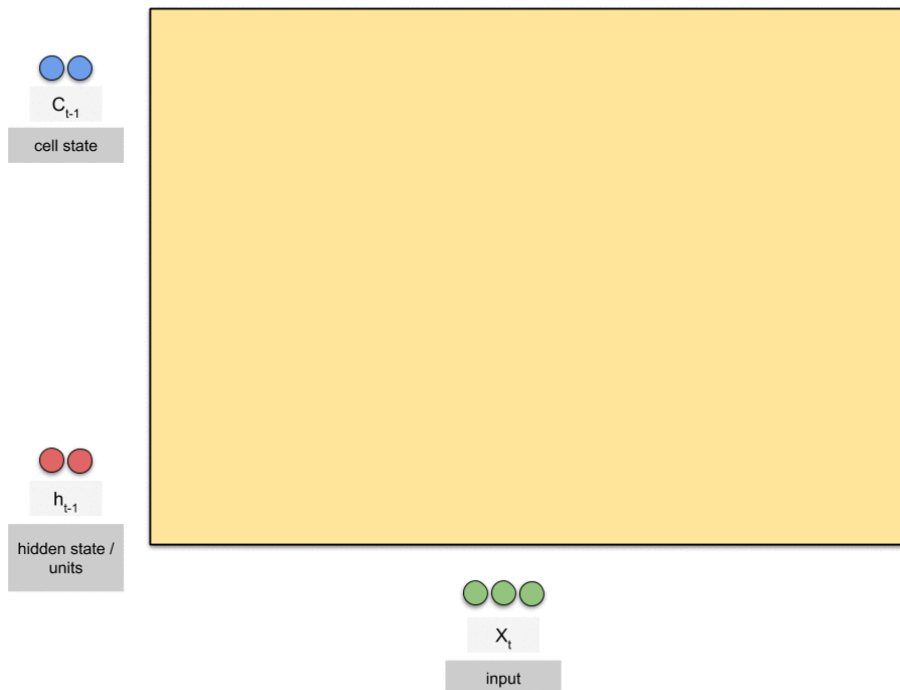
Recommended code to read for better understand this slide: <https://gist.github.com/karpathy/d4dee566867f8291f086>

## Truncated Backpropagation through time



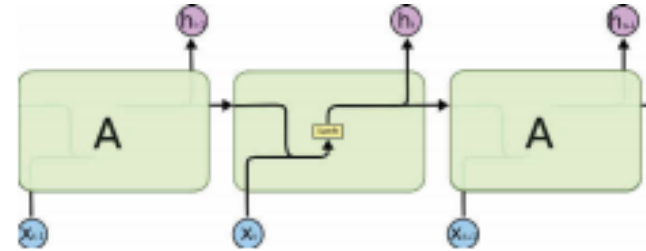
# Modern ways in RNN

- To solve the gradient issue, a better design of the architecture is important.
- Here are two successful networks that improve the performance by the way it design the gradient flow
  - Long Short-Term Memory (LSTM)
  - Gated Recurrent Unit (GRU)

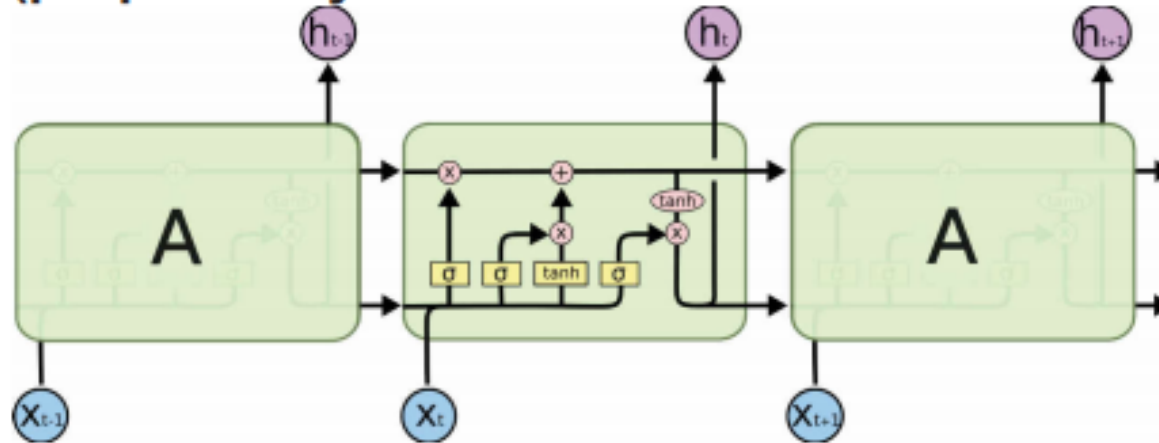




**Problem of *standard* RNNs =  
no actual LONG-TERM memory**



**LSTM = RNN variant for solving this issue**  
(proposed by Hochreiter & Schmidhuber in 1997)



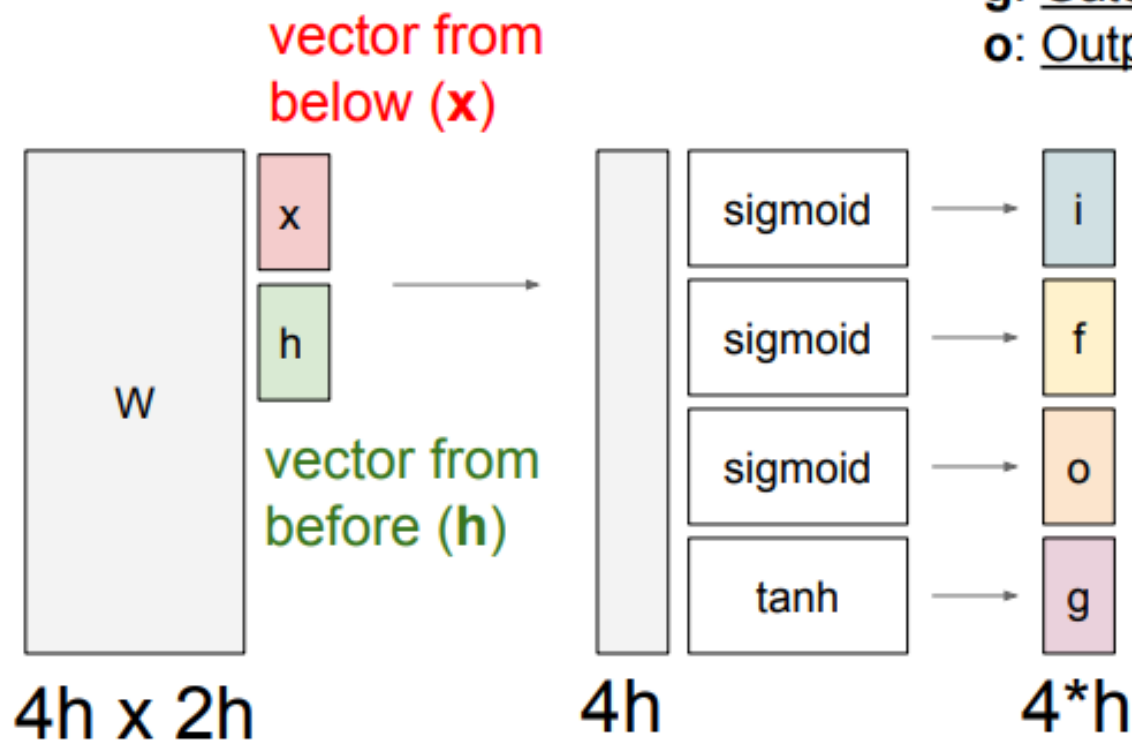
[Figures from <https://colah.github.io/posts/2015-08-Understanding-LSTMs/>]

- **Key idea = use “gates” that modulate respective influences of input and memory**

# Long Short Term Memory (LSTM)

[Hochreiter et al., 1997]

**f**: Forget gate, Whether to erase cell  
**i**: Input gate, whether to write to cell  
**g**: Gate gate (?), How much to write to cell  
**o**: Output gate, How much to reveal cell

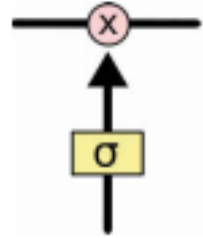


$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}$$

$$c_t = f \odot c_{t-1} + i \odot g$$

$$h_t = o \odot \tanh(c_t)$$

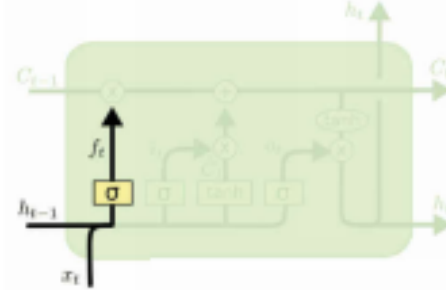




Gate = pointwise multiplication by  $\sigma$  in  $]0;1[$   
 → modulate between “*let nothing through*”  
 and “*let everything through*”

- FORGET gate

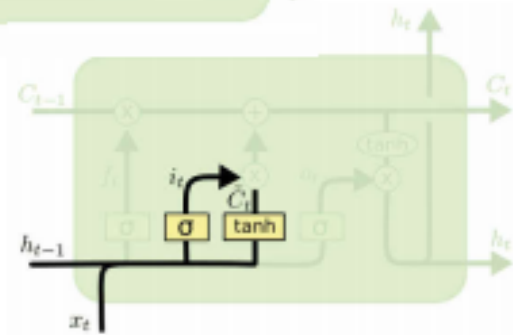
$$f_t = \sigma(W_f \cdot [h_{t-1}, x_t] + b_f)$$



- INPUT gate

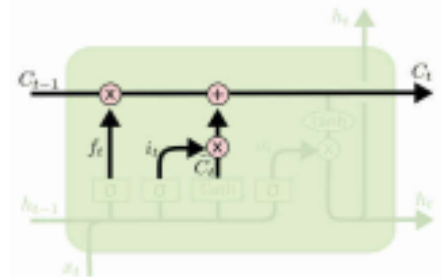
$$i_t = \sigma(W_i \cdot [h_{t-1}, x_t] + b_i)$$

$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] + b_C)$$



→ next state = mix between  
 pure memory or pure new

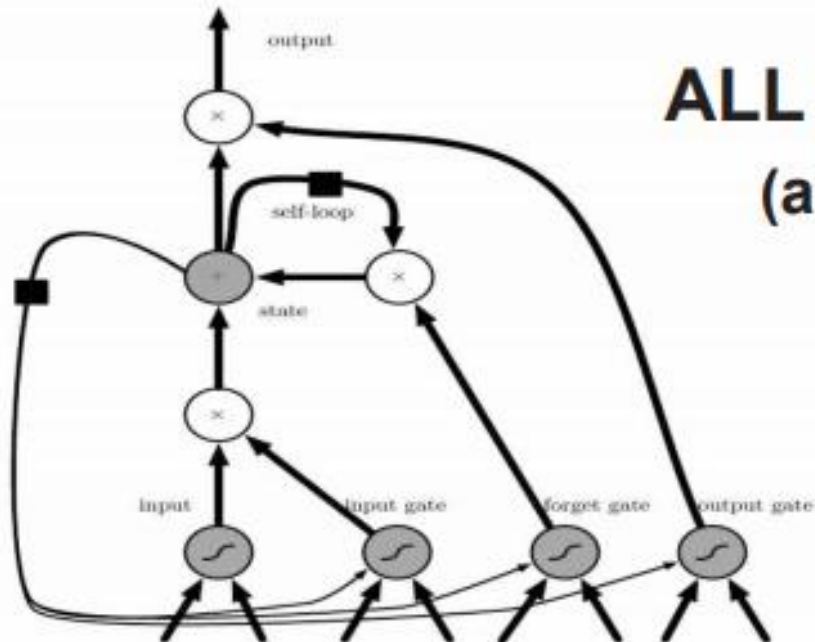
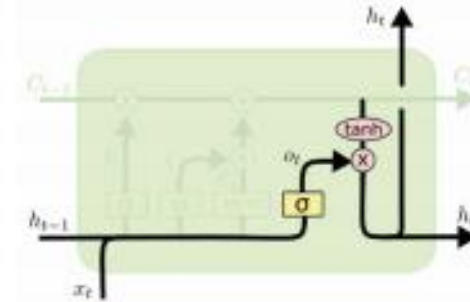
$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$



- OUTPUT gate**

$$o_t = \sigma(W_o [h_{t-1}, x_t] + b_o)$$

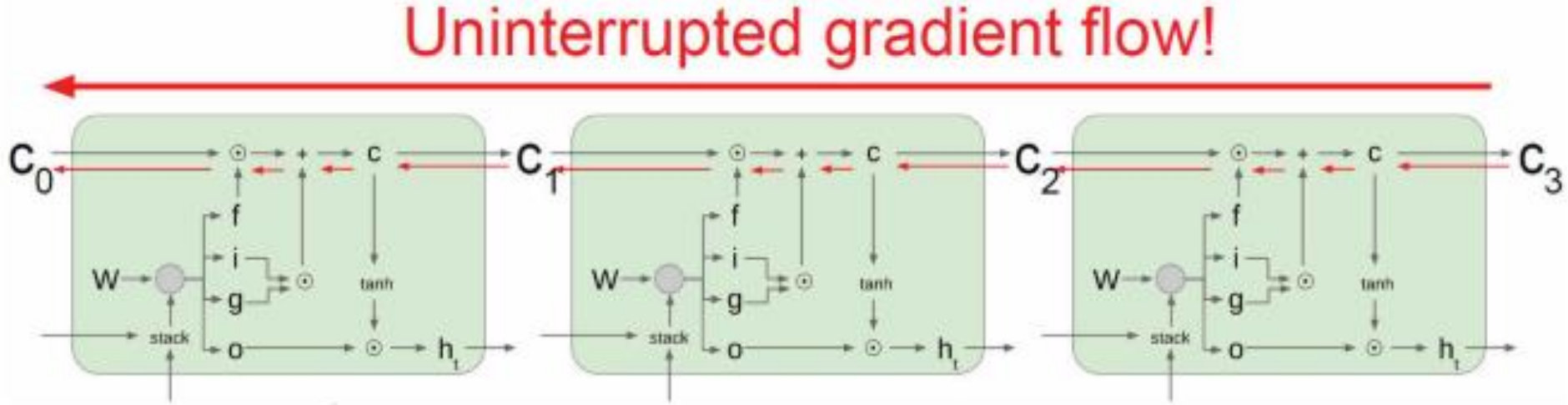
$$h_t = o_t * \tanh(C_t)$$



**ALL weights  $w_f$ ,  $w_i$ ,  $w_c$  and  $w_o$  (and biases) are LEARNED**

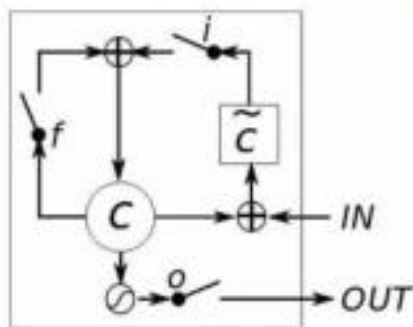
[Figure from *Deep Learning* book by I. Goodfellow, Y. Bengio & A. Courville]

# Why LSTM avoids vanishing gradients?



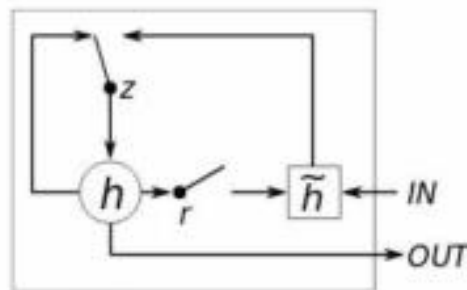
# Gated Recurrent Unit (GRU)

**Simplified variant of LSTM, with only 2 gates:  
a RESET gate & an UPDATE gate**  
(proposed by Cho, et al. in 2014)



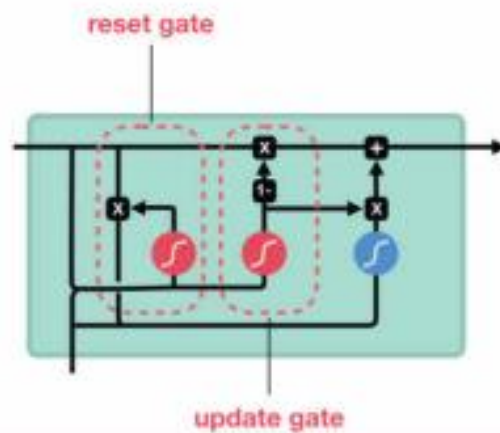
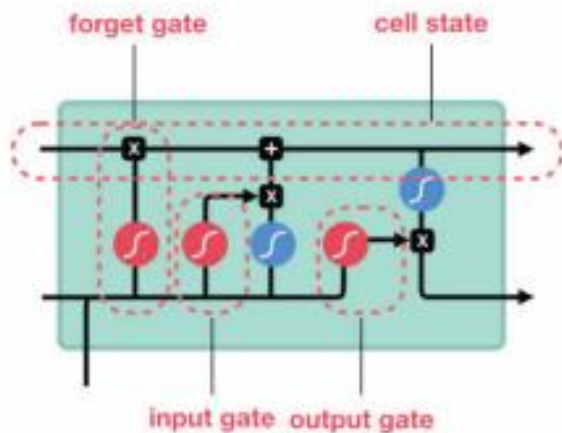
(a) Long Short-Term Memory

LSTM



(b) Gated Recurrent Unit

GRU



GRU [Learning phrase representations using rnn encoder-decoder for statistical machine translation, Cho et al. 2014]

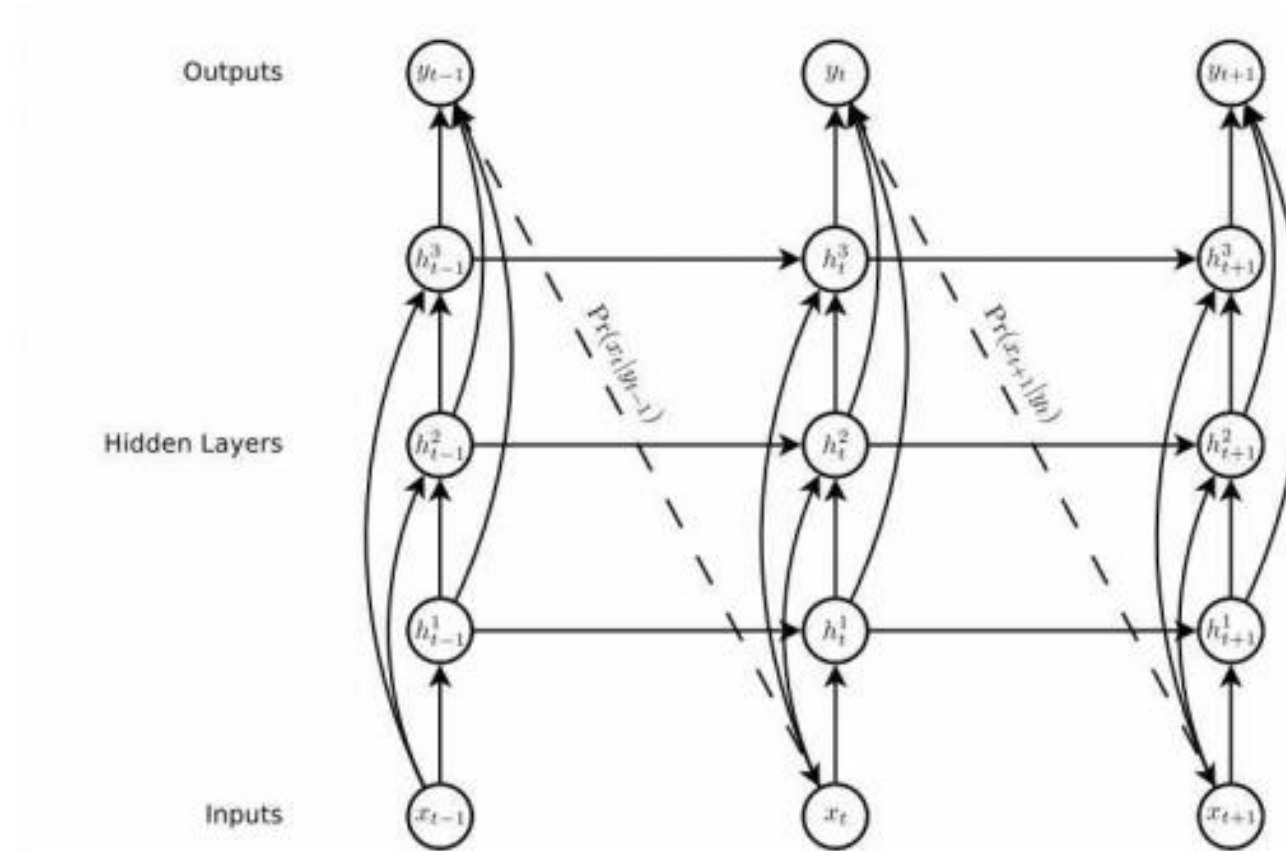
$$r_t = \sigma(W_{xr}x_t + W_{hr}h_{t-1} + b_r)$$

$$z_t = \sigma(W_{xz}x_t + W_{hz}h_{t-1} + b_z)$$

$$\tilde{h}_t = \tanh(W_{xh}x_t + W_{hh}(r_t \odot h_{t-1}) + b_h)$$

$$h_t = z_t \odot h_{t-1} + (1 - z_t) \odot \tilde{h}_t$$

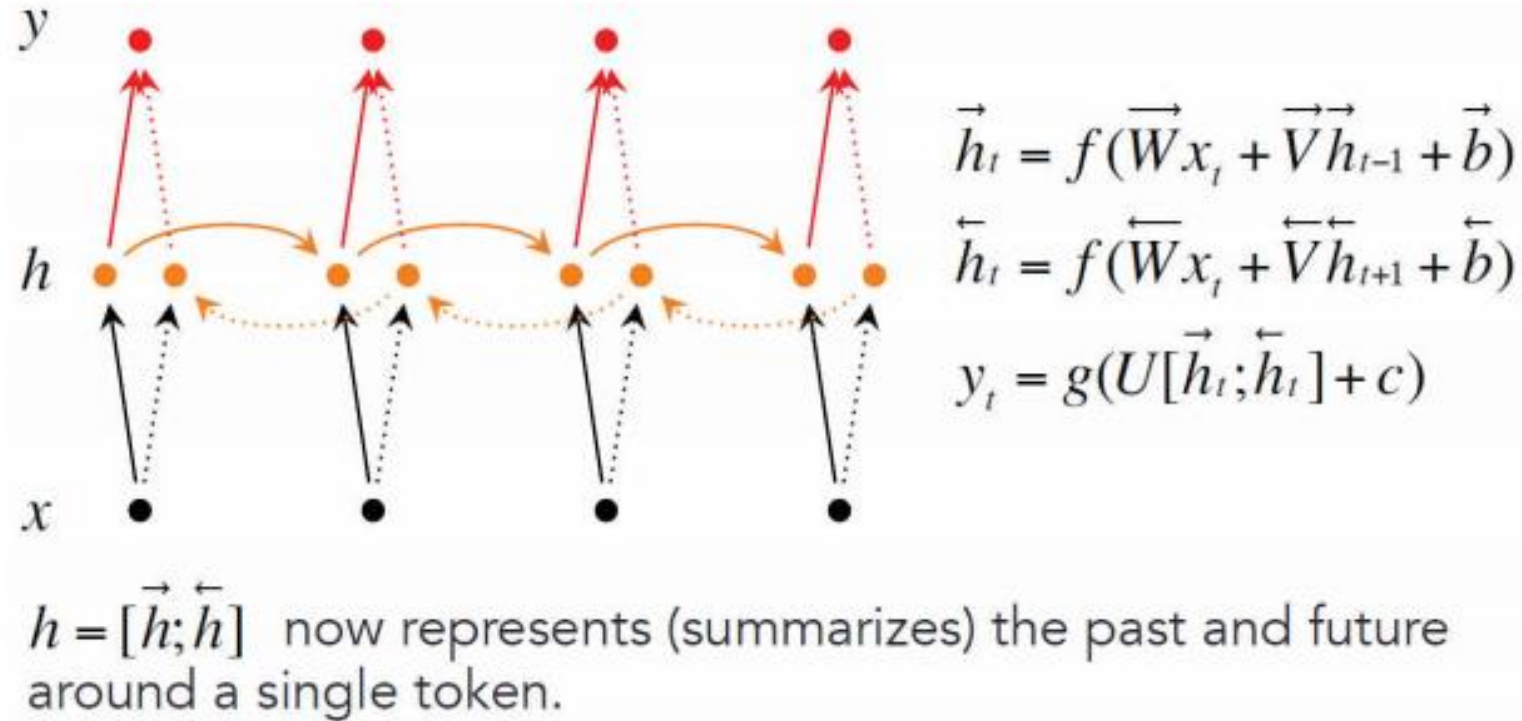
# Deeper RNN



**Several RNNs stacked (like layers in MLP)**

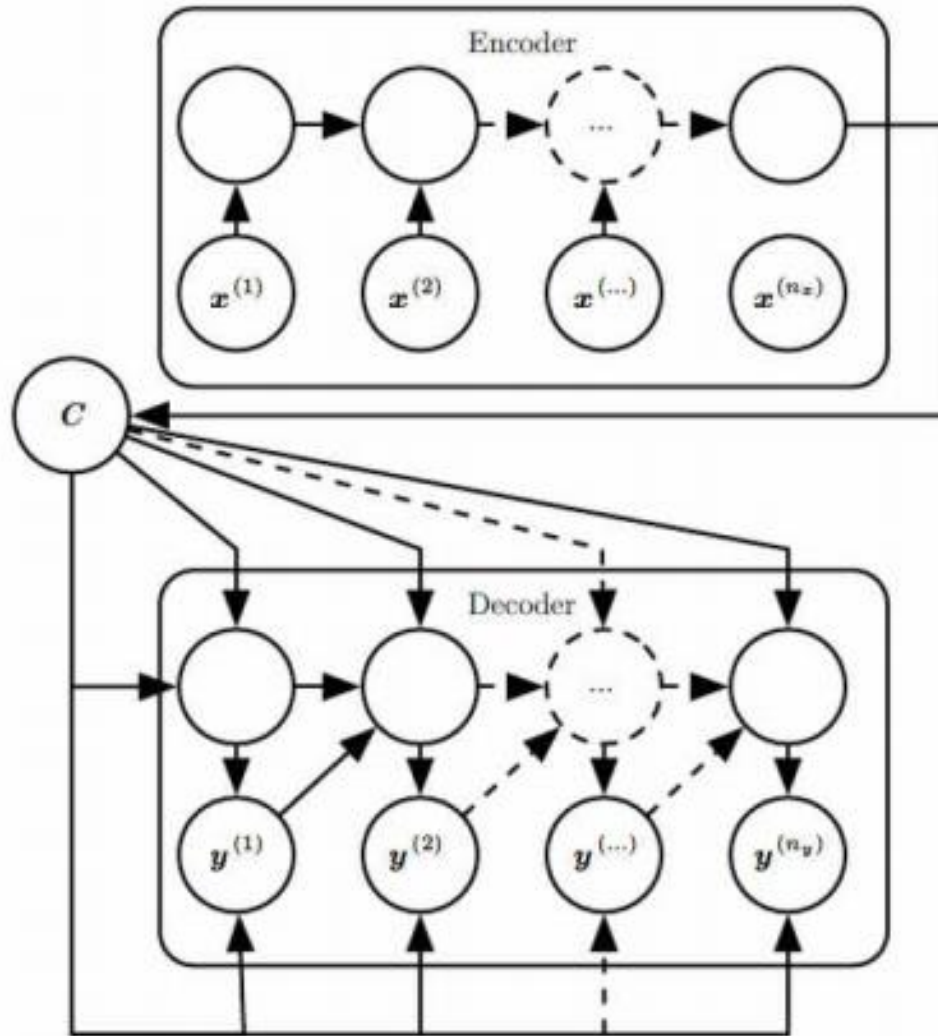


# Bi-directional RNNs



**(e.g. for offline classification of sequence of words)**

# Encoder-decoder RNN



We will play it in practical lesson

# Recommended reading: RNN variants

- [LSTM: A Search Space Odyssey, Greff et al., 2015] :
  - they play around the LSTM equations, swap out the non linearities at one point, do we need tanh, this paper made a lot of experiences in playing around different design
  - Conclusion is there is no significant difference.
- [An Empirical Exploration of Recurrent Network Architectures, Jozefowicz et al., 2015]:
  - Search over very large number of random RNN architecture, randomly permute these equations to see if there is a better one
  - Conclusion: No significant improvement with one specific version



## Image Captioning

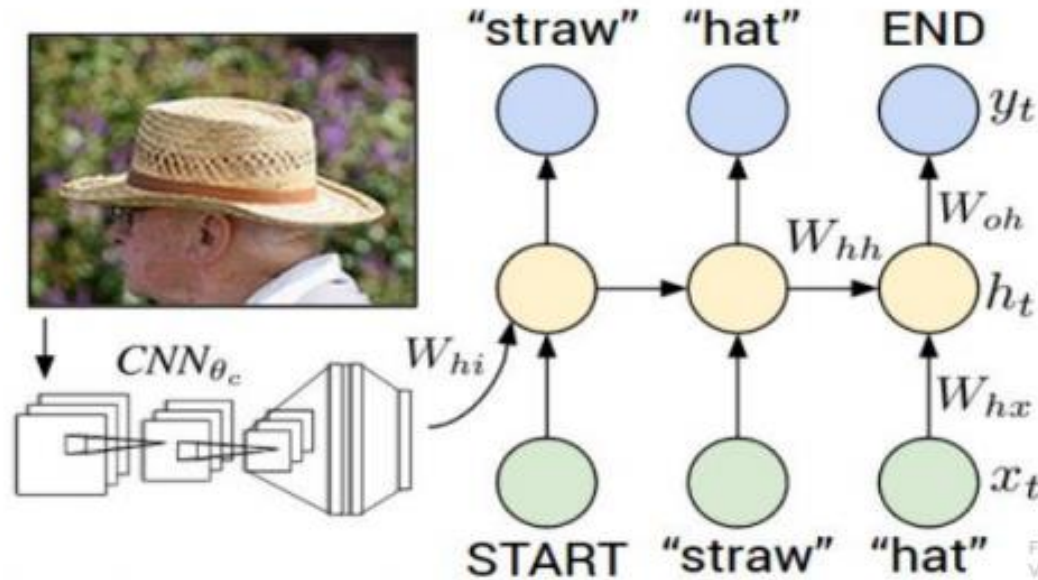


Figure from Karpathy et al., "Deep Visual-Semantic Alignments for Generating Image Descriptions", CVPR 2015; figure copyright IEEE, 2015.  
Reproduced for educational purposes.

Explain Images with Multimodal Recurrent Neural Networks, Mao et al.

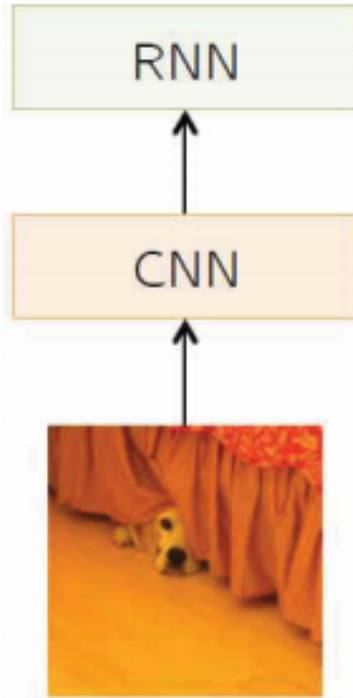
Deep Visual-Semantic Alignments for Generating Image Descriptions, Karpathy and Fei-Fei

Show and Tell: A Neural Image Caption Generator, Vinyals et al.

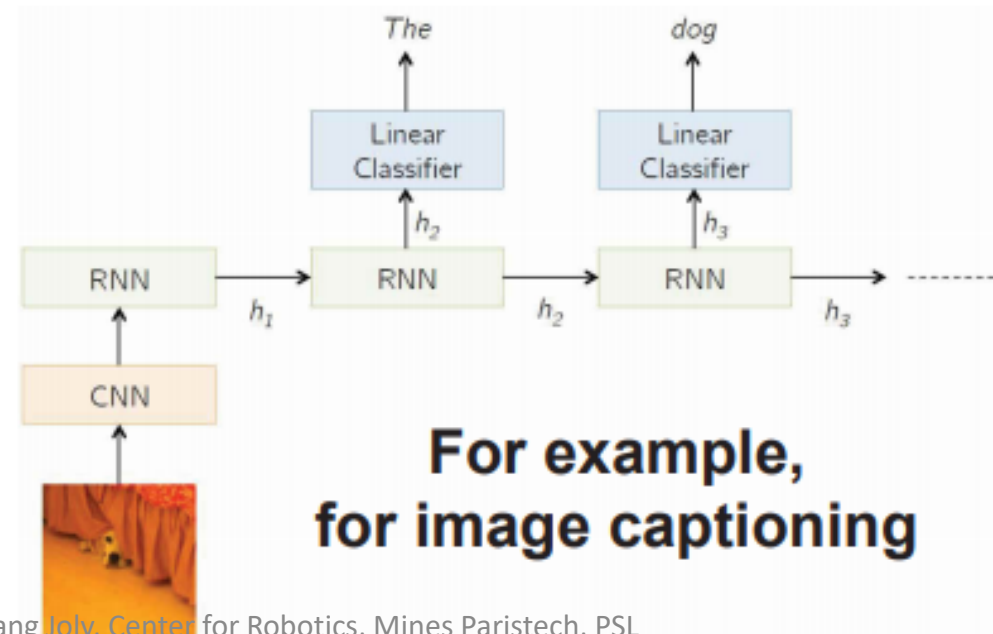
Long-term Recurrent Convolutional Networks for Visual Recognition and Description, Donahue et al.

Learning a Recurrent Visual Representation for Image Caption Generation, Chen and Zitnick

# Application mixed vision and text

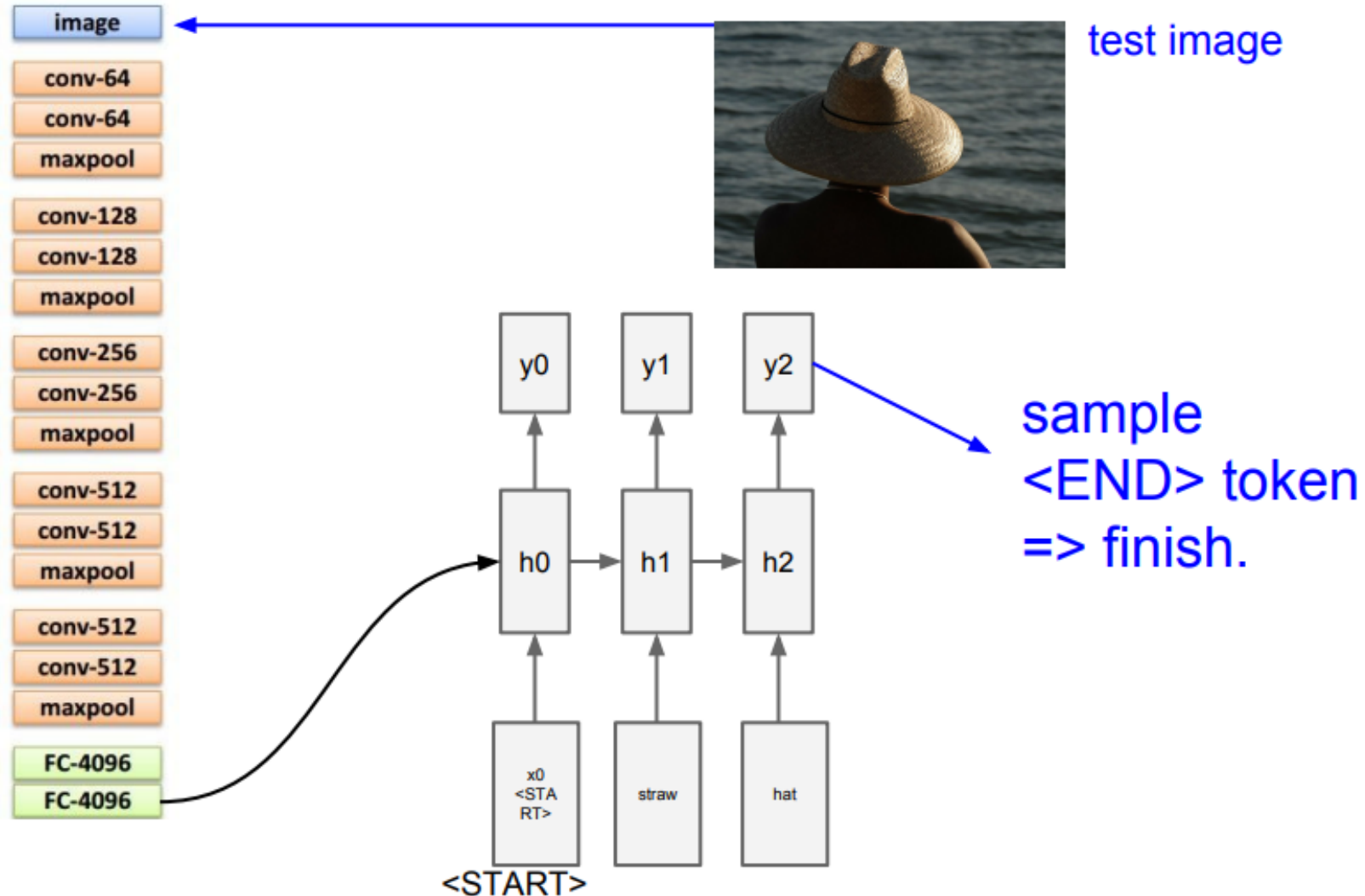


**Input into RNN the features from last convolutional layer**



**For example,  
for image captioning**

# Image captioning



## Image Captioning: Failure Cases

Captions generated using [neuraltalk2](#)  
All images are [CC0 Public domain](#): fur  
coat, handstand, spider web, baseball



*A woman is holding a cat in her hand*



*A person holding a computer mouse on a desk*



*A woman standing on a beach holding a surfboard*



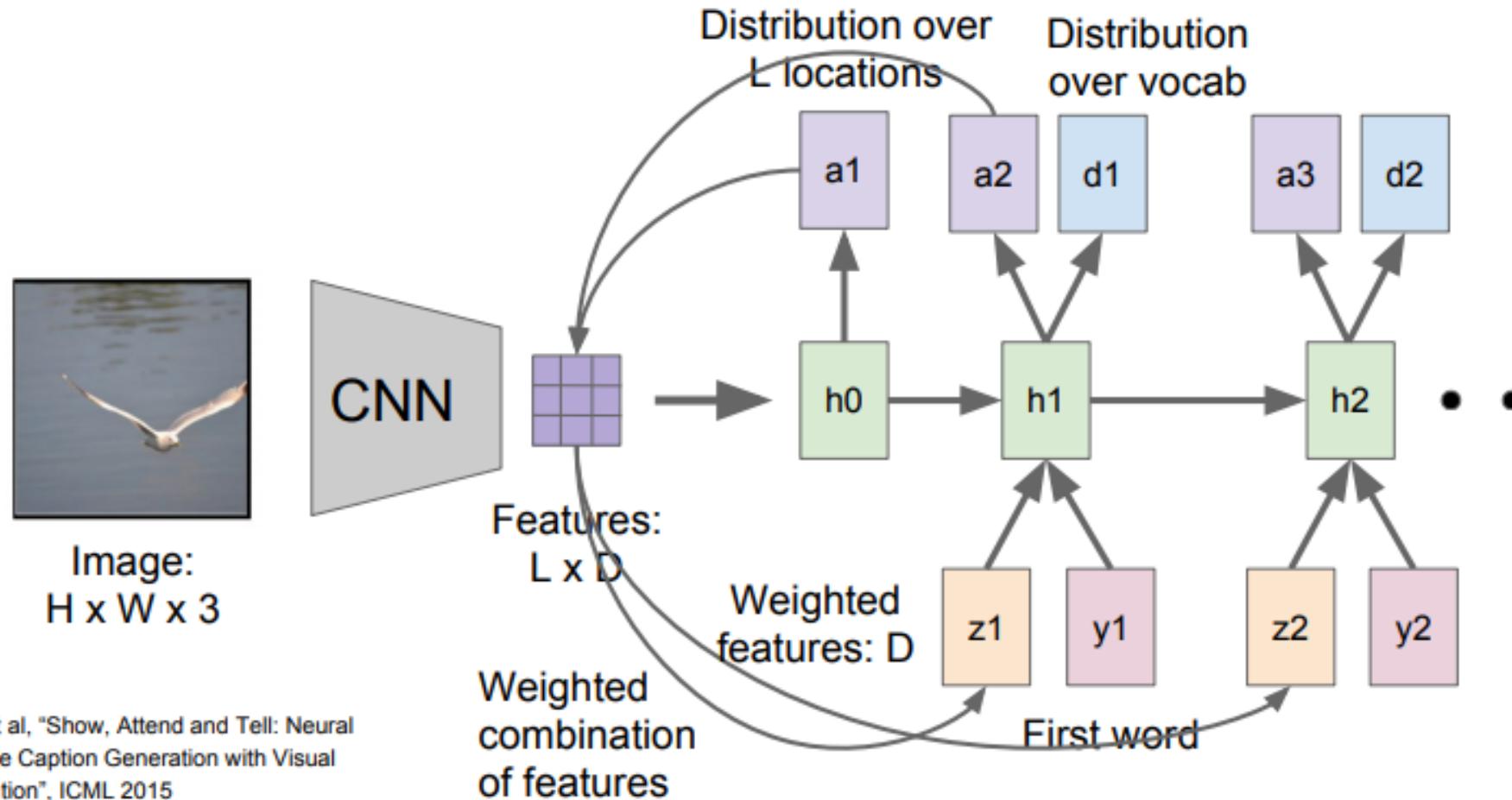
*A bird is perched on a tree branch*



*A man in a baseball uniform throwing a ball*



# Image captioning with attention



Xu et al, "Show, Attend and Tell: Neural Image Caption Generation with Visual Attention", ICML 2015

# Image captioning with attention



A woman is throwing a frisbee in a park.



A dog is standing on a hardwood floor.



A stop sign is on a road with a mountain in the background.



A little girl sitting on a bed with a teddy bear.



A group of people sitting on a boat in the water.



A giraffe standing in a forest with trees in the background.

Xu et al, "Show, Attend, and Tell: Neural Image Caption Generation with Visual Attention", ICML 2015

Figure copyright Kelvin Xu, Jimmy Lei Ba, Jamie Kiros, Kyunghyun Cho, Aaron Courville, Ruslan Salakhutdinov, Richard S. Zemel, and Yoshua Bengio, 2015. Reproduced with permission.

# Summary

- RNNs allow a lot of flexibility in architecture design
- Vanilla RNNs are simple but don't work very well
- Common to use LSTM or GRU: their additive interactions improve gradient flow
- Backward flow of gradients in RNN can explode or vanish
- Exploding is controlled with gradient clipping
- Vanishing is controlled with additive interactions (LSTM)
- Better/simpler architectures are a hot topic of current research
- Better understanding (both theoretical and empirical) is needed.