

The University of Melbourne
School of Computing and Information Systems

COMP90048

Declarative Programming

Semester 1, 2019

Section 0

Subject Introduction

Welcome to COMP90048 Declarative Programming

Lecturers: Peter Schachte and Michelle Blom

Contact information is available from the LMS.

Peter will mostly teach the functional programming part of the subject and Michelle will mostly teach the logic programming part. There will be one two-hour lecture per week on Tuesdays at 4:15pm (the timetable incorrectly says it is three hours). You are welcome to ask questions during or after a lecture.

There will be eleven one-hour workshops (labs), starting in week 2.

You should have already been allocated a workshop. Please check your personal timetable after the lecture.

- Please turn mobile phones OFF during lectures and workshops.
- Please do not use your laptop for anything distracting to others.

Workshops

The workshops will reinforce the material from lectures, partly by asking you to apply it to small scale programming tasks.

To get the most out of each workshop, you should read and attempt the exercises *before* your workshop.

Sample solutions for each set of workshop exercises are available on the LMS, but I strongly recommend you *not* look at them until you have completed them yourself.

Most programming questions have more than one correct answer; your answer may be correct even if it differs from the sample solution.

Textbooks

The lecture notes contain copies of the slides presented in lectures, plus some additional material.

All subject materials (lecture notes, workshop exercises, project specifications etc) will also be available online through the LMS.

The recommended text (which is available online) is

- Bryan O'Sullivan, John Goerzen and Don Stewart: *Real world Haskell*.

Other recommended resources are listed on the LMS.

Assessment

The subject has the following assessment components:

- 5%** short Haskell project, is due in Week 4
- 15%** larger Haskell project, due in Week 6 or 7
- 5%** short Prolog project, due in Week 9
- 15%** larger Prolog project, due in Week 11 or 12
- 60%** two-hour written final examination

To pass the subject (get 50%), you must pass *both* the project component and the exam component.

The exam is closed book, and will be held during the usual examination period after the end of the semester. Practice versions of the final exam are available on the LMS.

LMS Quizzes

Most lectures have quizzes available on the LMS. These are not assessed for credit, but are provided for you to test your understanding of the subject material. You can take each quiz as many times as you like, until you get everything right!

We'll try to get the quizzes posted before the lecture they apply to, so you can test your understanding of the material you've read before the lecture that covers it.

There is a discussion forum on the LMS specifically for discussion of the quizzes. Since these quizzes are not for credit, you can discuss them freely.

Please mention the quiz number and question number in your post subject line. I recommend you attempt a quiz yourself before reading discussions of its questions.

There are also “quizzes” in each lecture; these are completely different.

How to succeed

Declarative programming is substantially different from imperative programming.

Even after you can understand declarative code, it can take a while before you can master writing your own.

If you have been writing imperative code all your programming life, you will probably try to write imperative code even in a declarative language. This often does not work, and when it does work, it usually does not work *well*.

Writing declarative code requires a different mindset, which takes a while to acquire.

This is why attending the workshops, and practicing, practicing and practicing some more are *essential* for passing the subject.

Sources of help

During contact hours:

- Ask me during or after a lecture (not before).
- Ask the demonstrator in your workshop.

Outside contact hours:

- The LMS discussion board (preferred: everyone can see the discussion)
- Email (if not of interest to everyone)
- Attend my consultation hours (see LMS for schedule)
- Email to schedule an appointment

Announcements will be made both in lectures and on the LMS.

Please do not pose a question *both* on the LMS and in private email; if it is on the LMS, I *will* see it.

Do you want to represent this class?

Would you like to represent this class on the department's student-staff liaison committee?

The job is to collect the concerns of your classmates and present them to me and to the committee. I will post your name and email address on the LMS so classmates can contact you.

The bribe is that you get pizza, and something to put on your resumé, and the knowledge you've done something to help both your classmates and the university, and possibly make the subject better for yourself.

If you would like to volunteer, please email me.

Objectives

On completion of this subject, students should be able to:

- apply declarative programming techniques;
- write medium size programs in a declarative language;
- write programs in which different components use different languages;
- select appropriate languages for each component task in a project.

These objectives are not all of equal weight; we will spend almost all of our time on the first two objectives.

Content

- introduction to functional programming and Haskell
- declarative programming techniques
- introduction to logic programming and Prolog
- tools for declarative programming, such as debuggers
- interfacing to imperative language code
- introduction to constraint programming

This subject will teach you Haskell and Prolog, with an emphasis on Haskell.

Why Declarative Programming

Declarative programming languages are quite different from imperative and object oriented languages.

- They give you a different perspective: a focus on *what* is to be done, rather than *how*.
- They work at a higher level of abstraction.
- They make it easier to use some powerful programming techniques.
- Their clean semantics means you can do things with declarative programs that you can't do with conventional programs.

The ultimate objective of this subject is to widen your horizons and thus to make you better programmers, and not just when using declarative programming languages.

Imperative vs functional

Imperative languages are based on *commands*, in the form of *instructions* and *statements*.

- Commands are executed.
- Commands have an effect.

Assignment statements update *state*, and later code may depend on this update. Example: `x := 3 + 4 ; ...`

Functional languages are based on *expressions*.

- Expressions are evaluated.
- Expressions have no effect.

Side effects

Code is said to have a *side effect* if, in addition to producing a value, it also modifies some state or has an observable interaction with calling functions or the outside world. For example, a function might

- modify a global or a static variable,
- modify one of its arguments,
- raise an exception (e.g. divide by zero),
- write data to a display, file or network,
- read data from a keyboard, mouse, file or network, or
- call other side-effecting functions.

An example: destructive update

In imperative languages, the natural way to insert a new entry into a table is to modify the table in place: a side-effect. This effectively destroys the old table.

In declarative languages, you would instead create a new version of the table, but the old version (without the new entry) would still be there.

The price is that the language implementation has to work harder to recover memory and to ensure efficiency.

The benefit is that you don't need to worry what other code will be affected by the change. It also allows you to keep previous versions, for purposes of comparison, or for implementing undo.

The *immutability of data structures* also makes parallel programming *much* easier. Some people think that programming the dozens of cores that CPUs will have in future is the killer application of declarative programming languages.

Guarantees

- If you pass a pointer to a data structure to a function, can you guarantee that the function does not update the data structure?

If not, you will need to make a copy of the data structure, and pass a pointer to *that*.

- You add a new field to a structure. Can you guarantee that every piece of code that handles the structure has been updated to handle the new field?

If not, you will need many more test cases, and will need to find and fix more bugs.

- Can you guarantee that this function does not read or write global variables? Can you guarantee that this function does no I/O?

If the answer to either question is “no”, you will have much more work to do during testing and debugging, and parallelising the program will be a *lot* harder.

Some uses of declarative languages

- In a US Navy study in which several teams wrote code for the same task in several languages, declarative languages like Haskell were much more productive than imperative languages.
- Mission Critical used Mercury to build an insurance application in one third the time and cost of the next best quote (which used Java).
- Ericsson, one of the largest manufacturers of phone network switches, uses Erlang in some of their switches.
- The statistical machine learning algorithms behind Bing's advertising system are written in F#.
- Facebook used Haskell to build the system they use to fight spam. Haskell allowed them to increase power and performance over their previous system.

The Blub paradox

Consider Blub, a hypothetical average programming language right in the middle of the power continuum.

When a Blub programmer looks down the power continuum, he knows he is looking down. Languages below Blub are obviously less powerful, because they are missing some features he is used to.

But when a Blub programmer looks up the power continuum, he does not realize he is looking up. What he sees are merely weird languages. He thinks they are about equivalent in power to Blub, but with some extra hairy stuff. Blub is good enough for him, since he thinks in Blub.

When we switch to the point of view of a programmer using a language higher up the power continuum, however, we find that she in turn looks down upon Blub, because it is missing some things *she* is used to.

Therefore understanding the differences in power between languages requires understanding the most powerful ones.

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Section 1

Introduction to Functional Programming

Functional programming

The basis of functional programming is *equational reasoning*. This is a grand name for a simple idea:

- if two expressions have equal values, then one can be replaced by the other.

You can use equational reasoning to rewrite a complex expression to be simpler and simpler, until it is as simple as possible. Suppose $x = 2$ and $y = 4$, and you start with the expression $x + (3 * y)$:

step 0: $x + (3 * y)$

step 1: $2 + (3 * y)$

step 2: $2 + (3 * 4)$

step 3: $2 + 12$

step 4: 14

Lists

Of course, programs want to manipulate more complex data than just simple numbers.

Like most functional programming languages, Haskell allows programmers to define their own types, using a much more expressive type system than the type system of e.g. C.

Nevertheless, the most frequently used type in Haskell programs is probably the builtin *list* type.

The notation `[]` means the empty list, while `x:xs` means a nonempty list whose *head* (first element) is represented by the variable `x`, and whose *tail* (all the remaining elements) is represented by the variable `xs`.

The notation `["a", "b"]` is syntactic sugar for `"a":"b":[]`. As in most languages, `"a"` represents the string that consists of a single character, the first character of the alphabet.

Functions

A function definition consists of equations, each of which establishes an equality between the left and right hand sides of the equal sign.

```
len []      = 0
len (x:xs) = 1 + len xs
```

Each equation typically expects the input arguments to conform to a given *pattern*; `[]` and `(x:xs)` are two patterns.

The set of patterns should be *exhaustive*: at least one pattern should apply for any possible call.

It is good programming style to ensure that the set of patterns is also *exclusive*, which means that *at most one* pattern should apply for any possible call.

If the set of patterns is both exhaustive and exclusive, then *exactly one* pattern will apply for any possible call.

Aside: syntax

- In most languages, a function call looks like `f(fa1, fa2, fa3)`.
- In Haskell, it looks like `f fa1 fa2 fa3`.

If the second argument is not `fa2` but the function `g` applied to the single argument `ga1`, then in Haskell you would need to write

$$f \text{ fa1 } (g \text{ ga1}) \text{ fa3}$$

since Haskell would interpret `f fa1 g ga1 fa3` as a call to `f` with four arguments.

In Haskell, there are no parentheses around the whole argument list of a function call, but parentheses may be needed around *individual* arguments. This applies on the left as well the right hand sides of equations.

This is why the recursive call is `len xs` and not `len(xs)`, and why the left hand side of the second equation is `len (x:xs)` instead of `len x:xs`.

More syntax issues

Comments start with two minus signs and continue to the end of the line.

The names of functions and variables are sequences of letters, numbers and/or underscores that must start with a lower case letter.

Suppose line1 starts in column n , and the following nonblank line, line2, starts in column m . The *offside rule* says that

- if $m > n$, then line2 is a continuation of the construct on line1;
- if $m = n$, then line2 is the start of a new construct at the same level as line1;
- if $m < n$, then line2 is either the continuation of something else that line1 is part of, or a new item at the same level as something else that line1 is part of.

This means that the structure of the code as shown by indentation must match the structure of the code.

Recursion

The definition of a function to compute the length of a list, like many Haskell functions, reflect the structure of the data: a list is either empty, or has a head and a tail.

The first equation for `len` handles the empty list case.

$$\text{len } [] = 0$$

This is called the *base case*.

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len [] = 0
```

This is called the *base case*.

The second equation handles nonempty lists. This is called the *recursive case*, since it contains a recursive call.

```
len (x:xs) = 1 + len xs
```

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```
len (x:xs) = 1 + len xs
```

If you want to be a good programmer in a declarative language, you have to get comfortable with recursion, because most of the things you need to do involve recursion.

Using a function

```
len []      = 0
len (x:xs) = 1 + len xs
```

Given a function definition like this, the Haskell implementation can use it to replace calls to the function with the right hand side of an applicable equation.

```
step 0:  len ["a", "b"]  -- ("a":("b":[]))
step 1:  1 + len ["b"]   -- ("b":[])
step 2:  1 + 1 + len []
step 3:  1 + 1 + 0
step 4:  1 + 1
step 5:  2
```

Expression evaluation

To evaluate an expression, the Haskell runtime system conceptually executes a loop, each iteration of which consists of these steps:

- looks for a function call in the current expression,
- searches the list of equations defining the function from the top downwards, looking for a matching equation,
- sets the values of the variables in the matching pattern to the corresponding parts of the actual arguments, and
- replaces the left hand side of the equation with the right hand side.

The loop stops when the current expression contains no function calls, not even calls to such builtin “functions” as addition.

The actual Haskell implementation is more sophisticated than this loop, but the effect it achieves is the same.

Order of evaluation

The first step in each iteration of the loop, “look for a function call in the current expression”, can find more than one function call. Which one should we select?

```
len [] = 0
```

```
len (x:xs) = 1 + len xs
```

evaluation order A:

```
step 0: len ["a"] + len []
```

```
step 1: 1 + len [] + len []
```

```
step 2: 1 + 0 + len []
```

```
step 3: 1 + len []
```

```
step 4: 1 + 0
```

```
step 5: 1
```

evaluation order B:

```
step 0: len ["a"] + len []
```

```
step 1: len ["a"] + 0
```

```
step 2: 1 + len [] + 0
```

```
step 3: 1 + 0 + 0
```

```
step 4: 1 + 0
```

```
step 5: 1
```

Church-Rosser theorem

In 1936, Alonzo Church and J. Barkley Rosser proved a famous theorem, which says that for the rewriting system known as the *lambda calculus*, regardless of the order in which the original term's subterms are rewritten, the final result is always the same.

This theorem also holds for Haskell and for several other functional programming languages (though not for all).

This is not that surprising, since most modern functional languages are based on one variant or another of the lambda calculus.

We will ignore the order of evaluation of Haskell expressions for now, since in most cases it does not matter. We will come back to the topic later.

The Church-Rosser theorem is *not* applicable to imperative languages.

Order of evaluation: efficiency

```
all_pos [] = True  
all_pos (x:xs) = x > 0 && all_pos xs
```


Order of evaluation: efficiency

```
all_pos [] = True
all_pos (x:xs) = x > 0 && all_pos xs
```

evaluation order A:

```
0: all_pos [-1, 2]
1: -1 > 0 && all_pos [2]
2: False && all_pos [2]
3: False
```

Order of evaluation: efficiency

```
all_pos [] = True
all_pos (x:xs) = x > 0 && all_pos xs
```

evaluation order A:

```
0: all_pos [-1, 2]
1: -1 > 0 && all_pos [2]
2: False && all_pos [2]
3: False
```

evaluation order B:

```
0: all_pos [-1, 2]
1: -1 > 0 && all_pos [2]
2: -1 > 0 && 2 > 0 && all_pos []
3: -1 > 0 && 2 > 0 && True
4: -1 > 0 && True && True
5: -1 > 0 && True
6: False && True
7: False
```

QUIZ: Imperative code

What will this C code print out?

```
int glob = 0;

int inc(void)
{
    glob++;
    return glob;
}

printf("inc() - inc() = %d\n", inc() - inc());
```

QUIZ: Imperative code

What will this C code print out?

```
int glob = 0;

int inc(void)
{
    glob++;
    return glob;
}

printf("inc() - inc() = %d\n", inc() - inc());
```

It depends. If the first `inc()` is evaluated first, it will print $1 - 2 = -1$. If it is evaluated second, it will print out $2 - 1 = 1$. The C language specification does not specify which.

Imperative vs declarative languages

In the presence of side effects, a program's behavior depends on history; that is, the order of evaluation matters.

Because understanding an effectful program requires thinking about all possible histories, side effects often make a program harder to understand.

When developing larger programs or working in teams, managing side-effects is critical and difficult; Haskell guarantees the absence of side-effects.

What really distinguishes pure declarative languages from imperative languages is that they do not allow side effects.

There is only one benign exception to that: they do allow programs to generate exceptions.

We will ignore exceptions from now on, since in the programs we deal with, they have only one effect: they abort the program.

Referential transparency

The absence of side effects allows pure functional languages to achieve *referential transparency*, which means that an expression can be replaced with its value. This requires that the expression has no side effects and is pure, i.e. always returns the same results on the same input.

By contrast, in imperative languages such as C, functions in general are not pure and are thus not functions in a mathematical sense: two identical calls may return different results.

Impure functional languages such as Lisp are called impure precisely because they *do* permit side effects like assignments, and thus their programs are not referentially transparent.

Single assignment

One consequence of the absence of side effects is that assignment means something different in a functional language than in an imperative language.

- In conventional, imperative languages, even object-oriented ones (including C, Java, and Python), each variable has a current value (a garbage value if not yet initialized), and assignment statements can change the current value of a variable.
- In functional languages, variables are *single assignment*, and there are no assignment statements. You can define a variable's value, but you cannot redefine it. Once a variable has a value, it has that value until the end of its lifetime.

Giving variables values

Haskell programs can give a variable a value in one of two ways.

The explicit way is to use a *let clause*:

```
let pi = 3.14159 in ...
```

This defines `pi` to be the given value in the expression represented by the dots. It does *not* define `pi` anywhere else.

The implicit way is to put the variable in a pattern on the left hand side of an equation:

```
len (x:xs) = 1 + len xs
```

If `len` is called with a nonempty list, Haskell will bind `x` to its head and `xs` to its tail.

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Section 2

Builtin Haskell types

The Haskell type system

Haskell has a *strong*, *safe* and *static* type system.

The *strong* part means that the system has no loopholes; one cannot tell Haskell to e.g. consider an integer to be a pointer, as one can in C with `(char *) 42`.

The *safe* part means that a running program is guaranteed never to crash due to a type error. (A C program that dereferenced the above pointer would almost certainly crash.)

The *static* part means that types are checked when the program is compiled, not when the program is run.

This is partly what makes the *safe* part possible; Haskell will not even start to run a program with a type error.

Basic Haskell types

Haskell has the usual basic types. These include:

- The Boolean type is called `Bool`. It has two values: `True` and `False`.
- The native integer type is called `Int`. Values of this type are 32 or 64 bits in size, depending on the platform. Haskell also has a type for integers of unbounded size: `Integer`.
- The usual floating-point type is `Double`. (`Float` is also available, but its use is discouraged.)
- The character type is called `Char`.

There are also others, e.g. integer types with 8, 16, 32 and 64 bits regardless of platform.

There are more complex types as well.

The types of lists

In Haskell, list is not a type; it is a *type constructor*.

Given any type `t`, it constructs a type for lists whose elements are all of type `t`. This type is written as `[t]`, and it is pronounced as “list of `t`”.

You can have lists of any type. For example,

- `[Bool]` is the type of lists of Booleans,
- `[Int]` is the type of lists of native integers,
- `[[Int]]` is the type of lists of lists of native integers.

These are similar to `ArrayList<Boolean>`, `ArrayList<Integer>`, and `ArrayList<ArrayList<Boolean>>` in Java.

Haskell considers strings to be lists of characters, whose type is `[Char]`; `String` is a synonym for `[Char]`.

The names of types and type constructors should be identifiers starting with an upper case letter; the list type constructor is an exception.

ghci

The usual implementation of Haskell is `ghc`, the Glasgow Haskell Compiler. It also comes with an interpreter, `ghci`.

```
$ ghci
...
Prelude> let x = 2
Prelude> let y = 4
Prelude> x + (3 * y)
14
```

The *prelude* is Haskell's standard library.

`ghci` uses its name as the prompt to remind users that they can call its functions.

DEMO: Using ghci

```
cat len.hs  
ghci  
:load len  
len ["a", "b", "c"]
```

Types and `ghci`

You can ask `ghci` to tell you the type of an expression by prefacing that expression with `:t`. The command `:set +t` tells `ghci` to print the type as well as the value of every expression it evaluates.

```
Prelude> :t "abc"
"abc" :: [Char]
Prelude> :set +t
Prelude> "abc"
"abc"
...
it :: [Char]
```

The notation `x::y` says that expression `x` is of type `y`. In this case, it says `"abc"` is of type `[Char]`.

`it` is `ghci`'s name for the value of the expression just evaluated.

Function types

You can also ask `ghci` about the types of functions. Consider this function, which checks whether a list is empty:

```
isEmpty []      = True  
isEmpty (_:_)  = False
```

(`_` is a special pattern that matches anything.)

If you ask `ghci` about its type, you get

```
> :t isEmpty  
isEmpty :: [a] -> Bool
```

A function type lists the types of all the arguments and the result, all separated by arrows. We'll see what the `a` means a bit later.

Function types

Programmers should declare the type of each function. The syntax for this is similar to the notation printed by `ghci`: the function name, a double colon, and the type.

```
module Emptiness where

isEmpty :: [t] -> Bool
isEmpty [] = True
isEmpty _  = False
```

Declaring the type of functions is required only by good programming style. The Haskell implementation will infer the types of functions if not declared.

Haskell also infers the types of all the local variables.

Later in the subject, we will briefly introduce the algorithm Haskell uses for type inference.

Function type declarations

With type declarations, Haskell will report an error and refuse to compile the file if the declared type of a function is incompatible with its definition.

It's also an error if a *call* to the function is incompatible with its declared type.

Without declarations, Haskell will report an error if the types in any call to any function are incompatible with its definition. Haskell will never allow code to be run with a type error.

Type declarations improve Haskell's error messages, and make function definitions much easier to understand.

QUIZ: Summing a list

Write a type declaration for a Haskell function `sumlist` that adds a list of `Ints`.

Also define this function.

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```
sumlist :: [Int] -> Int
```

QUIZ: Summing a list

Write a type declaration for a Haskell function `sumlist` that adds a list of `Ints`.

Also define this function.

```
sumlist :: [Int] -> Int  
sumlist [] = 0
```

QUIZ: Summing a list

Write a type declaration for a Haskell function `sumlist` that adds a list of `Ints`.

Also define this function.

```
sumlist :: [Int] -> Int
sumlist [] = 0
sumlist (x:xs) = x + sumlist xs
```

QUIZ: Summing a list

Write a type declaration for a Haskell function `sumlist` that adds a list of `Ints`.

Also define this function.

```
sumlist :: [Int] -> Int
sumlist [] = 0
sumlist (x:xs) = x + sumlist xs
```

This same function definition can be made to handle all numeric types. We will see how to do this next.

Number types

Haskell has several numeric types, including `Int`, `Integer`, `Float`, and `Double`. A plain integer constant belongs to all of them. So what does Haskell say when asked what the type of e.g. `3` is?

```
Prelude> :t 3
3 :: Num p => p
Prelude> :t [1, 2]
[1, 2] :: Num a => [a]
```

In these messages, `a` and `p` are *type variables*; they are variables that stand for *types*, not *values*.

The notation `Num p` means “the type `p` is a member of type class `Num`”. `Num` is the class of numeric types, including the four types above.

The notation `3 :: Num p => p` means that “if `p` is a numeric type, then `3` is a value of that type”.

Number type flexibility

The usual arithmetic operations, such as addition, work for any numeric type:

```
Prelude> :t (+)
(+) :: (Num a) => a -> a -> a
```

The notation `a -> a -> a` denotes a function that takes two arguments and returns a result, all of which have to be of the same type (since they are denoted by the same type variable, `a`), which in this case must be a member of the `Num` type class.

This flexibility is nice, but it does result in confusing error messages:

```
Prelude> [1, True]
No instance for (Num Bool) arising from the literal '1'
...
```

QUIZ: Summing a list

Give a type declaration for the `sumlist` function that works for lists of any numeric type.

QUIZ: Summing a list

Give a type declaration for the `sumlist` function that works for lists of any numeric type.

```
sumlist :: Num a => [a] -> a
```

if-then-else

```
-- Definition A  
iota n = if n == 0 then [] else iota (n-1) ++ [n]
```

Definition A uses an if-then-else. If-then-else in Haskell differs from if-then-elses in imperative languages in that

- the else arm is not optional, and
- the then and else arms are expressions, not statements.

Guards

```
-- Definition B
iota n
  | n == 0  = []
  | n > 0  = iota (n-1) ++ [n]
```

Definition B uses *guards* to specify cases. Note the first line does not end with an “=”; each guard line specifies a case and the value for that case, much as in definition A.

Note that the second guard specifies $n > 0$. What should happen if you do `iota (-3)`? What do you expect to happen? What about for definition A?

Structured definitions

Some Haskell equations do not fit on one line, and even the ones that do fit are often better split across several. Guards are only one example of this.

```
-- Definition C
iota n =
    if n == 0
    then
        []
    else
        iota (n-1) ++ [n]
```

The offside rule says that

- the keywords `then` and `else`, if they start a line, must be at the same level of indentation as the corresponding `if`, and
- if the `then` and `else` expressions are on their own lines, these must be *more* indented than those keywords.

Parametric polymorphism

Here is a version of the code of `len` complete with type declaration:

```
len :: [t] -> Int
```

```
len [] = 0
```

```
len (_:xs) = 1 + len xs
```

This function, like many others in Haskell, is *polymorphic*. The phrase “poly morph” means “many shapes” or “many forms”. In this context, it means that `len` can process lists of type `t` *regardless* of what type `t` is, i.e. regardless of what the form of the elements is.

The *reason* why `len` works regardless of the type of the list elements is that it does not do anything with the list elements.

This version of `len` shows this in the second pattern: the underscore is a placeholder for a value you want to ignore.

The University of Melbourne
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COMP90048

Declarative Programming

Semester 1, 2019

Section 3

Defining Haskell types

Type definitions

Like most languages, Haskell allows programmers to define their own types. The simplest type definitions define types that are similar to enumerated types in C:

```
data Gender = Female | Male
data Role = Staff | Student
```

This defines two new types. The type called `Gender` has the two values `Female` and `Male`, while the type called `Role` has the two values `Staff` and `Student`.

Both types are also considered arity-0 type constructors; given zero argument types, they each construct a type.

The four values are also called *data constructors*. Given zero arguments, they each construct a value (a piece of data).

The names of type constructors and data constructors must be identifiers starting with upper-case letters.

Using Booleans

You do not have to use such types. If you wish, you can use the standard Boolean type instead, like this:

```
show1 :: Bool -> Bool -> String
-- intended usage: show1 isFemale isStaff
show1 True True    = "female staff"
show1 True False   = "female student"
show1 False True   = "male staff"
show1 False False  = "male student"
```

You can use such a function like this:

```
> let isFemale = True
> let isStaff = False
> show1 isFemale isStaff
```

Using defined types vs using Booleans

```
> show1 isFemale isStaff  
> show1 isStaff isFemale
```

The problem with using Booleans is that of these two calls to `show1`, only one matches the programmer's intention, but since both are type correct (both supply two Boolean arguments), Haskell cannot catch errors that switch the arguments.

```
show2 :: Gender -> Role -> String
```

With `show2`, Haskell can and *will* detect and report any accidental switch. This makes the program safer and the programmer more productive.

In general, you should use separate types for separate semantic distinctions. You can use this technique in any language that supports enumerated types.

DEMO: Defined types vs Booleans

```
cat uni1.hs
ghci
:load uni1
let isFemale = True
let isStaff = False
show1 isFemale isStaff
show1 isStaff isFemale
```

```
cat uni2.hs
ghci
:load uni2
show2 Female Student
show2 Student Female
```

Representing cards

Here is one way to represent standard western playing cards:

```
data Suit = Club | Diamond | Heart | Spade
data Rank
    = R2 | R3 | R4 | R5 | R6 | R7 | R8 | R9 | R10
    | Jack | Queen | King | Ace

data Card = Card Suit Rank
```

The types `Suit` and `Rank` would be enumerated types in C, while the type `Card` would be a structure type.

On the right hand side of the definition of the type `Card`, `Card` is the name of the *data constructor*, while `Suit` and `Rank` are the types of its two arguments.

QUIZ: Defining a type

Define a type to represent a naughts-and-crosses (tic-tac-toe) board.

QUIZ: Defining a type

Define a type to represent a naughts-and-crosses (tic-tac-toe) board.

There are many possibilities; here's one:

```
data SquareContent = X | O | Empty
```

```
data TicTacToeBoard =  
    TicTacToeBoard  
    SquareContent SquareContent SquareContent  
    SquareContent SquareContent SquareContent  
    SquareContent SquareContent SquareContent
```

Creating structures

```
data Card = Card Suit Rank
```

In this definition, `Card` is not just the name of the type (from its first appearance), but (from its second appearance) also the name of the data constructor which constructs the “structure” from its arguments.

In languages like C, creating a structure and filling it in requires a call to `malloc` or its equivalent, a check of its return value, and an assignment to each field of the structure. This typically takes several lines of code.

In Haskell, you can construct a structure just by writing down the name of the data constructor, followed by its arguments, like this: `Card Club Ace`. This typically takes only *part* of one line of code.

In practice, this seemingly small difference has a significant impact, because it removes much clutter (details irrelevant to the main objective).

Printing values

Many programs have code whose job it is to print out the values of a given type in a way that is meaningful to the programmer. Such functions are particularly useful during various forms of debugging.

The Haskell approach is use a function that returns a string. However, writing such functions by hand can be tedious, because each data constructor requires its own case:

```
showrank :: Rank -> String
showrank R2 = "R2"
showrank R3 = "R3"
...
```

Show

The Haskell prelude has a standard string conversion function called `show`. Just as the arithmetic functions are applicable to all types that are members of the type class `Num`, this function is applicable to all types that are members of the type class `Show`.

You can tell Haskell that the `show` function for values of the type `Rank` is `showrank`:

```
instance Show Rank where show = showrank
```

This of course requires defining `showrank`. If you don't want to do that, you can get Haskell to define the `show` function for a type by adding deriving `Show` to the type's definition, like this:

```
data Rank =
    R2 | R3 | R4 | R5 | R6 | R7 | R8 |
    R9 | R10 | Jack | Queen | King | Ace
    deriving Show
```

Eq and Ord

Another operation even more important than string conversion is comparison for equality.

To be able to use Haskell's `==` comparison operation for a type, it must be in the `Eq` type class. This can also be done automatically by putting `deriving Eq` at the end of a type definition.

To compare values of a type for order (using `<`, `<=`, *etc.*), the type must be in the `Ord` type class, which can also be done by putting `deriving Ord` at the end of a type definition. To be in `Ord`, the type must also be in `Eq`.

To derive multiple type classes, parenthesise them:

```
data Suit = Club | Diamond | Heart | Spade
          deriving (Show, Eq, Ord)
```

Disjunction and conjunction

```
data Suit = Club | Diamond | Heart | Spade
data Card = Card Suit Rank
```

A value of type `Suit` is *either* a Club *or* a Diamond *or* a Heart *or* a Spade. This *disjunction* of values corresponds to an enumerated type.

A value of type `Card` contains a value of type `Suit` *and* a value of type `Rank`. This *conjunction* of values corresponds to a structure type.

In most imperative languages, a type can represent either a disjunction or a conjunction, but not both at once.

Haskell and related languages do not have this limitation.

Discriminated union types

Haskell has *discriminated union* types, which can include both disjunction and conjunction at once.

Since disjunction and conjunction are operations in Boolean algebra, type systems that allows them to be combined in this way are often called algebraic type systems, and their types algebraic types.

```
data JokerColor = Red | Black
data JCard = NormalCard Suit Rank | JokerCard JokerColor
```

A value of type JCard is constructed

- *either* using the NormalCard constructor, in which case it contains a value of type Suit *and* a value of type Rank,
- *or* using the JokerCard constructor, in which case it contains a value of type JokerColor.

Discriminated vs undiscriminated unions

In C, you could try to represent JCard like this:

```
struct normalcard_struct { ... };
struct jokercard_struct { ... };
union card_union {
    struct normalcard_struct    normal;
    struct jokercard_struct     joker;
};
```

but you wouldn't know which field of the union is applicable in any given case. In Haskell, you do (the data constructor tells you), which is why Haskell's unions are said to be *discriminated*.

Note that unlike C's union types, C's enumeration types and structure types are special cases of Haskell's discriminated union types.

Discriminated union types allow programmers to define types that describe *exactly* what they mean.

QUIZ: Library Circulation Items

Define a type `LibraryItem` representing an item the library loans to patrons: either a book or a periodical. Books have a catalogue number, title, and author. Periodicals have a catalogue number, a title, and a publication frequency. A publication frequency is either a number of days or of months.

QUIZ: Library Circulation Items

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```
data LibraryItem
    = Book      Integer String String
    | Periodical Integer String Period

data Period = Days Integer | Months Integer
```


QUIZ: Getting the Title

Given the `LibraryItem` type, define a function `title` that will return the title of any `LibraryItem`

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Given the `LibraryItem` type, define a function `title` that will return the title of any `LibraryItem`

```
title (Book      _ title _) = title
title (Periodical _ title _) = title
```

Maybe

In languages like C, have a value of type $*T$ for some type T , or in languages like Java, if you have a value of some non-primitive type, can this value be null?

If not, the value represents a value of type T . If yes, the value *may* represent a value of type T , or it may represent nothing. The problem is, often the reader of the code has *no idea* whether it can be null.

And even if the value **must** not be null, there's no guarantee it won't be. This can lead to segfaults or `NullPointerException`s.

In Haskell, if a value is optional, you indicate this by using the `maybe` type defined in the prelude:

```
data Maybe t = Nothing | Just t
```

For any type t , a value of type `Maybe t` is either `Nothing`, or `Just x`, where x is a value of type t . This is a *polymorphic* type, like `[t]`.

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Section 4

Using Haskell Types

Representing expressions in C

```
typedef enum {  
    EXPR_NUM, EXPR_VAR, EXPR_BINOP, EXPR_UNOP  
} ExprKind;  
  
typedef struct expr_struct *Expr;  
struct expr_struct  
{  
    ExprKind kind;  
    int      value;      /* if EXPR_NUM */  
    char     *name;      /* if EXPR_VAR */  
    Binop    binop;      /* if EXPR_BINOP */  
    Unop     unop;       /* if EXPR_UNOP */  
    Expr     subexpr1; /* if EXPR_BINOP or EXPR_UNOP */  
    Expr     subexpr2; /* if EXPR_BINOP */  
};
```

Representing expressions in Java

```
public abstract class Expr {  
    ... abstract methods ...  
}  
  
public class NumExpr extends Expr {  
    int value;  
    ... implementation of abstract methods ...  
}  
  
public class VarExpr extends Expr {  
    String name;  
    ... implementation of abstract methods ...  
}
```

Representing expressions in Java (2)

```
public class BinExpr extends Expr {  
    Binop binop;  
    Expr arg1;  
    Expr arg2;  
    ... implementation of abstract methods ...  
}
```

```
public class UnExpr extends Expr {  
    Unop unop;  
    Expr arg;  
    ... implementation of abstract methods ...  
}
```

Representing expressions in Haskell

```
data Expr
  = Number Int
  | Variable String
  | Binop Binopr Expr Expr
  | Unop Unopr Expr

data Binopr = Plus | Minus | Times | Divide
data Unopr  = Negate
```

As you can see, this is a much more direct definition of the set of values that the programmer wants to represent.

It is also much shorter, and entirely free of notes that are meaningless to the compiler and understood only by humans.

Comparing representations: errors

By far the most important difference is that the C representation is quite error-prone.

- You can access a field when that field is not meaningful, e.g. you can access the `subexpr2` field instead of the `subexpr1` field when `kind` is `EXPR_UNOP`.
- You can forget to initialize some of the fields, e.g. you can forget to assign to the `name` field when setting `kind` to `EXPR_VAR`.
- You can forget to process some of the alternatives, e.g. when switching on the `kind` field, you may handle only three of the four enum values.

The first mistake is literally impossible to make with Haskell, and would be caught by the Java compiler. The second is guaranteed to be caught by Haskell, but not Java. The third will be caught by Java, and by Haskell if you ask `ghc` to be on the lookout for it.

Comparing representations: memory

The C representation requires more memory: seven words for every expression, whereas the Java and Haskell representations need a maximum of four (one for the kind, and three for arguments/members).

Using unions can make the C representation more compact, but only at the expense of more complexity, and therefore a higher probability of programmer error.

Even with unions, the C representation needs four words for all kinds of expressions. The Java and Haskell representations need only two for numbers and variables, and three for expressions built with unary operators.

This is an example where a Java or Haskell program can actually be *more efficient* than a C program.

Comparing representations: maintenance

Adding a new kind of expression requires:

Java: Adding a new class and implementing all the methods for it

C: Adding a new alternative to the enum and adding the needed members to the type, and adding code for it to all functions handling that type

Haskell Adding a new alternative, with arguments, to the type, and adding code for it to all functions handling that type

Adding a new operation for expressions requires:

Java: Adding a new method to the abstract Expr class, and implementing it for each class

C: Writing one new function

Haskell Writing one new function

Switching on alternatives

You do not have to have separate equations for each possible shape of the arguments. You can test the value of a variable (which may or may not be the value of an argument) in the body of an equation, like this:

```
is_static :: Expr -> Bool
is_static expr =
  case expr of
    Number _      -> True
    Variable _     -> False
    Unop _ expr1   -> is_static expr1
    Binop _ expr1 expr2 ->
      is_static expr1 && is_static expr2
```

This function figures out whether the value of an expression can be known statically, i.e. without having to know the values of variables.

Missing alternatives

If you specify the option `-fwarn-incomplete-patterns`, `ghc` and `ghci` will warn about any missing alternatives, both in case expressions and in sets of equations.

This option is particularly useful during program maintenance. When you add a new alternative to an existing type, all the switches on values of that type instantly become incorrect. To fix them, a programmer must add a case to each such switch to handle the new alternative.

If you always compile the program with this option, the compiler will tell you all the switches in the program that must be modified.

Without such help, programmers must look for such switches themselves, and they may not find them all.

The consequences of missing alternatives

If a Haskell program finds a missing alternative at runtime, it will throw an exception, which (unless caught and handled) will abort the program.

Without a default case, a C program would simply go on and silently compute an incorrect result. If a default case is provided, it is likely to just print an error message and abort the program. C programmers thus have to do more work than Haskell programmers just to get up to the level of safety offered by Haskell.

If an abstract method is used in Java, this gives the same safety as Haskell. However, if overriding is used alone, forgetting to write a method for a subclass will just inherit the (probably wrong) behaviour of the superclass.

Binary search trees

Here is one possible representation of binary search trees in C:

```
typedef struct bst_struct *BST;
struct bst_struct {
    char    *key;
    int     value;
    BST     left;
    BST     right;
};
```

Here it is in Haskell:

```
data Tree = Leaf | Node String Int Tree Tree
```

The Haskell version has two alternatives, one of which has no associated data. The C version uses a null pointer to represent this alternative.

QUIZ: Maybe tree

How would you represent “maybe a tree, maybe nothing” in Haskell?

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QUIZ: Maybe tree

How would you represent “maybe a tree, maybe nothing” in Haskell?

Maybe Tree

How would you represent “maybe a tree, maybe nothing” in C?

You can have a null pointer meaning “there is no tree”, you can have a null pointer meaning “there is a tree, and it is empty”, but it cannot mean both at once.

QUIZ: Maybe tree

How would you represent “maybe a tree, maybe nothing” in Haskell?

Maybe Tree

How would you represent “maybe a tree, maybe nothing” in C?

You can have a null pointer meaning “there is no tree”, you can have a null pointer meaning “there is a tree, and it is empty”, but it cannot mean both at once.

You need something like BST `*maybe_tree` (which means that `maybe_tree` is a pointer to a pointer to a `bst_struct`), with

- `maybe_tree == NULL` meaning there is no tree, and
- `maybe_tree != NULL && *maybe_tree == NULL` meaning there is a tree, and it is empty.

Counting nodes in a BST

```
countnodes :: Tree -> Int
countnodes Leaf = 0
countnodes (Node _ _ l r) =
    1 + (countnodes l) + (countnodes r)
```

```
int countnodes(BST tree)
{
    if (tree == NULL) {
        return 0;
    } else {
        return 1 +
            countnodes(tree->left) +
            countnodes(tree->right);
    }
}
```

Pattern matching vs pointer dereferencing

The left-hand-side of the second equation in the Haskell definition naturally gives names to each of the fields of the node that actually need names (because they are used in the right hand side).

These variables do not have to be declared, and Haskell infers their types.

The C version refers to these fields using syntax that dereferences the pointer tree and accesses one of the fields of the structure it points to.

The C code is longer, and using Haskell-like names for the fields would make it longer still:

```
BST l = tree->left;  
BST r = tree->right;  
...
```

Searching a BST in C (iteration)

```
int search_bst(BST tree, char *key, int *value_ptr)
{
    while (tree != NULL) {
        int cmp_result;

        cmp_result = strcmp(key, tree->key);
        if (cmp_result == 0) {
            *value_ptr = tree->value;
            return TRUE;
        } else if (cmp_result < 0) {
            tree = tree->left;
        } else {
            tree = tree->right;
        }
    }

    return FALSE;
}
```

Searching a BST in Haskell

```
search_bst :: Tree -> String -> Maybe Int
search_bst Leaf _ = Nothing
search_bst (Node k v l r) sk =
    if sk == k then
        Just v
    else if sk < k then
        search_bst l sk
    else
        search_bst r sk
```

- If the search succeeds, this function returns `Just v`, where `v` is the searched-for value.
- If the search fails, it returns `Nothing`.

Data structure and code structure

The Haskell definitions of `countnodes` and `search_bst` have similar structures:

- an equation handling the case where the tree is empty (a `Leaf`), and
- an equation handling the case where the tree is nonempty (a `Node`).

The type we are processing has two alternatives, so these two functions have two equations: one for each alternative.

This is quite a common occurrence:

- a function whose input is a data structure will need to process all or a selected part of that data structure, and
- what the function needs to do often depends on the shape of the data, so the structure of the code often mirrors the structure of the data.

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Section 5

Adapting to Declarative Programming

Writing code

Consider a C function with two loops, and some other code around and between the loops:

```
... somefunc(...)
{
    straight line code A
    loop 1
    straight line code B
    loop 2
    straight line code C
}
```

How can you get the same effect in Haskell?

The functional equivalent

```
loop1func base case  
loop1func recursive case
```

```
loop2func base case  
loop2func recursive case
```

```
somefunc =  
    let ... = ... in  
    let r1 = loop1func ... in  
    let ... = ... in  
    let r2 = loop2func ... in  
    ...
```

The only effect of the absence of iteration constructs is that instead of writing a loop inside `somefunc`, the Haskell programmer needs to write an auxiliary recursive function, usually outside `somefunc`.

Example: C

```
int f(int *a, int size)
{
    int i;
    int target;
    int first_gt_target;

    i = 0;
    while (i < size && a[i] <= 0) {
        i++;
    }

    target = 2 * a[i];
    i++;
    while (i < size && a[i] <= target) {
        i++;
    }
    first_gt_target = a[i];
    return 3 * first_gt_target;
}
```

Example: Haskell version

```
f :: [Int] -> Int
f list =
    let after_skip = skip_init_le_zero list in
    case after_skip of
        (x:xs) ->
            let target = 2 * x in
            let first_gt_target = find_gt xs target in
            3 * first_gt_target

skip_init_le_zero :: [Int] -> [Int]
skip_init_le_zero [] = []
skip_init_le_zero (x:xs) =
    if x <= 0 then skip_init_le_zero xs else (x:xs)

find_gt :: [Int] -> Int -> Int
find_gt (x:xs) target =
    if x <= target then find_gt xs target else x
```

QUIZ: Filtering a list

Write a Haskell function to filter out the negative numbers from a list. Begin with a type declaration.

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```
nonNegElts :: (Ord a, Num a) => [a] -> [a]
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Write a Haskell function to filter out the negative numbers from a list. Begin with a type declaration.

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nonNegElts :: (Ord a, Num a) => [a] -> [a]
nonNegElts [] = []
nonNegElts (e:es)
    | e < 0      = nonNegElts es
    | otherwise = e:nonNegElts es
```

QUIZ: Filtering a list

Write a Haskell function to filter out the negative numbers from a list. Begin with a type declaration.

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nonNegElts :: (Ord a, Num a) => [a] -> [a]
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  | e < 0      = nonNegElts es
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```

We could have defined the type as `[Int] -> [Int]`, but then it would only work for ints; this will work for any kind of number.

QUIZ: Filtering a list

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nonNegElts :: (Ord a, Num a) => [a] -> [a]
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    | e < 0      = nonNegElts es
    | otherwise = e:nonNegElts es
```

We could have defined the type as `[Int] -> [Int]`, but then it would only work for ints; this will work for any kind of number.

We need to specify both `Num a` *and* `Ord a`, because `Nums` are not necessarily `Ord`.

Recursion vs iteration

Functional languages do not have language constructs for iteration. What imperative language programs do with iteration, functional language programs do with recursion.

For a programmer who has known nothing but imperative languages, the absence of iteration can seem like a crippling limitation.

In fact, it is not a limitation at all. Any loop can be implemented with recursion, but some recursions are difficult to implement with iteration.

There are several viewpoints to consider:

- How does this affect the process of writing code?
- How does this affect the reliability of the resulting code?
- How does this affect the productivity of the programmers?
- How does this affect the efficiency of the resulting code?

C version vs Haskell versions

The Haskell versions use lists instead of arrays, since in Haskell, lists are the natural representation.

With `-fwarn-incomplete-patterns`, Haskell will warn you that

- there may not be a strictly positive number in the list;
- there may not be a number greater than the target in the list,

and that these situations need to be handled.

The C compiler cannot generate such warnings. If the Haskell code operated on an array, the Haskell compiler couldn't either.

The Haskell versions give meaningful names to the jobs done by the loops.

Reliability

The names of the auxiliary functions should remind readers of their tasks.

These functions should be documented like other functions. The documentation should give the meaning of the arguments, and describe the relationship between the arguments and the return value.

This description should allow readers to construct a correctness argument for the function.

The imperative language equivalent of these function descriptions are *loop invariants*, but they are as rare as hen's teeth in real-world programs.

The act of writing down the information needed for a correctness argument gives programmers a chance to notice situations where the (implicit or explicit) correctness argument doesn't hold water.

The fact that such writing down occurs much more often with functional programs is one factor that tends to make them more reliable.

Productivity

Picking a meaningful name for each auxiliary function and writing down its documentation takes time.

This cost imposed on the original author of the code is repaid manyfold when

- other members of the team read the code, and find it easier to read and understand,
- the *original author* reads the code much later, and finds it easier to read and understand.

Properly documented functions, whether created as auxiliary functions or not, can be reused. Separating the code of a loop out into a function allows the code of that function to be reused, requiring less code to be written overall.

In fact, modern functional languages come with large libraries of prewritten useful functions.

Efficiency

The recursive version of e.g. `search_bst` will allocate one stack frame for each node of the tree it traverses, while the iterative version will just allocate one stack frame period.

The recursive version will therefore be less efficient, since it needs to allocate, fill in and then later deallocate more stack frames.

The recursive version will also need more stack space. This should not be a problem for `search_bst`, but the recursive versions of some other functions can run out of stack space.

However, compilers for declarative languages put huge emphasis on the optimization of recursive code. In many cases, they can take a recursive algorithm in their source language (e.g. Haskell), and generate iterative code in their target language.

Efficiency in general

Overall, programs in declarative languages are typically slower than they would be if written in C. Depending on which declarative language and which language implementation you are talking about, and on what the program does, the slowdown can range from a few percent to huge integer factors, such as 10% to a factor of a 100.

However, popular languages like Python and Javascript typically also yield significantly slower programs than C. In fact, their programs will typically be significantly *slower* than corresponding Haskell programs.

In general, the higher the level of a programming language (the more it does for the programmer), the slower its programs will be on average. The price of C's speed is the need to handle all the details yourself.

The right point on the productivity vs efficiency tradeoff continuum depends on the project (and component of the project).

QUIZ: Sublists

Write a Haskell function

```
sublists :: [a] -> [[a]]
```

that returns a list of all the “sublists” of a list. A list a is a sublist of a list b iff every element of a appears in b in the same order, though some elements of b may be omitted from a . It does not matter in what order the sublists appear in the resulting list.

For example:

```
sublists "ABC" = ["ABC","AB","AC","A","BC","B","C",""]  
sublists "BC"  = ["BC","B","C",""]
```

QUIZ: Sublists

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For example:

```
sublists "ABC" = ["ABC","AB","AC","A","BC","B","C",""]  
sublists "BC"  = ["BC","B","C",""]
```

This is a bit tricky. We'll discuss how to approach this problem before presenting a solution.

Declarative thinking

Some problems are difficult to approach imperatively, and are much easier to think about declaratively.

The imperative approach is procedural: we devise a way to solve the problem step by step. As an afterthought we may think about grouping the steps into chunks (methods, procedures, functions, etc.)

The declarative approach breaks down the problem into chunks (functions), assembling the results of the chunks to construct the result.

You must be careful in imperative languages, because the chunks may not compose due to side-effects. The chunks always compose in purely declarative languages.

Recursive thinking

One especially useful approach is recursive thinking: use the function you are defining as one of the chunks.

To take this approach:

- 1 Determine how to produce the result for the whole problem from the results for the parts of the problem (recursive case);
- 2 Determine the solution for the smallest part of the input (base case).

Keep in mind the specification of the problem, but it also helps to think of concrete examples.

For lists, (1) usually means generating the result for the whole list from the list head and the result for the tail; (2) usually means the result for the empty list.

This works perfectly well in most imperative languages, if you're careful to ensure your function composes. But it takes practice to think this way.

QUIZ: Sublists again

Write a Haskell function:

```
sublists :: [a] -> [[a]]
sublists "ABC" = ["ABC","AB","AC","A","BC","B","C",""]
sublists "BC" = ["BC","B","C",""]
```

How can we produce ["ABC","AB","AC","A","BC","B","C",""] from ["BC","B","C",""]?

It is just ["BC","B","C",""] with A added to the front of each string, followed by ["BC","B","C",""] itself.

For the base case, the only sublist of [] is [] itself, so the list of sublists of [] is [[]].

QUIZ: Sublists again

Write a Haskell function:

```
sublists :: [a] -> [[a]]
```

that returns a list of all the “sublists” of a list. A list a is a sublist of a list b iff every element of a appears in b in the same order, though some elements of b may be omitted from a . It does not matter in what order the sublists appear in the resulting list.

QUIZ: Sublists again

Write a Haskell function:

```
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```

that returns a list of all the “sublists” of a list. A list a is a sublist of a list b iff every element of a appears in b in the same order, though some elements of b may be omitted from a . It does not matter in what order the sublists appear in the resulting list.

```
sublists [] = [[]]
sublists (e:es) = addToEach e restSeqs ++ restSeqs
  where restSeqs = sublists es
```

```
addToEach :: a -> [[a]] -> [[a]]
addToEach h [] = []
addToEach h (t:ts) = (h:t):addToEach h ts
```


Immutable data structures

In declarative languages, data structures are *immutable*: once created, they cannot be changed. So what do you do if you *do* need to update a data structure?

You create another version of the data structure, one which has the change you want to make, and use *that* version from then on.

However, if you want to, you can hang onto the old version as well. You will definitely want to do so if some part of the system still needs the old version (in which case imperative code must also make a modified copy).

The old version can also be used

- because both old and new are needed, as in `sublists`
- to implement undo
- to gather statistics, e.g. about how the size of a data structure changes over time

Updating a BST

```
insert_bst :: Tree -> String -> Int -> Tree
insert_bst Leaf ik iv = Node ik iv Leaf Leaf
insert_bst (Node k v l r) ik iv =
    if ik == k then
        Node ik iv l r
    else if ik < k then
        Node k v (insert_bst l ik iv) r
    else
        Node k v l (insert_bst r ik iv)
```

Note that *all* of the code of this function is concerned with the job at hand; there is no code concerned with memory management.

In Haskell, as in Java, memory management is automatic. Any unreachable cells of memory are recovered by the garbage collector.

The University of Melbourne
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Declarative Programming

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Section 6

Polymorphism

Polymorphic types

Our definition of the tree type so far was this:

```
data Tree = Leaf | Node String Int Tree Tree
```

This type assumes that the keys are strings and the values are integers. However, the functions we have written to handle trees (`countnodes` and `search_bst`) *do not really care about the types of the keys and values*.

We could also define trees like this:

```
data Tree k v = Leaf | Node k v (Tree k v) (Tree k v)
```

In this case, `k` and `v` are *type variables*, variables standing in for the types of keys and values, and `Tree` is a *type constructor*, which constructs a new type from two other types.

Using polymorphic types: countnodes

With the old, *monomorphic* definition of `Tree`, the type declaration or *signature* of `countnodes` was:

```
countnodes :: Tree -> Int
```

With the new, *polymorphic* definition of `Tree`, it will be

```
countnodes :: Tree k v -> Int
```

Regardless of the types of the keys and values in the tree, `countnodes` will count the number of nodes in it.

The *exact same code* works in these cases.

Using polymorphic types: `search_bst`

`countnodes` does not touch keys or values, but `search_bst` does perform some operations on keys. Replacing

```
search_bst :: Tree -> String -> Maybe Int
```

with

```
search_bst :: Tree k v -> k -> Maybe v
```

will not work; it will yield an error message.

The reason is that `search_bst` contains these two tests:

- a comparison for equality: `sk == k`, and
- a comparison for order: `sk < k`.

Comparing values for equality and order

Some types cannot be compared for equality. For example, two functions should be considered equal if for all sets of input argument values, they compute the same result. Unfortunately, it has been proven that testing whether two functions are equal is *undecidable*. This means that building an algorithm that is guaranteed to decide in finite time whether two functions are equal is *impossible*.

Some types that can be compared for equality cannot be compared for order. Consider a set of integers. It is obvious that $\{1, 5\}$ is not equal to $\{2, 4\}$, but using the standard method of set comparison (set inclusion), they are otherwise incomparable; neither can be said to be greater than the other.

Eq and Ord

In Haskell,

- comparison for equality can only be done on values of types that belong to the type class `Eq`, while
- comparison for order can only be done on values of types that belong to the type class `Ord`.

Membership of `Ord` implies membership of `Eq`, but not vice versa.

The declaration of `search_bst` should be this:

```
search_bst :: Ord k => Tree k v -> k -> Maybe v
```

The construct `Ord k =>` is a type class constraint; it says `search_bst` requires whatever type `k` stands for to be in `Ord`. This guarantees its membership of `Eq` as well.

QUIZ: Polymorphic list insertion

A monomorphic function to insert an element into a sorted list of `Ints`, with type declaration, looks like this:

```
listInsert :: Int -> [Int] -> [Int]
listInsert elt [] = [elt]
listInsert elt (e:es)
  | e < elt    = e:listInsert elt es
  | otherwise = elt:e:es
```

Write a type declaration for a polymorphic version (that inserts into a sorted list of *anything*).

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  | otherwise = elt:e:es
```

Write a type declaration for a polymorphic version (that inserts into a sorted list of *anything*).

```
listInsert :: Ord a => a -> [a] -> [a]
```

QUIZ: Polymorphic list insertion

A monomorphic function to insert an element into a sorted list of `Ints`, with type declaration, looks like this:

```
listInsert :: Int -> [Int] -> [Int]
listInsert elt [] = [elt]
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```

Write a type declaration for a polymorphic version (that inserts into a sorted list of *anything*).

```
listInsert :: Ord a => a -> [a] -> [a]
```

What modifications to the function definition are necessary to make it polymorphic?

QUIZ: Polymorphic list insertion

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listInsert elt [] = [elt]
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    | otherwise = elt:e:es
```

Write a type declaration for a polymorphic version (that inserts into a sorted list of *anything*).

```
listInsert :: Ord a => a -> [a] -> [a]
```

What modifications to the function definition are necessary to make it polymorphic?

None.

Deriving membership automatically

```
data Suit = Club | Diamond | Heart | Spade
    deriving (Show, Eq, Ord)
data Card = Card Suit Rank
    deriving (Show, Eq, Ord)
```

The automatically created comparison function takes the order of data constructors from the order in the declaration itself: a constructor listed earlier is less than a constructor listed later (e.g. `Club < Diamond`).

If the two values being compared have the same top level data constructor, the automatically created comparison function compares their arguments in turn, from left to right. This means the argument types must also be instances of `Ord`. If the corresponding arguments are not equal, the comparison stops (e.g. `Card Club Ace < Card Spade Jack`); if the corresponding argument are equal, it goes on to the next argument, if there is one (e.g. `Card Spade Ace > Card Spade Jack`). This is called *lexicographic* ordering.

Recursive vs nonrecursive types

```
data Tree = Leaf | Node String Int Tree Tree
data Card = Card Suit Rank
```

Tree is a recursive type because some of its data constructors have arguments of type Tree.

Card is a non-recursive type because none of its data constructors have an arguments of type Card.

A recursive type needs a nonrecursive alternative, because without one, all values of the type would have infinite size.

Mutually recursive types

Some types are recursive but not *directly* recursive.

```
data BoolExpr
  = BoolConst Bool
  | BoolOp BoolOp BoolExpr BoolExpr
  | CompOp CompOp IntExpr IntExpr
data IntExpr
  = IntConst Int
  | IntOp IntOp IntExpr IntExpr
  | IntIfThenElse BoolExpr IntExpr IntExpr
```

In a mutually recursive set of types, it is enough for one of the types to have a nonrecursive alternative.

These types represent Boolean- and integer-valued expressions in a program. They must be mutually recursive because comparison of integers returns a Boolean and integer-valued conditionals use a Boolean.

Structural induction

Code that follows the shape of a nonrecursive type tends to be simple.
Code that follows the shape of a directly or mutually recursive type tends to be more interesting.

Consider a recursive type with one nonrecursive data constructor (like Leaf in Tree) and one recursive data constructor (like Node in Tree).
A function that follows the structure of this type will typically have

- an equation for the nonrecursive data constructor, and
- an equation for the recursive data constructor.

Typically, recursive calls will occur only in the second equation, and the switched-on argument in the recursive call will be *strictly smaller* than the corresponding argument in the left hand side of the equation.

Proof by induction

You can view the function definition's structure as the outline of a correctness argument.

The argument is a proof by induction on n , the number of data constructors of the switched-on type in the switched-on argument.

- *Base case:* If $n = 1$, then the applicable equation is the base case. If the first equation is correct, then the function correctly handles the case where $n = 1$.
- *Induction step:* Assume the induction hypothesis: the function correctly handles all cases where $n \leq k$. This hypothesis implies that all the recursive calls are correct. If the second equation is correct, then the function correctly handles all cases where $n \leq k + 1$.

The base case and the induction step together imply that the function correctly handles all inputs.

Formality

If you want, you can use these kinds of arguments to formally *prove* the correctness of functions, and of entire functional programs.

This typically requires a formal specification of the expected relationship between each function's arguments and its result.

Typical software development projects do not do formal proofs of correctness, regardless of what kind of language their code is written in.

However, projects using functional languages do tend to use *informal* correctness arguments slightly more often.

The support for this provided by the original programmer usually consists of nothing more than a natural language description of the criterion of correctness of each function. Readers who want a correctness argument can then construct it for themselves from this and the structure of the code.

Structural induction for more complex types

If a type has nr nonrecursive data constructors and r recursive data constructors, what happens when $nr > 1$ or $r > 1$, like `BoolExpr` and `IntExpr`?

You can do structural induction on such types as well.

Such functions will typically have nr nonrecursive equations and r recursive equations, but not always. Sometimes you need more than one equation to handle a constructor, and sometimes one equation can handle more than one constructor. For example, sometimes all base cases need the same treatment.

Picking the right representation of the data is important in every program, but when the structure of the code follows the structure of the data, it is particularly important.

QUIZ: Writing an Interpreter

```

data BoolExpr
  = BoolConst Bool
  | BoolOp BoolOp BoolExpr BoolExpr
  | CompOp CompOp IntExpr IntExpr
data IntExpr
  = IntConst Int
  | IntOp IntOp IntExpr IntExpr
  | IntIfThenElse BoolExpr IntExpr IntExpr
data BoolOp = And
data CompOp = LessThan
data IntOp = Plus | Times

```

Write functions to *evaluate* a BoolExpr and an IntExpr. *E.g.*,
`intExprValue (IntOp Times (IntConst 6) (IntConst 7)) = 42`.
 Include type declarations for your functions.

QUIZ: Writing an Interpreter

```
boolExprValue :: BoolExpr -> Bool
boolExprValue (BoolConst b) = b
boolExprValue (BoolOp And e1 e2)
    = boolExprValue e1 && boolExprValue e2
boolExprValue (CompOp LessThan i1 i2)
    = intExprValue i1 < intExprValue i2
```

```
intExprValue :: IntExpr -> Int
intExprValue (IntConst i) = i
intExprValue (IntOp Plus i1 i2)
    = intExprValue i1 + intExprValue i2
intExprValue (IntOp Times i1 i2)
    = intExprValue i1 * intExprValue i2
intExprValue (IntIfThenElse b i1 i2)
    = if boolExprValue b
        then intExprValue i1
        else intExprValue i2
```

Let clauses and where clauses

```
assoc_list_to_bst ((hk, hv):kvs) =  
  let t0 = assoc_list_to_bst kvs  
  in insert_bst t0 hk hv
```

A `let` clause `let name = expr in mainexpr` introduces a name for a value to be used in the main expression.

```
assoc_list_to_bst ((hk, hv):kvs) = insert_bst t0 hk hv  
  where t0 = assoc_list_to_bst kvs
```

A `where` clause `mainexpr where name = expr` has the same meaning, but has the definition of the name *after* the main expression.

Which one you want to use depends on where you want to put the emphasis.

But you can only use `where` clauses *at the top level of a function*, while you can use a `let` for any expression.

Defining multiple names

You can define multiple names with a single `let` or `where` clause:

```
let name1 = expr1
    name2 = expr2
in mainexpr
```

```
mainexpr
where
    name1 = expr1
    name2 = expr2
```

The scope of each name includes the right hand sides of the definitions of the following names, as well as the main expression, *unless* one of the later definitions defines the same name, in which case the original definition is *shadowed* and not visible from then on.

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Section 7

Higher order functions

First vs higher order

First order values are data.

Second order values are functions whose arguments and results are first order values.

Third order values are functions whose arguments and results are first or second order values.

In general, n th order values are functions whose arguments and results are values of any order from first up to $n - 1$.

Values that belong to an order higher than first are higher order values.

Java 8, released mid-2014, supports higher order programming. C also supports it, if you work at it. Higher order programming is a central aspect of Haskell, often allowing Haskell programmers to avoid writing recursive functions.

A higher order function in C

```
IntList filter(Bool (*f)(int), IntList list)
{
    IntList filtered_tail, new_list;

    if (list == NULL) {
        return NULL;
    } else {
        filtered_tail = filter(f, list->tail);
        if ((*f)(list->head)) {
            new_list = checked_malloc(sizeof(*new_list));
            new_list->head = list->head;
            new_list->tail = filtered_tail;
            return new_list;
        } else {
            return filtered_tail;
        }
    }
}
```

A higher order function in Haskell

Haskell's syntax for passing a function as an argument is much simpler than C's syntax. All you need to do is wrap the type of the higher order argument in parentheses to tell Haskell it is one argument.

```
filter :: (a -> Bool) -> [a] -> [a]
filter _ [] = []
filter f (x:xs) =
    if f x then x:fxs else fxs
    where fxs = filter f xs
```

Even though it is significantly shorter, this function is actually more general than the C version, since it is polymorphic, and thus works for lists with *any* type of element.

`filter` is defined in the Haskell prelude.

Using higher order functions

You can call filter like this:

```
... filter is_even [1, 2, 3, 4] ...  
... filter is_pos [0, -1, 1, -2, 2] ...  
... filter is_long ["a", "abc", "abcde"] ...
```

given definitions like this:

```
is_even :: Int -> Bool  
is_even x = if (mod x 2) == 0 then True else False
```

```
is_pos :: Int -> Bool  
is_pos x = if x > 0 then True else False
```

```
is_long :: String -> Bool  
is_long x = if length x > 3 then True else False
```

QUIZ: Using higher order functions

Would these definitions work?

```
is_even :: Int -> Bool  
is_even x = (mod x 2) == 0
```

```
is_pos :: Int -> Bool  
is_pos x = x > 0
```

```
is_long :: String -> Bool  
is_long x = length x > 3
```

QUIZ: Using higher order functions

Would these definitions work?

```
is_even :: Int -> Bool  
is_even x = (mod x 2) == 0
```

```
is_pos :: Int -> Bool  
is_pos x = x > 0
```

```
is_long :: String -> Bool  
is_long x = length x > 3
```

Yes.

Backquote

Modulo is a built-in infix operator in many languages. For example, in C or Java, 5 modulo 2 would be written `5 % 2`.

Haskell uses `mod` for the modulo operation, but Haskell allows you to make any function an infix operator by surrounding the function name with backquotes (backticks, written `'`).

So a friendlier way to write the `is_even` function would be:

```
is_even :: Int -> Bool
is_even x = x `mod` 2 == 0
```

Operators written with backquotes have high precedence and associate to the left.

It's also possible to explicitly declare non-alphanumeric operators, and specify their associativity and fixity, but this feature should be used sparingly.

Anonymous functions

In some cases, the only thing you need a function for is to pass as an argument to a higher order function like `filter`. In such cases, readers may find it more convenient if the call contained the *definition* of the function, not its name.

In Haskell, anonymous functions are defined by *lambda expressions*, and you use them like this.

```
... filter (\x -> x `mod` 2 == 0) [1, 2, 3, 4] ...  
... filter (\s -> length s > 3) ["a", "abc", "abcde"] ...
```

This notation is based on the lambda calculus, the basis of functional programming.

In the lambda calculus, each argument is preceded by a lambda, and the argument list is followed by a dot and the expression that is the function body. For example, the function that adds together its two arguments is written as $\lambda a.\lambda b.a + b$.

Map

`map` is one of the most frequently used Haskell functions. (It is defined in the Haskell prelude.) Given a function and a list, `map` applies the function to every member of the list.

```
map :: (a -> b) -> [a] -> [b]
map _ [] = []
map f (x:xs) = (f x):(map f xs)
```

Many things that an imperative programmer would do with a loop, a functional programmer would do with a call to `map`. An example:

```
get_names :: [Customer] -> [String]
get_names customers = map customer_name customers
```

This assumes that `customer_name` is a function whose type is `Customer -> String`.

Partial application

Given a function with n arguments, *partially applying* that function means giving it its first k arguments, where $k < n$.

The result of the partial application is a *closure* that records the identity of the function and the values of those k arguments.

This closure behaves as a function with $n - k$ arguments. A call of the closure leads to a call of the original function with both sets of arguments.

```
is_longer :: Int -> String -> Bool  
is_longer limit x = length x > limit
```

```
... filter (is_longer 4) ["ab", "abcd", "abcdef"] ...
```

In this case, the function `is_longer` takes two arguments. The expression `is_longer 4` partially applies this function, and creates a closure which records 4 as the value of the first argument.

Calling a closure: an example

```
filter f (x:xs) =  
  if f x then x:fxs else fxs  
  where fxs = filter f xs
```

```
... filter (is_longer 4) ["ab", "abcd", "abcdef"] ...
```

In this case, the code of `filter` will call `is_longer` three times:

- `is_longer 4 "ab"`
- `is_longer 4 "abcd"`
- `is_longer 4 "abcdef"`

Each of these calls comes from the higher order call `f x` in `filter`. In this case `f` represents the closure `is_longer 4`. In each case, the first argument comes from the closure, with the second being the value of `x`.

QUIZ: get_names

Would Haskell accept `get_names2` as a definition equivalent to `get_names1`?

```
get_names1 :: [Customer] -> [String]
get_names1 customers = map customer_name customers
```

```
get_names2 :: [Customer] -> [String]
get_names2 = map customer_name
```

QUIZ: `get_names`

Would Haskell accept `get_names2` as a definition equivalent to `get_names1`?

```
get_names1 :: [Customer] -> [String]
get_names1 customers = map customer_name customers
```

```
get_names2 :: [Customer] -> [String]
get_names2 = map customer_name
```

The first says “for all values of `customer`, the value `get_names1 customers` is equal to the value `map customer_name customers`”.

QUIZ: `get_names`

Would Haskell accept `get_names2` as a definition equivalent to `get_names1`?

```
get_names1 :: [Customer] -> [String]
get_names1 customers = map customer_name customers
```

```
get_names2 :: [Customer] -> [String]
get_names2 = map customer_name
```

The first says “for all values of `customer`, the value `get_names1 customers` is equal to the value `map customer_name customers`”.

The second says “the function `get_names2` is equal to the function `map customer_name`”.

QUIZ: `get_names`

Would Haskell accept `get_names2` as a definition equivalent to `get_names1`?

```
get_names1 :: [Customer] -> [String]
get_names1 customers = map customer_name customers
```

```
get_names2 :: [Customer] -> [String]
get_names2 = map customer_name
```

The first says “for all values of `customer`, the value `get_names1 customers` is equal to the value `map customer_name customers`”.

The second says “the function `get_names2` is equal to the function `map customer_name`”.

Since two functions are equal if and only if they return the same value for all possible combinations of their arguments, the two definitions are indeed equivalent, and Haskell recognizes this.

Operators and sections

If you enclose an infix operator in parentheses, you can partially apply it by enclosing its left or right operand with it; this is called a *section*.

```
Prelude> map (*3) [1, 2, 3]  
[3,6,9]
```

You can use section notation to partially apply *either* of its arguments.

```
Prelude> map (5 'mod') [3, 4, 5, 6, 7]  
[2,1,0,5,5]  
Prelude> map ('mod' 3) [3, 4, 5, 6, 7]  
[0,1,2,0,1]
```


QUIZ: Using higher order functions

What would this return?

```
filter (<3) [1,2,3,4,5,6,7]
```

QUIZ: Using higher order functions

What would this return?

```
filter (<3) [1,2,3,4,5,6,7]
```

[1,2]

QUIZ: Using higher order functions

What would this return?

```
filter (<3) [1,2,3,4,5,6,7]
```

[1,2]

How about this?

```
filter (3<) [1,2,3,4,5,6,7]
```

QUIZ: Using higher order functions

What would this return?

```
filter (<3) [1,2,3,4,5,6,7]
```

[1,2]

How about this?

```
filter (3<) [1,2,3,4,5,6,7]
```

[4,5,6,7]

Types for partial application

In most languages, the type of a function with n arguments would be something like:

$$f :: (at1, at2, \dots atn) \rightarrow rt$$

where $at1$, $at2$ etc are the argument types, $(at1, at2, \dots atn)$ is the type of a tuple containing all the arguments, and rt is the result type.

To allow the function to be partially applied by supplying the first argument, you need a function with a different type:

$$f :: at1 \rightarrow ((at2, \dots atn) \rightarrow rt)$$

This function takes a single value of type $at1$, and returns as its result another function, which is of type $(at2, \dots atn) \rightarrow rt$.

Currying

You can keep transforming the function type until every single argument is supplied separately:

```
f :: at1 -> (at2 -> (at3 -> ... (atn -> rt)))
```

The transformation from a function type in which all arguments are supplied together to a function type in which the arguments are supplied one by one is called *currying*.

In Haskell, *all* function types are curried. This is why the syntax for function types is what it is. The arrow that makes function types is right associative, so the second declaration below just shows explicitly the parenthesization implicit in the first:

```
is_longer :: Int -> String -> Bool  
is_longer :: Int -> (String -> Bool)
```

Functions with all their arguments

Given a function with curried argument types, you can supply the function its first argument, then its second, then its third, and so on. What happens when you have supplied them all?

```
is_longer 3 ["a", "ab", "abc", "abcd"]
```

There are two things you can get:

- a closure that contains all the function's arguments, or
- the result of the evaluation of the function.

In C and in most other languages, these would be very different, but in Haskell, as we will see later, they are equivalent.

Composing functions

Any function that makes a higher order function call or creates a closure (e.g. by partially applying another function) is a second order function. This means that both `filter` and its callers are second order functions.

`filter` has a piece of data as an argument (the list to filter) as well as a function (the filtering function). Some functions do not take *any* piece of data as arguments; *all* their arguments are functions.

The builtin operator `'.'` composes two functions. The expression `f . g` represents a function which first calls `g`, and then invokes `f` on the result:

$$(f . g) x = f (g x)$$

If the type of `x` is represented by the type variable `a`, then the type of `g` must be `a -> b` for some `b`, and the type of `f` must be `b -> c` for some `c`. The type of `.` itself is therefore `(b -> c) -> (a -> b) -> (a -> c)`.

Composing functions: some examples

Suppose you already have a function that sorts a list and a function that returns the head of a list, if it has one. You can then compute the minimum of the list like this:

```
min = head . sort
```

If you also have a function that reverses a list, you can also compute the maximum with very little extra code:

```
max = head . reverse . sort
```

This shows that functions created by composition, such as `reverse . sort`, can themselves be part of further compositions.

This style of programming is sometimes called *point-free style*, though *value-free style* would be a better description, since its distinguishing characteristic is the absence of variables representing (first order) values.

Composition as sequence

Function composition is one way to express a sequence of operations. Consider the function composition $\text{step3f} \cdot \text{step2f} \cdot \text{step1f}$.

- 1 You start with the input, x .
- 2 You compute $\text{step1f } x$.
- 3 You compute $\text{step2f } (\text{step1f } x)$.
- 4 You compute $\text{step3f } (\text{step2f } (\text{step1f } x))$.

This idea is the basis of *monads*, which is the mechanism Haskell uses to do input/output.

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Declarative Programming

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Section 8

Functional design patterns

Higher order programming

Higher order programming is widely used by functional programmers. Its advantages include

- code reuse,
- a higher level of abstraction, and
- a set of canned solutions to frequently encountered problems.

In programs written by programmers who do not use higher order programming, you frequently find pieces of code that have the same structure but slot different pieces of code into that structure.

Such code typically qualifies as an instance of the copy-and-paste programming *antipattern*, a pattern that programmers should strive to avoid.

Folds

We have already seen the functions `map` and `filter`, which operate on and transform lists.

The other class of popular higher order functions on lists are the *reduction* operations, which reduce a list to a single value.

The usual reduction operations are *folds*. There are three main folds: left, right and balanced.

left $(((((I \odot X_1) \odot X_2) \dots) \odot X_n)$

right $(X_1 \odot (X_2 \odot (\dots (X_n \odot I))))$

balanced $((X_1 \odot X_2) \odot (X_3 \odot X_4)) \odot \dots$

Here \odot denotes a binary function, the folding operation, and I denotes the identity element of that operation. (The balanced fold also needs the identity element in case the list is empty.)

Foldl

```
foldl :: (v -> e -> v) -> v -> [e] -> v
```

```
foldl _ base [] = base
```

```
foldl f base (x:xs) =
```

```
    let newbase = f base x in
```

```
    foldl f newbase xs
```

```
suml :: Num a => [a] -> a
```

```
suml = foldl (+) 0
```

```
productl :: Num a => [a] -> a
```

```
productl = foldl (*) 1
```

```
concatl :: [[a]] -> [a]
```

```
concatl = foldl (++) []
```

Foldr

```
foldr :: (e -> v -> v) -> v -> [e] -> v
foldr _ base [] = base
foldr f base (x:xs) =
    let fxs = foldr f base xs in
    f x fxs
```

```
sumr = foldr (+) 0
productr = foldr (*) 1
concatr = foldr (++) []
```

You can define sum, product and concatenation in terms of both `foldl` and `foldr` because addition and multiplication on integers, and list append, are all associative operations.

Balanced fold

```
balanced_fold :: (e -> e -> e) -> e -> [e] -> e
balanced_fold _ b [] = b
balanced_fold _ _ (x:[]) = x
balanced_fold f b l@(_:_:_) =
  let
    len = length l
    (half1, half2) = splitAt (div len 2) l
    value1 = balanced_fold f b half1
    value2 = balanced_fold f b half2
  in
    f value1 value2
```

`splitAt n l` returns a pair of the first `n` elements of `l` and the rest of `l`.
It is defined in the standard Prelude.

QUIZ: Hypotenuse

Here is a definition of hypotenuse:

```
hypotenuse sides = sqrt (sum (map (^2) sides))
```

Rewrite the hypotenuse function in point-free style.

QUIZ: Hypotenuse

Here is a definition of hypotenuse:

```
hypotenuse sides = sqrt (sum (map (^2) sides))
```

Rewrite the hypotenuse function in point-free style.

```
hypotenuse = sqrt . sum . map (^2)
```

More folds

The Haskell prelude defines `sum`, `product`, and `concat`.

For maximum and minimum, there is no identity element, and it is an error if the list is empty. For such cases, the Haskell Prelude defines:

```
foldl1 :: (a -> a -> a) -> [a] -> a
```

```
foldr1 :: (a -> a -> a) -> [a] -> a
```

that compute

$$\text{foldl1 } o [X_1, X_2, \dots, X_n] = ((X_1 \odot X_2) \dots \odot X_n)$$

$$\text{foldr1 } o [X_1, X_2, \dots, X_n] = (X_1 \odot (X_2 \odot \dots X_n))$$

```
maximum = foldr1 max
```

```
minimum = foldr1 min
```

You could equally well use `foldl1` for these.

Folds are really powerful

You can compute the length of a list by summing 1 for each element, instead of the element itself. So if we can define a function that takes anything and returns 1, together with (+) we can use fold to define length.

```
const :: a -> b -> a
const a b = a
```

```
length = foldr ((+) . const 1) 0
```

You can map over a list with foldr:

```
map f = foldr ((:) . f) []
```

Fold can reverse a list

If we had a “backwards” $(:)$ operation, call it `snoc`, then `foldl` could reverse a list:

$$\text{reverse } [X_1, X_2, \dots X_n] = [] \text{ `snoc` } X_1 \text{ `snoc` } X_2 \dots \text{ `snoc` } X_n$$

```
snoc :: [a] -> a -> [a]
```

```
snoc tl hd = hd:tl
```

```
reverse = foldl snoc []
```

But the Haskell Prelude defines a function to flip the arguments of a binary function:

```
flip :: (a -> b -> c) -> b -> a -> c
```

```
flip f x y = f y x
```

```
reverse = foldl (flip (:)) []
```

Foldable

But what about types other than lists? Can we fold over them?

Actually, we can:

```
Prelude> sum (Just 7)
7
Prelude> sum Nothing
0
```

In fact we can fold over any type in the type class `Foldable`

```
Prelude> :t foldr
foldr :: Foldable t => (a -> b -> b) -> b -> t a -> b
```

We can declare our own types to be instances of `Foldable` by defining `foldr` for our type; then many standard functions, such as `length`, `sum`, etc. will work on that type, too.

List comprehensions

Haskell has special syntax for one class of higher order operations. These two implementations of quicksort do the same thing, with the first using conventional higher order code, and the second using list comprehensions:

```
qs1 [] = []
qs1 (x:xs) = qs1 littles ++ [x] ++ qs1 bigs
  where
    littles = filter (<x) xs
    bigs     = filter (>=x) xs
```

```
qs2 [] = []
qs2 (x:xs) = qs2 littles ++ [x] ++ qs2 bigs
  where
    littles = [l | l <- xs, l < x]
    bigs     = [b | b <- xs, b >= x]
```

List comprehensions

List comprehensions can be used for things other than filtering a *single* list.

In general, a list comprehension consists of

- a template (an expression, which is often just a variable)
- one or more generators (each of the form `var <- list`),
- zero or more tests (boolean expressions),
- zero or more `let` expressions defining local variables.

Some more examples:

```
columns = "abcdefgh"
rows    = "12345678"
chess_squares = [[c, r] | c <- columns, r <- rows]
```

```
pairs = [(a, b) | a <- [1, 2, 3], b <- [1, 2, 3]]
nums  = [10*a+b | a <- [1, 2, 3], b <- [1, 2, 3]]
```


Traversing HTML documents

Types to represent HTML documents:

```
type HTML = [HTML_element]
data HTML_element
  = HTML_text String
  | HTML_font Font_tag HTML
  | HTML_p HTML
data Font_tag = Font_tag (Maybe Int)
                                   (Maybe String)
                                   (Maybe Font_color)
data Font_color
  = Colour_name String
  | Hex Int
  | RGB Int Int Int
```

Collecting font sizes

```

font_sizes_in_html :: HTML -> Set Int -> Set Int
font_sizes_in_html elements sizes =
    foldr font_sizes_in_elt sizes elements

font_sizes_in_elt :: HTML_element -> Set Int -> Set Int
font_sizes_in_elt (HTML_text _) sizes = sizes
font_sizes_in_elt (HTML_font font_tag html) sizes =
    let
        Font_tag maybe_size _ _ = font_tag
        newsizes = case maybe_size of
            Nothing -> sizes
            Just fontsize -> Data.Set.insert fontsize sizes
    in
        font_sizes_in_html html newsizes
font_sizes_in_elt (HTML_p html) sizes =
    font_sizes_in_html html sizes

```

Collecting font names

```

font_names_in_html :: HTML -> Set String -> Set String
font_names_in_html elements names =
    foldr font_names_in_elt names elements

font_names_in_elt :: HTML_element -> Set String -> Set String
font_names_in_elt (HTML_text _) names = names
font_names_in_elt (HTML_font font_tag html) names =
    let
        Font_tag _ maybe_name _ = font_tag
        newnames = case maybe_name of
            Nothing -> names
            Just fontname -> Data.Set.insert fontname names
    in
        font_names_in_html html newnames
font_names_in_elt (HTML_p html) names =
    font_names_in_html html names

```

Collecting any font information

```
font_stuff_in_html :: (Font_tag -> a -> a) -> HTML -> a -> a
font_stuff_in_html f elements stuff =
  foldr (font_stuff_in_elt f) stuff elements
```

```
font_stuff_in_elt :: (Font_tag -> a -> a) ->
  HTML_element -> a -> a
font_stuff_in_elt f (HTML_text _) stuff = stuff
font_stuff_in_elt f (HTML_font font_tag html) stuff =
  let newstuff = f font_tag stuff in
  font_stuff_in_html f html newstuff
font_stuff_in_elt f (HTML_p html) stuff =
  font_stuff_in_html f html stuff
```

Collecting font sizes again

```
font_sizes_in_html' :: HTML -> Set Int -> Set Int
font_sizes_in_html' html sizes =
    font_stuff_in_html accumulate_font_sizes html sizes

accumulate_font_sizes font_tag sizes =
    let Font_tag maybe_size _ _ = font_tag in
    case maybe_size of
        Nothing ->
            sizes
        Just fontsize ->
            Data.Set.insert fontsize sizes
```

Using the higher order version avoids duplicating the code that traverses the data structure. The benefit you get from this scales linearly with the complexity of the data structure being traversed.

QUIZ: Types

What is the type of the `map` function?

QUIZ: Types

What is the type of the `map` function?

$(a \rightarrow b) \rightarrow [a] \rightarrow [b]$

QUIZ: Types

What is the type of the `map` function?

$(a \rightarrow b) \rightarrow [a] \rightarrow [b]$

What is the type of the `(+1)` function?

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What is the type of the `(+1)` function?

$\text{Num } a \Rightarrow a \rightarrow a$

QUIZ: Types

What is the type of the `map` function?

$(a \rightarrow b) \rightarrow [a] \rightarrow [b]$

What is the type of the `(+1)` function?

$\text{Num } a \Rightarrow a \rightarrow a$

What is the type of the `map (+1)` function?

QUIZ: Types

What is the type of the `map` function?

$(a \rightarrow b) \rightarrow [a] \rightarrow [b]$

What is the type of the `(+1)` function?

$\text{Num } a \Rightarrow a \rightarrow a$

What is the type of the `map (+1)` function?

$\text{Num } a \Rightarrow [a] \rightarrow [a]$

Comparison to the visitor pattern

The function `font_stuff_in_html` does a job that is very similar to the job that the visitor design pattern would do in an object-oriented language like Java: they both traverse a data structure, invoking a function at one or more selected points in the code. However, there are also differences.

- In the Haskell version, the type of the higher order function makes it clear whether the code executed at the selected points just gathers information, or whether it modifies the traversed data structure. In Java, the invoked code is imperative, so it can do either.
- The Java version needs an `accept` method in every one of the classes that correspond to Haskell types in the data structure (in this case, `HTML` and `HTML_element`).
- In the Haskell version, the functions that implement the traversal can be (and typically are) next to each other. In Java, the corresponding methods have to be dispersed to the classes to which they belong.

Libraries vs frameworks

The way typical libraries work in any language (including C and Java as well as Haskell) is that code written by the programmer calls functions in the library.

In some cases, the library function is a higher order function, and thus it can call back a function supplied to it by the programmer.

Application frameworks are libraries but they are not typical libraries, because they are intended to be the top layer of a program.

When a program uses a framework, the framework is in control, and it calls functions written by the programmer when circumstances call for it.

For example, a framework for web servers would handle all communication with remote clients. It would itself implement the event loop that waits for the next query to arrive, and would invoke user code only to generate the response to each query.

Frameworks: libraries vs application generators

Frameworks in Haskell can be done like this, with `framework` simply being a library function:

```
main = framework plugin1 plugin2 plugin3
```

```
plugin1 = ...
```

```
plugin2 = ...
```

```
plugin3 = ...
```

This approach could also be used in other languages, since even C and Java support callbacks, though sometimes clumsily.

Unfortunately, many frameworks instead just generate code (in C, C#, Java, ...) that the programmer is then expected to modify. This approach throws abstraction out the window, and is much more error-prone.

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Section 9

Exploiting the type system

Representation of C programs in gcc

The gcc compiler has one main data type to represent the code being compiled. The node type is a giant union which has different fields for different kinds of entities. A node can represent, amongst other things,

- a data type,
- a variable,
- an expression or
- a statement.

Every link to another part of a program (such as the operands of an operator) is a pointer to a tree node of this can-represent-everything type.

When Stallman chose this design in the 1980s, he was a Lisp programmer. Lisp does not have a static type system, so the Blub paradox applies here in reverse: even C has a better static type system than Lisp. It's up to the programmer to design types to exploit the type system.

Representation of if-then-elses

For if-then-else expressions such as $(x > y) ? x : y$, the relevant union field is a structure that has an array of operands.

For if-then-else expressions, the array should have exactly three elements (the condition, the then part and the else part), and all three should be expressions.

This representation is subject to two main kinds of error.

- The array of operands could have the wrong number of operands.
- Any operand in the array could point to the wrong kind of tree node.

gcc has extensive infrastructure designed to detect these kinds of errors, but this infrastructure itself has three problems:

- it makes the source code harder to read and write;
- if enabled, it slows down gcc by about 5 to 15%; and
- it detects violations only at runtime.

Exploiting the type system

A well designed representation using algebraic types is not vulnerable to either kind of error, and is not subject any of those three kinds of problems.

```
data Expr
  = Const Const
  | Var String
  | Binop Binop Expr Expr
  | Unop Unop Expr
  | Call String [Expr]
  | ITE Expr Expr Expr
```

```
data Binop = Add | Sub | ...
```

```
data Unop = Uminus | ...
```

```
data Const = IntConst Int | FloatConst Double | ...
```

Generic lists in C

```
typedef struct generic_list List;
struct generic_list {
    void    *head;
    List    *tail;
};

...

List    *int_list;
List    *p;
int      item;

for (p = int_list; p != NULL; p = p->tail) {
    item = (int) p->head;
    ... do something with item ...
}
```

Type system expressiveness

Programmers who choose to use generic lists in a C program need only one list type and therefore one set of functions operating on lists.

The downside is that every loop over lists needs to cast the list element to the right type, and this cast is a frequent source of bugs.

The other alternative in a C program is to define and use a separate list type for every different type of item that the program wants to put into a list. This is type safe, but requires repeated duplication of the functions that operate on lists. Any bugs in those those functions must be fixed in each copy.

Haskell has a very expressive type system that is increasingly being copied by other languages. Some OO/procedural languages now support generics. A few such languages (Rust, Swift, Java 8) support *option types*, like Haskell's Maybe. No well-known such languages support full algebraic types.

Encoding semantic information in types

When designing data structures, programmers sometimes know that a given list of items may not be empty. For example, multimaps map a key to a *list* of values. If this list is empty, the key shouldn't be in the map.

```
-- obvious definition
```

```
type MultiMap k v = Map k [v]
```

```
-- semantically tighter definition
```

```
type MultiMap k v = Map k (OneOrMore v)
```

```
data OneOrMore v = OneOrMore v [v]
```

Using encoded semantic information

Suppose `map` is a `MultiMap`. Using the first definition, finding the maximum value associated with `key` requires code like this:

```
case lookup key map of
  [] -> error "empty item list"
  (head:tail) -> foldr max head tail
```

You need the test for `[]`, because sooner or later someone *will* create an empty value list, even if the documentation of `MultiMap` says they shouldn't.

If you are using the second definition, then you do not need to consider error handling at all.

```
let OneOrMore head tail = lookup key map in
  foldr max head tail
```

Units

One typical bug type in programs that manipulate physical measurements is unit confusion, such as adding 2 meters and 3 feet, and thinking the result is 5 meters. Mars Climate Orbiter was lost because of such a bug.

Such bugs can be prevented by wrapping the number representing the length in a data constructor giving its unit.

```
data Length = Meters Double
```

```
meters_to_length :: Double -> Length  
meters_to_length m = Meters m
```

```
feet_to_length :: Double -> Length  
feet_to_length f = Meters (f * 0.3048)
```

```
add_lengths :: Length -> Length -> Length  
add_lengths (Meters a) (Meters b) = Meters (a+b)
```

Different uses of one unit

Sometimes, you want to prevent confusion even between two kinds of quantities measured in the same units.

For example, many operating systems represent time as the number of seconds elapsed since a fixed *epoch*. For Unix, the epoch is 0:00am on 1 Jan 1970.

```
data Duration = Seconds Int
```

```
data Time = SecondsSinceEpoch Int
```

```
add_durations :: Duration -> Duration -> Duration
```

```
add_durations (Seconds a) (Seconds b) = Seconds (a+b)
```

```
add_duration_to_time :: Time -> Duration -> Time
```

```
add_duration_to_time (SecondsSinceEpoch sse) (Seconds t) =  
  SecondsSinceEpoch (sse + t)
```


Different units in one type

Sometimes, you cannot apply a fixed conversion rate between different units. In such applications, each operation may need to do conversion on demand at whatever rate is applicable at the time of its execution.

```
data Money
  = USD_dollars Double
  | AUD_dollars Double
  | GBP_pounds Double
```

For financial applications, using Doubles would not be a good idea, since accounting rules that precede the use of computers specify rounding methods (e.g. for interest calculations) that binary floating point numbers do not satisfy.

One workaround is to use fixed-point numbers, such as integers in which 1 represents not one dollar, but one one-thousandth of one cent.

Fat lists

A potential problem with lists is their memory consumption, which is typically two words (each of 32 or 64 bits) per element: one for the head element, and one for the tail, which is typically represented as a pointer to the next node, if any. The second word is overhead.

Using *fat lists*, in which a node can contain more than one item, can amortize this overhead over those items. In this example, we store three elements for each next pointer, and keep the leftover elements at the *front* of the fat list. This allows an element added to the front of the fat list to be combined with the left over elements.

Fat lists

The fat list type:

```
data FatList a
  = FCons0 (FatRest a)
  | FCons1 a (FatRest a)
  | FCons2 a a (FatRest a)
  deriving (Show, Eq)

data FatRest a
  = FatNil
  | FatRest a a a (FatRest a)
  deriving (Show, Eq)
```

This type ensures that all but one nodes in a fat list will contain exactly three elements.

Using fat lists

One price for using fat lists is that functions defined for fat lists are much more complex than for standard lists.

```
length :: [a] -> Int
length [] = 0
length (_:lst) = 1 + length lst
```

```
fatLength :: FatList a -> Int
fatLength (FCons0 fr)      = fatLength' fr
fatLength (FCons1 _ fr)    = 1 + fatLength' fr
fatLength (FCons2 _ _ fr) = 2 + fatLength' fr
```

```
fatLength' :: FatRest a -> Int
fatLength' FatNil          = 0
fatLength' (FatRest _ _ _ fr) = 3 + fatLength' fr
```

Converting fat lists to normal lists

```
fromFatList :: FatList a -> [a]
fromFatList (FCons0 fr)          =      fromFatList' fr
fromFatList (FCons1 x1 fr)       =      x1:fromFatList' fr
fromFatList (FCons2 x1 x2 fr) = x1:x2:fromFatList' fr

fromFatList' :: FatRest a -> [a]
fromFatList' FatNil = []
fromFatList' (FatRest e1 e2 e3 fr)
  = e1:e2:e3:fromFatList' fr
```

Converting normal lists to fat lists

```

toFatList :: [a] -> FatList a
toFatList []                = FCons0 FatNil
toFatList [x1]              = FCons1 x1 FatNil
toFatList [x1,x2]           = FCons2 x1 x2 FatNil
toFatList (x1:x2:x3:xt) = fcons3 x1 x2 x3 (toFatList xt)

```

```

fcons3 :: a -> a -> a -> FatList a -> FatList a
fcons3 e1 e2 e3 (FCons0 fr)
  = FCons0 (FatRest e1 e2 e3 fr)
fcons3 e1 e2 e3 (FCons1 e4 fr)
  = FCons1 e1 $ FatRest e2 e3 e4 fr
fcons3 e1 e2 e3 (FCons2 e4 e5 fr)
  = FCons2 e1 e2 $ FatRest e3 e4 e5 fr

```

Appending fat lists

```
fatAppend :: FatList a -> FatList a -> FatList a
```

```
fatAppend (FCons0 xr) ys
```

```
  = fatAppend' xr ys
```

```
fatAppend (FCons1 x1 xr) ys
```

```
  = fcons x1 $ fatAppend' xr ys
```

```
fatAppend (FCons2 x1 x2 xr) ys
```

```
  = fcons x1 $ fcons x2 $ fatAppend' xr ys
```

```
fatAppend' FatNil ys = ys
```

```
fatAppend' (FatRest x1 x2 x3 xr) ys
```

```
  = fcons3 x1 x2 x3 $ fatAppend' xr ys
```

```
fcons :: a -> FatList a -> FatList a
```

```
fcons x1 (FCons0 xr)      = FCons1 x1 xr
```

```
fcons x1 (FCons1 x2 xr)   = FCons2 x1 x2 xr
```

```
fcons x1 (FCons2 x2 x3 xr) = FCons0 $ FatRest x1 x2 x3 xr
```

Defining a typeclass

A second shortcoming of fat lists is that none of the standard list manipulation functions work for them. This cannot be helped.

However, it is possible to arrange for newly defined functions to work for both ordinary and fat lists. This is done by defining a typeclass for lists.

However, because list is a type constructor, not a type, we must use the `-XMultiParamTypeClasses` and `-XFlexibleInstances` GHC extensions.

This can be specified by adding the special comment form:

```
{-# OPTIONS_GHC -XMultiParamTypeClasses -XFlexibleInstances #-}
```

near the top of the file

Declaring a type class

A type class is declared by specifying all the functions that must be defined for all instances of that class. Our list typeclass will need functions to construct and deconstruct a list. We shall use the names given to these operations by LISP: `nil`, `cons`, `car`, and `cdr`. We'll also specify functions to append and compute length, and to convert to and from ordinary lists.

```
class List l a where
  nil      :: l a
  cons     :: a -> l a -> l a
  car      :: l a -> a
  cdr      :: l a -> l a
  len      :: l a -> Int
  append   :: l a -> l a -> l a
  fromList :: [a] -> l a
  toList   :: l a -> [a]
```

Declaring an instance

Then we can declare our fat list type to be an instance of the List class just by specifying the definition of these functions for fat lists.

```
instance List FatList a where
```

```
  nil      = FCons0 FatNil
```

```
  cons     = fcons
```

```
  len      = fatLength
```

```
  append   = fatAppend
```

```
  fromList = toFatList
```

```
  toList   = fromFatList
```

```
car (FCons0 (FatRest x1 _ _)) = x1
```

```
car (FCons1 x1 _)             = x1
```

```
car (FCons2 x1 _ _)          = x1
```

```
cdr (FCons0 (FatRest _ x1 x2 xr)) = FCons2 x1 x2 xr
```

```
cdr (FCons1 _ xr)                  = FCons0 xr
```

```
cdr (FCons2 _ x1 xr)               = FCons1 x1 xr
```

Declaring an instance

Similarly to make ordinary list types instances of this class:

```
instance List [] a where
  nil      = []
  cons     = (:)
  len      = length
  append   = (++)
  fromList = id
  toList   = id
  car      = head
  cdr      = tail
```

With these declarations, functions defined to operate on instances of `List` can be applied to both kinds of lists (and any other instances).

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Section 10

Introduction to Logic Programming

Logic programming

Functional programming languages are based on the lambda calculus of Alonzo Church and the concept of the *function*: a thing that maps inputs to outputs.

Logic programming languages are based on the predicate calculus of Gottlob Frege and the concept of the *relation*, the relationship among a number of individuals, and the *predicate* that relates them.

A function is a special kind of relation that can only be used in one direction (inputs to outputs), and can only have one result. Relations do not have these limitations. In this sense, logic programming generalises functional programming.

While the first functional programming language was Lisp, implemented by John McCarthy's group at MIT in 1958, the first logic programming language was Prolog, implemented by Alain Colmerauer's group at Marseille in 1971.

Relations

A *relation* specifies a relationship; for example, a family relationship. In Prolog syntax,

```
parent(queen_elizabeth, prince_charles).
```

specifies (a small part of the) parenthood relation, which relates parents to their children. This says that Queen Elizabeth is a parent of Prince Charles.

The name of a relation is called a *predicate*. Predicates have no directionality: it makes just as much sense to ask of whom is Queen Elizabeth a parent as to ask who is Prince Charles's parent. There is also no promise that there is a unique answer to either of these questions.

Facts

A statement such as:

```
parent(queen_elizabeth, prince_charles).
```

is called a *fact*. It may take many facts to define a relation:

```
% (A small part of) the British Royal family
parent(queen_elizabeth, prince_charles).
parent(prince_philip, prince_charles).
parent(prince_charles, prince_william).
parent(prince_charles, prince_harry).
parent(princess_diana, prince_william).
parent(princess_diana, prince_harry).
:
```

Text between a percent sign (%) and end-of-line is treated as a comment.

Using Prolog

Most Prolog systems have an environment similar to GHCi. A file containing facts like this should be written in a file whose name begins with a lower-case letter and contains only letters, digits, and underscores, and ends with “.pl”.

A source file can be loaded into Prolog by typing its filename (without the .pl extension) between square brackets at the Prolog prompt (?-). Prolog prints a message to say the file was compiled, and true to indicate it was successful (user input looks [like this](#)):

```
?- [royals].  
% royals compiled 0.00 sec, 8 clauses  
true.  
  
?-
```


Queries

Once your code is loaded, you can use or test it by issuing *queries* at the Prolog prompt. A Prolog query looks just like a fact. When written in a source file and loaded into Prolog, it is treated as a true statement. At the Prolog prompt, it is treated as a query, asking if the statement is true.

```
?- parent(prince_charles, prince_william).  
true .  
  
?- parent(prince_william, prince_charles).  
false.
```

Variables

Each predicate argument may be a *variable*, which in Prolog begins with a capital letter or underscore and follows with letters, digits, and underscores. A query containing a variable asks if there exists a value for that variable that makes that query true, and prints the value that makes it true.

If there is more than one answer to the query, Prolog prints them one at a time, pausing to see if more solutions are wanted. Typing semicolon asks for more solutions; just hitting enter (return) finishes without more solutions.

This query asks: *of whom Prince Charles is a parent?*

```
?- parent(prince_charles, X).  
X = prince_william ;  
X = prince_harry.
```

Multiple modes

The same parenthood relation can be used just as easily to ask who is a parent of Prince Charles or even who is a parent of whom. Each of these is a different *mode*, based on which arguments are *bound* (inputs; non-variables) and which are *unbound* (outputs; variables).

```
?- parent(X, prince_charles).  
X = queen_elizabeth ;  
X = prince_philip.  
?- parent(X, Y).  
X = queen_elizabeth,  
Y = prince_charles ;  
X = prince_philip,  
Y = prince_charles ;  
:
```

Compound queries

Queries may use multiple predicate applications (called *goals* in Prolog and *atoms* in predicate logic). The simplest way to combine multiple goals is to separate them with a comma. This asks Prolog for all bindings for the variables that satisfy both (or all) of the goals. The comma can be read as “and”. In relational algebra, this is called an *inner join* (but do not worry if you do not know what that is).

```
?- parent(queen_elizabeth, X), parent(X, Y).  
X = prince_charles,  
Y = prince_william ;  
X = prince_charles,  
Y = prince_harry.
```

QUIZ: Query

Give a Prolog query to find the grandparent(s) of Prince Harry.

QUIZ: Query

Give a Prolog query to find the grandparent(s) of Prince Harry.

```
?- parent(Y, prince_harry), parent(X, Y).  
Y = prince_charles,  
X = queen_elizabeth ;  
Y = prince_charles,  
X = prince_philip.
```

or:

```
?- parent(X, Y), parent(Y, prince_harry).  
X = queen_elizabeth,  
Y = prince_charles ;  
X = prince_philip,  
Y = prince_charles ;  
false.
```

Rules

Predicates can be defined using *rules* as well as facts. A rule has the form

$$\textit{Head} \text{ :- } \textit{Body},$$

where *Head* has the form of a fact and *Body* has the form of a (possibly compound) query. The :- is read “if”, and the clause means that the *Head* is true if the *Body* is. For example

$\text{grandparent}(X,Z) \text{ :- } \text{parent}(X, Y), \text{parent}(Y, Z).$

means “ X is grandparent of Z if X is parent of Y and Y is parent of Z .”

Rules and facts are the two different kinds of *clauses*. A predicate can be defined with any number of clauses of either or both kinds, intermixed in any order.

Recursion

Rules can be recursive. Like Haskell, Prolog has no looping constructs, so recursion is widely used. Prolog does not have as well-developed a library of higher-order operations as Haskell, so recursion is used more in Prolog than in Haskell.

A person's ancestors are their parents and the ancestors of their parents.

```
ancestor(Anc, Desc) :-  
    parent(Anc, Desc).  
ancestor(Anc, Desc) :-  
    parent(Parent, Desc),  
    ancestor(Anc, Parent).
```


Equality

Equality in Prolog, written “=” and used as an infix operator, can be used both to bind variables and to check for equality. Like Haskell, Prolog is a *single-assignment* language: once bound, a variable cannot be reassigned.

```
?- X = 7.
```

```
X = 7.
```

```
?- a = b.
```

```
false.
```

```
?- X = 7, X = a.
```

```
false.
```

```
?- X = 7, Y = 8, X = Y.
```

```
false.
```

Disjunction

Goals can be combined with disjunction (or) as well as conjunction (and). Disjunction is written “;” and used as an infix operator. Conjunction (“,”) has higher precedence (binds tighter) than disjunction, but parentheses can be used to achieve the desired precedence.

Who are the children of Queen Elizabeth or Princess Diana?

```
?- parent(queen_elizabeth, X) ; parent(princess_diana, X).  
X = prince_charles ;  
X = prince_william ;  
X = prince_harry.
```

Negation

Negation in Prolog is written “\+” and used as a prefix operator. Negation has higher (tighter) precedence than both conjunction and disjunction. Be sure to leave a space between the \+ and an open parenthesis.

Who are the parents of Prince William other than Prince Charles?

```
?- parent(X, prince_william), \+ X = prince_charles.
X = princess_diana.
```

Disequality in Prolog is written as an infix “\=” . $X \neq Y$ is the same as $\neg X = Y$.

```
?- parent(X, prince_william), X \= prince_charles.
X = princess_diana.
```

QUIZ: Siblings

Define a Prolog predicate `siblings(Sib1,Sib2)` that holds when `Sib1` and `Sib2` are siblings (or half-siblings). That is, they share at least one parent.

Remember: no one is their own sibling!

QUIZ: Siblings

Define a Prolog predicate `siblings(Sib1,Sib2)` that holds when `Sib1` and `Sib2` are siblings (or half-siblings). That is, they share at least one parent.

Remember: no one is their own sibling!

```
siblings(Sib1, Sib2) :-  
    parent(P, Sib1),  
    parent(P, Sib2),  
    Sib1 \= Sib2.
```

The Closed World Assumption

Prolog assumes that all true things can be derived from the program. This is called the *closed world assumption*. Of course, this is not true for our parent relation (that would require tens of billions of clauses!).

```
?- \+ parent(queen_elizabeth, princess_anne).  
true.
```

but Princess Anne *is* a daughter of Queen Elizabeth. Our program simply does not know about her.

So use negation with great care on predicates that are not complete, such as parent.

Negation as failure

Prolog executes $\text{\textbackslash+}G$ by first trying to prove G . If this fails, then $\text{\textbackslash+}G$ succeeds; if it succeeds, then $\text{\textbackslash+}G$ fails. This is called *negation as failure*.

In Prolog, failing goals can never bind variables, so any variable bindings made in solving G are thrown away when $\text{\textbackslash+}G$ fails. Therefore, $\text{\textbackslash+}G$ cannot solve for any variables, and goals such as these cannot work properly.

Is there anyone of whom Queen Elizabeth is not a parent?

Is there anyone who is not Queen Elizabeth?

```
?- \+ parent(queen_elizabeth, X).  
false.
```

```
?- X \= queen_elizabeth.  
false.
```

Execution Order

The solution to this problem is simple: ensure all variables in a negated goal are bound before the goal is executed.

Prolog executes goals in a query (and the body of a clause) from first to last, so put the goals that will bind the variables in a negation before the negation (or `\=`).

In this case, we can generate all people who are either parents or children, and ask whether any of them is different from Queen Elizabeth.

```
?- (parent(X,_) ; parent(_,X)), X \= queen_elizabeth.  
X = prince_philip ;  
:  
:
```


Datalog

The fragment of Prolog discussed so far, which omits data structures, is called *Datalog*. It is a generalisation of what is provided by relational databases. Many modern databases now provide Datalog features or use Datalog implementation techniques.

```
capitol(australia, canberra).  
capitol(france, paris).  
:  
continent(australia, australia).  
continent(france, europe).  
:  
population(australia, 22_680_000).  
population(france, 65_700_000).  
:  
:
```

Datalog Queries

What is the capitol of France?

?- capitol(france, Capitol).

Capitol = paris.

What are capitols of European countries?

?- continent(Country, europe), capitol(Country, Capitol).

Country = france,

Capitol = paris.

What European countries have populations > 50,000,000?

?- continent(Country, europe), population(Country, Pop),
| Pop > 50_000_000.

Country = france,

Pop = 65700000.

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Section 11

Beyond Datalog

Terms

In Prolog, all data structures are called *terms*. A term can be *atomic* or *compound*, or it can be a variable. Datalog has only atomic terms and variables.

Atomic terms include integers and floating point numbers, written as you would expect, and atoms.

An atom begins with a lower case letter and follows with letters, digits and underscores, for example `a`, `queen_elizabeth`, or `banana`.

An atom can also be written beginning and ending with a single quote, and have any intervening characters. The usual character escapes can be used, for example `\n` for newline, `\t` for tab, and `\'` for a single quote. For example: `'Queen Elizabeth'` or `'Hello, World!\n'`.

Compound Terms

In the syntax of Prolog, each compound term is a *functor* (sometimes called *function symbol*) followed by zero or more arguments; if there are any arguments, they are shown in parentheses, separated by commas. Functors are Prolog's equivalent of data constructors, and have the same syntax as atoms.

For example, the small tree that in Haskell syntax would be written as

```
Node Leaf 1 (Node Leaf 2 Leaf)
```

would be written in Prolog syntax as the term

```
node(leaf, 1, node(leaf, 2, leaf))
```

Because Prolog is dynamically typed, each argument of a term can be *any* term, and there is no need to declare types.

Prolog has special syntax for some functors, such as infix notation.

Variables

A variable is also a term. It denotes a single unknown term.

A variable name begins with an upper case letter or underscore, followed by any number of letters, digits, and underscores.

A single underscore `_` is special: it specifies a different variable each time it appears, much like `_` in Haskell pattern matching.

Like Haskell, Prolog is a *single-assignment* language: a variable can only be bound (assigned) once.

Because the arguments of a compound term can be any terms, and variables are terms, variables can appear in terms.

For example `f(A,A)` denotes a term whose functor is `f` and whose two arguments can be anything, as long as they are the same; `f(_,_)` denotes a term whose functor is `f` and has any two arguments.

List syntax

Like Haskell, Prolog has a special syntax for lists.

Both denote the empty list by `[]`.

Both denote the list with the three elements 1, 2 and 3 by `[1, 2, 3]`.

While Haskell uses `x:xs` to denote a list whose head is `x` and whose tail is `xs`, the Prolog syntax is `[X | Xs]` (not to be confused with list comprehensions, which Prolog lacks).

The Prolog syntax for what Haskell would represent with `x1:x2:xs` is `[X1, X2 | Xs]`.

Ground vs nonground terms

A term is a *ground term* if it contains no variables, and it is a *nonground term* if it contains at least one variable.

3 and $f(a, b)$ are ground terms.

Since Name and $f(a, X)$ each contain at least one variable, they are *nonground terms*.

Substitutions

A *substitution* is a mapping from variables to terms.

Applying a substitution to a term means consistently replacing all occurrences of each variable in the map with the term it is mapped to.

Note that a substitution only replaces variables, never atomic or compound terms.

For example, applying the substitution $\{X1 \mapsto \text{leaf}, X2 \mapsto 1, X3 \mapsto \text{leaf}\}$ to the term `node(X1,X2,X3)` yields the term `node(leaf,1,leaf)`.

Since you can get `node(leaf,1,leaf)` from `node(X1,X2,X3)` by applying a substitution to it, `node(leaf,1,leaf)` is an *instance* of `node(X1,X2,X3)`.

Any ground Prolog term has only one instance, while a nonground Prolog terms has an infinite number of instances.

Unification

The term that results from applying a substitution θ to a term t is denoted $t\theta$.

A term u is therefore an instance of term t if there is some substitution θ such that $u = t\theta$.

A substitution θ *unifies* two terms t and u if $t\theta = u\theta$.

Consider the terms $f(X, b)$ and $f(a, Y)$.

Applying a substitution $\{X \mapsto a\}$ to those two terms yields $f(a, b)$ and $f(a, Y)$, which are not syntactically identical, so this substitution is not a unifier.

On the other hand, applying the substitution $\{X \mapsto a, Y \mapsto b\}$ to those terms yields $f(a, b)$ in both cases, so this substitution *is* a unifier.

QUIZ: Unification

What substitution unifies the terms

$\text{branch}(\text{branch}(\text{leaf}(X), \text{leaf}(b)), \text{leaf}(Z))$ and
 $\text{branch}(\text{branch}(\text{leaf}(a), \text{leaf}(Y)), \text{leaf}(Z))$?

QUIZ: Unification

What substitution unifies the terms

`branch(branch(leaf(X), leaf(b)), leaf(Z))` and
`branch(branch(leaf(a), leaf(Y)), leaf(Z))`?

$\{X \mapsto a, Y \mapsto b\}$

QUIZ: Unification

What substitution unifies the terms

`branch(branch(leaf(X), leaf(b)), leaf(Z))` and
`branch(branch(leaf(a), leaf(Y)), leaf(Z))`?

$\{X \mapsto a, Y \mapsto b\}$

What substitution unifies the terms

`branch(branch(leaf(X), leaf(X)), leaf(Z))` and
`branch(branch(leaf(a), leaf(b)), leaf(c))`

QUIZ: Unification

What substitution unifies the terms

`branch(branch(leaf(X), leaf(b)), leaf(Z))` and
`branch(branch(leaf(a), leaf(Y)), leaf(Z))`?

$\{X \mapsto a, Y \mapsto b\}$

What substitution unifies the terms

`branch(branch(leaf(X), leaf(X)), leaf(Z))` and
`branch(branch(leaf(a), leaf(b)), leaf(c))`

The two terms are not unifiable, since a substitution cannot map X to both a and b .

QUIZ: Unification

What substitution unifies the terms

`branch(branch(leaf(X), leaf(b)), leaf(Z))` and
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What substitution unifies the terms

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`branch(leaf(a), branch(leaf(Y), leaf(Z)))`?

QUIZ: Unification

What substitution unifies the terms

`branch(branch(leaf(X), leaf(b)), leaf(Z))` and
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$\{X \mapsto a, Y \mapsto b\}$

What substitution unifies the terms

`branch(branch(leaf(X), leaf(X)), leaf(Z))` and
`branch(branch(leaf(a), leaf(b)), leaf(c))`

The two terms are not unifiable, since a substitution cannot map X to both a and b .

What substitution unifies the terms

`branch(branch(leaf(X), leaf(b)), leaf(Z))` and
`branch(leaf(a), branch(leaf(Y), leaf(Z)))`?

The two terms are not unifiable, since the first arguments of the top-level `branch` have different function symbols in the two terms.

Recognising proper lists

A proper list is either empty (`[]`) or not (`[X|Y]`), in which case, the tail of the list must be a proper list. We can define a predicate to recognise these.

```
proper_list([]).
proper_list([Head|Tail]) :-
    proper_list(Tail).
```

```
?- [list].
Warning: list.pl:3:
    Singleton variables: [Head]
% list compiled 0.00 sec, 1 clauses
true.
```

Detour: singleton variables

```
Warning: list.pl:3:  
    Singleton variables: [Head]
```

The variable `Head` appears only once in this clause:

```
proper_list([Head|Tail]) :-  
    proper_list(Tail).
```

This often indicates a typo in the source code. For example, if `Tail` were spelled `Tial` in one place, this would be easy to miss. But Prolog's singleton warning would alert us to the problem.

Detour: singleton variables

In this case, there is no problem; to avoid the warning, we should begin the variable name `Head` with an underscore, or just name the variable `_`.

```
proper_list([]).
proper_list([_Head|Tail]) :-
    proper_list(Tail).
```

```
?- [list].
% list compiled 0.00 sec, 1 clauses
true.
```

General programming advice: always fix compiler warnings (if possible). Some warnings may indicate a real problem, and you will not see them if they're lost in a sea of unimportant warnings. It is easier to fix a problem when the compiler points it out than when you have to find it yourself.

Append

Appending two lists is a common operation in Prolog. This is a built in predicate in most Prolog systems, but could easily be implemented as:

```
append([], C, C).
append([A|B], C, [A|BC]) :-
    append(B, C, BC).
```

```
?- append([a,b,c],[d,e],List).
List = [a, b, c, d, e].
```

This is similar to ++ in Haskell.

append is like proper_list

Compare the code for `proper_list` to the code for `append`:

<code>proper_list([]).</code>	<code>append([], C, C).</code>
<code>proper_list([Head Tail]) :-</code>	<code>append([A B], C, [A BC]) :-</code>
<code>proper_list(Tail).</code>	<code>append(B, C, BC).</code>

This is common: code for a predicate that handles a term often follows the structure of that term (as we saw in Haskell).

While the `proper_list` predicate is not very useful itself, it was worth designing, as it gives a hint at the structure of other code that traverses lists. Since types are not declared in Prolog, predicates like `proper_list` can serve to indicate the notional type.

Appending backwards

Unlike ++ in Haskell, append in Prolog can work in other modes:

```
?- append([1,2,3], Rest, [1,2,3,4,5]).
```

```
Rest = [4, 5].
```

```
?- append(Front, [3,4], [1,2,3,4]).
```

```
Front = [1, 2] ;
```

```
false.
```

```
?- append(Front,Back,[a,b,c]).
```

```
Front = [],
```

```
Back = [a, b, c] ;
```

```
Front = [a],
```

```
Back = [b, c] ;
```

```
Front = [a, b],
```

```
Back = [c] ;
```

```
Front = [a, b, c],
```

```
Back = [] ;
```

```
false.
```

Length

The `length` built-in predicate relates a list to its length:

```
?- length([a,b,c], Len).  
Len = 3.  
  
?- length(List, 3).  
List = [_G273, _G276, _G279].
```

The `_G...` terms are how Prolog prints out unbound variables. The number part reflects when the variable was created; because these variables are all printed differently, we can tell they are all distinct variables.

`[_G273, _G276, _G279]` is a list of three distinct unbound variables, and each unbound variable can be any term, so this can be any three-element list, as specified by the query.

Putting them together

How would we implement `take` in Prolog?

`take(N,List,Front)` should hold if `Front` is the first `N` elements of `List`. So `length(Front,N)` should hold.

Also, `append(Front, _, List)` should hold. Then:

```
take(N, List, Front) :-  
    length(Front,N),  
    append(Front, _, List).
```

Prolog coding hint: sometimes it is easier to write code if you think about *checking* if the result is correct rather than *computing* it. That is, *think declaratively*.

Then you need to think about whether your code will work the ways you want it to. We will return to that.

QUIZ: Drop

Implement `drop(N, List, Back)`, such that `Back` is the list that remains after the first `N` elements of `List` are dropped.

QUIZ: Drop

Implement `drop(N, List, Back)`, such that `Back` is the list that remains after the first `N` elements of `List` are dropped.

```
drop(N, List, Back) :-  
    length(Front, N),  
    append(Front, Back, List).
```

Member

Here is list membership, two ways:

```
member1(Elt, List) :- append(_, [Elt|_], List).
```

```
member2(Elt, [Elt|_]).
```

```
member2(Elt, [_|Rest]) :- member2(Elt, Rest).
```

These behave the same, but the second is a bit more efficient because the first builds and ignores the list of elements before `Elt` in `List`, and the second does not.

Note the recursive version does not exactly match the structure of our earlier `proper_list` predicate. This is because `Elt` is never a member of the empty list, so we do not need a clause for `[]`. In Prolog, we do not need to specify when a predicate should fail; only when it should succeed. We also have two cases to consider when the list is non-empty (like Haskell in this respect).

Arithmetic

In Prolog, terms like $6 * 7$ are just data structures, and $=$ does not evaluate them, it just unifies them.

The built-in predicate `is` (an infix operator) evaluates expressions.

```
?- X = 6 * 7.
```

```
X = 6*7.
```

```
?- X is 6 * 7.
```

```
X = 42.
```

Arithmetic modes

Use `is/2` to evaluate expression

```
square(N, N2) :- N2 is N * N.
```

Unfortunately, `square` only works when the first argument is bound. This is because `is` only works if its second argument is ground.

```
?- square(5, X).  
X = 25.  
?- square(X, 25).  
ERROR: is/2: Arguments are not sufficiently instantiated  
?- 25 is X * X.  
ERROR: is/2: Arguments are not sufficiently instantiated
```

Later we shall see how to write code to do arithmetic in different modes.

Arithmetic

Prolog provides the usual arithmetic operators, including:

<code>+</code>	<code>-</code>	<code>*</code>	add, subtract, multiply
<code>/</code>			division (may return a float)
<code>//</code>			integer division (rounds toward 0)
<code>mod</code>			modulo (result has same sign as second argument)
<code>-</code>			unary minus (negation)
<code>integer float</code>			coersions (not operators)

More arithmetic predicates (infix operators; both arguments must be ground expressions):

<code><</code>	<code>=<</code>	less, less or equal (note!)
<code>></code>	<code>>=</code>	greater, greater or equal
<code>:=</code>	<code>=\=</code>	equal, not equal (only numbers)

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Section 12

Understanding and Debugging Prolog code

List Reverse

To reverse a list, put the first element of the list at the end of the reverse of the tail of the list.

```
rev1([], []).  
rev1([A|BC], CBA) :-  
    rev1(BC, CB),  
    append(CB, [A], CBA).
```

reverse/2 is an SWI Prolog built-in, so we use a different name to avoid conflict.

List Reverse

The *mode* of a Prolog goal says which arguments are bound (inputs) and which are unbound (outputs) when the predicate is called.

`rev1/2` works as intended when the first argument is ground and the second is free, but not for the opposite mode.

```
?- rev1([a,b,c], Y).
```

```
Y = [c, b, a].
```

```
?- rev1(X, [c,b,a]).
```

```
X = [a, b, c] ;
```

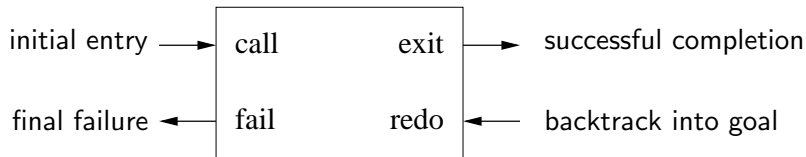
Prolog hangs at this point. We will use the Prolog debugger to understand why. For now, hit control-C and then 'a' to abort.

The Prolog Debugger

To understand the debugger, you will need to understand the Byrd box model. Think of goal execution as a box with a port for each way to enter and exit.

A conventional language has only one way to enter and one way to exit; Prolog has two of each.

The four debugger ports are:



Turn on debugger with `trace`, and off with `nodebug`, at the Prolog prompt.

Using the Debugger

The debugger prints the current port, execution depth, and goal (with the current variable bindings) at each step.

```
Call: (7) rev1([a, b], _G12717) ? creep
Call: (8) rev1([b], _G12834) ? creep
Call: (9) rev1([], _G12834) ? creep
Exit: (9) rev1([], []) ? creep
Call: (9) lists:append([], [b], _G12838) ? creep
Exit: (9) lists:append([], [b], [b]) ? creep
Exit: (8) rev1([b], [b]) ? creep
Call: (8) lists:append([b], [a], _G12717) ? creep
Exit: (8) lists:append([b], [a], [b, a]) ? creep
Exit: (7) rev1([a, b], [b, a]) ? creep
Y = [b, a].
```

“lists:” in front of append is a module name.

Reverse backward

Now try the “backwards” mode of rev1/2. We shall use a smaller test case to keep it manageable.

```
Call: (7) rev1(_G11553, [a]) ? creep
Call: (8) rev1(_G11661, _G11671) ? creep
Exit: (8) rev1([], []) ? creep
Call: (8) lists:append([], [_G11660], [a]) ? creep
Exit: (8) lists:append([], [a], [a]) ? creep
Exit: (7) rev1([a], [a]) ? creep
```

```
X = [a] ;
```

Reverse backward, continued

after showing the first solution, Prolog goes on forever like this:

```

Redo: (8) rev1(_G11661, _G11671) ? creep
Call: (9) rev1(_G11664, _G11674) ? creep
Exit: (9) rev1([], []) ? creep
Call: (9) lists:append([], [_G11663], _G11678) ? creep
Exit: (9) lists:append([], [_G11663], [_G11663]) ? creep
Exit: (8) rev1([_G11663], [_G11663]) ? creep
Call: (8) lists:append([_G11663], [_G11660], [a]) ? creep
Fail: (8) lists:append([_G11663], [_G11660], [a]) ? creep
Redo: (9) rev1(_G11664, _G11674) ? creep
Call: (10) rev1(_G11667, _G11677) ? creep
Exit: (10) rev1([], []) ? creep
Call: (10) lists:append([], [_G11666], _G11681) ? creep
Exit: (10) lists:append([], [_G11666], [_G11666]) ? creep
Exit: (9) rev1([_G11666], [_G11666]) ? creep
Call: (9) lists:append([_G11666], [_G11663], _G11684) ? cr
Exit: (9) lists:append([_G11666], [_G11663], [_G11666, _G1
Exit: (8) rev1([_G11663, _G11666], [_G11666, _G11663]) ? c
Call: (8) lists:append([_G11666, _G11663], [_G11660], [a])
Fail: (8) lists:append([_G11666, _G11663], [_G11660],
:
:

```

Infinite backtracking loop

```

rev1([], []).
rev1([A|BC], CBA) :-
    rev1(BC, CB),
    append(CB, [A], CBA).

```

The problem is that the goal `rev1(X, [a])`, resolves to the goal `rev1(BC, CB)`, `append(CB, [A], [a])`. The call `rev1(BC, CB)` produces an infinite backtracking sequence of solutions $\{BC \mapsto [], CB \mapsto []\}$, $\{BC \mapsto [Z], CB \mapsto [Z]\}$, $\{BC \mapsto [Y, Z], CB \mapsto [Z, Y]\}$, For each of these solutions, we call `append(CB, [A], [a])`.

`append([], [A], [a])` succeeds, with $\{A \mapsto [a]\}$. However, `append([Z], [A], [a])` fails, as does this goal for all following solutions for `CB`. This is an infinite backtracking loop.

Infinite backtracking loop

We could fix this problem by executing the body goals in the other order:

```
rev2([], []).
rev2([A|BC], CBA) :-
    append(CB, [A], CBA),
    rev2(BC, CB).
```

But this definition does not work in the forward direction:

```
?- rev2(X, [a,b]).
X = [b, a] ;
false.
```

```
?- rev2([a,b], Y).
Y = [b, a] ;
^CAction (h for help) ? abort
% Execution Aborted
```

Working in both directions

The solution is to ensure that when `rev1` is called, the first argument is always bound to a list. We do this by observing that the length of a list must always be the same as that of its reverse. When `samelength/2` succeeds, both arguments are bound to lists of the same fixed length.

```
rev3(ABC, CBA) :-
    samelength(ABC, CBA),
    rev1(ABC, CBA).
```

```
samelength([], []).
samelength(_|Xs, _|Ys) :-
    samelength(Xs, Ys).
```

```
?- rev3(X, [a,b]).
```

```
X = [b, a].
```

```
?- rev3([a,b], Y).
```

```
Y = [b, a].
```


QUIZ: Modes

`append/3` and `length/2` both work in all possible modes. Both predicates generate a finite number of solutions as long as either the first or last argument is ground; otherwise they generate an infinite number of solutions.

In which modes will these definitions of `take/3` work (without hanging)?

```
take(N, List, Front) :-  
    length(Front, N),  
    append(Front, _, List).
```

```
take(N, List, Front) :-  
    append(Front, _, List),  
    length(Front, N).
```

QUIZ: Modes

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In which modes will these definitions of `take/3` work (without hanging)?

```
take(N, List, Front) :-
    length(Front, N),
    append(Front, _, List).
```

All modes but `<out,in,out>`.

```
take(N, List, Front) :-
    append(Front, _, List),
    length(Front, N).
```

All modes but `<in,out,out>`.

This shows that it is often quite difficult to make Prolog code behave properly in all modes.

More on the Debugger

Some useful debugger commands:

- h** display debugger help
- c** creep to the next port (also space, enter)
- s** skip over goal; go straight to exit or fail port
- r** back to initial call port of goal, undoing all bindings done since starting it;
- a** abort whole debugging session
- +** set spypoint (like breakpoint) on this pred
 - remove spypoint from this predicate
- l** leap to the next spypoint
- b** pause this debugging session and enter a “break level,” giving a new Prolog prompt; end of file reenters debugger

More on the Debugger

Built-in predicates for controlling the debugger:

`spy(Predspec)` Place a spy point on Predspec, which can be a Name/Arity pair, or just a predicate name.

`nospy(Predspec)` Remove the spy point from Predspec.

`trace` Turn on the debugger

`debug` Turn on the debugger and leap to first spy point

`nodebug` Turn off the debugger

A “Predspec” is a predicate *name* or *name/arity*

Using the debugger

Note the `r` (retry) debugger command restarts a goal from the beginning, “time travelling” back to the time when starting to execute that goal.

The `s` (skip) command skips forward in time, over the whole execution of a goal, to its exit or fail port.

This leads to a quick way of tracking down most bugs:

- 1 When you arrive at a call or redo port: skip.
- 2 If you come to an exit port with the correct results (or a correct fail port): creep.
- 3 If you come to an incorrect exit or fail port: retry, then creep.

Eventually you will find a clause that has the right input and wrong output (or wrong failure); this is the bug. This will not help find infinite recursion, though.

Spypoints

For larger computations, it may take some time to get to the part of the computation where the bug lies. Usually, you will have a good idea, or at least a few good guesses, which predicates you suspect of being buggy (usually the predicates you have edited most recently). In cases of infinite recursion you may suspect certain predicates of being involved in the loop.

In these cases, spypoints will be helpful. Like a breakpoint in most debuggers, when Prolog reaches any port of a predicate with a spypoint set, Prolog stops and shows the port. The `l (leap)` command tells Prolog to run quietly until it reaches a spypoint. Use the `spy(pred)` goal at the Prolog prompt to set a spypoint on the named predicate, `nospy(pred)` to remove one. You can also add a spypoint on the predicate of the current debugger port with the `+` command, and remove it with `-`.

Managing nondeterminism

This is a common mistake in defining factorial:

```
fact(0, 1).  
fact(N, F) :-  
    N1 is N - 1,  
    fact(N1, F1),  
    F is N * F1.
```

Managing nondeterminism

This is a common mistake in defining factorial:

```
fact(0, 1).  
fact(N, F) :-  
    N1 is N - 1,  
    fact(N1, F1),  
    F is N * F1.
```

```
?- fact(5, F).  
F = 120 ;  
ERROR: Out of local stack
```

`fact(5,F)` has only one solution, why was Prolog looking for another?

Correctness

The second clause promises that for all n , $n! = n \times (n - 1)!$. This is wrong for $n < 1$.

Prolog is not like Haskell: even if one clause applies, later clauses are still tried. After finding $0! = 1$, Prolog thinks $0! = 0 \times -1!$; tries to compute $-1!$, $-2!$, \dots

The simple solution is to ensure each clause is a correct (part of the) definition.

```
fact(0, 1).
fact(N, F) :-
    N > 0,
    N1 is N - 1,
    fact(N1, F1),
    F is F1 * N.
```

Choicepoints

This definition is correct, but it could be more efficient.

When a clause succeeds but there are later clauses that could possibly succeed, Prolog will leave a choicepoint so it can later backtrack and try the later clause.

In this case, backtracking to the second clause will fail unless $N > 0$. This test is quick. However, as long as the choicepoint exists, it inhibits the very important last call optimisation (discussed later). Therefore, where efficiency matters, it is important to make your recursive predicates not leave choicepoints when they should be deterministic.

In this case, $N = 0$ and $N > 0$ are mutually exclusive, so at most one clause can apply, so `fact/2` should not leave a choicepoint.

If-then-else

We can avoid the choicepoint with Prolog's if-then-else construct:

```
fact(N, F) :-
    ( N == 0 ->
      F = 1
    ;   N > 0,
      N1 is N - 1,
      fact(N1, F1),
      F is F1 * N
    ).
```

The `->` is treated like a conjunction (`,`), except that when it is crossed, any alternative solutions of the goal before the `->`, as well as any alternatives following the `;` are forgotten. Conversely, if the goal before the `->` fails, then the goal after the `;` is tried. So this is deterministic whenever both the code between `->` and `;`, and the code after the `;`, are.

If-then-else caveats

However, you should prefer indexing (discussed next time) and avoid if-then-else, when you have a choice. If-then-else usually leads to code that will not work smoothly in multiple modes. For example, `append` could be written with if-then-else:

```
ap(X, Y, Z) :-
    ( X = [] ->
      Z = Y
    ;   X = [U|V],
      ap(V, Y, W),
      Z = [U|W]
    ).
```

This may appear correct, and may follow the logic you would use to code it in another language, but it is not appropriate for Prolog.

If-then-else caveats

With that definition of `ap`:

```
?- ap([a,b,c], [d,e], L).
```

```
L = [a, b, c, d, e].
```

```
?- ap(L, [d,e], [a,b,c,d,e]).
```

```
false.
```

```
?- ap(L, M, [a,b,c,d,e]).
```

```
L = [],
```

```
M = [a, b, c, d, e].
```

Because the if-then-else commits to binding the first argument to `[]` when it can, this version of `append` will not work correctly unless the first argument is bound when `append` is called.

The University of Melbourne
School of Computing and Information Systems

COMP90048

Declarative Programming

Semester 1, 2019

Section 13

Logic and Resolution

Interpretations

In the mind of the person writing a logic program,

- each constant (atomic term) stands for an entity in the “*domain of discourse*” (world of the program);
- each functor (function symbol of arity n where $n > 0$) stands for a function from n entities to one entity in the domain of discourse; and
- each predicate of arity n stands for a particular relationship between n entities in the domain of discourse.

This mapping from the symbols in the program to the world of the program (which may be the real world or some imagined world) is called an *interpretation*.

The obvious interpretation of the atomic formula `parent(queen_elizabeth, prince_charles)` is that Queen Elizabeth II is a parent of Prince Charles, but other interpretations are also possible.

Two views of predicates

As the name implies, the main focus of the predicate calculus is on *predicates*.

You can think of a predicate with n arguments in two equivalent ways.

- You can view the predicate as a function from all possible combinations of n terms to a truth value (i.e. true or false).
- You can view the predicate as a set of tuples of n terms. Every tuple in this set is implicitly mapped to true, while every tuple not in this set is implicitly mapped to false.

The task of a predicate definition is to define the mapping in the first view, or equivalently, to define the set of tuples in the second view.

The meaning of clauses

The meaning of the clause

$\text{grandparent}(A, C) :- \text{parent}(A, B), \text{parent}(B, C).$

is: for all the terms that A and C may stand for, A is the grandparent of C if there is a term B such that A is the parent of B and B is the parent of C.

In mathematical notation:

$\forall A \forall C : \text{grandparent}(A, C) \leftarrow \exists B : \text{parent}(A, B) \wedge \text{parent}(B, C)$

The variables appearing in the head are universally quantified over the entire clause, while variables appearing only in the body are existentially quantified over the body.

The meaning of predicate definitions

A predicate is defined by a finite number of clauses, each of which is in the form of an implication. A fact such as `parent(queen_elizabeth, prince_charles)` represents this implication:

$$\forall A \forall B : \text{parent}(A, B) \leftarrow \\ (A = \text{queen_elizabeth} \wedge B = \text{prince_charles})$$

To represent the meaning of the predicate, create a disjunction of the bodies of all the clauses:

$$\forall A \forall B : \text{parent}(A, B) \leftarrow \\ (A = \text{queen_elizabeth} \wedge B = \text{prince_charles}) \vee \\ (A = \text{prince_philip} \wedge B = \text{prince_charles}) \vee \\ (A = \text{prince_charles} \wedge B = \text{prince_william}) \vee \\ (A = \text{prince_charles} \wedge B = \text{prince_harry}) \vee \\ (A = \text{princess_diana} \wedge B = \text{prince_william}) \vee \\ (A = \text{princess_diana} \wedge B = \text{prince_harry})$$

The closed world assumption

To implement the *closed world assumption*, we only need to make the implication arrow go both ways (*if and only if*):

$$\begin{aligned} \forall A \forall B : \text{parent}(A, B) \leftrightarrow & \\ & (A = \text{queen_elizabeth} \wedge B = \text{prince_charles}) \vee \\ & (A = \text{prince_philip} \wedge B = \text{prince_charles}) \vee \\ & (A = \text{prince_charles} \wedge B = \text{prince_william}) \vee \\ & (A = \text{prince_charles} \wedge B = \text{prince_harry}) \vee \\ & (A = \text{princess_diana} \wedge B = \text{prince_william}) \vee \\ & (A = \text{princess_diana} \wedge B = \text{prince_harry}) \end{aligned}$$

This means that A is not a parent of B unless they are one of the listed cases.

Adding the reverse implication this way creates the *Clark completion* of the program.

Semantics of logic programs

A logic program P consists of a set of predicate definitions. The *semantics* of this program (its *meaning*) is the set of its *logical consequences* as ground atomic formulas.

A ground atomic formula a is a logical consequence of a program P if P makes a true.

A negated ground atomic formula $\neg a$, written in Prolog as `\+ a`, is a logical consequence of P if a is not a logical consequence of P .

For most logic programs, the set of ground atomic formulas it entails is infinite (as is the set it does not entail). As logicians, we do not worry about this any more than a mathematician worries that there are an infinite number of solutions to $a + b = c$.

Finding the semantics

You can find the semantics of a logic program by working backwards. Instead of reasoning from a query to find a satisfying substitution, you reason from the program to find what ground queries will succeed.

The immediate consequence operator T_P takes a set of ground unit clauses C and produces the set of ground unit clauses implied by C together with the program P .

This always includes all ground instances of all unit clauses in P . Also, for each clause $H : -G_1, \dots, G_n$ in P , if C contains instances of G_1, \dots, G_n , then the corresponding instance of H is also in the result.

Eg, if $P = \{q(X, Z) : -p(X, Y), p(Y, Z)\}$ and $C = \{p(a, b).p(b, c).p(c, d).\}$, then $T_P(C) = \{q(a, c).q(b, d).\}$.

The semantics of program P is always $T_P(T_P(T_P(\dots(\emptyset)\dots)))$ (T_P applied infinitely many times to the empty set).

QUIZ: Semantics

What is the semantics of this logic program:

$p(1, 2).$ $p(2, 3).$ $p(2, 6).$ $p(3, 4).$

$q(A, B) :- p(A, B).$

$q(A, C) :- q(A, B), q(B, C).$

$r(A, C) :- r(A, B), r(B, C).$

QUIZ: Semantics

What is the semantics of this logic program:

$p(1, 2).$ $p(2, 3).$ $p(2, 6).$ $p(3, 4).$

$q(A, B) :- p(A, B).$

$q(A, C) :- q(A, B), q(B, C).$

$r(A, C) :- r(A, B), r(B, C).$

$p(1, 2).$ $p(2, 3).$ $p(2, 6).$ $p(3, 4).$

$q(1, 2).$ $q(2, 3).$ $q(2, 6).$ $q(3, 4).$

$q(1, 3).$ $q(1, 6).$ $q(2, 4).$ $q(1, 4).$

There can be no clauses for r .

Procedural Interpretation

The *logical* reading of the clause

```
grandparent(X, Z) :-  
    parent(X, Y), parent(Y, Z).
```

says “*for all* X, Y, Z , if X is parent of Y and Y is parent of Z , then X is grandparent of Z ”.

The *procedural* reading says “to show that X is a grandparent of Z , it is sufficient to show that X is a parent of Y and Y is a parent of Z ”.

SLD resolution, used by Prolog, implements this strategy.

SLD Resolution

The consequences of a logic program are determined through a simple but powerful deduction strategy called *resolution*.

SLD resolution is an efficient version of resolution. The basic idea is: given this program, to show this goal is true

```
q :- b1a, b1b.  
q :- b2a, b2b.  
⋮
```

```
?- p, q, r.
```

it is sufficient to show any of

```
?- p, b1a, b1b, r.  
?- p, b2a, b2b, r.  
⋮
```

SLD resolution in action

E.g., to determine if Queen Elizabeth is Prince Harry's grandparent:

```
?- grandparent(queen_elizabeth, prince_harry).
```

with this program

```
grandparent(X, Z) :-  
    parent(X, Y), parent(Y, Z).
```

we unify query goal `grandparent(queen_elizabeth, prince_harry)` with clause head `grandparent(X, Z)`, apply the resulting substitution to the clause, yielding the *resolvent*. Since the goal is identical to the resolvent head, we can replace it with the resolvent body, leaving:

```
?- parent(queen_elizabeth, Y), parent(Y, prince_harry).
```

SLD resolution can fail

Now we must pick one of these goals to resolve; we select the second.

The program has several clauses for `parent`, but only two can successfully resolve with `parent(Y, prince_harry)`:

```
parent(prince_charles, prince_harry).  
parent(princess_diana, prince_harry).
```

We choose the second. After resolution, we are left with the query (note the unifying substitution is applied to both the selected clause and the query):

```
?- parent(queen_elizabeth, princess_diana).
```

No clause unifies with this query, so resolution fails. Sometimes, it may take many resolution steps to fail.

SLD resolution can succeed

Selecting the second of these matching clauses led to failure:

```
parent(prince_charles, prince_harry).  
parent(princess_diana, prince_harry).
```

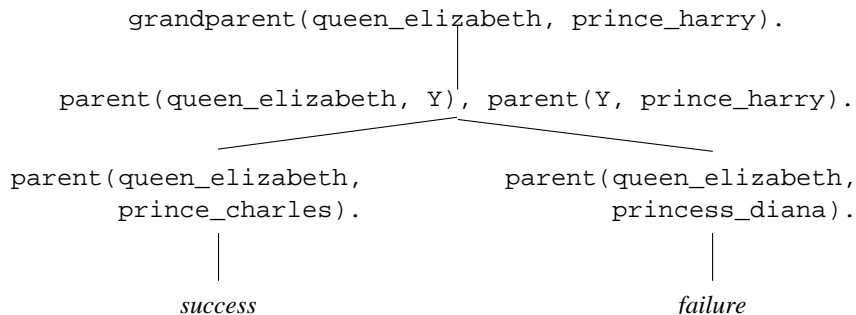
This does not mean we are through: we must *backtrack* and try the first matching clause. This leaves

```
?- parent(queen_elizabeth, prince_charles).
```

There is one matching program clause, leaving nothing more to prove. The query succeeds.

Resolution

This derivation can be shown as an SLD tree:



Order of execution

The order in which goals are resolved and the order in which clauses are tried does not matter for correctness (in pure Prolog), but it does matter for efficiency. In this example, resolving `parent(queen_elizabeth, Y)` before `parent(Y, prince_harry)` is more efficient, because there is only one clause matching the former, and two matching the latter.

```
grandparent(queen_elizabeth, prince_harry).
```

```
parent(queen_elizabeth, Y), parent(Y, prince_harry).
```

```
parent(prince_charles, prince_harry).
```

SUCCESS

QUIZ: Order and efficiency

Which of these queries will be more efficient, and why?

1 X = prince_charles, parent(X, Y).

2 parent(X, Y), X = prince_charles.

QUIZ: Order and efficiency

Which of these queries will be more efficient, and why?

1 X = prince_charles, parent(X, Y).

2 parent(X, Y), X = prince_charles.

The first query, because when `parent/2` is called, the first argument is bound, there are only two matching clauses. In the second query, both arguments are unbound, so all clauses for `parent/2` match.

SLD resolution in Prolog

At each resolution step we must make two decisions:

- 1 which goal to resolve
- 2 which clauses matching the selected goal to pursue

(though there may only be one choice for either or both).

Our procedure was somewhat haphazard when decisions needed to be made. For pure logic programming, this does not matter for correctness. All goals will need to be resolved eventually; which order they are resolved in does not change the answers. All matching clauses may need to be tried; the order in which we try them determines the order solutions are found, but not which solutions are found.

Prolog always selects the first goal to resolve, and always selects the first matching clause to pursue first. This gives the programmer more certainty, and control, over execution.

Backtracking

When there are multiple clauses matching a goal, Prolog must remember which one to go back to if necessary. It must be able to return the computation to the state it was in when the first matching clause was selected, so that it can return to that state and try the next matching clause. This is all done with a *choicepoint*.

When a goal fails, Prolog *backtracks* to the most recent choicepoint, removing all variable bindings made since the choicepoint was created, returning those variables to their unbound state. Then Prolog begins resolution with the next matching clause, repeating the process until Prolog detects that there are no more matching clauses, at which point it removes that choicepoint. Subsequent failures will then backtrack to the next most recent choicepoint.

Indexing

Indexing can greatly improve Prolog efficiency

Most Prolog systems will automatically create an index for a predicate such as `parent/2` (Prolog uses *name/arity* to refer to predicates) with multiple clauses the heads of which have distinct constants or functors. This means that, for a call with the first argument bound, Prolog will immediately jump to the first clause that matches. If backtracking occurs, the index allows Prolog to jump straight to the next clause that matches, and so on.

If the first argument is unbound, then all clauses will have to be tried.

Indexing

If some clauses have variables in the first argument of the head, those clauses will be tried at the appropriate time regardless of the call. Indexing changes performance, not behaviour. Consider:

$p(a, z).$

$p(b, y).$

$p(X, X).$

$p(a, x).$

For the call $p(I, J)$, all clauses will be tried, in order. For $p(a, J)$, the first clause will be tried, then the third, then fourth. For $p(b, J)$, the second, then third, clause will be tried. For $p(c, J)$, only the third clause will be tried.

Indexing

Some Prolog systems, such as SWI Prolog, will construct indices for arguments other than the first. For `parent/2`, SWI Prolog will index on both arguments, so finding the children of a parent or parents of a child both benefit from indexing.

Just as important as jumping directly to the first matching clause, indexing tells Prolog when no further clauses could possibly match the goal, allowing it to remove the choicepoint, or even to avoid creating the choicepoint in the first place. Even with only two clauses, such as for `append/3`, indexing can substantially improve performance.

The University of Melbourne
School of Computing and Information Systems

COMP90048

Declarative Programming

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Section 14

Tail Recursion

Tail recursion

A predicate (or function, or procedure, or method, or ...) is *tail recursive* if the only recursive call on any execution of that predicate is the last code executed before returning to the caller. For example, the usual definition of `append/3` is tail recursive, but `rev1/2` is not:

```
append([], C, C).
append([A|B], C, [A|BC]) :-
    append(B, C, BC).
```

```
rev1([], []).
rev1([A|BC], CBA) :-
    rev1(BC, CB),
    append(CB, [A], CBA).
```

Tail recursion optimisation

Like most declarative languages, Prolog performs *tail recursion optimisation* (TRO). This is important for declarative languages, since they use recursion more than non-declarative languages. TRO makes recursive predicates behave as if they were loops.

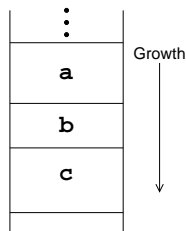
Note that TRO is more often directly applicable in Prolog than other languages because more Prolog code is tail recursive. For example, while `append/3` in Prolog is tail recursive, `++` in Haskell is not, because the last operation performed is `(:)`, not `++`.

```
[] ++ lst = lst
(h:t) ++ lst = h:(t++lst)
```

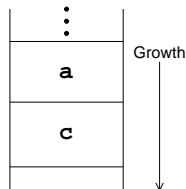
However another optimisation can permit TRO for this code.

The stack

To understand TRO, it is important to understand how programming languages (not just Prolog) implement call and return using a stack (Haskell is a rare exception). While *a* is executing, it stores its local variables, and where to return to when finished, in a *stack frame* or *activation record*. When *a* calls *b*, it creates a fresh stack frame for *b*'s local variables and return address, preserving *a*'s frame, and similarly when *b* calls *c*, as shown to the right.



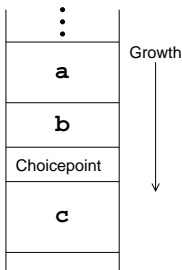
But if all *b* will do after calling *c* is return to *a*, then there is no need to preserve its local variables. Prolog can release *b*'s frame before calling *c*, as shown to the right. When *c* is finished, it will return directly to *a*. This is called *last call optimisation*, and can save significant stack space.



TRO and choicepoints

Tail recursion optimisation is a special case of last call optimisation where the last call is recursive. This is especially beneficial, since recursion is used to replace looping. Without TRO, this would require a stack frame for each iteration, and would quickly exhaust the stack. With TRO, tail recursive predicates execute in constant stack space, just like a loop.

However, if **b** leaves a choicepoint, it sits on the stack above **b**'s frame, “freezing” that and all earlier frames so that they are not reclaimed. This is necessary because when Prolog backtracks to that choicepoint, **b**'s arguments must be ready to try the next matching clause for **b**. The same is true if **c** or any predicate called later leaves a choicepoint, but choicepoints before the call to **b** do not interfere.



Making code tail recursive

Our factorial predicate was not tail recursive, as the last thing it does is perform arithmetic.

```
fact(N, F) :-  
    ( N == 0 ->  
        F = 1  
    ;   N > 0,  
        N1 is N - 1,  
        fact(N1, F1),  
        F is F1 * N  
    ).
```

Note that Prolog's if-then-else construct does not leave a choicepoint. A choicepoint is created, but is removed as soon as the condition succeeds or fails. So `fact` would be subject to TRO, if only it were tail recursive.

Adding an accumulator

We make factorial tail recursive by introducing an **accumulating parameter**, or just an **accumulator**. This is an extra parameter to the predicate that holds a partially computed result.

Usually the base case for the recursion will specify that the partially computed result is actually the result. The recursive clause usually computes more of the partially computed result, and passes this in the recursive goal.

The key to getting the implementation correct is specifying what the accumulator *means* and how it relates to the final result. To see how to add an accumulator, determine what is done after the recursive call, and then respecify the predicate so it performs this task, too.

Adding an accumulator

For factorial, we compute `fact(N1, F1)`, `F` is `F1 * N` last, so the tail recursive version will need to perform the multiplication too. We must define a predicate `fact1(N, A, F)` so that `F` is `A` times the factorial of `N`. In most cases, it is not difficult to see how to transform the original definition to the tail recursive one.

<pre>fact(N, F) :- (N == 0 -> F = 1 ; N > 0, N1 is N - 1, fact(N1, F1), F is F1 * N).</pre>	<pre>fact(N, F) :- fact1(N, 1, F). fact1(N, A, F) :- (N == 0 -> F = A ; N > 0, N1 is N - 1, A1 is A * N, fact1(N1, A1, F)).</pre>
---	---

Adding an accumulator

Finally, define the original predicate in terms of the new one. Again, it is usually easy to see how to do that.

Another way to think about writing a tail recursive implementation of a predicate is to realise that it will essentially be a loop, so think of how you would write it as a `while` loop, and then write that loop in Prolog.

```
A = 1;
while (N > 0) {
    A *= N;
    N--;
}
if (N == 0) return A;
else FAIL;
```

```
fact(N, F) :- fact1(N, 1, F).
fact1(N, A, F) :-
    ( N > 0 ->
        A1 is A * N,
        N1 is N - 1,
        fact1(N1, A1, F)
    ; N == 0 ->
        F = A
    ).
```

QUIZ: Tail recursive multiply

If multiplication were not built into Prolog, we could define it as:

```
multiply(X, Y, XY) :-  
    ( X = 0 ->  
        XY = 0  
    ;   X1 is X - 1,  
        multiply(X1, Y, X1Y),  
        XY is X1Y + Y  
    ).
```

(This definition only works when the first two arguments are bound, and does not handle negative X values. This is just an exercise.)

Give a tail recursive definition of multiply/3.

QUIZ: Tail recursive multiply (answer)

```
multiply(X, Y, XY) :-  
    multiply1(X, Y, 0, XY).
```

```
multiply1(X, Y, A, XY) :-  
    ( X = 0 ->  
        XY = A  
    ;   X1 is X - 1,  
        A1 is A + Y,  
        multiply(X1, Y, A1, XY)  
    ).
```


Transformation

Another approach is to systematically transform the non-tail recursive version into an equivalent tail recursive predicate. Start by defining a predicate to do the work of the recursive call to `fact/2` and everything following it. Then replace the call to `fact(N, F2)` by the definition of `fact/2`. This is called *unfolding*.

```
fact1(N, A, F) :-  
    fact(N, F2),  
    F is F2 * A
```

```
fact1(N, A, F) :-  
    ( N == 0 ->  
        F2 = 1  
    ;   N > 0,  
        N1 is N - 1,  
        fact(N1, F1),  
        F2 is F1 * N  
    ),  
    F is F2 * A.
```

Transformation

Next we move the final goal into both the *then* and *else* branches.

<pre>fact1(N, A, F) :- (N == 0 -> F2 = 1 ; N > 0, N1 is N - 1, fact(N1, F1), F2 is F1 * N), F is F2 * A.</pre>	<pre>fact1(N, A, F) :- (N == 0 -> F2 = 1, F is F2 * A ; N > 0, N1 is N - 1, fact(N1, F1), F2 is F1 * N, F is F2 * A).</pre>
--	---

Transformation

The next step is to simplify the arithmetic goals.

<pre>fact1(N, A, F) :- (N == 0 -> F2 = 1, F is F2 * A ; N > 0, N1 is N - 1, fact(N1, F1), F2 is F1 * N, F is F2 * A).</pre>	<pre>fact1(N, A, F) :- (N == 0 -> F = A ; N > 0, N1 is N - 1, fact(N1, F1), F is (F1 * N) * A).</pre>
---	---

Transformation

Now we utilise the associativity of multiplication. This is the insightful step that is necessary to be able to make the next step.

```
fact1(N, A, F) :-
    ( N == 0 ->
        F = A
    ;   N > 0,
        N1 is N - 1,
        fact(N1, F1),
        F is (F1 * N) * A
    ).
```

```
fact1(N, A, F) :-
    ( N == 0 ->
        F = A
    ;   N > 0,
        N1 is N - 1,
        fact(N1, F1),
        F is F1 * (N * A)
    ).
```

Transformation

Now part of the computation can be moved before the call to fact/2.

```
fact1(N, A, F) :-
    ( N == 0 ->
        F = A
    ;   N > 0,
        N1 is N - 1,
        fact(N1, F1),
        F is F1 * (N * A)
    ).
```

```
fact1(N, A, F) :-
    ( N == 0 ->
        F = A
    ;   N > 0,
        N1 is N - 1,
        A1 is N * A,
        fact(N1, F1),
        F is F1 * A1
    ).
```

Transformation

The final step is to recognise that the last two goals look very much like the body of the original definition of `fact1/3`, with the substitution $\{N \mapsto N1, F2 \mapsto F1, A \mapsto A1\}$. So we replace those two goals with the clause head with that substitution applied. This is called *folding*.

<pre>fact1(N, A, F) :- (N == 0 -> F = A ; N > 0, N1 is N - 1, A1 is N * A, fact(N1, F1), F is F1 * A1).</pre>	<pre>fact1(N, A, F) :- (N == 0 -> F = A ; N > 0, N1 is N - 1, A1 is N * A, fact1(N1, A1, F)).</pre>
---	---

Accumulating Lists

The tail recursive version of `fact` is a constant factor more efficient, because it behaves like a loop. Sometimes accumulators can make an order difference, if it can replace an operation with a computation of lower asymptotic complexity, for example replacing `append/3` (linear time) with list construction (constant time).

```
rev1([], []).
rev1([A|BC], CBA) :-
    rev1(BC, CB),
    append(CB, [A], CBA).
```

This definition of `rev1/2` is of quadratic complexity, because for the n^{th} element from the end of the first argument, we append a list of length $n - 1$ to a singleton list. Doing this for each of the n elements gives time proportional to $\frac{n(n-1)}{2}$.

Tail recursive rev1/2

The first step in making a tail recursive version of rev1/2 is to specify the new predicate. It must combine the work of rev1/2 with that of append/3. The specification is:

```
% rev(BCD, A, DCBA)
% DCBA is BCD reversed, with A appended
```

We could develop this by transformation as we did for fact1/3, but we implement it directly here. We begin with the base case, for $BCD = []$:

```
rev([], A, A).
```


Tail recursive rev1/2

For the recursive case, take $BCD = [B|CD]$:

```
rev([B|CD], A, DCBA) :-
```

the result, DCBA, must be the reverse of CD, with [B] appended to the end, and A appended after that. In Haskell notation this is

$$(\text{rev } cd ++ [b]) ++ a.$$

Because append is associative, this is the same as

$$\text{rev } cd ++ ([b] ++ a) \equiv \text{rev } cd ++ (b:a).$$

We can use our rev/3 predicate to compute that:

```
rev([B|CD], A, DCBA) :-  
  rev(CD, [B|A], DCBA).
```

Tail recursive rev1/2

```
rev([], A, A).  
rev([B|CD], A, DCBA) :-  
    rev(CD, [B|A], DCBA).
```

At each recursive step, this code removes an element from the head of the input list and adds it to the head of the accumulator. The cost of each step is therefore a constant, so the overall cost is linear in the length of the list.

Accumulator lists work like a stack: the last element of the accumulator is the first element that was added to it, and so on. Thus at the end of the input list, the accumulator is the reverse of the original input.

Difference pairs

The trick used for a tail recursive `reverse` predicate is often used in Prolog: a predicate that generates a list takes an extra argument specifying what should come after the list. This avoids the need to append to the list.

In Prolog, if you do not know what will come after at the time you call the predicate, you can pass an unbound variable, and bind that variable when you do know what should come after. Thus many predicates intended to produce a list have two arguments, the first is the list produced, and the second is what comes after. This is called a *difference pair*, because the predicate generates the *difference* between the first and second list.

```
flatten(empty, List, List).  
flatten(node(L,E,R), List, List0) :-  
    flatten(L, List, [E|List1]),  
    flatten(R, List1, List0).
```

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Declarative Programming

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Section 15

All solutions and impurity

All solutions

Sometimes one would like to bring together all solutions to a goal. Prolog's *all solutions* predicates do exactly this.

`setof(Template, Goal, List)` binds `List` to sorted list of all distinct instances of `Template` satisfying `Goal`

`Template` can be any term, usually containing some of the variables appearing in `Goal`. On completion, `setof/3` binds its `List` argument, but does not further bind any variables in the `Goal`.

```
?- setof(P-C, parent(P, C), List).
```

```
List = [duchess_kate-prince_george, prince_charles-prince_harry,  
prince_charles-prince_william, prince_philip-prince_charles,  
prince_william-prince_george, princess_diana-prince_harry,  
princess_diana-prince_william, queen_elizabeth-prince_charles].
```

All solutions

If Goal contains variables not appearing in Template, setof/3 will backtrack over each distinct binding of these variables, for each of them binding List to the list of instances of Template for that binding.

```
?- setof(C, parent(P, C), List).  
P = duchess_kate,  
List = [prince_george] ;  
P = prince_charles,  
List = [prince_harry, prince_william] ;  
P = prince_philip,  
List = [prince_charles] ;  
P = prince_william,  
List = [prince_george] ;  
P = princess_diana,  
List = [prince_harry, prince_william] ;  
P = queen_elizabeth,  
List = [prince_charles].
```

Existential quantification

Use existential quantification, written with infix caret (\sim), to collect solutions for a template regardless of the bindings of some of the variables not in the Template.

E.g., to find all the people in the database who are parents of any child:

```
?- setof(P, C $\sim$ parent(P, C), Parents).
```

```
Parents = [duchess_kate, prince_charles, prince_philip,  
prince_william, princess_diana, queen_elizabeth].
```

Unsorted solutions

The `bagof/3` predicate is just like `setof/3`, except that it does not sort the result or remove duplicates.

```
?- bagof(P, C^parent(P, C), Parents).
```

```
Parents = [queen_elizabeth, prince_philip, prince_charles,  
prince_charles, princess_diana, princess_diana, prince_william,  
duchess_kate].
```

Solutions are collected in the order they are produced. This is not purely logical, because the order of solutions should not matter, nor should the number of times a solution is produced.

Higher Order Programming

The `call/1` built-in predicate executes a term as a goal. This capitalises on the fact that Prolog terms look just like Prolog goals. Many Prologs, including SWI Prolog, will `call` a variable used as a goal.

```
?- X=append([1,2],[3],L), call(X).
```

```
X = append([1, 2], [3], [1, 2, 3]),
```

```
L = [1, 2, 3].
```

```
?- X=append(A, B, [1]), X.
```

```
X = append([], [1], [1]),
```

```
A = [],
```

```
B = [1] ;
```

```
X = append([1], [], [1]),
```

```
A = [1],
```

```
B = [] ;
```

```
false.
```

Currying

Currying in Prolog is done by simply omitting some final arguments of a higher-order goal. To support this, many Prologs, including SWI, support versions of `call` of higher arity. All arguments to `call/n` after the goal (first) are added as extra arguments to the goal.

```
?- X=append([1,2],[3]), call(X, L).  
X = append([1, 2], [3]),  
L = [1, 2, 3].
```

Writing higher-order code

It is fairly straightforward to write higher order predicates using `call/N`. For example, here is `map/3` in Prolog:

```
map(_, [], []).
map(P, [X|Xs], [Y|Ys]) :-
    call(P, X, Y),
    map(P, Xs, Ys).
```

```
?- map(plus(1), [3,4,5], L).
L = [4, 5, 6] ;
false.
?- map(plus(1), L, [3,4,5]).
L = [2, 3, 4] ;
false.
```

This is in the SWI Prolog library with the name `maplist/2`, 3, 4, and 5.

QUIZ: Higher-order code

Write a predicate `filter(Pred, List, Filtered)` such that `Filtered` is a list of the elements of `List` (in the same order) that satisfy the predicate `Pred`.

QUIZ: Higher-order code

Write a predicate `filter(Pred, List, Filtered)` such that `Filtered` is a list of the elements of `List` (in the same order) that satisfy the predicate `Pred`.

```
filter(_, [], []).
filter(P, [X|Xs], Filtered) :-
    ( call(P, X) ->
        Filtered = [X|Filtered1]
    ;   Filtered = Filtered1
    ),
    filter(P, Xs, Filtered1).
```

```
?- filter(<(0), [-5,5,-2,2], L).
L = [5, 2] ;
false.
```

Input/Output

Prolog's Input/Output facility does not even try to be pure. I/O operations are executed when they are reached according to Prolog's simple execution order. I/O is not “undone” on backtracking.

Prolog has builtin predicates to read and write arbitrary Prolog terms. Prolog also allows users to define their own operators. This makes Prolog very convenient for applications involving structured I/O.

```
?- op(800, xfy, wibble).
true.
?- read(X).
|: p(x, [1, 2], X>Y wibble z).
X = p(x, [1, 2], _G280>_G281 wibble z).
?- write(p(x, [1, 2], X>Y wibble z)).
p(x, [1, 2], _G207>_G208 wibble z)
true.
```

Input/Output

`write/1` is handy for printing messages:

```
?- write('hello '), write('world!').  
hello world!  
true.  
?- write('world!'), write('hello ').  
world!hello  
true.
```

This demonstrates that Prolog's input/output predicates are non-logical. These should be equivalent, because conjunction *should* be commutative.

Code that performs I/O must be handled carefully — you must be aware of the modes. It is recommended to isolate I/O in a small part of the code, and keep the bulk of your code I/O-free. (This is a good idea in any language.)

Comparing terms

All Prolog terms can be compared for ordering using the built-in predicates `@<`, `@=<`, `@>`, and `@>=`. Prolog, somewhat arbitrarily, uses the ordering

Variables < Numbers < Atoms < CompoundTerms

but most usefully, within these classes, terms are ordered as one would expect: numbers by value and atoms are sorted alphabetically. Compound terms are ordered first by arity, then alphabetically by functor, and finally by arguments, left-to-right. It is best to use these only for ground terms.

```
?- hello @< hi.
true.
?- X @< 7, X = foo.
X = foo.
?- X = foo, X @< 7.
false.
```


Sorting

There are three SWI Prolog builtins for sorting ground lists according to the @< ordering: `sort/2` sorts a list, removing duplicates, `msort/2` sorts a list, without removing duplicates, and `keysort/2` stably sorts list of X-Y terms, only comparing X parts:

```
?- sort([h,e,l,l,o], L).  
L = [e, h, l, o].  
?- msort([h,e,l,l,o], L).  
L = [e, h, l, l, o].  
?- keysort([7-a, 3-b, 3-c, 8-d, 3-a], L).  
L = [3-b, 3-c, 3-a, 7-a, 8-d].
```

Determining term types

`integer/1` holds for integers and fails for anything else. It also fails for variables.

```
?- integer(3).  
true.  
?- integer(a).  
false.  
?- integer(X).  
false.
```

Similarly, `float/1` recognises floats, `number` recognises either kind of number, `atom/1` recognises atoms, and `compound/1` recognises compound terms. All of these fail for variables, so must be used with care.

Recognising variables

`var/1` holds for unbound variables, `nonvar/1` holds for any term other than an unbound variable, and `ground/1` holds for ground terms (this requires traversing the whole term). Using these or the predicates on the previous slide can make your code behave differently in different modes.

But they can also be used to write code that works in multiple modes.

Here is a tail-recursive version of `len/2` that works whenever the length is known:

```
len2(L, N) :-
    (   N == 0
    ->  L = []
    ;   N1 is N - 1,
        L = [_|L1],
        len2(L1, N1)
    ).
```

Recognising variables

This version works when the length is unknown:

```
len1([], N, N).
len1(_|L, N0, N) :-
    N1 is N0 + 1,
    len1(L, N1, N).
```

This code chooses between the two:

```
len(L, N) :-
    (   integer(N)
    -> len2(L, N)
    ;   nonvar(N)
    -> throw(error(type_error(integer, N),
                    context(len/2, '')))
    ;   len1(L, 0, N)
    ).
```

Mercury

The Mercury language was developed at The University of Melbourne as a purely declarative successor to Prolog. Mercury is strongly typed, with a type system very similar to Haskell's. It is also strongly moded, which means that the binding state of all variables is determined at compile-time. Mercury has a strong determinism system, as well, which allows the compiler to determine, for each predicate mode, whether it will always be deterministic.

All these properties, along with aggressive compiler optimisations, make Mercury among the highest-performing declarative languages. Although Mercury's learning curve is somewhat steeper than Prolog's, it is a more realistic choice for serious development.

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Section 16

Constraint (logic) programming

Constraint (logic) programming

An imperative program specifies the exact sequence of actions to be executed by the computer.

A functional program specifies how the result of the program is to be computed at a more abstract level. One can read function definitions as suggesting an order of actions, but the language implementation can and sometimes will deviate from that order, due to laziness, parallel execution, and various optimizations.

A logic program is in some ways more declarative, as it specifies a set of equality constraints that the terms of the solution must satisfy, and then searches for a solution.

A constraint program is more declarative still, as it allows more general constraints than just equality constraints. The search for a solution will typically follow an algorithm whose relationship to the specification can be recognized only by experts.

Problem specification

The specification of a constraint problem consists of

- a set of variables, each variable having a known domain,
- a set of constraints, with each constraint involving one or more variables, and
- an optional objective function.

The job of the constraint programming system is to find a *solution*, a set of assignments of values to variables (with the value of each variable being drawn from its domain) that satisfies all the constraints.

The objective function, if there is one, maps each solution to a number. If this number represents a cost, you want to pick the cheapest solution; if this number represents a profit, you want to pick the most profitable solution.

Kinds of constraint problems

There are several kinds of constraints, of which the following four are the most common. Most CP systems handle only one or two kinds.

In *Herbrand* constraint systems, the variables represent terms, and the basic constraints are unifications, i.e. they have the form $term_1 = term_2$. This is the constraint domain implemented by Prolog.

In *finite domain* or *FD* constraint systems, each variable's domain has a finite number of elements.

In *boolean satisfiability* or SAT systems, the variables represent booleans, and each constraint asserts the truth of an expression constructed using logical operations such as AND, OR, NOT and implication.

In *linear inequality* constraint systems, the variables represent real numbers (or sometimes integers), and the constraints are of the form $ax + by \leq c$ (where x and y are variables, and a , b and c are constants).

Type inference

One application of constraint programming using Herbrand terms is type inference for the Hindley-Milner type system (the Haskell type system). The process of inferring the types of variables inside functions can be viewed as a matter of solving a set of constraints over variable types. Herbrand terms can be used to neatly represent types. and (if not declared) the types of the functions and predicates themselves can be viewed as a Herbrand constraint problem.

This approach represents the type of each variable in the program as a Herbrand term, and applies rules to produce equality constraints for each subexpression in the function definitions. To simplify the process, we assume the program has been “normalised” into an equivalent definition expressed in a restricted form. In particular, all nested expressions have been replaced by let-bound temporary variables, and deconstructions have been replaced by case constructs.

Type inference

For example, the rule for handling function application states that if the type of f is $a \rightarrow b$ and the type of x is t then the type of $f\ x$ is b , where $t = a$. The case construct is handled by unifying the type of the expression with the type of all of the cases.

Consider the normalised definition of the Haskell `map` function:

```
map f l = case l of
    []      -> []
    (e:es) -> let t = map f es
                h = f e
                in  h:t
```

The form of the equation tells you that the type of `map`, t_{map} , is a type of the form $t_1 \rightarrow t_2 \rightarrow t_r$.

t_{map} , t_1 , t_2 and t_r are all constraint variables whose domain is the set of possible Haskell types, and $t_{map} = t_1 \rightarrow t_2 \rightarrow t_r$ is a constraint on those variables.

Type inference: an example

We can use Prolog's Herbrand terms to infer types:

```
map f l = case l of []      -> []
              (e:es) -> let t = map f es
                        h = f e
                        in  h:t
```

```
?- op(200,xfy,'=>'). % define arrow operator for convenience
```

```
?- Map = F => L => Result,
   |   L = [_], Result = [_],
   |   L = [E], Es = [E],
   |   Map = F => Es => T,
   |   F = E => H,
   |   Result = [H], Result = T.
```

```
Map = (E=>H)=>[E]=>[H],
```

```
F = E=>H,
```

```
L = Es, Es = [E],
```

```
Result = T, T = [H].
```

QUIZ: Type inference

Give the set of Herbrand type constraints for this function definition:

```
filter f l = case l of
    []      -> []
    (x:xs) -> let fxs = filter f xs
               in if f x then x:fxs else fxs
```

QUIZ: Type inference

Give the set of Herbrand type constraints for this function definition:

```
filter f l = case l of
    []      -> []
    (x:xs) -> let fxs = filter f xs
               in if f x then x:fxs else fxs
```

```
?- Filter = F => L => Result,
| L = [],
| L = [X], Xs = [X],
| Filter = F => Xs => Fxs,
| F = X => bool,
| Result = [X], Fxs = [X],
| Result = Fxs.
Filter = (X=>bool)=>[X]=>[X],
F = X=>bool,
L = Result, Result = Xs, Xs = Fxs, Fxs = [X].
```

Linear inequality constraints

Suppose you want to make banana and/or chocolate cakes for a bake sale, and you have 10 kg of flour, 30 bananas, 1.2 kg of sugar, 1.5 kg of butter, and 700 grams of cocoa on hand. You can charge \$4.00 for a banana cake and \$6.50 for a chocolate one. Each kind of cake requires a certain quantity of each ingredient. How do you determine how many of each cake to make so as to maximise your profit?

To solve such a problem, you need to set up a system of constraints saying, for example, that the number of each kind of cake times the amount of flour needed for that kind of cake must add to no more than the amount of flour you have.

You also need to specify that the number of each kind of cake must be non-negative. Finally, you need to define your revenue as the sum of the number of each kind of cake times its price, and specify that you would like to maximise revenue.

Linear inequality constraints

We can use SWI Prolog's `library(clpr)` to solve such problems. This library requires constraints to be enclosed in curly braces.

```
?- {250*B + 200*C =< 10000},
|   {2*B =< 30},
|   {75*B + 150*C =< 1200},
|   {100*B + 150*C =< 1500},
|   {75*C =< 700},
|   {B>=0}, {C>=0},
|   {Revenue = 4*B + 6.5*C}, maximize(Revenue).
B = 12.0,
C = 2.0,
Revenue = 61.0
```

So we can make \$61.00 by making 12 Banana and 2 Chocolate cakes.

MiniZinc

The MiniZinc language is a *modelling* language, allowing concise specification of constraint satisfaction problems. This same problem could be expressed in MiniZinc as:

```
% b is the number of banana cakes to make
% c is the number of chocolate cakes to make

constraint 250*b + 200*c <= 10000;
constraint 2*b          <= 30;
constraint 75*b + 150*c <= 1200;
constraint 100*b + 150*c <= 1500;
constraint          75*c <= 700;
constraint b >= 0;
constraint c >= 0;

solve maximize 400*b + 650*c;
```

Sudoku

Sudoku is a class of puzzles played on a 9x9 grid. Each grid position should hold a number between 1 and 9. The same integer may not appear twice

- in a single row,
- in a single column, or
- in one of the nine 3x3 boxes.

The puzzle creator provides some of the numbers; the challenge is to fill in the rest.

This is a classic finite domain constraint satisfaction problem.

r1	5	3			7				
r2	6			1	9	5			
r3		9	8				6		
r4	8				6			3	
r5	4			8		3		1	
r6	7				2			6	
r7		6					2	8	
r8				4	1	9		5	
r9					8		7	9	
	c1	c2	c3	c4	c5	c6	c7	c8	c9

Sudoku as finite domain constraints

You can represent the rules of sudoku as a set of 81 constraint variables ($r1c1$, $r1c2$ etc) each with the domain $1..9$, and 27 all-different constraints: one for each row, one for each column, and one for each box. For example, the constraint for the top left box would be `all_different([r1c1, r1c2, r1c3, r2c1, r2c2, r2c3, r3c1, r3c2, r3c3])`.

Initially, the domain of each variable is $[1..9]$. If you fix the value of a variable e.g. by setting $r1c1 = 5$, this means that the other variables that share a row, column or box with $r1c1$ (and that therefore appear in an all-different constraint with it) cannot be 5, so their domain can be reduced to $[1..4, 6..9]$.

This is how the variables fixed by our example puzzle reduce the domain of $r3c1$ to only $[1..2]$, and the domain of $r5c5$ to only $[5]$.

Fixing $r5c5$ to be 5 gives us a chance to further reduce the domains of the other variables linked to $r5c5$ by constraints, e.g. $r7c5$.

Sudoku in SWI Prolog

Using SWI's library(`clpfd`), Sudoku problems can be solved:

```
sudoku(Rows) :-
    length(Rows, 9), maplist(same_length(Rows), Rows),
    append(Rows, Vs), Vs ins 1..9,
    maplist(all_distinct, Rows),
    transpose(Rows, Columns),
    maplist(all_distinct, Columns),
    Rows = [A,B,C,D,E,F,G,H,I],
    blocks(A, B, C), blocks(D, E, F), blocks(G, H, I).

blocks([], [], []).
blocks([A,B,C|Bs1], [D,E,F|Bs2], [G,H,I|Bs3]) :-
    all_distinct([A,B,C,D,E,F,G,H,I]),
    blocks(Bs1, Bs2, Bs3).
```

Sudoku in SWI Prolog

```
?- Puzzle=[[5,3,_,_,7,_,_,_,_],
|          [6,_,_,1,9,5,_,_,_],
|          [_,9,8,_,_,_,_,6,_],
|
|          [8,_,_,_,6,_,_,_,3],
|          [4,_,_,8,_,3,_,_,1],
|          [7,_,_,_,2,_,_,_,6],
|
|          [_,6,_,_,_,_,2,8,_],
|          [_,_,_,4,1,9,_,_,5],
|          [_,_,_,_,8,_,_,7,9]],
|  sudoku(Puzzle),
|  write(Puzzle).
```

Sudoku solution

In less than $\frac{1}{20}$ of a second, this produces the solution:

```
Puzzle=[[5,3,4, 6,7,8, 9,1,2],  
        [6,7,2, 1,9,5, 3,4,8],  
        [1,9,8, 3,4,2, 5,6,7],  
  
        [8,5,9, 7,6,1, 4,2,3],  
        [4,2,6, 8,5,3, 7,9,1],  
        [7,1,3, 9,2,4, 8,5,6],  
  
        [9,6,1, 5,3,7, 2,8,4],  
        [2,8,7, 4,1,9, 6,3,5],  
        [3,4,5, 2,8,6, 1,7,9]]
```

Search

Prolog normally employs a strategy known as “*generate and test*” to search for variable bindings that satisfy constraints. Nondeterministic goals *generate* potential solutions; later goals *test* those solutions, imposing further constraints and rejecting some candidate solutions.

For example, in

```
?- between(1,9,X), 0 ::= X mod 2, X ::= X * X mod 10.
```

The first goal generates single-digit numbers, the second tests that it is even, and the third that its square ends in the same digit.

Constraint logic programming uses the more efficient “*constrain and generate*” strategy. In this approach, constraints on variables can be more sophisticated than simply binding to a Herbrand term. This is generally accomplished in Prolog systems with *attributed variables*, which allow constraint domains to control unification of constrained variables.

Propagation

The usual algorithm for solving a set of FD constraints consists of two steps: *propagation* and *labelling*.

In the propagation step, we try to reduce the domain of each variable as much as possible.

For each constraint, we check whether the constraint rules out any values in the current domains of any of the variables in that constraint. If it does, then we remove that value from the domain of that variable, and schedule the constraints involving that variable to be looked at again.

The propagation step ends

- if every variable has a domain of size one, which represents a fixed value, since this represents a solution;
- if some variable has an empty domain, since this represents failure; or
- if there are no more constraints to look at, in which case propagation can do no more.

Labelling

If propagation cannot do any more, we go on to the *labelling* step, which

- picks a not-yet-fixed variable,
- partitions its current domain (of size n) into k parts d_1, \dots, d_k , where usually $k = 2$ but may be any value satisfying $2 \leq k \leq n$, and
- recursively invokes the whole constraint solving algorithm k times, with invocation i restricting the domain of the chosen variable to d_i .

Each recursive invocation also consists of a propagation step and (if needed) a labelling step. The whole computation therefore consists of alternating propagation and labelling steps.

The labelling steps generate a search tree. The size of the tree depends on the effectiveness of propagation: the more effective propagation is at removing values from domains, the smaller the tree will be, and the less time searching it will take.

Timetabling

Timetabling is a finite domain problem of great practical importance.

To timetable university lectures, you need two variables for every lecture:

- The domain of *timeCOMP30020L1* represents all the available lecture time slots from *mon8am* to *fri7pm*.
- The domain of *locCOMP30020L1* represents all the available lecture theatres that are big enough for COMP30020's enrolment.

There are many kinds of constraints, including:

- all the lectures for a subject must be at different times
- all the lectures taught by a single lecturer must be at different times
- all the rooms of lectures taught at the same time must be different
- the room in which each lecture is taught must be big enough for its anticipated enrolment

The search space

Timetabling happens to be very hard to do well, partly because it generates a huge search space if you impose only absolutely necessary constraints such as the previous two.

Imposing the constraint that *timeCOMP30020L1* must be earlier in the week than *timeCOMP30020L2* breaks a symmetry and thus halves the number of solutions. Imposing such a constraint on each of say 1000 subjects reduces the number of solutions by a factor of 2^{1000} .

Despite this *astronomical* reduction, the search space is still *very* large.

Most constraint problems are NP-complete, so in the worst case, their solution requires time that is exponential in the size of the problem.

Nevertheless, heuristics can often reduce the solution time to manageable levels, and new and better heuristics are being developed all the time.

Optional constraints

Another reason why timetabling is very hard to do well is because there are many objectives that are desirable but not essential:

- Different lectures in the same subject should not be on the same day.
- Schedule as few lectures for 8am, 1pm, 6pm and 7pm as possible.
- If a lecturer or a cohort of students have lectures in consecutive time slots, it should be possible to walk from the first lecture theatre to the second in less than ten minutes. (This *should* be essential, but apparently isn't.)
- If a lecturer or a cohort of students spend most of their time in a building, then their lectures should be in that building.

Requiring all these constraints to be fulfilled would result in a horribly overconstrained problem and thus have no solution.

However, if you have several solutions that satisfy the essential constraints, you can choose the solution that satisfies the most optional constraints.

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Section 17

Monads

any and all

Other useful higher-order functions are

```
any :: (a -> Bool) -> [a] -> Bool
```

```
all :: (a -> Bool) -> [a] -> Bool
```

For example, to see if every word in a list contains the letter 'e':

```
Prelude> all (elem 'e') ["eclectic", "elephant", "legion"]  
True
```

To check if a word contains any vowels:

```
Prelude> any (\x -> elem x "aeiou") "sky"  
False
```

flip

If the order of arguments of `elem` were reversed, we could have used currying rather than the bulky `\x -> elem x "aeiou"`. The `flip` function takes a function and returns a function with the order of arguments flipped.

```
flip :: (a -> b -> c) -> (b -> a -> c)
flip f x y = f y x
```

```
Prelude> any (flip elem "aeiou") "hmmmm"
False
```

The ability to write functions to construct other functions is one of Haskell's strengths.

Monads

Monads build on this strength. A *monad* is a type constructor that represents a computation. These computations can then be composed to create other computations, and so on. The power of monads lies in the programmer's ability to determine *how* the computations are composed. Phil Wadler, who introduced monads to Haskell, describes them as “programmable semicolons”.

A monad M is a type constructor that supports two operations:

- A sequencing operation, denoted $>>=$, whose type is $M\ a \rightarrow (a \rightarrow M\ b) \rightarrow M\ b$.
- An identity operation, denoted `return`, whose type is $a \rightarrow M\ a$.

Monads

Think of the type $M\ a$ as denoting a computation that produces an a , and possibly carries something extra. For example, if M is the `Maybe` type constructor, that something extra is an indication of whether an error has occurred so far.

- You can take a value of type a and use the (misnamed) identity operation to wrap it in the monad's type constructor.
- Once you have such a wrapped value, you can use the sequencing operation to perform an operation on it. The `>>=` operation will unwrap its first argument, and then typically it will invoke the function given to it as its second argument, which will return a wrapped up result.

The Maybe and MaybeOK monads

The obvious ways to define the monad operations for the Maybe and MaybeOK type constructors are these (MaybeOK is not in the library):

```
-- monad ops for Maybe      -- monad ops for MaybeOK
data Maybe t = Just t
                      | Nothing
return x = Just x
(Just x) >>= f = f x
Nothing >>= _ = Nothing

data MaybeOK t = OK t
              | Error String
return x      = OK x
(OK x) >>= f   = f x
(Error m) >>= _ = Error m
```

In a sequence of calls to `>>=`, as long as all invocations of `f` succeed, (returning `Just x` or `OK x`), you keep going.

Once you get a failure indication, (returning `Nothing` or `Error m`), you keep that failure indication and perform no further operations.

Why you may want these monads

Suppose you want to encode a sequence of operations that each may fail. Here are two such operations:

```
maybe_head :: [a] -> MaybeOK a
maybe_head [] = Error "head of empty list"
maybe_head (x:_) = OK x
```

```
maybe_sqrt :: Int -> MaybeOK Double
maybe_sqrt x =
    if x >= 0 then
        OK (sqrt (fromIntegral x))
    else
        Error "sqrt of negative number"
```

How can you encode a sequence of operations such as taking the head of a list and computing its square root?

Simplifying code with monads

```
maybe_head :: [a] -> MaybeOK a
maybe_sqrt :: Int -> MaybeOK Double
maybe_sqrt_of_head :: [Int] -> MaybeOK Double

-- definition not using monads
maybe_sqrt_of_head l =
    let mh = maybe_head l in
        case mh of
            Error msg -> Error msg
            OK h -> maybe_sqrt h

-- simpler definition using monads
maybe_sqrt_of_head l =
    maybe_head l >>= maybe_sqrt
```

DEMO: Using the MaybeOK monad

```
cat maybeok.hs
ghci
:load maybeok
maybe_sqrt_of_head []
maybe_sqrt_of_head [-2, 4]
maybe_sqrt_of_head [2, 4]
```

I/O actions in Haskell

Haskell has a type constructor called `IO`. A function that returns a value of type `IO t` for some `t` will return a value of type `t`, but can also do input and/or output. Such functions can be called I/O functions or I/O actions.

Haskell has several functions for reading input, including

```
getChar :: IO Char  
getLine :: IO String
```

Haskell has several functions for writing output, including

```
putChar  :: Char -> IO ()  
putStr   :: String -> IO ()  
putStrLn :: String -> IO ()  
print    :: (Show a) => a -> IO ()
```

The type `()`, called `unit`, is the type of 0-tuples (tuples containing zero values). This is similar to the `void` type in C or Java. There is only one value of this type, the empty tuple, which is also denoted `()`.

DEMO: I/O actions

```
ghci  
putStr "abc"  
putStrLn "abc"  
print "abc"  
print [1, 2+3]
```

Operations of the I/O monad

The type constructor `IO` is a monad.

The identity operation: `return val` just returns `val` (inside `IO`) without doing any I/O.

The sequencing operation: `f >>= g`

- 1 calls `f`, which may do I/O, and which will return a value `rf` that may be meaningful or may be `()`,
- 2 calls `g rf` (passing the return value of `f` to `g`), which may do I/O, and which will return a value `rg` that also may be meaningful or may be `()`,
- 3 returns `rg` (inside `IO`) as the result of `f >>= g`.

You can use the sequencing operation to create a chain of any number of I/O actions.

Example of monadic I/O: hello world

```
hello :: IO ()  
hello =  
    putStr "Hello, "  
    >>=  
    \_ -> putStrLn "world!"
```

This code has two I/O actions connected with `>>=`.

- 1 The first is a call to `putStr`. This prints the first half of the message, and returns `()`.
- 2 The second is an anonymous function. It takes as argument and ignores the result of the first action, and then calls `putStrLn` to print the second half of the message, adding a newline at the end.

The result of the action sequence is the result of the last action.

Example of monadic I/O: greetings

```
greet :: IO ()
greet =
  putStr "Greetings! What is your name? "
  >>=
  \_ -> getLine
  >>=
  \name -> (
    putStr "Where are you from? "
    >>=
    \_ -> getLine
    >>=
    \town ->
      let msg = "Welcome, " ++ name ++
                " from " ++ town
      in putStrLn msg
  )
```

do blocks

Code written using monad operations is often ugly, and writing it is usually tedious. To address both concerns, Haskell provides `do` blocks. These are merely syntactic sugar for sequences of monad operations, but they make the code much more readable and easier to write.

A *do block* starts with the keyword `do`, like this:

```
hello = do
    putStr "Hello, "
    putStrLn "world"
```

do block components

Each element of a do block can be

- an I/O action that returns an ignored value, usually of type `()`, such as the calls to `putStr` and `putStrLn` below (just call the function);
- an I/O action whose return value is used to bind a variable, (use `var <- expr` to bind the variable);
- bind a variable to a non-monadic value (use `let var = expr` (no `in`)).

```
greet :: IO ()
```

```
greet = do
```

```
    putStr "Greetings! What is your name? "
```

```
    name <- getLine
```

```
    putStr "Where are you from? "
```

```
    town <- getLine
```

```
    let msg = "Welcome, " ++ name ++ " from " ++ town
```

```
    putStrLn msg
```

DEMO: Greetings

```
cat greetings.hs  
ghci  
:load greetings  
greet
```

...

```
cat greetings4.hs  
ghci  
:load greetings4  
greet
```

Operator priority problem

Unfortunately, the following line of code does not work:

```
putStrLn "Welcome, " ++ name ++ " from " ++ town
```

The reason is that due to its system of operator priorities, Haskell thinks that the main function being invoked here is not `putStrLn` but `++`, with its left argument being `putStrLn "Welcome, "`.

This is also the reason why Haskell accepts only the second of the following equations. It parses the left hand side of the first equation as `(len x):xs`, not as `len (x:xs)`.

```
len x:xs = 1 + len xs
```

```
len (x:xs) = 1 + len xs
```

Working around the operator priority problem

There are two main ways to fix this problem:

```
putStrLn ("Welcome, " ++ name ++ " from " ++ town)
putStrLn $ "Welcome, " ++ name ++ " from " ++ town
```

The first simply uses parentheses to delimit the possible scope of the ++ operator.

The second uses another operator, \$, which has lower priority than ++, and thus binds less tightly.

The main function invoked on the line is thus \$. Its first argument is its left operand: the function putStrLn, which is of type `String -> IO ()`. Its second argument is its right operand: the expression `"Welcome, " ++ name ++ " from " ++ town`, which is of type `String`.

\$ is of type `(a -> b) -> a -> b`. It applies its first argument to its second argument, so in this case it invokes putStrLn with the result of the concatenation.

QUIZ: `countDown`

Give a type declaration for a function to count down from its argument to 0, printing the count as it goes, and print “Done.” at the end.

Then define the function.

QUIZ: countDown

Give a type declaration for a function to count down from its argument to 0, printing the count as it goes, and print “Done.” at the end.

Then define the function.

```
countDown :: Int -> IO ()
```

QUIZ: countdown

Give a type declaration for a function to count down from its argument to 0, printing the count as it goes, and print “Done.” at the end.

Then define the function.

```
countDown :: Int -> IO ()  
  
countDown n  
  | n < 0 = putStrLn "Done."  
  | otherwise = do  
    print n  
    countDown $ n - 1
```

See `countDown.hs` for alternatives.

return

If a function does I/O and returns a value, and the code that computes the return value does not do I/O, you will need to invoke the `return` monad operation as the last operation in the `do` block.

```
main :: IO ()
main = do
    putStrLn "Please input a string"
    len <- readlen
    putStrLn $ "The length of that string is " ++ show len

readlen :: IO Int
readlen = do
    str <- getLine
    return (length str)
```

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Section 18

More on monads

I/O actions as descriptions

Haskell programmers usually think of functions that return values of type $\text{IO } t$ as doing I/O as well as returning a value of type t . While this is usually correct, there are some situations in which it is not accurate enough.

The correct way to think about such functions is that they return two things:

- a value of type t , and
- a *description* of an I/O operation.

The monadic operator $\gg=$ can then be understood as taking descriptions of two I/O operations, and returning a description of those two operations being executed in order.

The monadic operator `return` simply associates a description of a do-nothing I/O operation with a value.

Description to execution: theory

Every complete Haskell program must have a function named `main`, whose signature should be

```
main :: IO ()
```

As in C, this is where the program starts execution. Conceptually,

- the OS starts the program by invoking the Haskell runtime system;
- the runtime system calls `main`, which returns a description of a sequence of I/O operations; and
- the runtime system executes the described sequence of I/O operations.

Description to execution: practice

In actuality, the compiler and the runtime system together ensure that each I/O operation is executed as soon as its description has been computed,

- *provided* that the description is created in a context which guarantees that the description will end up in the list of operation descriptions returned by `main`, and
- *provided* that all the previous operations in that list have also been executed.

The provisions are necessary since

- you don't want to execute an I/O operation that the program does not actually call for, and
- you don't want to execute I/O operations out of order.

Example: printing a table of squares directly

```
main :: IO ()
main = do
    putStrLn "Table of squares:"
    print_table 1 10

print_table :: Int -> Int -> IO ()
print_table cur max
    | cur > max = return ()
    | otherwise = do
        putStrLn (table_entry cur)
        print_table (cur+1) max

table_entry :: Int -> String
table_entry n = (show n) ++ "^2 = " ++ (show (n*n))
```


Non-immediate execution of I/O actions

Just because you have created a description of an I/O action, does not mean that this I/O action will eventually be executed.

Haskell programs can pass around descriptions of I/O operations. They cannot peer into a description of an I/O operation, but they can nevertheless do things with them, such as

- build up lists of I/O actions, and
- put I/O actions into binary search trees as values.

Those lists and trees can then be processed further, and programmers can, if they wish, take the descriptions of I/O actions out of those data structures, and have them executed by including them in the list of actions returned by `main`.

QUIZ: I/O actions in trees

Why can't you put descriptions of I/O actions into binary search trees as keys?

QUIZ: I/O actions in trees

Why can't you put descriptions of I/O actions into binary search trees as keys?

Because doing so would require comparing two descriptions for equality and for order, which cannot be done without peering into them.

Example: printing a table of squares indirectly

```
main = do
  putStrLn "Table of squares:"
  let row_actions = map show_entry [1..15]
  execute_actions (take 10 row_actions)

table_entry :: Int -> String
table_entry n = (show n) ++ "^2 = " ++ (show (n*n))

show_entry :: Int -> IO ()
show_entry n = do putStrLn (table_entry n)

execute_actions :: [IO ()] -> IO ()
execute_actions [] = return ()
execute_actions (x:xs) = do
  x
  execute_actions xs
```

DEMO: Print tables

```
cat sq1.hs  
ghci  
:load sq1  
main
```

```
cat sq2.hs  
ghci  
:load sq2  
main
```

Input, process, output

A typical batch program reads in its input, does the required processing, and prints its output.

A typical interactive program goes through the same three stages once for each interaction.

In most programs, the vast majority of the code is in the middle (processing) stage.

In programs written in imperative languages like C, Java, and Python, the type of a function (or procedure, subroutine or method) does not tell you whether the function does I/O.

In Haskell, it does.

I/O in Haskell programs

In most Haskell programs, the vast majority of the functions are not I/O functions and they do no input or output. They merely build, access and transform data structures, and do calculations. The code that does I/O is a thin veneer on top of this bulk.

This approach has several advantages.

- A unit test for a non-IO function is a record of the values of the arguments and the expected value of the result. The test driver can read in those values, invoke the function, and check whether the result matches. The test driver can be a human.
- Code that does no I/O can be rearranged. Several optimizations exploit this fact.
- Calls to functions that do no I/O can be done in parallel. Selecting the best calls to parallelize is an active research area.

Debugging Haskell functions

Suppose you are working on a function f , and tests reveal the function to have a bug.

Ideally, you would have unit tested all the other functions f calls and found them to be correct, which would narrow the potential scope of the bug down to f itself. In practice, you probably didn't do this.

You can try to debug f by calling the functions f calls with the arguments that the failed test case would have invoked them with, and see what happens. This is effectively unit testing on demand.

Unfortunately, sometimes you need lots of information before you can select a promising debugging approach.

Debugging printf

In a program written in C, you could get this information by adding debugging printf's to `f` and/or the functions it calls.

In a program written in Haskell, things are not always so simple.

- If `f` does I/O, which means that it returns a description of an I/O operation that you know will be executed, then you can simply print the diagnostics you need as part of that operation.
- If `f` does not do I/O, or if the I/O operation whose description it returns is not executed, this will not work.

This is where the function `unsafePerformIO` comes in.

unsafePerformIO

The type of `unsafePerformIO` is `IO t -> t`.

- You give it as argument an I/O operation, which means a function of type `IO t`. `unsafePerformIO` calls this function.
- The function will return a value of type `t` and a description of an I/O operation.
- `unsafePerformIO` executes the described I/O operation and returns the value.

Here is an example:

```
sum :: Int -> Int -> Int
sum x y = unsafePerformIO $ do
  putStrLn ("summing " ++ (show x) ++ " and " ++ (show y))
  return (x + y)
```

The State monad

The State monad is useful for computations that need to thread information throughout the computation. It allows such information to be transparently passed around a computation, and accessed and replaced when needed. That is, it allows an imperative style of programming without losing Haskell's declarative semantics.

This code adds 1 to each element of a tree, and does not need a monad:

```
data Tree a = Empty | Node (Tree a) a (Tree a)
              deriving Show
type IntTree = Tree Int

incTree :: IntTree -> IntTree
incTree Empty = Empty
incTree (Node l e r) =
    Node (incTree l) (e + 1) (incTree r)
```

Threading state

If we instead wanted to add 1 to the leftmost element, 2 to the next element, and so on, we would need to pass an integer into our function saying what to add, but also we need to pass an integer out, saying what to add to the *next* element. This requires more complex code:

```
incTree1 :: IntTree -> IntTree
incTree1 tree = fst (incTree1' tree 1)

incTree1' :: IntTree -> Int -> (IntTree, Int)
incTree1' Empty n = (Empty, n)
incTree1' (Node l e r) n =
    let (newl, n1) = incTree1' l n
        (newr, n2) = incTree1' r (n1 + 1)
    in (Node newl (e+n1) newr, n2)
```

Introducing the State monad

The State monad abstracts the type $s \rightarrow (v, s)$, hiding away the s part. Haskell's `do` notation allows us to focus on the v part of the computation while ignoring the s part where not relevant.

```
incTree2 :: IntTree -> IntTree
incTree2 tree = fst (runState (incTree2' tree) 1)
```

```
incTree2' :: IntTree -> State Int IntTree
incTree2' Empty = return Empty
incTree2' (Node l e r) = do
    newl <- incTree2' l
    n <- get      -- gets the current state
    put (n + 1)   -- sets the current state
    newr <- incTree2' r
    return (Node newl (e+n) newr)
```

Abstracting the state operations

In this case, we do not need the full generality of being able to update the integer state in arbitrary ways; the only update operation we need is an increment. We can therefore provide a version of the state monad that is specialized for this task. Such specialization provides useful documentation, and makes the code more robust.

```
type Counter = State Int
```

```
withCounter :: Int -> Counter a -> a
```

```
withCounter init f = fst (runState f init)
```

```
nextCount :: Counter Int
```

```
nextCount = do
```

```
    n <- get
```

```
    put (n + 1)
```

```
    return n
```

Using the counter

Now the code that uses the monad is even simpler:

```
incTree3 :: IntTree -> IntTree
incTree3 tree = withCounter 1 (incTree3' tree)
```

```
incTree3' :: IntTree -> Counter IntTree
incTree3' Empty = return Empty
incTree3' (Node l e r) = do
    newl <- incTree3' l
    n <- nextCount
    newr <- incTree3' r
    return (Node newl (e+n) newr)
```

DEMO: Using the counter

```
ghci statemonad.hs
testTree
incTree testTree
incTree1 testTree
incTree2 testTree
incTree3 testTree
... change incTree3': move nextCount ...
incTree3 testTree
```


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Section 19

Lazyness

Eager vs lazy evaluation

In a programming language that uses *eager evaluation*, each expression is evaluated as soon as it gets bound to a variable, either explicitly in an assignment statement, or implicitly during a call. (A call implicitly assigns each actual parameter expression to the corresponding formal parameter variable.)

In a programming language that uses *lazy evaluation*, an expression is not evaluated until its value is actually needed. Typically, this will be when

- the program wants the value as input to an arithmetic operation, or
- the program wants to match the value against a pattern, or
- the program wants to output the value.

Almost all programming languages use eager evaluation. Haskell uses lazy evaluation.

Lazyness and infinite data structures

Lazyness allows a program to work with data structures that are conceptually infinite, as long as the program looks at only a finite part of the infinite data structure.

For example, `[1..]` is a list of all the positive numbers. If you attempt to print it out, the printout will be infinite, and will take infinite time, unless you interrupt it.

On the other hand, if you want to print only the first `n` positive numbers, you can do that with `take n [1..]`.

Even though the second argument of the call to `take` is infinite in size, the call takes finite time to execute.

DEMO: Infinite data

```
ghci  
[1..]  
take 30 [1..]  
take 30 [10..]
```

The sieve of Eratosthenes

```
-- returns the (infinite) list of all primes
all_primes :: [Integer]
all_primes = prime_filter [2..]

prime_filter :: [Integer] -> [Integer]
prime_filter [] = []
prime_filter (x:xs) =
    x:prime_filter (filter (not . ('divisibleBy' x)) xs)

-- n 'divisibleBy' d means n is evenly divisible by d
divisibleBy n d = n 'mod' d == 0
```

Using `all_primes`

To find the first n primes:

```
take n all_primes
```

To find all primes up to n :

```
takeWhile (<= n) all_primes
```

Lazyness allows the programmer of `all_primes` to concentrate on the function's task, *without* having to also pay attention to exactly *how* the program wants to decide how many primes are enough.

Haskell automatically interleaves the computation of the primes with the code that determines how many primes to compute.

DEMO: Primes up to n

```
ghci
:load primes
takeWhile (<= 100) all_primes
take 100 all_primes
all_primes
```

Representing unevaluated expressions

In a lazy programming language, expressions are not evaluated until you need their value. However, until then, you do need to remember the code whose execution will compute that value.

In Haskell implementations that compile Haskell to C (this includes GHC), the data structure you need for that is a pointer to a C function, together with all the arguments you will need to give to that C function.

This representation is sometimes called a *suspension*, since it represents a computation whose evaluation is temporarily suspended.

It can also be called a *promise*, since it also represents a promise to carry out a computation if its result is needed.

Historically inclined people can also call it a *thunk*, because that was the name of this construct in the first programming language implementation that used it. That language was Algol-60.

Parametric polymorphism

Parametric polymorphism is the name for the form of polymorphism in which types like `[a]` and `Tree k v`, and functions like `length` and `insert_bst`, include type variables, and the types and functions work identically regardless of what types the type variables stand for.

The implementation of parametric polymorphism requires that the values of all types be representable in the same amount of memory. Without this, the code of e.g. `length` wouldn't be able to handle lists with elements of all types.

That “same amount of memory” will typically be the word size of the machine, which is the size of a pointer. Anything that does not fit into one word is represented by a pointer to a chunk of memory on the heap.

Given this fact, the arguments of the function in a suspension can be stored in an array of words, and we can arrange for all functions in suspensions to take their arguments from a single array of words.

Evaluating lazy values only once

Many functions use the values of some variables more than once. This includes `takeWhile`, which uses `x` twice:

```
takeWhile _ [] = []
takeWhile p (x:xs)
  | p x      = x : takeWhile p xs
  | otherwise = []
```

You need to know the value of `x` to do the test `p x`, which requires calling the function in the suspension representing `x`; if the test succeeds, you will again need to know the value of `x` to put it at the front of the output list.

To avoid redundant work, you want the first call to `x`'s suspension to record the result of the call, and you want all references to `x` after the first to get its value from this record.

Therefore once you know the result of the call, you don't need the function and its arguments anymore.

QUIZ: Using lazyness

Write a function

```
recycle :: [t] -> [t]
```

to generate an infinite list of repetitions of the elements of its input

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```
recycle l = l ++ recycle l
```

QUIZ: Using lazyness

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recycle l = l ++ recycle l
```

Now rewrite it without using ++

QUIZ: Using lazyness

Write a function

```
recycle :: [t] -> [t]
```

to generate an infinite list of repetitions of the elements of its input

```
recycle l = l ++ recycle l
```

Now rewrite it without using ++

```
recycle l = recycle' l l
```

```
recycle' [] l = recycle' l l
```

```
recycle' (x:xs) l = x:recycle' xs l
```

Call by need

Operations such as printing, arithmetic and pattern matching start by ensuring their argument is at least partially evaluated.

They will make sure that at least the top level data constructor of the value is determined. However, the arguments of that data constructor may remain suspensions.

For example, consider the match of the second argument of `takeWhile` against the patterns `[]` and `(p:ps)`. If the original second argument is a suspension, it must be evaluated enough to ensure its top-level constructor is determined. If it is `x:xs`, then the first argument must be applied to `x`. Whether `x` needs to be evaluated will depend on what the first argument (function) does.

This is called “call by need”, because function arguments (and other expressions) are evaluated only when their value is needed.

Control structures and functions

(a) ... if (x < y) f(x); else g(y); ...

(b) ... ite(x < y, f(x), g(y)); ...

```
int ite(bool c, int t, int e)
{ if (c) then return t; else return e; }
```

In C, (a) will generate a call to only one of f and g, but (b) will generate a call to both.

(c) ... if x < y then f x else g y ...

(d) ... ite (x < y) (f x) (g y) ...

```
ite :: Bool -> a -> a -> a
ite c t e = if c then t else e
```

In Haskell, (c) will execute a call to only one of f and g, and thanks to laziness, this is also true for (d).

Implementing control structures as functions

Without lazyness, using a function instead of explicit code such as a sequence of if-then-elses could get unnecessary non-termination or at least unnecessary slowdowns.

Lazyness' guarantee that an expression will not be evaluated if its value is not needed allows programmers to define their own control structures as functions.

For example, you can define a control structure that returns the value of one of three expressions, with the expression chosen based on whether an expression is less than, equal to or greater than zero like this:

```
ite3 :: (Ord a, Num a) => a -> b -> b -> b -> b
ite3 x lt eq gt
  | x < 0  = lt
  | x == 0 = eq
  | x > 0  = gt
```

Using lazyness to avoid unnecessary work

```
min = head . sort
```

On the surface, this looks like a very wasteful method for computing the minimum, since sorting is usually done with an $O(n^2)$ or $O(n \log n)$ algorithm, and `min` should be doable with an $O(n)$ algorithm.

However, in this case, the evaluation of the sorted list can stop after the materialization of the first element.

If `sort` is implemented using selection sort, this is just a somewhat higher overhead version of the direct code for `min`.

QUIZ: Using lazyness to avoid unnecessary work

```
min = head . sort
```

What is the complexity of `min` if `sort` is implemented using quicksort?

QUIZ: Using lazyness to avoid unnecessary work

```
min = head . sort
```

What is the complexity of `min` if `sort` is implemented using quicksort?

Quadratic (worst case). Linear in the best and average cases.

QUIZ: Using lazyness to avoid unnecessary work

```
min = head . sort
```

What is the complexity of `min` if `sort` is implemented using quicksort?

Quadratic (worst case). Linear in the best and average cases.

```
max = head . reverse . sort
```

What is the complexity of `max` if `sort` is implemented using quicksort?

QUIZ: Using lazyness to avoid unnecessary work

```
min = head . sort
```

What is the complexity of `min` if `sort` is implemented using quicksort?

Quadratic (worst case). Linear in the best and average cases.

```
max = head . reverse . sort
```

What is the complexity of `max` if `sort` is implemented using quicksort?

Quadratic (worst case). $O(n \log n)$ in the best and average cases.

Multiple passes

```
output_prog chars = do
  let anno_chars = annotate_chars 1 1 chars
  let tokens = scan anno_chars
  let prog = parse tokens
  let prog_str = show prog
  putStrLn prog_str
```

This function takes as input one data structure (`chars`) and calls for the construction of four more (`anno_chars`, `tokens`, `prog` and `prog_str`).

This kind of pass structure occurs frequently in real programs.

The effect of laziness on multiple passes

With eager evaluation, you would completely construct each data structure before starting construction of the next.

The maximum memory needed at any one time will be the size of the largest data structure (say pass n), plus the size of any part of the previous data structure (pass $n - 1$) needed to compute the last part of pass n . All other memory can be garbage collected before then.

With lazy evaluation, execution is driven by `putStrLn`, which needs to know what the next character to print (if any) should be. For each character to be printed, the program will materialize the parts of those data structures needed to figure that out.

The memory demand at a given time will be given by the tree of suspensions from earlier passes that you need to materialize the rest of the string to be printed. The maximum memory demand can be significantly less than with eager evaluation.

Lazy input

In Haskell, even input is implemented lazily.

Given a filename, `readFile` returns the contents of the file as a string, but it returns the string lazily: it reads the next character from the file only when the rest of the program needs that character.

```
parse_prog_file filename = do
  fs <- readFile filename
  let tokens = scan (annotate_chars 1 1 fs)
  return (parse_prog [] tokens)
```

When the main module calls `parse_prog_file`, it gets back a tree of suspensions.

Only when those suspensions start being forced will the input file be read, and each call to `evaluate_suspension` on that tree will cause only as much to be read as is needed to figure out the value of the forced data constructor.

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Section 20

Performance

Effect of laziness on performance

Laziness adds two sorts of overhead that slow down programs.

- The execution of a Haskell program creates a lot of suspensions, and most of them are evaluated, so eventually they also need to be unpacked.
- Every access to a value must first check whether the value has been materialized yet.

However, laziness can also speed up programs by avoiding the execution of computations that take a long time, or do not terminate at all.

Whether the dominant effect is the slowdown or the speedup will depend on the program and what kind of input it typically gets.

The usual effect is something like lotto: in most cases you lose a bit, but sometimes you win a little, and in some rare cases you win a lot.

Strictness

Theory calls the value of an expression whose evaluation loops infinitely or throws an exception “bottom”, denoted by the symbol \perp .

A function is *strict* if it always needs the values of all its arguments. In formal terms, this means that if any of its arguments is \perp , then its result will also be \perp .

The addition function $+$ is strict. The function `ite` from earlier in the last lecture is nonstrict.

Some Haskell compilers including GHC include *strictness analysis*, which is a compiler pass whose job is to analyze the code of the program and figure out which of its functions are strict and which are nonstrict.

When the Haskell code generator sees a call to a strict function, instead of generating code that creates a suspension, it can generate the code that an imperative language compiler would generate: code that evaluates all the arguments, and then calls the function.

Unpredictability

Besides generating a slowdown for most programs, laziness also makes it harder for the programmer to understand where the program is spending most of its time and what parts of the program allocate most of its memory.

This is because small changes in exactly where and when the program demands a particular value can cause great changes in what parts of a suspension tree are evaluated, and can therefore cause great changes in the time and space complexity of the program. (Lazy evaluation is also called *demand driven* computation.)

The main problem is that it is very hard for programmers to be simultaneous aware of *all* the relevant details in the program.

Modern Haskell implementations come with sophisticated profilers to help programmers understand the behavior of their programs. There are profilers for both time and for memory consumption.

Memory efficiency

(Revised) BST insertion code:

```
insert_bst :: Ord k => Tree k v -> k -> v -> Tree k v
insert_bst Leaf ik iv = Node ik iv Leaf Leaf
insert_bst (Node k v l r) ik iv
  | k == ik    = Node ik iv l r
  | k > ik     = Node k v (insert_bst l ik iv) r
  | otherwise = Node k v l (insert_bst r ik iv)
```

As discussed earlier, this creates new data structures instead of destructively modifying the old structure.

The advantage of this is that the old structure can still be used.

The disadvantage is new memory is allocated and written. This takes time, and creates garbage that must be collected.

Memory efficiency

Insertion into a BST replaces one node on each level of the tree: the node on the path from the root to the insertion site.

In (mostly) balanced trees with n nodes, the height of the tree tends to be about $\log_2(n)$.

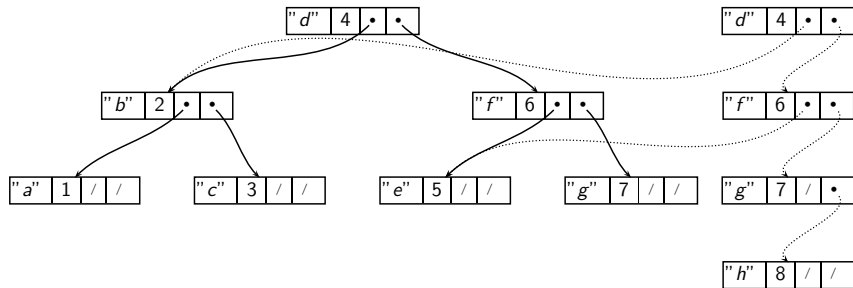
Therefore the number of nodes allocated during an insertion tends to be logarithmic in the size of the tree.

- If the old version of the tree is not needed, imperative code can do better: it must allocate only the new node.
- If the old version of the tree is needed, imperative code will do worse: it must copy the entire tree, since without that, later updates to the new version would update the old one as well.

Reusing memory

When `insert_bst` inserts a new node into the tree, it allocates new versions of every node on the path from the root to the insertion point. However, *every other node in the tree* will become part of the new tree as well as the old one.

This shows what happens when you insert the key "h" into a binary search tree that already contains "a" to "g".



Deforestation

As we discussed earlier, many Haskell programs have code that follows this pattern:

- You start with the first data structure, `ds1`.
- You traverse `ds1`, generating another data structure, `ds2`.
- You traverse `ds2`, generating yet another data structure, `ds3`.

If the programmer can restructure the code to compute `ds3` directly from `ds1`, this should speed up the program, for two reasons:

- the new version does not need to create `ds2`, and
- the new version does one traversal instead of two.

Since the eliminated intermediate data structures are often trees of one kind or another, this optimization idea is usually called *deforestation*.

filter_map

```
filter_map :: (a -> Bool) -> (a -> b) -> [a] -> [b]
filter_map _ _ [] = []
filter_map f m (x:xs) =
    let newxs = filter_map f m xs in
    if f x then (m x):newxs else newxs
```

```
one_pass xs = filter_map is_even triple xs
two_pass xs = map triple (filter is_even xs)
```

The `one_pass` function performs exactly the same task as the `two_pass` function, but it does the job with one list traversal, not two, and does not create an intermediate list.

One can also write similarly deforested combinations of many other pairs of higher order functions, such as `map` and `foldl`.

Computing standard deviations

```
four_pass_stddev :: [Double] -> Double
four_pass_stddev xs =
  let
    count = fromIntegral (length xs)
    sum = foldl (+) 0 xs
    sumsq = foldl (+) 0 (map square xs)
  in
    (sqrt (count * sumsq - sum * sum)) / count

square :: Double -> Double
square x = x * x
```

This is the simplest approach to writing code that computes the standard deviation of a list. However, it traverses the input list three times, and it also traverses a list of that same length (the list of squares) once.

Computing standard deviations in one pass

```
data StddevData = SD Double Double Double

one_pass_stddev :: [Double] -> Double
one_pass_stddev xs =
  let
    init_sd = SD 0.0 0.0 0.0
    update_sd (SD c s sq) x =
      SD (c + 1.0) (s + x) (sq + x*x)
    SD count sum sumsq = foldl update_sd init_sd xs
  in
    (sqrt (count * sumsq - sum * sum)) / count
```

Cords

Repeated appends to the end of a list take time that is quadratic in the final length of the list.

In imperative languages, you would avoid this quadratic behavior by keeping a pointer to the tail of the list, and destructively updating that tail.

In declarative languages, the usual solution is to switch from lists to a data structure that supports appends in constant time. These are usually called *cords*. This is one possible cord design; there are several.

```
data Cord a = Nil | Leaf a | Branch (Cord a) (Cord a)
```

```
append_cords :: Cord a -> Cord a -> Cord a
```

```
append_cords a b = Branch a b
```

Converting cords to lists

The obvious algorithm to convert a cord to a list is

```
cord_to_list :: Cord a -> [a]
cord_to_list Nil = []
cord_to_list (Leaf x) = [x]
cord_to_list (Branch a b) =
    (cord_to_list a) ++ (cord_to_list b)
```

Unfortunately, it suffers from the exact same performance problem that cords were designed to avoid.

The cord `Branch (Leaf 1) (Leaf 2)` that the last equation converts to a list may itself be one branch of a bigger cord, such as `Branch (Branch (Leaf 1) (Leaf 2)) (Leaf 3)`.

The list `[1]`, converted from `Leaf 1`, will be copied twice by `++`, once for each `Branch` data constructor in whose first operand it appears.

Accumulators

With one exception, all leaves in a cord are followed by another item, but the second equation puts an empty list behind all leaves, which is why all but one of the lists it creates will have to be copied again. The other two equations make the same mistake for empty and branch cords.

Fixing the performance problem requires telling the conversion function what list of items follows the cord currently being converted. This is easy to arrange using an accumulator.

```
cord_to_list :: Cord a -> [a]
cord_to_list c = cord_to_list' c []
```

```
cord_to_list' :: Cord a -> [a] -> [a]
cord_to_list' Nil rest = rest
cord_to_list' (Leaf x) rest = x:rest
cord_to_list' (Branch a b) rest =
    cord_to_list' a (cord_to_list' b rest)
```

Sortedness check

The obvious way to write code that checks whether a list is sorted:

```
sorted1 :: (Ord a) => [a] -> Bool
sorted1 []           = True
sorted1 [_]          = True
sorted1 (x1:x2:xs) = x1 <= x2 && sorted1 (x2:xs)
```

However, the code that looks at each list element handles three alternatives (lists of length zero, one and more).

It does this because each sortedness comparison needs *two* list elements, not one.

A better sortedness check

```
sorted2 :: (Ord a) => [a] -> Bool
sorted2 []      = True
sorted2 (x:xs) = sorted_lag x xs
```

```
sorted_lag :: (Ord a) => a -> [a] -> Bool
sorted_lag _ []      = True
sorted_lag x1 (x2:xs) = x1 <= x2 && sorted_lag x2 xs
```

In this version, the code that looks at each list element handles only two alternatives. The value of the previous element, the element that the current element should be compared with, is supplied separately.

Optimisation

You can use `:set +s` in GHCi to time execution.

```
Prelude> :l sorted
[1 of 1] Compiling Sorted      ( sorted.hs, interpreted )
Ok, modules loaded: Sorted.
*Sorted> :set +s
*Sorted> sorted1 [1..100000000]
True
(50.11 secs, 32,811,594,352 bytes)
*Sorted> sorted2 [1..100000000]
True
(40.76 secs, 25,602,349,392 bytes)
```

The `sorted2` version is about 20% faster and uses 22% less memory.

Optimisation

However, the Haskell compiler is very sophisticated. After doing `ghc -dynamic -c -O3 sorted.hs`, we get this:

```
Prelude> :l sorted
Ok, modules loaded: Sorted.
Prelude Sorted> :set +s
Prelude Sorted> sorted1 [1..100000000]
True
(2.89 secs, 8,015,369,944 bytes)
Prelude Sorted> sorted2 [1..100000000]
True
(2.91 secs, 8,002,262,840 bytes)
```

Compilation gives a *factor* of 17 speedup and a factor of 3 memory savings. It also removes the difference between `sorted1` and `sorted2`. *Always benchmark your compiled code when trying to speed it up.*

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Section 21

Interfacing with foreign languages

Foreign language interface

Many applications involve code written in a number of different languages; declarative languages are no different in this respect. There are many reasons for this:

- to interface to existing code (especially libraries) written in another language;
- to write performance-critical code in a lower-level language (typically C or C++);
- to write each part of an application in the most appropriate language;
- as a way to gracefully translate an application from one language to another, by replacing one piece at a time.

Any language that hopes to be successful must be able to work with other languages. This is generally done through what is called a *foreign language interface* or *foreign function interface*.

Application binary interfaces

In computer science, a *platform* is a combination of an instruction set architecture (ISA) and an operating system, such as x86/Windows 10, x86/Linux or SPARC/Solaris.

Each platform typically has an *application binary interface*, or ABI, which dictates such things as where the callers of functions should put the function parameters and where the callee function should put the result.

By compiling different files to the same ABI, functions in one file can call functions in a separately compiled file, even if compiled with different compilers.

The traditional way to interface two languages, such as C and Fortran, or Ada and Java, is for the compilers of both languages to generate code that follows the ABI.

Beyond C

ABIs are typically designed around C's simple calling pattern, where each function is compiled to machine language, and each function call passes some number of inputs, calls another known function, possibly returns one result, and is then finished.

This model does not work for lazy languages like Haskell, languages like Prolog or Mercury that support nondeterminism, languages like Prolog, Python, and Java that are implemented through an abstract machine, or even languages like C++ where function (method) calls may invoke different code each time they are executed.

In such languages, code is not compiled to the normal ABI. Then it becomes necessary to provide a mechanism to call code written in other languages. Typically, calling C code through the normal ABI is supported, but interfacing to other languages may also be supported.

Boolean functions

One application of a foreign interface is to use specialised data structures and algorithms that would be difficult or inefficient to implement in the host language.

Some applications need to be able to efficiently manipulate Boolean formulas (Boolean functions). This includes the following primitive values and operations:

- *true, false*
- Boolean variables: *eg: a, b, c, ...*
- Operations: and (\wedge), or (\vee), not (\neg), implies (\rightarrow), iff (\leftrightarrow), *etc.*
- Tests: satisfiability (is there any binding for the variables that makes a formula true?), equivalence (are two formulas the same for every set of variable bindings?)

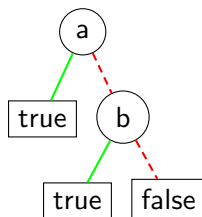
For example, is $a \leftrightarrow b$ equivalent to $\neg((a \wedge \neg b) \vee (\neg a \wedge b))$?

BDDs

Deciding satisfiability or equivalence of Boolean functions is NP-complete, so we need an efficient implementation.

BDDs are *decision graphs*, based on *if-then-else (ite)* nodes, where each node is labeled by a boolean variable and each leaf is a truth value.

With a *truth assignment* for each variable, the value of the formula can be determined by traversing from the root, following *then* branch for true variables and *else* branch for false variables.



a	b	the BDD
T	T	T
T	F	T
F	T	T
F	F	F

BDDs in Haskell

We could represent BDDs in Haskell with this type:

```
data BDD label = BTrue | BFalse
               | Itc label (BDD label) (BDD label)
```

The meaning of a BDD is given by:

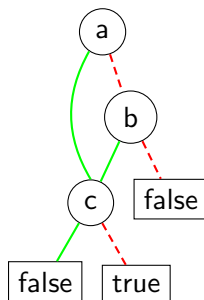
$$\begin{aligned} \text{meaning } \text{BTrue} &= \text{true} \\ \text{meaning } \text{BFalse} &= \text{false} \\ \text{meaning } (\text{Ite } l \ t \ e) &= (l \wedge \text{meaning } t) \vee (\neg l \wedge \text{meaning } e) \end{aligned}$$

So for example, $\text{meaning } (\text{Ite } a \ \text{BTrue} \ (\text{Ite } b \ \text{BTrue} \ \text{BFalse}))$

$$\begin{aligned} &= (a \wedge \text{true}) \vee (\neg a \wedge (b \wedge \text{true} \vee (\neg b \wedge \text{false}))) \\ &= a \vee (\neg a \wedge b \vee \text{false}) \\ &= a \vee b \end{aligned}$$

ROBDDs

Reduced Ordered Binary Decision Diagrams (ROBDDs) are BDDs where labels are in increasing order from root to leaf, no node has two identical children, and no two distinct nodes have the same semantics.



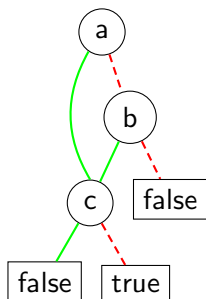
a	b	c	the BDD
T	T	T	F
T	T	F	T
T	F	T	F
T	F	F	T
F	T	T	F
F	T	F	T
F	F	T	F
F	F	F	F

By sharing the c node, the ROBDD is smaller than it would be if it were a tree. For larger ROBDDs, this can be a big savings.

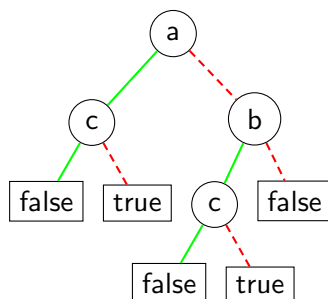
Object Identity

ROBDD algorithms traverse DAGs, and often meet the same subgraphs repeatedly. They greatly benefit from *caching*: recording results of past operations to reuse without repeating the computation.

Caching requires efficiently recognizing when a node is seen again. Haskell does not have the concept of *object identity*, so it cannot distinguish



from



Structural Hashing

Building a new ROBDD node $ite(v, thn, els)$ must ensure that:

- ❶ the two children of every node are different; and
- ❷ for any Boolean formula, there is only one ROBDD node with that semantics.

The first is achieved by checking if $thn = els$, and if so, returning thn

The second is achieved by *structural hashing* (AKA *hash-consing*): maintaining a hash table of past calls $ite(v, thn, els)$ and their results, and always returning the past result when a call is repeated.

Because of point 1 above, satisfiability of an ROBDD can be tested in constant time (*meaning* r is satisfiable iff $r \neq false$)

Because of point 2 above, equality of two ROBDDs is also a constant time test (*meaning* $r_1 = meaning\ r_2$ iff $r_1 = r_2$)

Impure implementation of pure operations

(We haven't cheated NP-completeness: we have only shifted the cost to building ROBDDs.)

Yet ROBDD operations are purely declarative. Constructing ROBDDs, conjunction, disjunction, negation, implication, checking satisfiability and equality, etc., are all pure.

This is an example of using impurity to build purely declarative code.

In fact, all declarative code running on a commodity computer does that: these CPUs work through impurity. Even adding two numbers on a modern CPU works by destructively adding one register to another.

If you are required to work in an imperative or object-oriented language, you can still use such languages to build declarative abstractions, and work with them instead of working directly with impure constructs.

robdd.h: C interface for ROBDD implementation

```
extern robdd *true_rep(void);          /* ROBDD true */
extern robdd *false_rep(void);         /* ROBDD false */
extern robdd *variable_rep(int var);    /* ROBDD for var */
extern robdd *conjoin(robdd *a, robdd *b);
extern robdd *disjoin(robdd *a, robdd *b);
extern robdd *negate(robdd *a);
extern robdd *implies(robdd *a, robdd *b);

extern int is_true(robdd *f);           /* is ROBDD == true */
extern int is_false(robdd *f);          /* is ROBDD == false */

extern int robdd_label(robdd *f);       /* label of f */
extern robdd *robdd_then(robdd *f);     /* then branch of f */
extern robdd *robdd_else(robdd *f);     /* else branch of f */
```

Interfacing Haskell to C

For simple cases, the Haskell foreign function interface is fairly simple. You can interface to a C function with a declaration of the form:

```
foreign import ccall "C name" Haskell name :: Haskell type
```

C primitive types convert to and from natural Haskell types, eg,

$$\text{C int} \longleftrightarrow \text{Haskell Int}$$

In Haskell, we want to treat ROBDDs as an *opaque type*: a type we cannot peer inside, we can only pass it to, and receive it as output from, foreign functions.

The Haskell `Word` type represents a word of memory, much like an `Int`. However `Word` is not opaque, as we can confuse it with an integer, or any `Word` type.

newtype

Declaring type `BoolFn = Word` would not make `BoolFn` opaque; it would just be an alias for `Word`, and could be passed as a `Word`.

We can make it opaque with a `data BoolFn = BoolFn Word` declaration. We could convert a `Word w` to a `BoolFn` with `BoolFn w`, and convert a `BoolFn b` to a `Word` with

```
let BoolFn w = b in ...
```

But this would *box* the word, adding an extra indirection to operations. Instead we declare:

```
newtype BoolFn = BoolFn Word deriving Eq
```

We can only use `newtype` to declare types with only one constructor, with exactly one argument. This avoids the indirection, makes the type opaque, and allows it to be used in the foreign interface.

The interface

```
foreign import ccall "true_rep"    true    :: BoolFn
foreign import ccall "false_rep"   false   :: BoolFn
foreign import ccall "variable_rep" variable :: Int->BoolFn

foreign import ccall "is_true"     isTrue   :: BoolFn->Bool
foreign import ccall "is_false"    isFalse  :: BoolFn->Bool
foreign import ccall "robdd_label" minVar   :: BoolFn->Int
foreign import ccall "robdd_then"  minThen  :: BoolFn->BoolFn
foreign import ccall "robdd_else"  minElse  :: BoolFn->BoolFn

type BoolBinOp = BoolFn -> BoolFn -> BoolFn
foreign import ccall "conjoin"      conjoin :: BoolBinOp
foreign import ccall "disjoin"      disjoin :: BoolBinOp
foreign import ccall "negate"        negation :: BoolFn->BoolFn
foreign import ccall "implies"      implies :: BoolBinOp
```

Using it

To make C code available, compile it and pass the object file on the `ghc` or `ghci` command line.

(There is also code to show `BoolFns` in disjunctive normal form; all code is in `BoolFn.hs`, `robdd.c` and `robdd.h` in the `examples` directory.)

```
nomad% gcc -c -Wall robdd.c
nomad% ghci robdd.o
GHCi, version 8.4.3: http://www.haskell.org/ghc/
Prelude> :l BoolFn.hs
[1 of 1] Compiling BoolFn          ( BoolFn.hs, interpreted )
Ok, one module loaded.
*BoolFn> (variable 1) 'disjoin' (variable 2)
((1) | (~1 & 2))
*BoolFn> it 'conjoin' (negation $ variable 3)
((1 & ~3) | (~1 & 2 & ~3))
```

Interfacing to Prolog

The Prolog standard does not standardise a foreign language interface. Each Prolog system has its own approach.

The SWI Prolog approach does most of the work on the C side, rather than in Prolog. This is powerful, but inconvenient.

In an SWI Prolog source file, the declaration

```
:- use_foreign_library(swi_robdd).
```

will load a compiled C library file that links the code in `swi_robdd.c`, which forms the interface to Prolog, and the `robdd.c` file.

These are compiled and linked with the shell command:

```
swipl-ld -shared -o swi_robdd swi_robdd.c robdd.c
```

Connecting C code to Prolog

The `swi_robdd.c` file contains C code to interface to Prolog:

```
install_t install_swi_robdd() {
    PL_register_foreign("boolfn_node", 4, pl_bdd_node, 0);
    PL_register_foreign("boolfn_true", 1, pl_bdd_true, 0);
    PL_register_foreign("boolfn_false", 1, pl_bdd_false, 0);
    PL_register_foreign("boolfn_conjoin", 3, pl_bdd_and, 0);
    PL_register_foreign("boolfn_disjoin", 3, pl_bdd_or, 0);
    PL_register_foreign("boolfn_negation", 2, pl_bdd_negate, 0);
    PL_register_foreign("boolfn_implies", 3, pl_bdd_implies, 0);
}
```

This tells Prolog that a call to `boolfn_node/4` is implemented as a call to the C function `pl_bdd_node`, *etc.*

Marshalling data

A C function that implements a Prolog predicate needs to convert between Prolog terms and C data structures. This is called *marshalling* data.

```
static foreign_t
pl_bdd_and(term_t f, term_t g, term_t result_term) {
    void *f_nd, *g_nd;
    if (PL_is_integer(f)
        && PL_is_integer(g)
        && PL_get_pointer(f, &f_nd)
        && PL_get_pointer(g, &g_nd)) {
        robdd *result = conjoin((robdd *)f_nd, (robdd *)g_nd);
        return PL_unify_pointer(result_term, (void *)result);
    } else {
        PL_fail;
    }
}
```

Making Boolean functions abstract in Prolog

To keep Prolog code from confusing an ROBDD (address) from a number, we wrap the address in a `boolfn/1` term, much like we did in Haskell. We must do this manually; it is most easily done in Prolog code.

```
%  conjoin(+BFn1, +BFn2, -BFn)
%  BFn is the conjunction of BFn1 and BFn2.
conjoin(boolfn(F), boolfn(G), boolfn(FG)) :-
    boolfn_conjoin(F, G, FG).
```

We can make Prolog print BDDs (or anything) nicely by adding a clause for `user:portray/1`:

```
:- multifile(user:portray/1).
user:portray(boolfn(BDD)) :- !,
    % definition is in boolfn.pl ...
```

Using it

```
nomad% swipl-ld -shared -o swi_robdd swi_robdd.c robdd.c
nomad% pl
Welcome to SWI-Prolog (threaded, 64 bits, version 7.7.19)
1 ?- [boolfn].
true.

2 ?- variable(1,A), variable(2,B), variable(3,C),
    |   disjoin(A,B,AB), negation(C,NotC), conjoin(AB,NotC,X).
A = ((1)),
B = ((2)),
C = ((3)),
AB = ((1) | (~1 & 2)),
NotC = ((~3)),
X = ((1 & ~3) | (~1 & 2 & ~3)).
```


Impedance mismatch

Declarative languages like Haskell and Prolog typically use different representations for similar data. For example, what would be represented as a list in Haskell or Prolog would most likely be represented as an array in C or Java.

The consequence of this is that in each language (declarative and imperative) it is difficult to write code that works on data structures defined in the other language.

This problem, usually called *impedance mismatch*, is the reason why most cross-language interfaces are low level, and operate only or mostly on values of primitive types.

Comparative strengths of declarative languages

- Programmers can be significantly more productive because they can work at a significantly higher level of abstraction. They can focus on the big picture without getting lost in details, such as whose responsibility it is to free a data structure.
- Processing of symbolic data is significantly easier due to the presence of algebraic data types and parametric polymorphism.
- Programs can be significantly more reliable, because
 - you cannot make a mistake in an aspect of programming that the language automates (e.g. memory allocation), and
 - the compiler can catch many more kinds of errors.
- What debugging is still needed is easier because you can jump backward in time.
- Maintenance is significantly easier, because
 - the type system helps to locate what needs to be changed, and
 - the typeclass system helps avoid unwanted coupling in the first place.
- You can automatically parallelize declarative programs.

Comparative strengths of imperative languages

- If you are willing to put in the programming time, you can make the final program significantly faster.
- Most existing software libraries are written in imperative languages. Using them in declarative languages is harder than using them in another imperative language (due to dissimilarity of basic concepts), while using them is easiest in the language they are written in. If the bulk of a program interfaces to an existing library, this argues for writing the program in the language of the library:
 - Java for Swing
 - Visual Basic.NET or C# for ASP.NET
- There is a much greater variety of programming tools to choose from (debuggers, profilers, IDEs etc).
- It is much easier to find programmers who know or can quickly learn the language.