2020 Numerical Analysis Homework #3, 2D Gaussian Quadrature

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IV. Therefore, we will generate 9 results which are produced by using 3 meshes and 3 sets of sample points.

2Dgauss - 記事本

檔案(F) 編輯(E) 格式(O) 檢視(V) 說

```
No. of Gaussian points(k*k) = 2*2
   No. of interval = 1*1
       Integral= 0.070596160023016
   No. of interval = 2*2
       Integral= 0.310942133110673
   No. of interval = 4*4
       Integral= 0.158579352723380
No. of Gaussian points(k*k) = 3*3
   No. of interval = 1*1
       Integral= 0.675487858475693
   No. of interval = 2*2
       Integral= 0.145319812135534
   No. of interval = 4*4
       Integral= 0.160515878210937
No. of Gaussian points(k*k) = 4*4
   No. of interval = 1*1
       Integral= 0.023902045217801
   No. of interval = 2*2
       Integral= 0.161459799068613
   No. of interval = 4*4
       Integral= 0.160427823321380
```

Sample points(2*2)

	intergral
Interval = 1*1	0.070596160023016
Interval = 2*2	0.310942133110673
Interval = 4*4	0.158579352723380

Sample points(3*3)

	intergral
Interval = 1*1	0.675487858475693
Interval = 2*2	0.145319812135534
Interval = 4*4	0.160515878210937

Sample points(4*4)

	intergral
Interval = 1*1	0.023902045217801
Interval = 2*2	0.161459799068613
Interval = 4*4	0.160427823321380

V.

A. 0.160429671298544

```
>> f = @(x,y) sin(pi*x.*2) ./ (2*pi*x)* sin (pi*y.*3) ./ (3*pi*y)
f =

@(x, y) sin (pi * x .* 2) ./ (2 * pi * x) * sin (pi * y .* 3) ./ (3 * pi * y)

>> format long e
>> dblquad(f, -1, 1, -1, 1, 1.0e-15)
warning: division by zero
warning: called from
     at line -1 column -1
    __dblquad_inner__> at line -1 column -1
    dblquad>__dblquad_inner__ at line 73 column 10
    dblquad> at line -1 column -1
    dblquad at line 66 column 5
ans =    1.60429671298544e-001
```

#Using octave.

```
f = @(x,y) sin(pi*x.*2) ./ (2*pi*x) * sin(pi*y.*3) ./ (3*pi*y)
format long e
dblquad(f, -1, 1, -1, 1, 1.0e-15)
```

B.

2Dgauss&relative_err - 記事本

檔案(F) 編輯(E) 格式(O) 檢視(V) 說明

```
No. of Gaussian points(k*k) = 2*2
  No. of interval = 1*1
     Integral= 0.070596160023016
     relative_err= 0.559955714852503
  No. of interval = 2*2
     Integral= 0.310942133110673
     relative_err= 0.938183445704627
  No. of interval = 4*4
     Integral= 0.158579352723380
     relative_err= 0.011533518458194
No. of Gaussian points(k*k) = 3*3
  No. of interval = 1*1
     Integral= 0.675487858475693
     relative_err= 3.210492068008265
  No. of interval = 2*2
     Integral= 0.145319812135534
     relative_err= 0.094183694579113
  No. of interval = 4*4
     Integral= 0.160515878210937
     relative_err= 0.000537350177780
No. of Gaussian points(k*k) = 4*4
  No. of interval = 1*1
     Integral= 0.023902045217801
     relative_err= 0.851012315712335
  No. of interval = 2*2
     Integral= 0.161459799068613
     relative_err= 0.006421055168477
  No. of interval = 4*4
     Integral= 0.160427823321380
     relative_err= 0.000011518923829
```

Sample points(2*2)

	Intergral	relative_error
Interval = 1*1	0.070596160023016	0.559955714852503
Interval = 2*2	0.310942133110673	0.938183445704627
Interval = 4*4	0.158579352723380	0.011533518458194

Sample points(3*3)

	Intergral	relative_error
Interval = 1*1	0.675487858475693	3.210492068008265
Interval = 2*2	0.145319812135534	0.094183694579113
Interval = 4*4	0.160515878210937	0.000537350177780

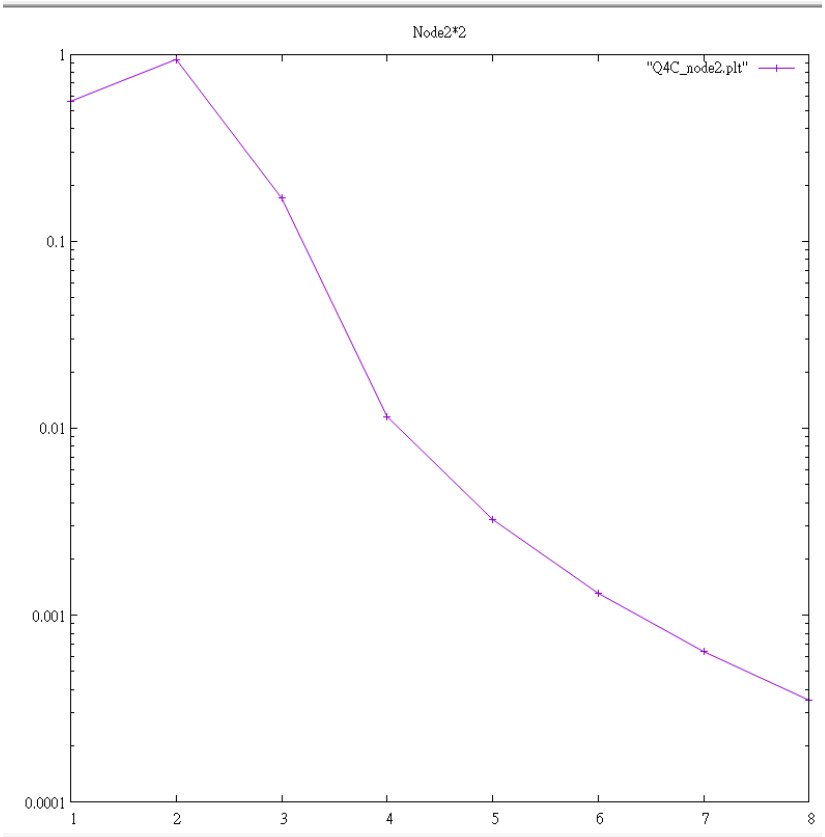
Sample points(4*4)

	Intergral	relative_error
Interval = 1*1	0.023902045217801	0.851012315712335
Interval = 2*2	0.161459799068613	0.006421055168477
Interval = 4*4	0.160427823321380	0.000011518923829

C. YES! We can improve the accuracy by dividing D into a finer mesh.

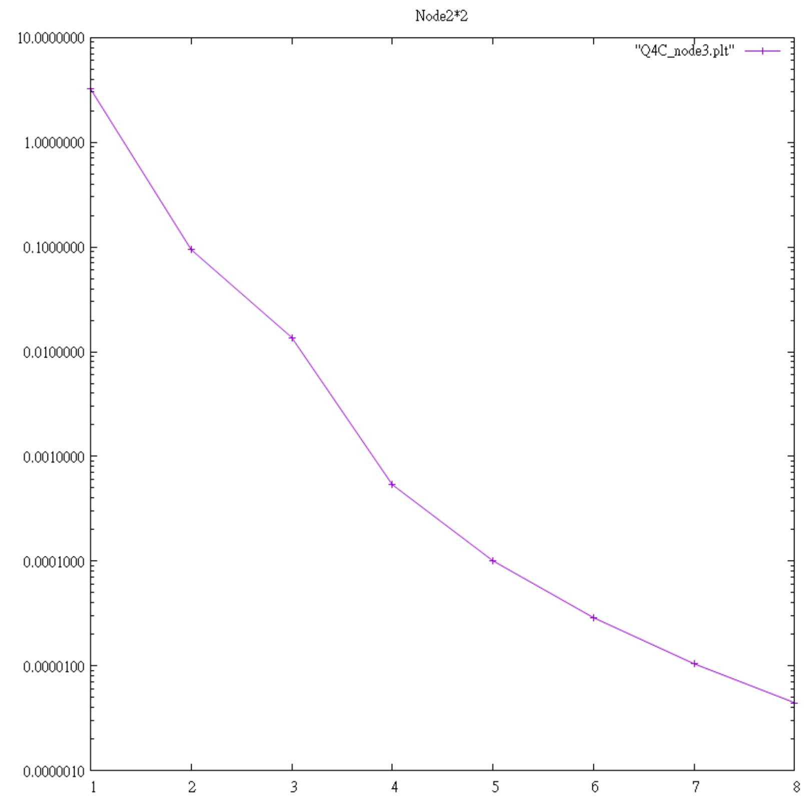
Take sample points2*2 for example:

```
No. of Gaussian points(k*k) = 2*2
  No. of interval = 1*1
     Integral= 0.070596160023016
     relative_err= 0.559955714852503
  No. of interval = 2*2
     Integral= 0.310942133110673
     relative_err= 0.938183445704627
  No. of interval = 3*3
     Integral= 0.133150921757051
     relative_err= 0.170035563376116
  No. of interval = 4*4
     Integral= 0.158579352723380
     relative_err= 0.011533518458194
  No. of interval = 5*5
     Integral= 0.159907902934333
     relative_err= 0.003252318352258
  No. of interval = 6*6
     Integral= 0.160219319559179
     relative_err= 0.001311177275763
  No. of interval = 7*7
     Integral= 0.160327157423636
     relative_err= 0.000638995729895
  No. of interval = 8*8
     Integral= 0.160373322645670
     relative_err= 0.000351235855675
```



Take sample points3*3 for example

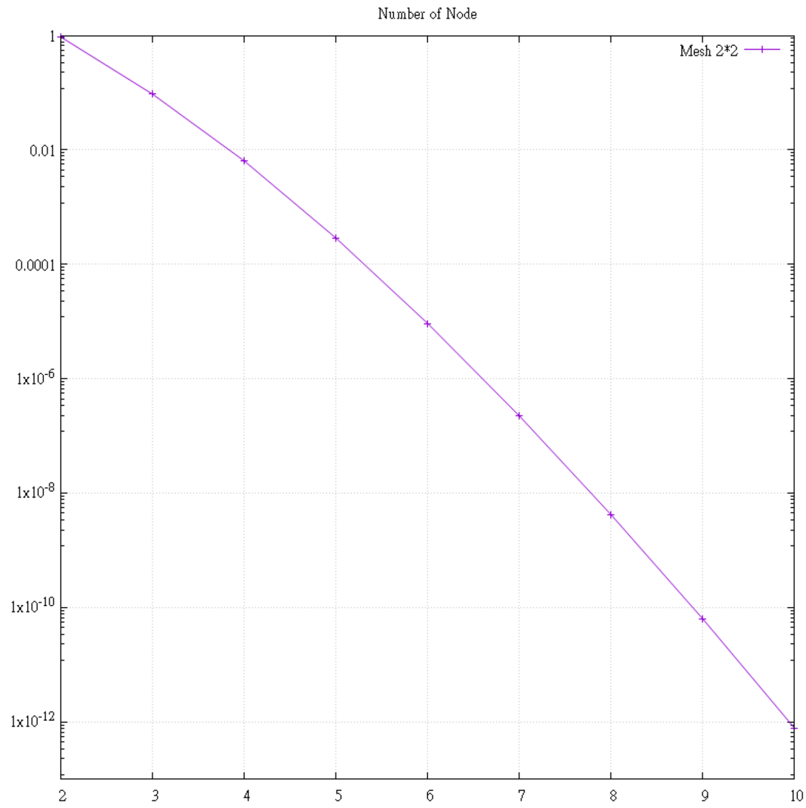
```
No. of Gaussian points(k*k) = 3*3
  No. of interval = 1*1
     Integral= 0.675487858475693
     relative_err= 3.210492068008265
  No. of interval = 2*2
     Integral= 0.145319812135534
     relative_err= 0.094183694579113
  No. of interval = 3*3
     Integral= 0.162620713416678
     relative_err= 0.013657337202021
  No. of interval = 4*4
     Integral= 0.160515878210937
     relative_err= 0.000537350177780
  No. of interval = 5*5
     Integral= 0.160445862599408
     relative_err= 0.000100924602869
  No. of interval = 6*6
     Integral= 0.160434310764520
     relative_err= 0.000028919001941
  No. of interval = 7*7
     Integral= 0.160431356645928
     relative_err= 0.000010505209979
  No. of interval = 8*8
     Integral= 0.160430387376437
     relative_err= 0.000004463500342
```



D. YES! We can improve the accuracy by using more sample points.

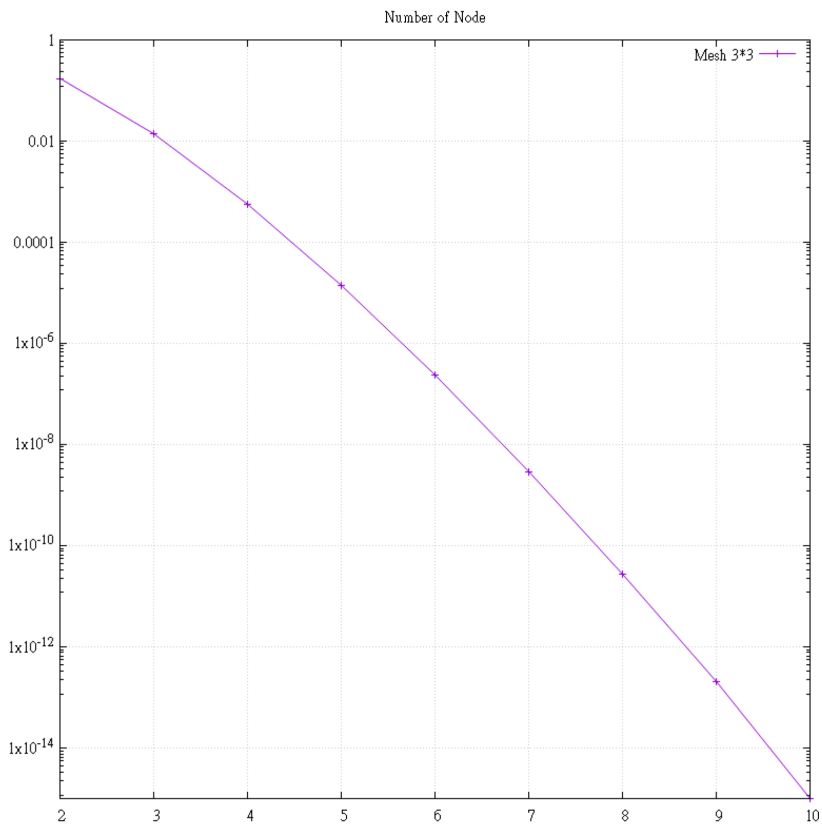
Take 2*2 meshes f or example:

```
No. of interval = 2*2
  No. of Gaussian points(k*k) = 2*2
    Integral= 0.310942133110673
    relative_err= 0.938183445704627
  No. of Gaussian points(k*k) = 3*3
    Integral= 0.145319812135534
    relative_err= 0.094183694579113
  No. of Gaussian points(k*k) = 4*4
    Integral= 0.161459799068613
    relative_err= 0.006421055168477
  No. of Gaussian points(k*k) = 5*5
    Integral= 0.160383490346669
    relative_err= 0.000287857922420
  No. of Gaussian points(k*k) = 6*6
    Integral= 0.160431155699868
    relative_err= 0.000009252660759
  No. of Gaussian points(k*k) = 7*7
    Integral= 0.160429635420403
    relative_err= 0.000000223637813
  No. of Gaussian points(k*k) = 8*8
    Integral= 0.160429671974007
    relative_err= 0.000000004210336
  No. of Gaussian points(k*k) = 9*9
    Integral= 0.160429671288363
    relative_err= 0.000000000063460
  No. of Gaussian points(k*k) = 10*10
    Integral= 0.160429671298670
    relative_err= 0.000000000000785
```



Take 3*3 meshe s for example:

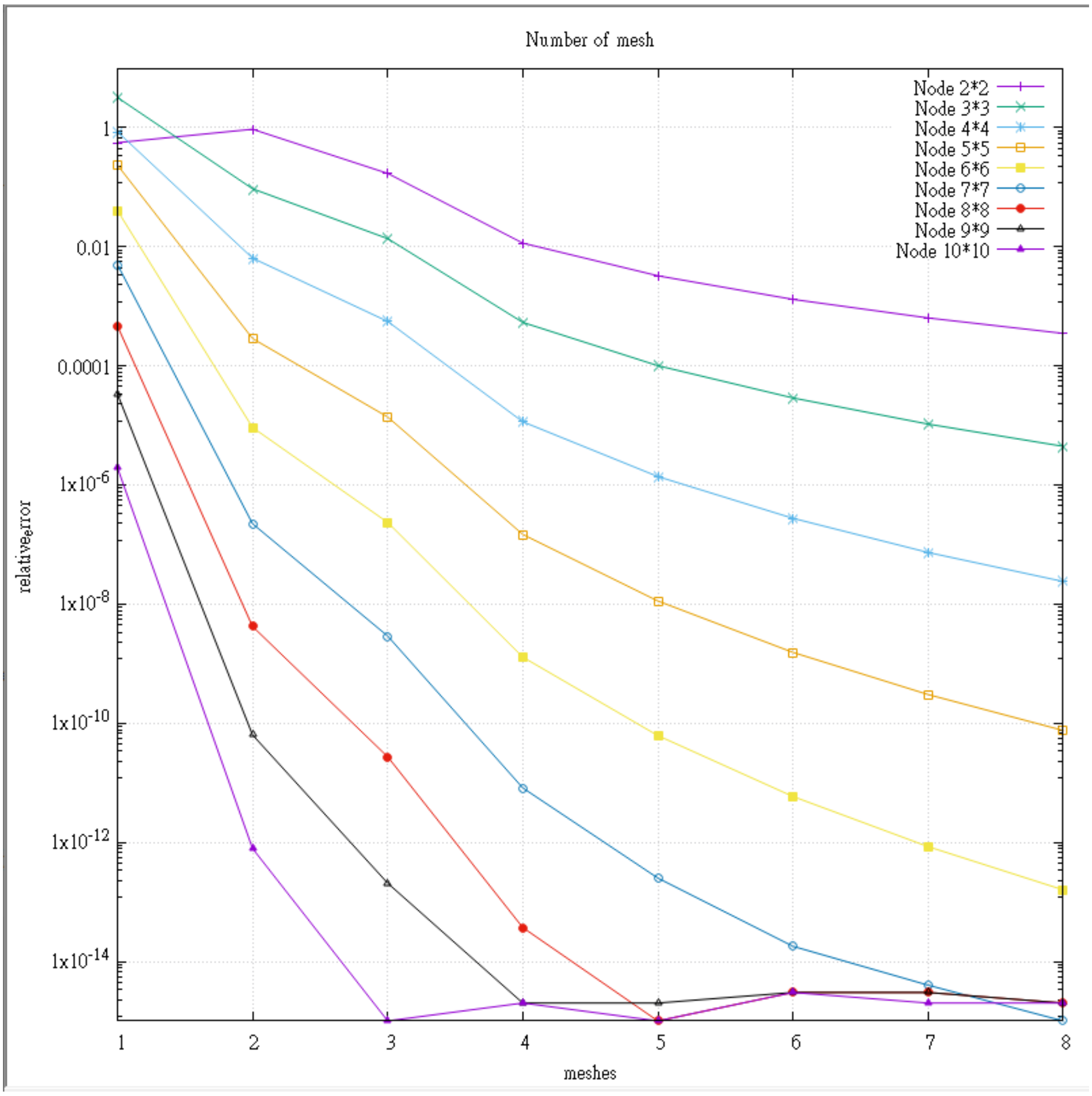
```
No. of interval = 3*3
  No. of Gaussian points(k*k) = 2*2
    Integral= 0.133150921757051
    relative_err= 0.170035563376116
  No. of Gaussian points(k*k) = 3*3
    Integral= 0.162620713416678
    relative_err= 0.013657337202021
  No. of Gaussian points(k*k) = 4*4
    Integral= 0.160340427870108
    relative_err= 0.000556277574549
  No. of Gaussian points(k*k) = 5*5
    Integral= 0.160431892890198
    relative_err= 0.000013847760427
  No. of Gaussian points(k*k) = 6*6
    Integral= 0.160429633653600
    relative_err= 0.000000234650758
  No. of Gaussian points(k*k) = 7*7
    Integral= 0.160429671762906
    relative_err= 0.000000002894492
  No. of Gaussian points(k*k) = 8*8
    Integral= 0.160429671294178
    relative_err= 0.000000000027217
  No. of Gaussian points(k*k) = 9*9
    Integral= 0.160429671298577
    relative_err= 0.000000000000205
  No. of Gaussian points(k*k) = 10*10
    Integral= 0.160429671298544
    relative_err= 0.000000000000001
```

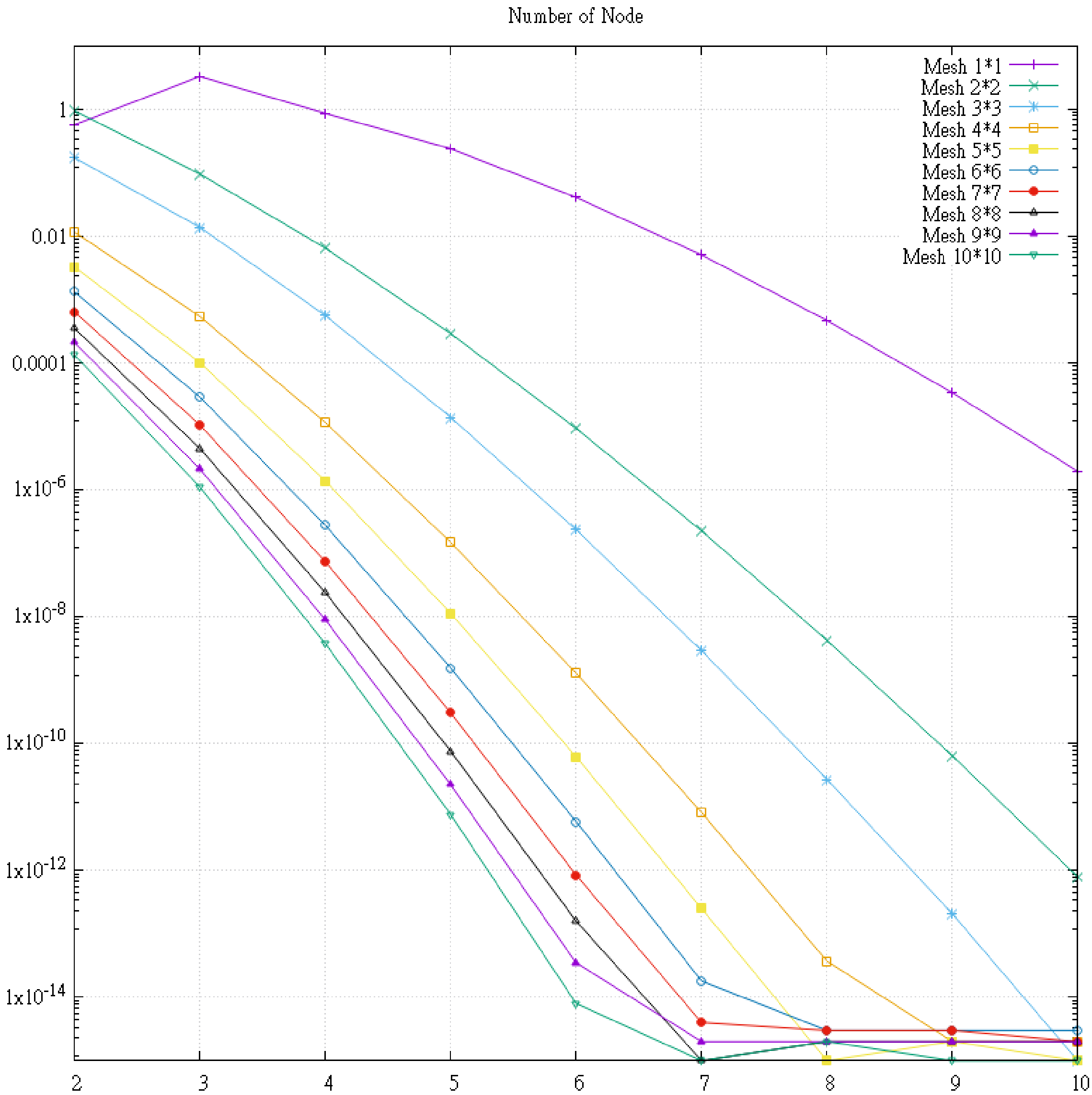


E. which factor is more important, the number of sample point or the mesh resolution?

從 C,D 的結果可以感覺到，若是要找最後的結果應是 the number of sample point 較為重要，因為其誤差值最終會比較小，但是剛開始下降的速度沒有 the number of mesh 快，所以若是要在”比較少的計算”中有比較準確的值的話，the number of mesh 較為重要。

F.





Number of Node
Mesh 1*1
Mesh 2*2
Mesh 3*3
Mesh 4*4
Mesh 5*5
Mesh 6*6
Mesh 7*7
Mesh 8*8
Mesh 9*9
Mesh 10*10
1
0.01
0.0001
1x10-6
1x10-8
1x10-10
1x10-12
1x10-14
2
3
4
5
6
7
8
9
10