概统　第四周作业

习题2.1

3、(1)

$P(X=1) = \frac{7}{10}$

$P(X=2) = \frac{3}{10} \times \frac{7}{9} = \frac{7}{30}$

$P(X=3) = \frac{3}{10} \times \frac{2}{9} \times \frac{7}{8} = \frac{7}{120}$

$P(X=4) = \frac{3}{10} \times \frac{2}{9} \times \frac{1}{8} = \frac{1}{120}$

X分布列:

X	1	2	3	4
P	$\frac{7}{10}$	$\frac{7}{30}$	$\frac{7}{120}$	$\frac{1}{120}$

(2)

$P(X=1) = \frac{7}{10}$

$P(X=2) = \frac{3}{10} \times \frac{8}{10} = \frac{6}{25}$

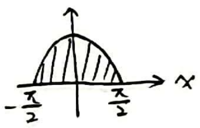
$P(X=3) = \frac{3}{10} \times \frac{2}{10} \times \frac{9}{10} = \frac{27}{500}$

$P(X=4) = \frac{3}{10} \times \frac{2}{10} \times \frac{1}{10} = \frac{3}{500}$

X分布列

X	1	2	3	4
P	$\frac{7}{10}$	$\frac{6}{25}$	$\frac{27}{500}$	$\frac{3}{500}$

14、(1)



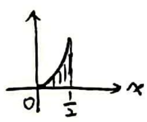
$$\int_{-\infty}^{+\infty} p(x)\,dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} A\cos x\,dx = A\sin x\Big|_{x=-\frac{\pi}{2}}^{\frac{\pi}{2}} = 2A = 1$$

$$\Rightarrow A = \frac{1}{2}$$

(2) $P(0 < X < \frac{\pi}{4}) = \int_0^{\frac{\pi}{4}} p(x)dx = \int_0^{\frac{\pi}{4}} \frac{1}{2}\cos x\,dx = \frac{1}{2}\sin x\Big|_{x=0}^{\frac{\pi}{4}} = \frac{\sqrt{2}}{4}$

16、

(1)



$$\int_{-\infty}^{+\infty} p(x)\,dx = \int_0^{\frac{1}{2}} cx^2 + x = \frac{1}{3}cx^3 + \frac{1}{2}x^2\Big|_{x=0}^{\frac{1}{2}} = \frac{1}{24}c + \frac{1}{8} = 1$$

$$\Rightarrow c = 21$$

(2)

$X \le 0$ 时　$F(x) = 0$

$0 < X \le \frac{1}{2}$ 时　$F(x) = \int_{-\infty}^{x} p(t)dt = \int_0^x 21t^2 + t\,dt = 7x^3 + \frac{1}{2}x^2$

$X > \frac{1}{2}$ 时　$F(x) = 1$

综上: $F(x) = \begin{cases} 0, & x \le 0 \\ 7x^3 + \frac{1}{2}x^2, & 0 < x \le \frac{1}{2} \\ 1, & x > \frac{1}{2} \end{cases}$

(3)

$P(X \le \frac{1}{3}) = F(\frac{1}{3}) = \frac{7}{27} + \frac{1}{18} = \frac{17}{54}$

(4)

$P(X \ge \frac{1}{6}) = 1 - P(X < \frac{1}{6}) = 1 - F(\frac{1}{6}) = 1 - (\frac{7}{216} + \frac{1}{72}) = \frac{216-7-3}{216} = \frac{103}{108}$

18. $P(A) = P(X > a) = \int_a^{+\infty} p(x)dx = \int_a^{+\infty} \frac{3}{8}x^2 = \frac{1}{8}x^3\Big|_a^2 = 1 - \frac{1}{8}a^3$

考虑到X、Y同分布,故　$P(B) = P(Y > a) = P(X > a) = 1 - \frac{1}{8}a^3$

由　$P(A \cup B) = P(A) + P(B) - P(AB) = P(A) + P(B) - P(A)P(B)$

$= 2P(A) - P(A)^2 = \frac{3}{4}$　得

$P(A) = \frac{1}{2}$　$\Rightarrow 1 - \frac{1}{8}a^3 = \frac{1}{2}$　$\Rightarrow a = \sqrt[3]{4}$.

19. (1) $F(-a)=\int_{-\infty}^{-a}p(x)dx=\int_{a}^{+\infty}p(x)dx=\int_{-\infty}^{+\infty}p(x)dx-\int_{-\infty}^{a}p(x)dx=1-F(a)$ ①

而 $\int_{-\infty}^{+\infty}p(x)dx=\int_{-\infty}^{0}p(x)dx+\int_{0}^{+\infty}p(x)dx=2\int_{0}^{+\infty}p(x)dx=1$ 则 $\int_{0}^{+\infty}p(x)dx=0.5$

因此 $F(-a)=\int_{a}^{+\infty}p(x)dx=\int_{0}^{+\infty}p(x)dx-\int_{0}^{a}p(x)dx=0.5-\int_{0}^{a}p(x)dx$ ②

结合①② $F(-a)=1-F(a)=0.5-\int_{0}^{a}p(x)dx$ ③

(2) $P(|X|<a)=P(-a<X<a)=\int_{-a}^{+a}p(x)dx=2\int_{0}^{a}p(x)dx$

(由③式) $=2(0.5-1+F(a))=2F(a)-1$

(3) $P(|X|>a)=P(X<-a\cup X>a)=P(X<-a)+P(X>a)$

$=F(-a)+\int_{a}^{+\infty}p(x)dx$

$=F(-a)+1-F(a)$ (由①式)

$=1-F(a)+1-F(a)$ (由①式)

$=2[1-F(a)]$

习题2.2

6、设X是取出的不合格品只数

$P(X=0)=\frac{8}{10}=\frac{4}{5}$

$P(X=1)=\frac{2}{10}\times\frac{8}{9}=\frac{8}{45}$

$P(X=2)=\frac{2}{10}\times\frac{1}{9}=\frac{1}{45}$

则 $EX=0\times\frac{4}{5}+1\times\frac{8}{45}+2\times\frac{1}{45}=\frac{2}{9}$

7、设X为每批产品要查的件数

$P(X=i)=(1-p)^{i-1}p\quad(1\le i<a)$

$P(X=a)=(1-p)^{a}+(1-p)^{a-1}p=(1-p)^{a-1}$

则 $EX=\sum_{i=1}^{a-1}i(1-p)^{i-1}p+a(1-p)^{a-1}$

求 $\sum_{i=1}^{a-1}i(1-p)^{i-1}=S$

$S=(1-p)^{0}+2(1-p)^{1}+3(1-p)^{2}+\cdots+(a-1)(1-p)^{a-2}$

$(1-p)S=(1-p)^{1}+2(1-p)^{2}+3(1-p)^{3}+\cdots+(a-1)(1-p)^{a-1}$

$+pS=(1-p)^{0}+(1-p)^{1}+\cdots+(1-p)^{a-2}-(a-1)(1-p)^{a-1}$

$=\frac{1(1-(1-p)^{a-1})}{p}-(a-1)(1-p)^{a-1}$

$S=\frac{1}{p}\left[\frac{1-(1-p)^{a-1}}{p}-(a-1)(1-p)^{a-1}\right]$

$EX=\frac{1-(1-p)^{a-1}}{p}-(a-1)(1-p)^{a-1}+a(1-p)^{a-1}$

$=\frac{1-(1-p)^{a-1}+p(1-p)^{a-1}}{p}=\frac{1-(1-p)^{a}}{p}$

14. 在 $x<0, 0\le x<1, x\ge 1$ 三段中 $p=F'$ 故

$$p(x)=\begin{cases}\frac{e^x}{2}, & x<0\\ 0, & 0\le x<1\\ \frac{1}{4}e^{-\frac{1}{2}(x-1)}, & x>1\end{cases}$$

则 $EX=\int_{-\infty}^{+\infty} x\,p(x)dx=\int_{-\infty}^{0} x\frac{e^x}{2}dx+\int_1^{+\infty}\frac{1}{4}xe^{-\frac{1}{2}(x-1)}dx$

$=\left[x\frac{e^x}{2}\Big|_{-\infty}^{0}-\int_{-\infty}^{0}\frac{e^x}{2}dx\right]+\left[-\frac{1}{2}xe^{-\frac{1}{2}(x-1)}\Big|_1^{+\infty}+\int_1^{+\infty}\frac{1}{2}e^{-\frac{1}{2}(x-1)}dx\right]$

$=\left[0-\frac{e^x}{2}\Big|_{-\infty}^{0}\right]+\left[\frac{1}{2}-e^{-\frac{1}{2}(x-1)}\Big|_1^{+\infty}\right]$

$=0-\frac{1}{2}+0+\frac{1}{2}+1=1$

17. $P(X\ge\frac{\pi}{3})=1-P(X<\frac{\pi}{3})=1-\int_{-\infty}^{\frac{\pi}{3}}p(x)dx=1-\int_0^{\frac{\pi}{3}}\frac{1}{2}\cos\frac{x}{2}dx=1-\sin\frac{x}{2}\Big|_0^{\frac{\pi}{3}}=\frac{1}{2}$

则 $P(Y=0)=(\frac{1}{2})^4=\frac{1}{16}$

$P(Y=1)=C_4^1(\frac{1}{2})^4=\frac{4}{16}=\frac{1}{4}$

$P(Y=2)=C_4^2(\frac{1}{2})^4=\frac{6}{16}=\frac{3}{8}$

$P(Y=3)=C_4^3(\frac{1}{2})^4=\frac{4}{16}=\frac{1}{4}$

$P(Y=4)=(\frac{1}{2})^4=\frac{1}{16}$

则 $E(Y^2)=\sum_{i=0}^{4} i^2P(Y=i)=0^2\times\frac{1}{16}+1^2\times\frac{1}{4}+2^2\times\frac{3}{8}+3^2\times\frac{1}{4}+4^2\times\frac{1}{16}$

$=0+\frac{1}{4}+\frac{3}{2}+\frac{9}{4}+1=5$

18. $E(\frac{1}{X^2})=\int_{-\infty}^{+\infty}\frac{1}{x^2}p(x)dx=\int_0^2\frac{1}{x^2}\frac{3}{8}x^2dx=\frac{3}{8}\times 2=\frac{3}{4}$