Ido 308343474

Q 4

1. $f: \mathbb{R}^d \rightarrow \mathbb{R}, \quad f(w) = w^T x + b$

by def:

$$\nabla_w f = \left[\frac{\partial f}{\partial w_1}, \dots \frac{\partial f}{\partial w_n}\right]$$

$$f(w) = w_1 x_1 + w_2 x_2 \dots w_n x_n + b$$

$$\Downarrow$$

$$\frac{\partial f}{\partial w_i} = x_i$$

$$\Downarrow$$

$$\nabla_w f = [x_1, x_2 \dots x_n] = x$$

2. $\forall x_i$:

$$\frac{\partial f}{\partial w_i} x_i = 0$$

$$\Downarrow$$

$$\frac{\partial f}{\partial^2 w_i} = 0$$

$$\Downarrow$$

$$\nabla_w^2 f = \mathbb{O}$$

3. Yes, $\mathbb{O}$ matrix is PSD by definition.

$\forall x \in \mathbb{R}^n$

$$x^T \mathbb{O} x = 0$$

4.

$$\nabla_w g = \nabla_w \frac{1}{2}\lambda ||w||^2 = \lambda w$$

5

$$\nabla_w^2 g = \nabla_w \lambda w = \lambda I$$

6. PD.

λI is a symetric matrix with eignvalues greater than 0.
In diagonal matrix, eigenvalues are the objects in the main diagonal.

7.

$$h(w_1,w_2) = 12w_1^3 - 36w_1w_2 - 2w_2^3 + 9w_2^2 - 72w_1 + 60w_2 + 5$$

$$\boxed{\begin{aligned} \frac{\partial h}{\partial w_1} &= 36w_1^2 - 36w_2 - 72 = 0 \\ \frac{\partial h}{\partial w_2} &= -36w_1 - 6w_2^2 + 18w_2 + 60 = 0 \end{aligned}}$$

$$\Downarrow$$

$$w_2 = w_1^2 - 2$$

$$-36w_1 - 6(w_1^2-2)^2 + 18w_2 + 60 = 0$$

$$\Downarrow$$

$$-36w_1 - 6\left(w_1^3 - 4w_1^2 + \cancel{4}\right) + 18w_1^2 - \cancel{36} + \cancel{40}$$

$$-36w_1 - 6w_1^4 + 42w_1^2 = 0$$

$$w_1\left[-6w_1^3 + 42w_1 - 36\right] = 0$$

$$w_1\left(-6\left[w_1^3 - 7w_1 + 6\right]\right) = 0$$

"guess"

$$w_1\left(-6\left[(w_1 - 1)\left(w_1^2 + w_1 - 6\right)\right]\right) = 0$$

$$w_1\left(-6\left[(w_1 - 1)(w_1 - 2)(w_1 + 3)\right]\right) = 0$$

$$\Downarrow$$

$$\begin{array}{l} \quad w_1^2 + w_1 - 6 \\ \hline w_1^3 - 7w_1 + 6 \;\big|\; w_1 - 1 \\ -\;(w_1^3 - w_1^2) \\ \quad w_1^2 - 7w_1 + 6 \\ -\;(w_1^2 - w_1) \\ \quad\quad -6w_1 + 6 \\ \hline \end{array}$$

$\boxed{\text{Critical points: } (0,-2)\ (1,-1)\ (2,2)\ (-3,7)}$

8.

$$\frac{dh^2}{dw_1^2} = 72w_1 \qquad \frac{dh^2}{dw_2^2} = -12w_2 + 18 \qquad \frac{dh^2}{dw_1 dw_2} = -36$$

$(0,-2)$:

$$D(0,-2) = 72 \cdot 0 \cdot (-12 \cdot -2 + 18) - (-36)^2 < 0$$

$(0,-2)$ Saddle

$(1,-1)$:

$$D(1,-1) = 72 \cdot 1 \cdot (-12 \cdot -1 + 18) - (-36)^2 > 0$$

$$\frac{dh^2}{dw_1^2} = 72 > 0$$

$$\Downarrow$$

$(1,-1)$ local minimum

$(2,2)$:

$$D(2,2) = 72\cdot 2\left(-12\cdot 2+18\right) - (-36)^2 < 0$$

$(2,2)$ Saddle

$(-3,7)$:

$$D(-3,7) = 72\cdot -3\left(-12\cdot 7+18\right) - (-36)^2 > 0$$

$$\frac{dh^2}{dw_1^2} = -3\cdot 72 < 0$$

$(-3,7)$ local maximum

9.

No!

lets assume a global max does exist.

$h(w_1, v_2) = M$.

if $M < 5$

$h(0,0) = 5 > M$.

if $M > 5$

$$h(3M,0) = 324\underbrace{M^3}_{\substack{\text{for } M>5 \\ M^3 > M}} - 216M + 5 > 324M - 216M$$

$$= 108M > M$$

$\Downarrow$

M isn't a global max!

less assume a global min does exist

$h(w_1, w_2) = m$

if $m \geq -1$

$h(-3\ 0) = 12 \cdot (-3)^3 - 72 \cdot (-3) = 108 < m$

if $m \leq -1$

$h(3m, 0) = 324m^3 - 216m + 5$

$m \leq -1\ ,\ m^3 \leq m$

$< 324m - 216m + 5 = 108m + 5 \leq m$

$m \leq -1$

m isn't a global min