习题 3.1.1

b) $(0+1)^* 1 (0+1)^9$

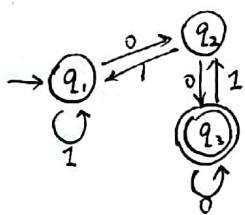c) $(0+10)^* 11 (0+01)^* + (1+\varepsilon)(0+01)^*$

习题 3.1.2

b) $(1^*01^*01^*01^*01^*)^* + 1^*$

习题 3.1.3

a) $0^* ((1^*\cdot 00\cdot 0^*1^*)^* + 1^*) 0^*$

习题 3.2.1

c)

$R_{11}^{(0)} = \varepsilon+1$

$R_{12}^{(0)} = 0$

$R_{13}^{(0)} = \phi$

$R_{21}^{(0)} = 1$

$R_{22}^{(0)} = \varepsilon$

$R_{23}^{(0)} = 0$

$R_{31}^{(0)} = \phi$

$R_{32}^{(0)} = 1$

$R_{33}^{(0)} = \varepsilon+0$

$R_{11}^{(1)} = R_{11}^{(0)} + R_{11}^{(0)} R_{11}^{(0)*} R_{11}^{(0)} = (\varepsilon+1) + (\varepsilon+1)(\varepsilon+1)^*(\varepsilon+1) = 1^*$

$R_{12}^{(1)} = R_{12}^{(0)} + R_{11}^{(0)} R_{11}^{(0)*} R_{12}^{(0)} = 0 + (\varepsilon+1)(\varepsilon+1)^*0 = 1^*0$

$R_{13}^{(1)} = R_{13}^{(0)} + R_{11}^{(0)} R_{11}^{(0)*} R_{13}^{(0)} = \phi + (\varepsilon+1)(\varepsilon+1)^* \phi = \phi$

$R_{21}^{(1)} = R_{21}^{(0)} + R_{21}^{(0)} R_{11}^{(0)*} R_{11}^{(0)} = 1 + 1(\varepsilon+1)^*(\varepsilon+1) = 1\cdot 1^*$

$R_{22}^{(1)} = R_{22}^{(0)} + R_{21}^{(0)} R_{11}^{(0)*} R_{12}^{(0)} = \varepsilon + 1(\varepsilon+1)^*0 = \varepsilon + 1\cdot 1^*\cdot 0$

$R_{23}^{(1)} = R_{23}^{(0)} + R_{21}^{(0)} R_{11}^{(0)*} R_{13}^{(0)} = 0 + 1(\varepsilon+1)^*\phi = 0$

$R_{31}^{(1)} = R_{31}^{(0)} + R_{31}^{(0)} R_{11}^{(0)*} R_{11}^{(0)} = \phi$

$R_{32}^{(1)} = R_{32}^{(0)} + R_{31}^{(0)} R_{11}^{(0)*} R_{12}^{(0)} = 1 + \phi = 1$

$R_{33}^{(1)} = R_{33}^{(0)} + R_{31}^{(0)} R_{11}^{(0)*} R_{13}^{(0)} = \varepsilon+0+\phi = \varepsilon+0$

$R_{11}^{(2)} = R_{11}^{(1)} + R_{12}^{(1)} R_{22}^{(1)*} R_{21}^{(1)} = 1^* + (1^*0)(\varepsilon+1\cdot 1^*\cdot 0)^*(1\cdot 1^*) = 1^* + 1^*0(1\cdot 1^*\cdot 0)^*1\cdot 1^*$

$R_{12}^{(2)} = R_{12}^{(1)} + R_{12}^{(1)} R_{22}^{(1)*} R_{22}^{(1)} = 1^*0 + 1^*0(\varepsilon+1\cdot 1^*\cdot 0)^*(\varepsilon+1\cdot 1^*\cdot 0) = 1^*0 + 1^*0(1\cdot 1^*\cdot 0)^*$

$R_{13}^{(2)} = R_{13}^{(1)} + R_{12}^{(1)} R_{22}^{(1)*} R_{23}^{(1)} = \phi + 1^*0(\varepsilon+1\cdot 1^*0)^*0 = 1^*0(\cdot 1\cdot 1^*\cdot 0)^*0$

$R_{21}^{(2)} = R_{21}^{(1)} + R_{22}^{(1)} R_{22}^{(1)*} R_{21}^{(1)} = 1\cdot 1^* + (\varepsilon+1\cdot 1^*0)(\varepsilon+1\cdot 1^*0)^*1\cdot 1^* = (1\cdot 1^*0)^*1\cdot 1^*$

$R_{22}^{(2)} = R_{22}^{(1)} + R_{22}^{(1)} R_{22}^{(1)*} R_{22}^{(1)} = (\varepsilon+1\cdot 1^*\cdot 0) + (\varepsilon+1\cdot 1^*\cdot 0)(\varepsilon+1\cdot 1^*\cdot 0)^*(\varepsilon+1\cdot 1^*0) = (1\cdot 1^*0)^*$

$R_{23}^{(2)} = R_{23}^{(1)} + R_{22}^{(1)} R_{22}^{(1)*} R_{23}^{(1)} = 0 + (\varepsilon+1\cdot 1^*0)^*0 = (1\cdot 1^*0)^*0$

$R_{31}^{(2)} = R_{31}^{(1)} + R_{32}^{(1)} R_{22}^{(1)*} R_{21}^{(1)} = \phi + 1(1\cdot 1^*0)^*\cdot 1\cdot 1^* = 1(1\cdot 1^*0)^*1\cdot 1^*$

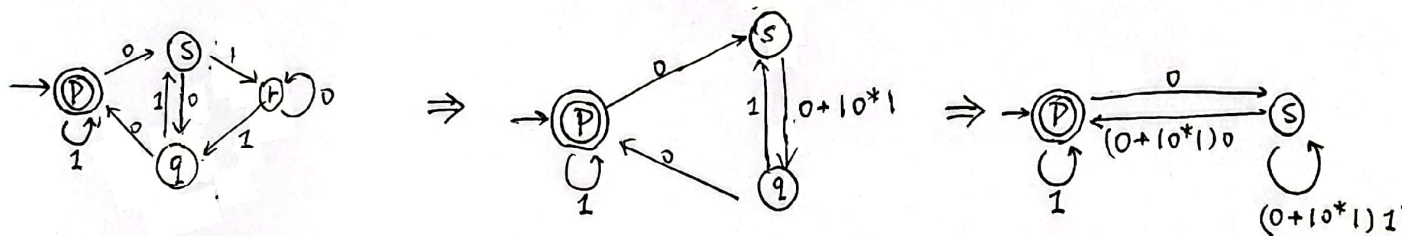$R_{32}^{(2)} = R_{32}^{(1)} + R_{32}^{(1)} R_{22}^{(1)*} R_{22}^{(1)} = 1 + 1(1\cdot 1^*0)^* = 1(1\cdot 1^*0)^*$

$R_{33}^{(2)} = R_{33}^{(1)} + R_{32}^{(1)} R_{22}^{(1)*} R_{23}^{(1)} = \varepsilon+0 + 1(1\cdot 1^*0)^*\cdot 0$

d) $R_{13}^{(3)} = R_{13}^{(2)} + R_{13}^{(2)} R_{33}^{(2)*} R_{33}^{(2)} = 1^*0(1\cdot 1^*\cdot 0)^*0 + 1^*0(1\cdot 1^*0)^*0(\cdot 0 + 1(1\cdot 1^*0)^*0)^*$

$= \underline{1^*0(1\cdot 1^*0)^*0(0 + 1(1\cdot 1^*0)^*0)^*}$

这即自动机语言的正则表达式

习题 3.2.3

$$r = \left(1 + 0\left((0+10^*1)1\right)^*(0+10^*1)0\right)^*$$

习题 3.4.2

b)

$$(RS+R)^*R = \left(\varepsilon + (RS+R) + (RS+R)(RS+R) + \cdots\right)R$$
$$= \left(\varepsilon + R(S+\varepsilon) + R(S+\varepsilon)R(S+\varepsilon) + \cdots\right)R$$
$$= \left(R + R(S+\varepsilon)R + R(S+\varepsilon)R(S+\varepsilon)R + \cdots\right)$$
$$= R\left(\varepsilon + (S+\varepsilon)R + (S+\varepsilon)R(S+\varepsilon)R + \cdots\right)$$
$$= R\left(\varepsilon + (SR+R) + (SR+R)(SR+R) + \cdots\right)$$
$$= R(SR+R)^*$$

d) 不正确

$$\varepsilon \in L\left((R^*S)^*\right) \text{ 但 } \varepsilon \notin L\left((R+S)^*S\right)$$
$$\text{故 } L\left((R^*S)^*\right) \neq L\left((R+S)^*S\right)$$
$$\text{证明表达式不等价}$$