

2020 Fall Semester, Numerical Analysis, Homework #4

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9.

a.

i	X	Y
1	2.000000	255.000000
2	2.200000	456.465613
3	2.400000	785.537958
4	2.600000	1304.544154
5	2.800000	2098.344435
6	3.000000	3280.000000
7	3.200000	4997.325581
8	3.400000	7440.391270
9	3.600000	10850.038106
10	3.800000	15527.471923
11	4.000000	21845.000000

b. result1,2,3 所求出的向量 c (即是 a_0, a_1, \dots, a_{10})

result1:

```
2.0659809991 -1.6394873026 3.7727927868 -0.6021168521 1.5499708351 0.8878144809 1.0125938706 0.9993996402
```

result2:

```
52.3148395012 -125.5187616362 133.3378721229 -75.1369635234 27.0250617748 -4.2864887100 1.5910248157 0.9719379180
```

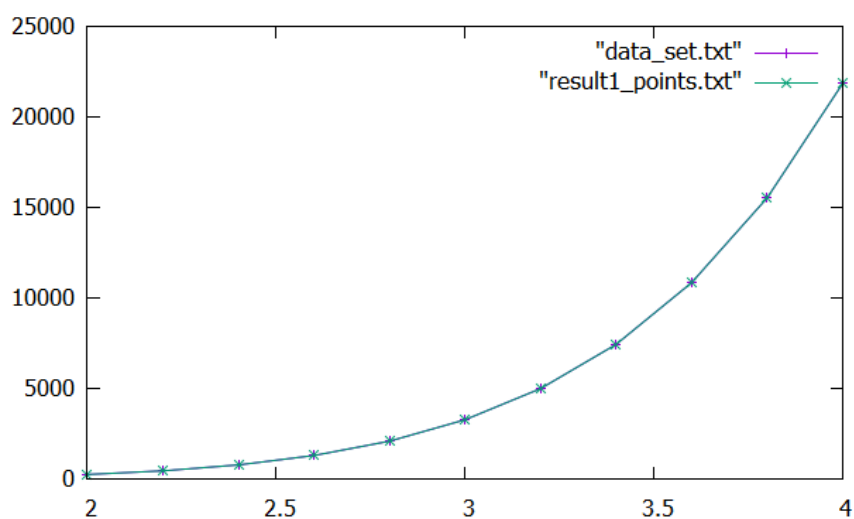
result3:

```
1.00000000241 0.99999999485 1.00000000458 0.99999999783 1.00000000058 0.99999999991 1.00000000001 1.00000000000
```

c.

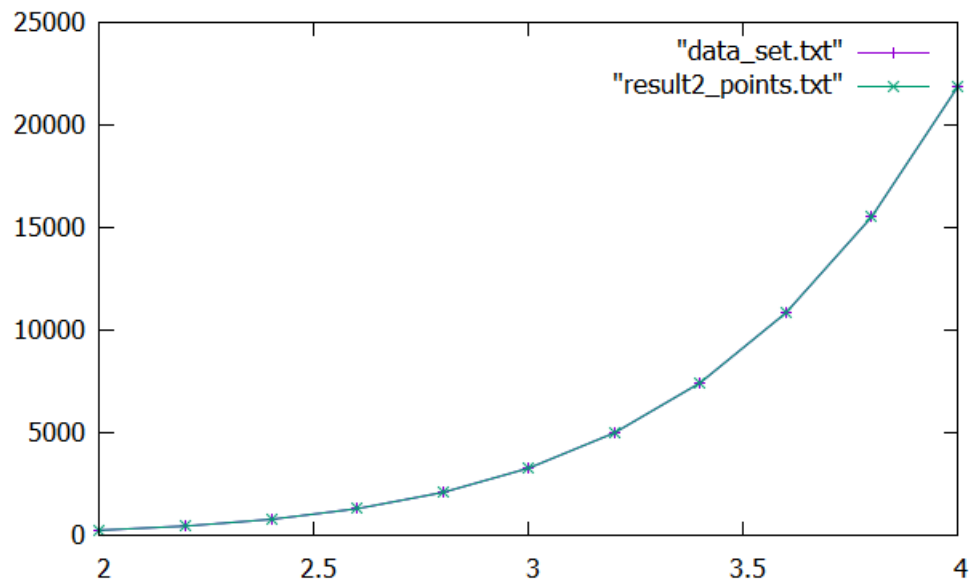
result1:

```
two_norm_residual: 0.000019  
one_norm_residual: 0.000050
```



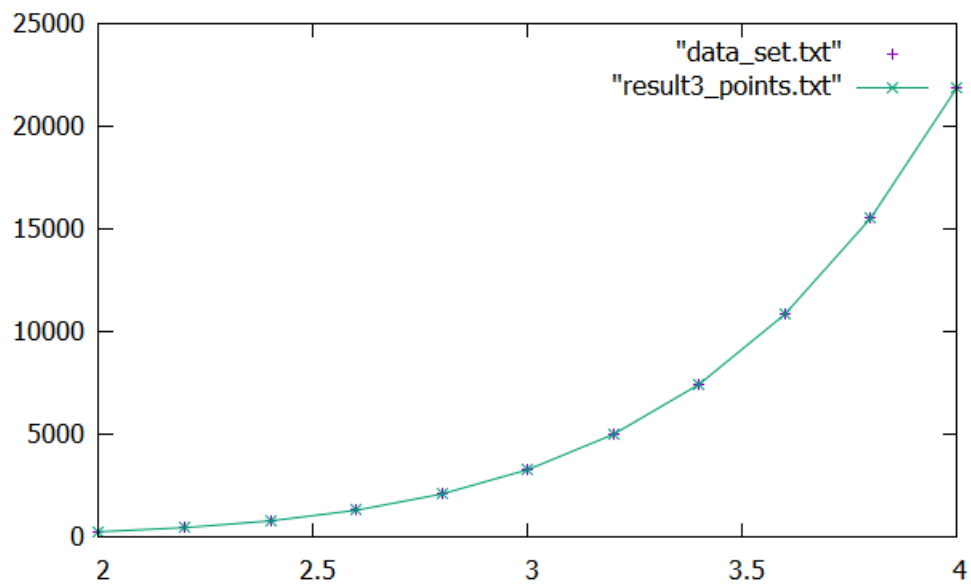
result2:

```
two_norm_residual: 0.0008921119  
one_norm_residual: 0.0024000637
```



result3:

```
two_norm_residual: 0.0000000000  
one_norm_redisual: 0.0000000000
```



d.

新系統的誤差較大，因為 A 本身就已經存在誤差了，但是新系統又

$A_T * A$ 這誤差就更大了

e.
用三種方法的測試結果，polynomial of degree 大概都到 9 或 10 就差不多了，就
算 polynomial of degree 再往上加，算出來的向量 c 後面的元素都差不多接近
0(即 $a_{11}, a_{12}, a_{13}, \dots$ 都接近 0)。

result1

```
-----  
-419.8419984323 968.1229075609 -922.6340246319 464.6262070624 -121.1769107025 11.7831921480 3.6216963121 0.1949192085 0.0643155362 0.0024122261 -0.0006355246 0.0000358523 0.0000149838 -0.0000051058 0.0000000751 0.0000001257 -0.0000000079 -0.0000000013 -0.0000000000 0.0000000000 0.0000000000 -0.0000000000 -0.0000000000 0.0000000000 0.0000000000 -0.0000000000 0.0000000000  
-----  
two_norm_residual: 0.031842  
one_norm_residual: 0.142294
```

result2:

```
-----  
5695.2153602842 -9489.0089285196 6053.8164066113 -1639.6199569528 14.5786153813 106.2552293578 -25.2350234557 3.4983456506 -0.0773144993 0.0110904363 -0.0024899442 0.0001006304 8 -0.0000052386 0.0000012350 0.0000000342 -0.0000000252 0.0000000002 0.0000000001 0.0000000000 0.0000000000 -0.0000000000 -0.0000000000 -0.0000000000 0.0000000000 0.0000000000 0.0000000000 -0.0000000000  
-----  
two_norm_residual: 11.3448298755  
one_norm_residual: 23.5339176222
```

result3:

```
-----  
1.0708597268 0.6873776251 1.6498511708 0.1498891664 1.7886918561 0.4462656348 1.3055562553 0.8648660621 0.0481026657 -0.0136367205 0.0029921262 -0.0004783474 0.0000478464 -0.0000000003132 0.0000000164 0.0000000040 0.0000000008 -0.0000000005 0.0000000001 -0.0000000000 0.0000000000 0.0000000000 -0.0000000000 0.0000000000 -0.0000000000 -0.0000000000 0.0000000000  
-----
```