

HW4

Problem 1 谓词变换

1-1

$$\begin{aligned}
 \textcircled{1} \quad & \text{wlp} (b[m] := b[n]; b[n] := t, b[m] < b[n]) \\
 &= \text{wlp} (b[m] := b[n], \text{wlp} (b[n] := t, b[m] < b[n])) \\
 &= \text{wlp} (b[m] := b[n], b[m] < b[n] [b \mapsto b \triangleleft t]) \\
 &= \text{wlp} (b[m] := b[n], (m=n \wedge t < t) \vee (m \neq n \wedge b[m] < t)) \\
 &= \text{wlp} (b[m] := b[n], m \neq n \wedge b[m] < t) \\
 &= (m \neq n \wedge b[m] < t) [b \mapsto b \triangleleft b[n]] \\
 &= (m \neq n \wedge b[n] < t)
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{2} \quad & \text{wlp} (\text{if } y > 2 \text{ then } x := y - 5 \text{ else } x := -y, x \geq 0) \\
 &= (y > 2 \rightarrow \text{wlp} (x := y - 5, x \geq 0)) \wedge \\
 &\quad (\neg(y > 2) \rightarrow \text{wlp} (x := -y, x \geq 0)) \\
 &= ((y > 2) \rightarrow (x \geq 0) [x \mapsto y - 5]) \wedge (\neg(y > 2) \rightarrow (x \geq 0) [x \mapsto -y]) \\
 &= ((y > 2) \rightarrow (y - 5 \geq 0)) \wedge (\neg(y > 2) \rightarrow (-y \geq 0)) \\
 &= (y \geq 5 \vee y \leq 0)
 \end{aligned}$$

1-2

设 $I: x = n \times n$, $st = \{n: n+1; x := x + 2 \times n - 1; r := r - 1;\}$

验证条件:

- ① $I \wedge \neg(r \neq 0) \rightarrow \psi$
- ② $I \wedge r \neq 0 \rightarrow \text{wlp}(st, I)$
- ③ $\text{true} \rightarrow \text{wlp}(n := 0; x := 0, I)$

$$① \quad I \wedge \neg (r \neq 0) \rightarrow \psi$$

即 为 $x = n \times n \wedge \neg (r \neq 0) \rightarrow x = n \times n$
显然 是有效式

$$② \quad I \wedge r \neq 0 \rightarrow wp(st, I)$$

$$\begin{aligned} wp(st, I) &= wp(n := n+1; x := x+2 \times n-1; r := r-1, x = n \times n) \\ &= wp(n := n+1; x := x+2 \times n-1, wp(r := r-1, x = n \times n)) \\ &= wp(n := n+1; x := x+2 \times n-1, (x = n \times n)[r \mapsto r-1]) \\ &= wp(n := n+1; x := x+2 \times n-1, x = n \times n) \\ &= wp(n := n+1, wp(x := x+2 \times n-1, x = n \times n)) \\ &= wp(n := n+1, (x = n \times n)[x \mapsto x+2 \times n-1]) \\ &= wp(n := n+1, x+2 \times n-1 = n \times n) \\ &= (x+2 \times n-1 = n \times n)[n \mapsto n+1] \\ &= (x+2 \times (n+1)-1 = (n+1)) \\ &= (x = n \times n) \end{aligned}$$

$$I \wedge r \neq 0 \rightarrow wp(st, I) \text{ 也 } PP$$

$$(x = n \times n) \wedge (r \neq 0) \rightarrow (x = n \times n)$$

显然是有效式

$$③ \quad true \rightarrow wp(n := 0; x := 0, x = n \times n)$$

$$\begin{aligned} wp(n := 0; x := 0, x = n \times n) &= wp(n := 0, wp(x := 0, x = n \times n)) \\ &= wp(n := 0, (x = n \times n)[x \mapsto 0]) \\ &= wp(n := 0, 0 = n \times n) \\ &= (0 = n \times n)[n \mapsto 0] \\ &= (0 = 0 \times 0) \\ &= (0 = 0) = true \end{aligned}$$

则 $true \rightarrow wp(n := 0; x := 0, x = n \times n)$ 是有效式

综合①②③：所有验证条件均为有效式
故程序属性正确性得到验证

Problem 2 基本路径

路径 1 (从过程入口至 $v := \text{Proc_M}(y)$ 之前)

```
{y > 0}  
assume y > 10;  
{y > 0}
```

路径 2 (从过程入口至 $\text{assert}(v \geq 0)$ 之前)

```
{y > 0}  
assume y > 10;  
assume  $v_1 = 0$ ;  
 $v := v_1$ ;  
{v > 0}
```

路径 3 (从过程入口至 $\text{return } v$ 之前)

```
{y > 0}  
assume y > 10;  
assume  $v_1 = 0$ ;  
 $v := v_1$ ;
```

```
rV := v;  
{rV > 0}
```

路径 4 (从过程入口至 $\text{while}(y > 0)$ 之前)

```
{y > 0}  
assume  $y \leq 10$ ;  
{y > 0}
```

路径 5 (从外层循环开始跳到 return 0)

```
{ y ≥ 0 }  
assume y ≤ 0;  
rv := 0;  
{ rv ≥ 0 }
```

路径 6 (从外层循环开始至内层循环)

```
{ y ≥ 0 }  
assume y > 0;  
t := y;  
{ t ≥ 0 ∧ y ≥ t }
```

路径 7 (从内层循环开始跳到外层循环)

```
{ t ≥ 0 ∧ y ≥ t }  
assume t ≤ 0;  
y := y - 1;  
{ y ≥ 0 }
```

路径 8 (从内层循环开始继续循环)

```
{ t ≥ 0 ∧ y ≥ t }  
assume t > 0;  
t := t - 1;  
{ t ≥ 0 ∧ y ≥ t }
```