离散数学2 第八周作业.

20. (1) 将 B_S 进行列交换

$$B_S = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & -1 & 0 & -1 \\ 0 & 0 & 0 & -1 & 1 & -1 & 0 & 0 \end{bmatrix}$$
$$\quad e_1 \; e_2 \; e_5 \; e_8 \quad e_3 \; e_4 \; e_6 \; e_7$$

$$B_{11} = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \qquad B_{12} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \qquad B_{12}^{-1} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & -1 \end{bmatrix}$$

$$C_{f12} = -B_{11}^{T} B_{12}^{-T} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & -1 & 1 & 1 \\ 0 & 0 & -1 & -1 \\ -1 & -1 & 0 & 1 \\ 1 & 0 & 0 & -1 \end{bmatrix}$$

则 $C_f = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & -1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 1 & 0 & -1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & -1 \end{bmatrix}$
$$\quad e_1 \; e_2 \; e_5 \; e_8 \quad e_3 \; e_4 \; e_6 \; e_7$$

(2) 将 B_S 进行列交换

$$B_S = \begin{bmatrix} -1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & -1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$
$$\quad e_1 \; e_3 \; e_4 \; e_7 \quad e_2 \; e_5 \; e_6 \; e_8$$

$$B_{11} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & -1 \\ 0 & 1 & -1 & 0 \end{bmatrix} \qquad B_{12} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \qquad B_{12}^{-1} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$S_{f11} = B_{12}^{-1} B_{11} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & -1 \\ 0 & 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ -1 & -1 & 0 & -1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

则 $S_f = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ -1 & -1 & 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$
$$\quad e_1 \; e_3 \; e_4 \; e_7 \quad e_2 \; e_5 \; e_6 \; e_8$$

统计字符

c	e	s	t	□	a
1	2	3	4	4	5

① c+e 3

② ce+s 6

③ t+□ 8
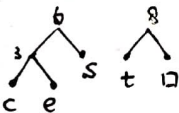
④ ces+a 11
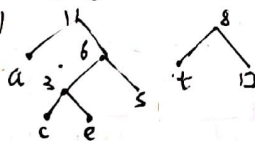
⑤ cesa+t□
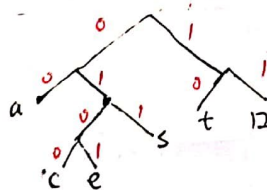
$a \leftarrow 00$
$c \leftarrow 0100$
$e \leftarrow 0101$
$s \leftarrow 011$
$t \leftarrow 10$
$□ \leftarrow 11$

则   state act as a seat 对应

0111 0.00 100.10 11.00 010.0 1011.000 11.11 001.10 110.10 100.10

4.7个字符

16. 采用 Kruskal 算法思路，将边按权排序

	边	权		选			
e_1	(V_4, V_3)	2		e_1	$V_4、V_3$	2	$V_3 V_4$
e_2	(V_8, V_7)	3		e_2	$V_8、V_7$	2	$V_7 V_8, V_3 V_4$
e_3	(V_1, V_8)	3		e_3	$V_1、V_8$	3	$V_1 V_7 V_8, V_3 V_4$
e_4	(V_7, V_6)	4		e_4	$V_7、V_6$	3	$V_1 V_6 V_7 V_8, V_3 V_4$
e_5	(V_2, V_4)	4		e_5	$V_2、V_4$	4	$V_1 V_6 V_7 V_8, V_2 V_3 V_4$
e_6	(V_5, V_7)	4		e_6	$V_5 V_7$	4	$V_1 V_5 V_6 V_7 V_8, V_2 V_3 V_4$
e_7	(V_4, V_8)	4		e_7	$V_4 V_8$	4	$V_1 V_2 V_3 V_4 V_5 V_6 V_7 V_8$
e_8	(V_4, V_1)	4					
e_9	(V_4, V_6)	5					
e_{10}	(V_6, V_5)	5					
e_{11}	(V_8, V_5)	5					
e_{12}	(V_1, V_2)	6					
e_{13}	(V_3, V_1)	7					
e_{14}	(V_2, V_3)	7					
e_{15}	(V_5, V_4)	7					

已选边

最短树由 $e_1 \sim e_7$ 构成
总长 22