

Session 3

Recurrent Neural Network

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PSL Acknowledgements

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 - Fei-Fei Li + J.Johnson + S.Yeung: slides on "Recurrent Neural Networks" from the "Convolutional Neural Networks for Visual Recognition" course at Stanford

http://cs231n.stanford.edu/slides/2019/cs231n 2019 lecture10.pdf

 Yingyu Liang: slides on "Recurrent Neural Networks" from the "Deep Learning Basics" course at Princeton

https://www.cs.princeton.edu/courses/archive/spring16/cos495/slides/DL_lecture9_RNN.pdf

- Arun Mallya: slides "Introduction to RNNs" from the "Trends in Deep Learning and Recognition" course of Svetlana LAZEBNIK at University of Illinois at Urbana-Champaign
- Tingwu Wang: slides on "Recurrent Neural Network" for a course at University of Toronto

https://www.cs.toronto.edu/%7Ftingwuwang/rnp_tutorial.pdf

 Christopher Olah: online tutorial "Understanding LSTM Networks" https://colah.github.io/posts/2015-08-Understanding-LSTMs/

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PSL ★ Introduction

- Recurrent neural networks have been an important focus of research and development during the 1990's. → It is much older than ConvNet!
- They are designed to learn sequential or time-varying patterns.
- A recurrent net is a neural network with feedback (closed loop) connections
 - BAM
 - Associative memory networks, A.K.A. Hopfield [John Hopfield, 1982]
 - Boltzmann machine
 - Recurrent backpropagation nets [Hecht-Nielsen, 1990].
- From the computational perspective, a dynamic neural network that contains the feedback loop may provide more computational advantages than a static neural network, which contains only feed-forward architecture
- Applications: natural language processing (NLP), forecasting, signal processing, and control require the treatment of dynamics associated with the unknown model.

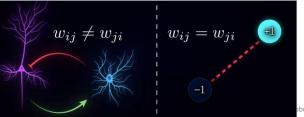
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PSL★ Hopfield network (1)

- It is a foundational model of associative memory that underlies many important ideas in neuroscience and machine learning, such as Boltzmann machines and Dense associative memory
- Neurons are binary units (on or off)
- Unlike biological system with asymmetric connections of the synapse, the Hopfield network simplifies and stabilizes the calculation to symmetric connections.



 $x_i x_j > 0$: agree with each other $x_i x_j < 0$: disagree with each other happiness of $edge_{ij} = w_{ij} x_i x_j$

 $Network\ happiness = \sum_{ij}^{eages} w_{ij} x_i x_j$

Degree of agreement between states and connection weights: goal is to maximin it

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PSL★ Hopfield network (2)

- The energy is the opposite of happiness and the goal becomes to minimize the energy
 - Adjusting weights: sculping energy landscape, creating local minima at memory locations → learning
 - Adjusting states: evolve the system towards local minima by descending along the surface → inference
- The energy of state vector, v, is defined as

$$E(v) = -\left(\sum_{i} s_i^{\nu} b_i + \sum_{i < j} s_i^{\nu} s_j^{\nu} w_{ij}\right)$$

- w_{ij} : weight on the connection between unit I and j.
- s_i^v : given state vector v, 1 if unit j is on and 0 otherwise
- b_i : bias of unit i

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PSL★ Hopfield network (3)

- Inference is like a voting that the neuron are updated based on the voting rule.
- Randomly pick neuron x and calculate the result of voting from all the connection: $h_i = \sum_{j \neq i} w_{ij} x_i$
- Update: $x_i = \begin{cases} +1 & \text{if } h_i > 0 \\ -1 & \text{if } h_i < 0 \end{cases}$
- These gradually decrease the energy → expected to reach the minimum energy (steady state) when no neuron is flipped
- Is this steady state guarantee the global energy descend? Mathematically, it will be when the connection w_{ij} is symmetric

Slide 6

HC0 Cite paper

HsiuWen Chang, 2024-11-22T16:16:27.120

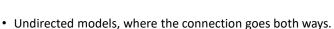


PSL★ Boltzman Machine (1)

• BM is a network of symmetrically connected, neuron-like units that make stochastic decisions about whether to be on or off

$$z_i = b_i + \sum_j s_j w_{ij}$$

$$prob(s_i = 1) = \frac{1}{1 + e^{-z_i}}$$



- · When there is no more change during the update of units in any order, stationary distribution is reached

$$P(v) = \frac{e^{-E(v)}}{\sum_{u} e^{-E(u)}}$$

- Purpose is to optimize the solution: search problem (fixed weights as cost function, find best state) and learning problem (use set of data vectors and learn weights
- · Discover features from datasets composed of binary vectors

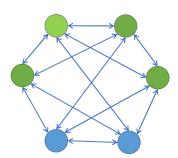
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PSL® Boltzman Machine (2)

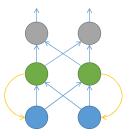
 Restricted Boltzman machine removes the connections between hidden neurons and those between input neurons.

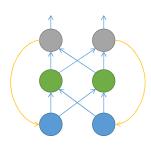




PSL★ Old style of RNN

- Elman introduced feedback from the hidden layer to the context portion of the input layer.
 - This approach pays more attention to the sequence of input values.
- Jordan recurrent neural networks use feedback from the output layer to the context nodes of the input layer and give more emphasis to the sequence of output values.

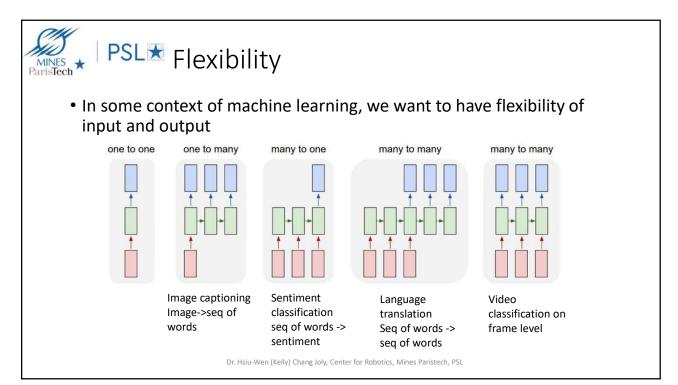




However, these methods did not succeed in bigger data set due to the design of gradient flow

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PSL Gradient flow

- Gradient flow is very important in network
- We already saw a lots in the last section
- Risk to have feed-back connection:
 - stability
 - Controllability
 - Observability

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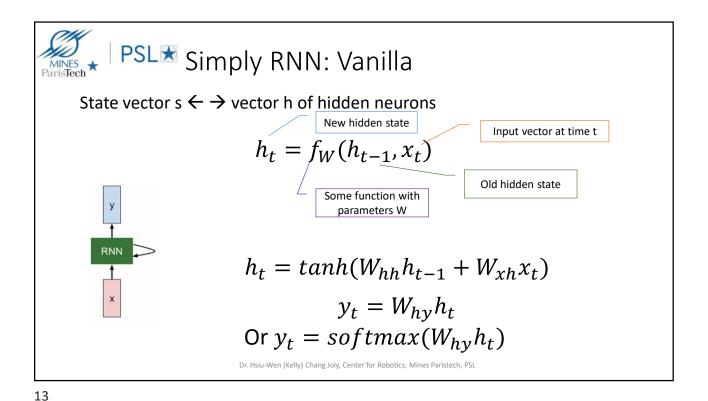
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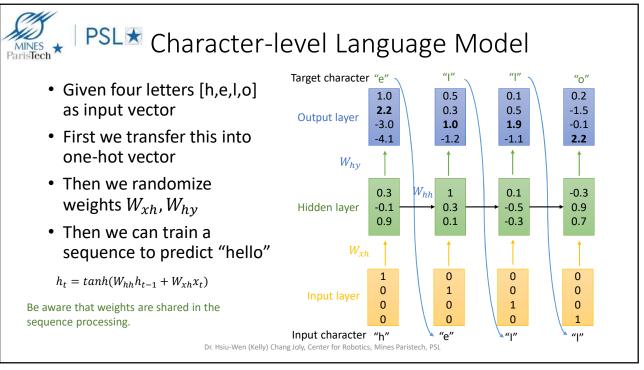


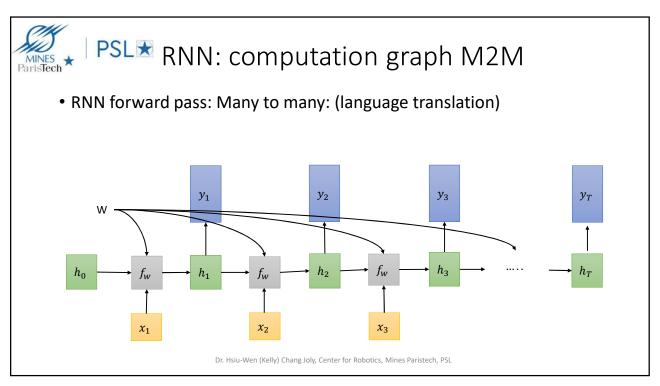
PSL★ Advantages of RNN

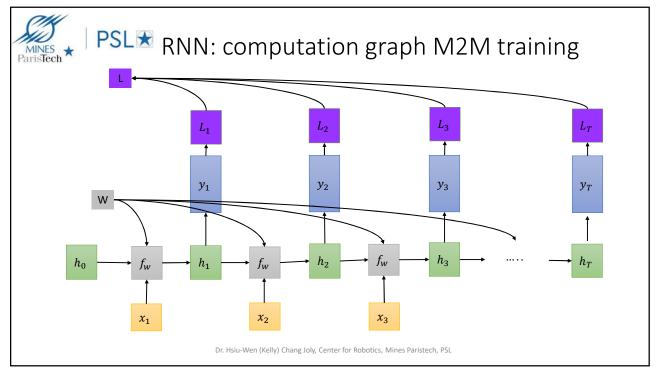
- The hidden state s of the RNN builds a kind of lossy summary of the past
- RNN totally adapted to processing SEQUENTIAL data (same computation formula applied at each time step, but modulated by the evolving "memory" contained in state s)
- Universality of RNNs: any function computable by a Turing Machine can be computed by a finite-size RNN (Siegelmann and Sontag, 1995)

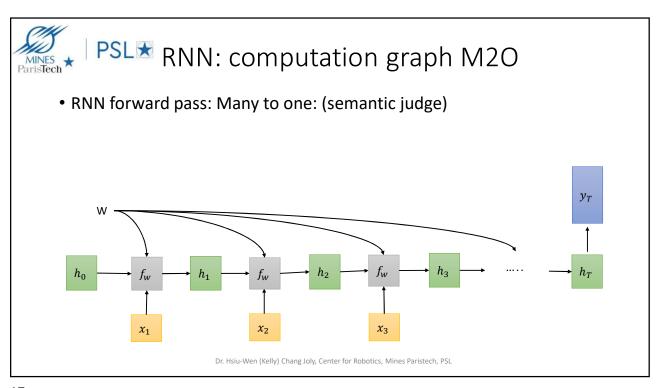
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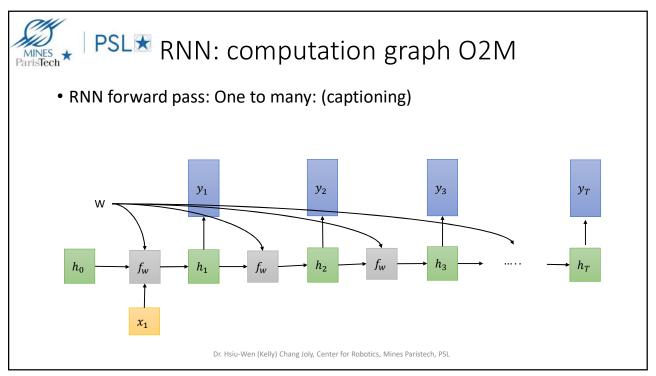


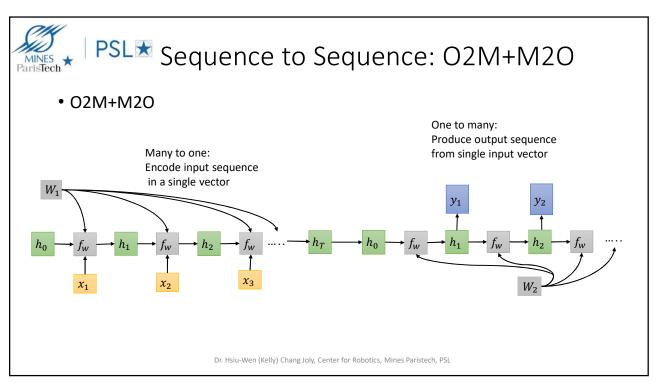














- Applying backpropagation in RNNs is called backpropagation through time [Werbos.1990].
- This procedure requires us to expand (or unroll) the computational graph of an RNN one time step at a time.
- The unrolled RNN is essentially a feedforward neural network with the special property that the same parameters are repeated throughout the unrolled network, appearing at each time step.
- Then, like in feedforward neural network, we apply the chain rule to backpropagate gradients through the unrolled net.
- The gradient with respect to each parameter must be summed across all places that the parameter occurs in the unrolle d net.
- Handling such weight tying should be familiar from our chapters on convolutional neural networks.

Werbos, P. J. (1990). Backpropagation through time: what it does and how to do it. Proceedings of the IEEE, 78(10), 1550-1560.



PSL★ Analysis of gradients

$$h_t = f(x_t, h_{t-1}, w_h)$$

o_t = $g(h_t, w_o)$

- where f and g are transformations of the hidden layer and the output layer, respectively.
- · Hence, we have a chain of values that depend on each other via recurrent computation

$$\{..., (x_{t-1}, h_{t-1}, o_{t-1}), (x_t, h_t, o_t), ...\}$$

The forward pass of this model is to loop through the (x_t, h_t, o_t) triples one time step at a time. The discrepancy between output o_t and the desired target y_t is then evaluated by an objective function across all the T time steps

$$L(x_1, ..., x_T, y_1, ..., y_T, w_h, w_o) = \frac{1}{T} \sum_{t=1}^{T} l(y_t, o_t)$$

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PSL ★ Analysis of gradients

 Here are the tricky way to derive the gradients regarding the parameters w_h of the objective function L.

$$\frac{\partial L}{\partial w_h} = \frac{1}{T} \sum_{t=1_T}^T \frac{\partial l(y_t, o_t)}{\partial w_h}$$

$$= \frac{1}{T} \sum_{t=1}^T \frac{\partial l(y_t, o_t)}{\partial o_t} \frac{\partial (o_t)}{\partial w_h} = \frac{1}{T} \sum_{t=1}^T \frac{\partial l(y_t, o_t)}{\partial o_t} \frac{\partial g(h_t, w_o)}{\partial h_t} \frac{\partial h_t}{\partial w_h}$$

• The third term in this equation is tricky because the computation of h_t depends on both h_{t-1} , w_h where computation of h_{t-1} also depends on w_h . Applying total differential of $df(x,y) = f_x dx + f_y dy$, we have: $\frac{\partial h_t}{\partial w_h} = \frac{\partial f(x_t,h_{t-1},w_h)}{\partial w_h} + \frac{\partial f(x_t,h_{t-1},w_h)}{\partial h_{t-1}} \frac{\partial h_{t-1}}{\partial w_h}$

$$\frac{\partial h_t}{\partial w_h} = \frac{\partial f(x_t, h_{t-1}, w_h)}{\partial w_h} + \frac{\partial f(x_t, h_{t-1}, w_h)}{\partial h_{t-1}} \frac{\partial h_{t-1}}{\partial w_h}$$



PSL Analysis of gradients

$$\frac{\partial h_t}{\partial w_h} = \frac{\partial f(x_t, h_{t-1}, w_h)}{\partial w_h} + \sum_{i=1}^{t-1} \left(\prod_{j=i+1}^{t} \frac{\partial f(x_j, h_{j-1}, w_h)}{\partial h_{j-1}} \right) \frac{\partial f(x_i, h_{i-1}, w_h)}{\partial w_h}$$

- While we can use the chain rule to compute $\frac{\partial h_t}{\partial w_h}$ recursively, this chain can get very long whenever t is large.
 - Full computation: very slow and gradients can blow up, since subtle changes in the initial conditions can potentially affect the outcome a lot
 - Truncating time steps [Jaeger, 2002]
 - Randomized Truncation [Tallec and Ollivier, 2017]



Jaeger, H. (2002). Tutorial on training recurrent neural networks, covering BPPT, RTRL, EKF and the echo state network approach. Vol. 5. GMD-Forschungszentrum Informationstechnik Bonn

Tallec, C., & Ollivier, Y. (2017). Unbiasing truncated backpropagation through time. arXiv preprint arXiv:1705.08209.

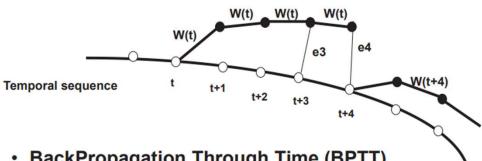
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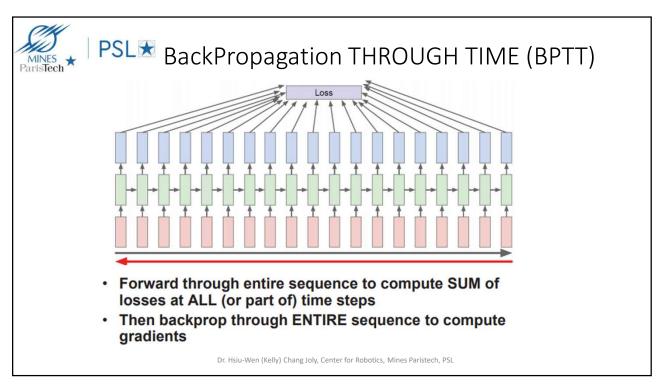


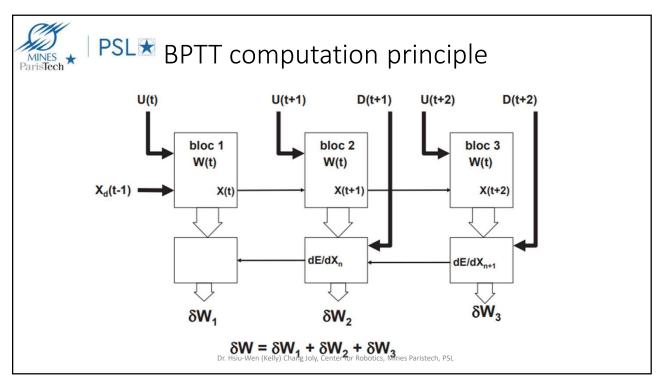
PSL★ RNN training

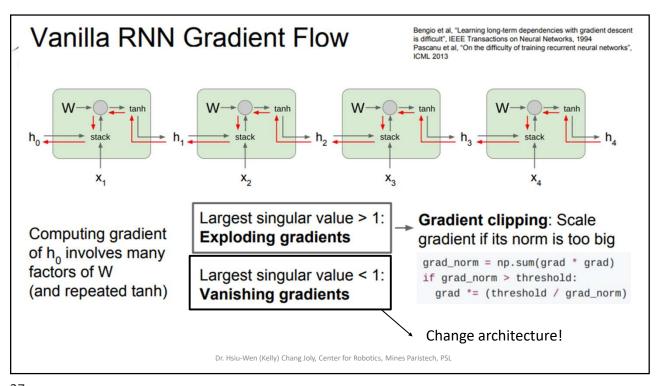
Horizon $N_t = 4$



- BackPropagation Through Time (BPTT) gradients update for a whole sequence
- or Real Time Recurrent Learning (RTRL) gradients update for each frame in a sequence



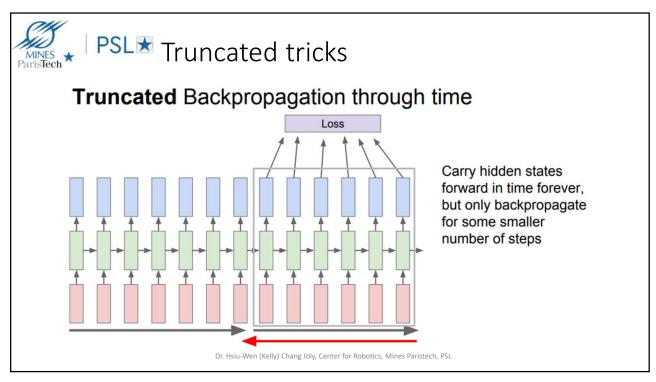


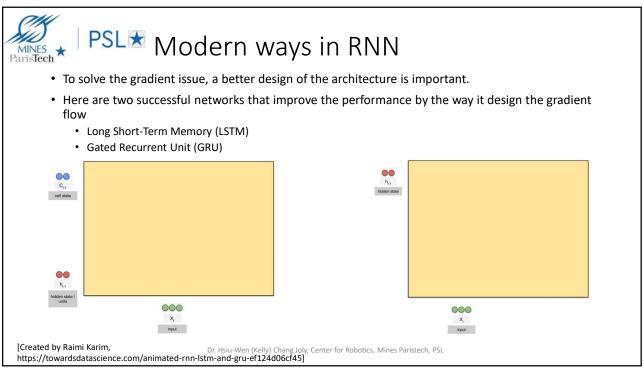




- If eigenvalues of Jacobian matrix >1, then gradients tend to EXPLODE
 → Learning will never converge.
- Conversely, if eigenvalues of Jacobian matrix < 1, then gradients tend to VANISH
 - → Error signals can only affect small time lags
 - →short-term memory.
- Possible solutions for exploding gradient: CLIPPING trick (limited values in an array, see numpy.clip), truncated.
- Possible solutions for vanishing gradient:
 - · use ReLU instead of tanh
 - change what is inside the RNN!

Recommended code to read for better understand this slide: https://gist.github.com/karpathy/d4dee566867f8291f086







PSL™ Long short-term memory

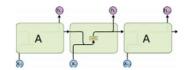
- The term "long short-term memory" comes from the following intuition. Simple recurrent neural networks have long-term memory in the form of weights. The weights change slowly during training, encoding general knowledge about the data. They also have short-term memory in the form of ephemeral activations, which pass from each node to successive nodes. The LSTM model introduces an intermediate type of storage via the memory cell. A memory cell is a composite unit, built from simpler nodes in a specific connectivity pattern, with the novel inclusion of multiplicative nodes
- Gated memory cell is equipped with an internal state and a number of multiplicative gates that determine
 - a given input should impact the internal state (the *input gate*): $i \in [0,1]$
 - Weather the internal state should be flushed to 0 (the forget gate): $f \in [0,1]$
 - Weather the internal state of a given neuron should be allowed to impact the cell's output (the *output* gate): $o \in [0,1]$

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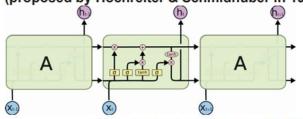
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Problem of standard RNNs = no actual LONG-TERM memory

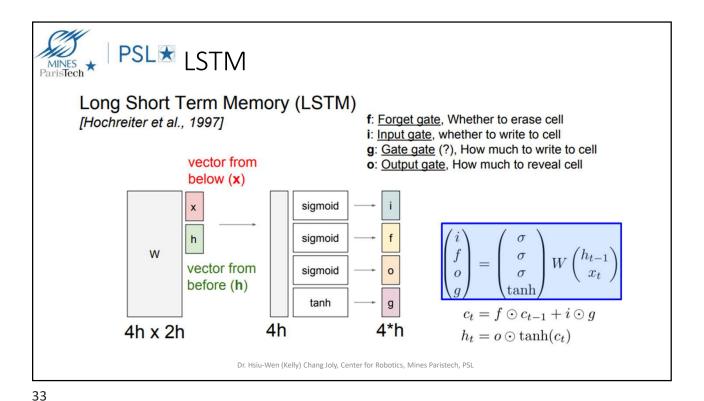


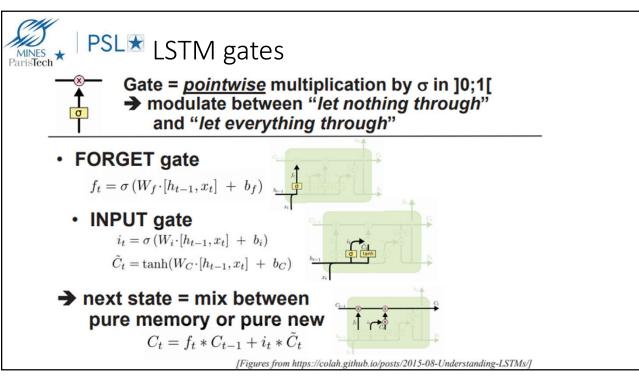
LSTM = RNN variant for solving this issue (proposed by Hochreiter & Schmidhuber in 1997)

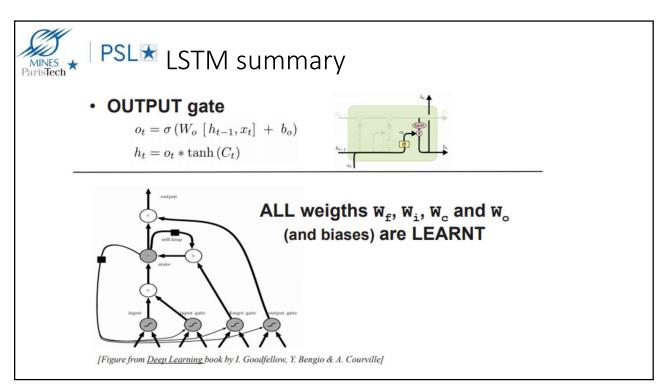


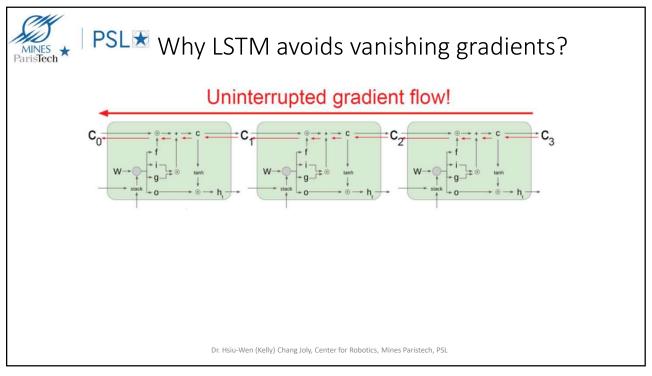
[Figures from https://colah.github.io/posts/2015-08-Understanding-LSTMs/]

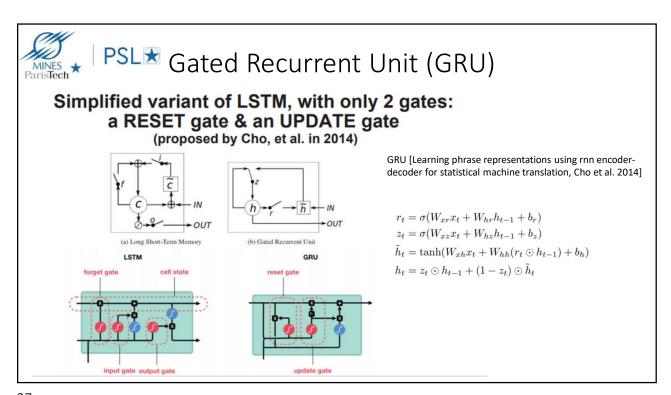
 Key idea = use "gates" that modulate respective influences of input and memory

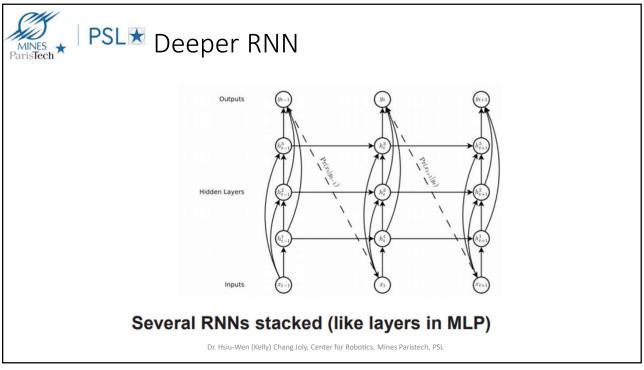


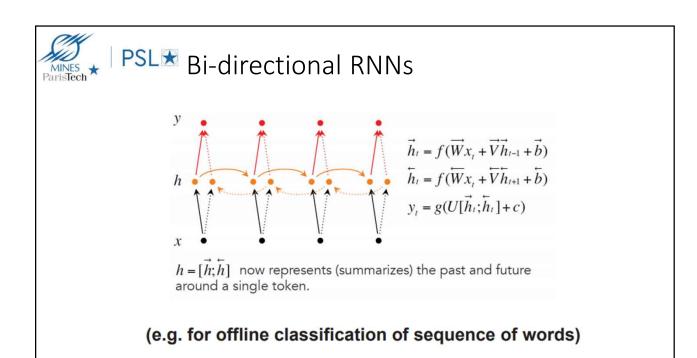




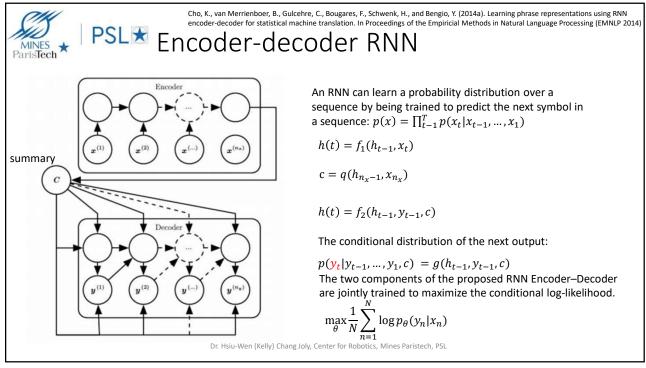


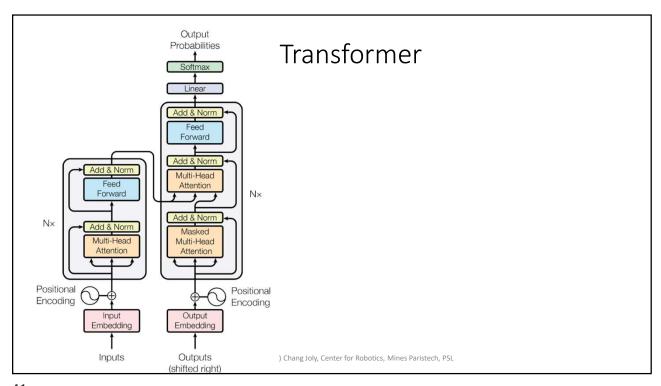






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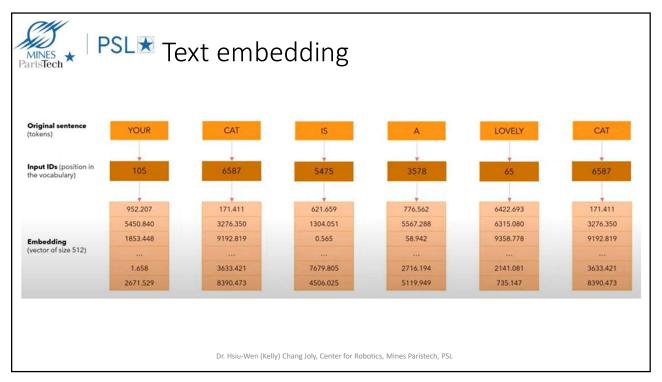


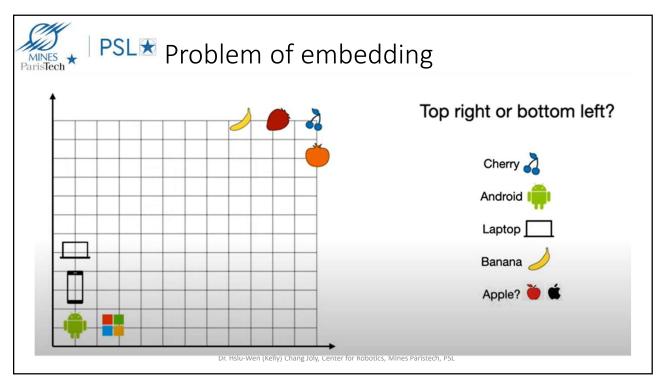


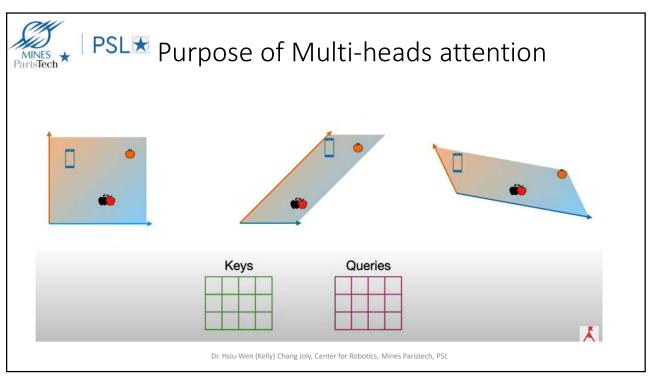
- Embedding layer maps input information from high-dimensional to a lower-dimensional space, allowing the network to learn more about the relationship between inputs and to process the data more efficiently.
- When the data is not numerical values, such as in natural language processing (NLP), finding a meaningful representation and inter-word semantics numeric vectors plays a critical role in machine learning.
- In the previous example, one-hot vectors are high-dimensional and sparse and without the meaning of each word.
 - Text embedding
 - · Image embedding
 - · Graph embedding and others



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- The attention mechanism is a way to "align" the input and output in natural language process application (NLP) where the RNN is used to build the temporal information in a sequence but can't properly link the component of inputs and outputs in a good way. Vision applications provide the network a way to "pay attention" to specific pixels that are important for the output.
- · Generally, there are two kinds of attention mechanisms:
 - A soft attention map is a fully differentiable deterministic mechanism that can be
 plugged into an existing system, and the gradients are propagated through the
 attention mechanism at the same time they are propagated through the rest of the
 network.
 - Hard attention is a stochastic process: instead of using all the hidden states as an
 input for the decoding, the system samples a hidden state yi with the probabilities si.
 To propagate a gradient through this process, we estimate the gradient by Monte
 Carlo sampling.



PSL Recommended reading: RNN variants

- [LSTM: A Search Space Odyssey, Greff et al., 2015]:
 - they play around with the LSTM equations, and swap out the non-linearities at one point, do we need tanh? This paper made many experiences in playing with different design
 - Conclusion: there is no significant difference.
- [An Empirical Exploration of Recurrent Network Architectures, Jozefowicz et al., 2015]:
 - Search over vast number of random RNN architectures, randomly permute these equations to see if there is a better one
 - Conclusion: No significant improvement with one specific version

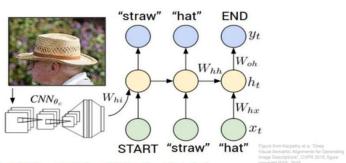
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PSL Applications

Image Captioning



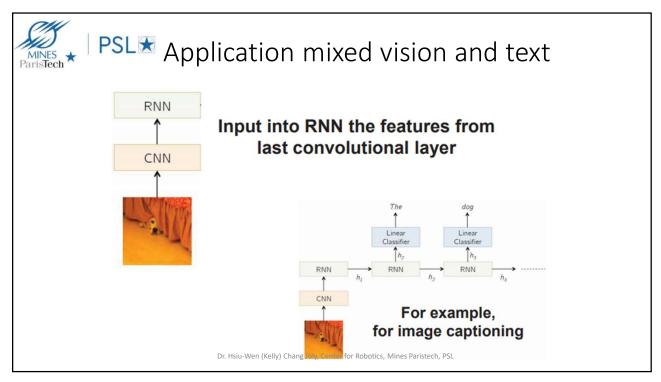
Explain Images with Multimodal Recurrent Neural Networks, Mao et al.

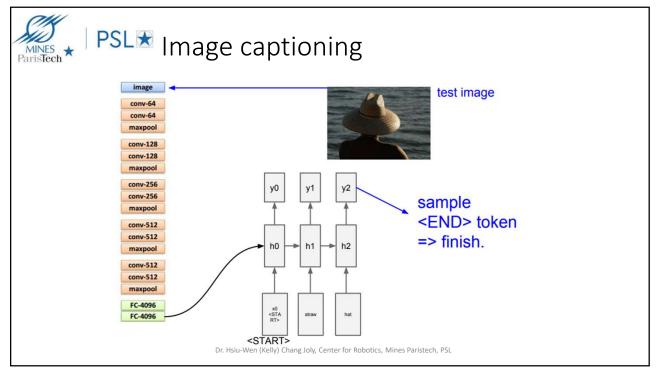
Deep Visual-Semantic Alignments for Generating Image Descriptions, Karpathy and Fei-Fei
Show and Tell: A Neural Image Caption Generator, Vinyals et al.

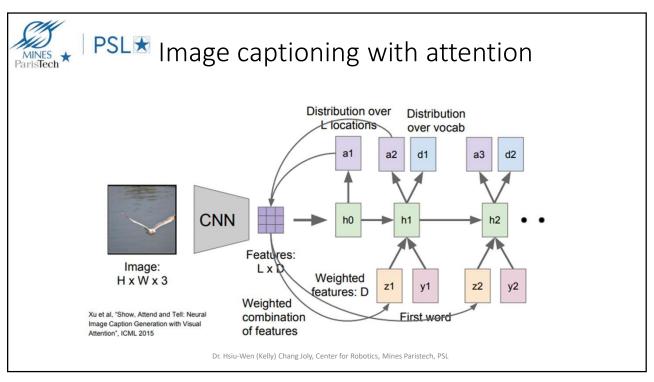
Long-term Recurrent Convolutional Networks for Visual Recognition and Description, Donahue et al.

Learning a Recurrent Visual Representation for Image Caption Generation, Chen and Zitnick

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- RNNs allow a lot of flexibility in architecture design
- Vanilla RNNs are simple but don't work very well
- Common to use LSTM or GRU: their additive interactions improve gradient flow
- Backward flow of gradients in RNN can explode or vanish
- Exploding is controlled with gradient clipping
- Vanishing is controlled with additive interactions (LSTM)
- Better/simpler architectures are a hot topic of current research
- Better understanding (both theoretical and empirical) is needed.

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