Homework For GP1 (Optical Part)

1. A electro-magnetic wave is specified by the following function (in SI unit):

$$\vec{E} = (-3\hat{i} + 3\sqrt{3}\hat{j})(10^4 V/m)e^{i[1/3(2\sqrt{3}x+2y)\pi\times10^7 - 9.42\times10^{15}t]}$$

Find (a) the direction along which the electric field oscillates, (b) the scalar value of the amplitude of the E-field, (c) the direction of the propagation of the wave, (d) the propagation number (or wave vector value) and wavelength, (e) the frequency and angular frequency, (f) the speed. (g) the irradiance *I*.

2. Given 3 harmonic waves, $Ae^{i(kx+\varphi)}e^{-i\omega t}$, where ϕ are 0, π/3, 2/3π. Find the resultant wave which is a superposition of the three harmonic wave.

3. Given two waves:

$E_1 = Ae^{i(kx-\omega t)}$ and $E_2 = \sqrt{i}Ae^{i(kx-\omega t)}$ (Here $i^2 = -1$, not the unit vector long x-direction)

Please find out the phase difference between the two waves and which one is leading.

4. $E_0\cos(kx-\omega t)$ and $E_0\cos(k'x-\omega' t)$ are two E-M waves in vacuum and $\omega' = \omega + \Delta\omega$ where $\Delta\omega$ is much smaller than ω. Show that $k' = k + \Delta k$ where $\frac{\Delta k}{k} = \frac{\Delta\omega}{\omega}$.

5. (Hecht' 3.10) The time average of some function over an interval of time is given by:

$$< f(t) >_T = \frac{1}{T}\int_t^{t+T} f(t')dt'$$

if τ=2π/ω is the period of a harmonic function, show that:

$$< \cos^2(\vec{k}\cdot\vec{r}-\omega t) >= 1/2, < \sin^2(\vec{k}\cdot\vec{r}-\omega t) >= 1/2$$

and

$< \sin(\vec{k}\cdot\vec{r}-\omega t)\cos(\vec{k}\cdot\vec{r}-\omega t) >= 0$ when T=τ, and T>>τ.

6. Hecht's 3.14:

3.14* A light bulb puts out 20 W of radiant energy (most of it IR). Assume it to be a point source and calculate the irradiance 1.00 m away.

7. Hecht's 3.23:

3.23* How many photons per second are emitted from a 100-W yellow lightbulb if we assume negligible thermal losses and a quasi-monochromatic wavelength of 550 nm? In actuality only about 2.5% of the total dissipated power emerges as visible radiation in an ordinary 100-W lamp.

(some more for your practice purpose, no need to hand in these: Hecht's 2.14, 2.23, 2.32, 2.37, 2.41, 2.42, 3.16, 3.17)