1、

(1)
$$S \underset{rm}{\overset{(1)}{\Longrightarrow}} (L) \underset{rm}{\overset{(2)}{\Longrightarrow}} (L,S) \underset{rm}{\overset{(2)}{\Longrightarrow}} (L,(L)) \underset{rm}{\overset{(2)}{\Longrightarrow}} (L,(L,S)) \underset{rm}{\overset{(1)}{\Longrightarrow}} (L,(L,a))$$
$$\underset{rm}{\overset{(2)}{\Longrightarrow}} (L,(S,a)) \underset{rm}{\overset{(1)}{\Longrightarrow}} (L,(a,a)) \underset{rm}{\overset{(2)}{\Longrightarrow}} (S,(a,a)) \underset{rm}{\overset{(1)}{\Longrightarrow}} (a,(a,a))$$

(2)
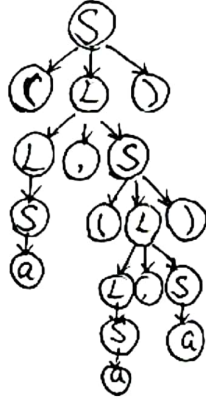
(3) 消除 ε 产生式   A 可空符号，消除 A→ε

$$S \to (L)|a \quad L \to L,S|S \quad A \to ab|aAb$$
$$B \to bB|b \quad C \to cC|D \quad D \to d$$

消除单-产生式   除自身与自身构成微单-偶对
                还有以下单-偶对： (L,S),(C,D)
$$S \to (L)|a \quad L \to L,S|(L)|a \quad A \to ab|aAb$$
$$B \to bB|b \quad C \to cC|d \quad D \to d$$

消除无用符号
    1) 非产生符： 无非产生符
    2) 非可达符： A、B、C、D 均为非可达符
            消去后得
                $$S \to (L)|a \ ; \ L \to L,S|(L)|a$$

2. (1) 对任何正整数 m, 设 w 满足 $w \in L$ 且 $|w| \geq m$.

取 $w = a^m b^m$, w 可写成 $w = xyz$ 且 $|xy| \leq m$, $|y| \geq 1$

因此 $y = a^k$ 且 $k \geq 1$

$$xyz = \underbrace{\overbrace{a \cdots a}^{x} \overbrace{a \cdots a}^{y}}_{m \text{个} a} \underbrace{a \cdots a \, b \cdots b}_{m \text{个} b}$$

则 $xy^2z = a^{m+k}b^m \notin L$

因此由泵引理, L 不是正则语言

(2) $S \rightarrow S_1 | S_2$

$S_1 \rightarrow CaS_1bS_1 | CbS_1aS_1 | C \qquad C \rightarrow cC | \varepsilon$

$S_2 \rightarrow BaS_2aS_2cS_2 | BaS_2cS_2aS_2 | BcS_2aS_2aS_2 | B \qquad B \rightarrow bB | \varepsilon$

则 $G = (V = \{S, S_1, S_2, C, B\}, T = \{a,b,c\}, S, P)$ 即为所求

(3)

构造一个空栈型 PDA

$\delta(q, \varepsilon, S) = \{(q, S_1), (q, S_2)\}$

$\delta(q, \varepsilon, S_1) = \{(q, CaS_1bS_1), (q, CbS_1aS_1), (q, C)\}$

$\delta(q, \varepsilon, S_2) = \{(q, BaS_2aS_2cS_2), (q, BaS_2cS_2aS_2), (q, BcS_2aS_2aS_2), (q, B)\}$

$\delta(q, \varepsilon, C) = \{(q, cC), (q, \varepsilon)\} \qquad \delta(q, a, a) = \{(q, \varepsilon)\}$

$\delta(q, \varepsilon, B) = \{(q, bB), (q, \varepsilon)\} \qquad \delta(q, b, b) = \{(q, \varepsilon)\} \quad \delta(q, c, c) = \{(q, \varepsilon)\}$

则 $M = (\{q\}, T, V \cup T, \delta, q, S)$ 即为所求

（转化为终态型 P, 需加入

$\delta(p, \varepsilon, S) = \{(q, SX)\}$

$\delta(q, \varepsilon, X) = \{(f, \varepsilon)\}$

则 $M' = (\{p, q, f\}, T, V \cup T \cup \{X\}, \delta, p, S, \{f\})$ 即为所求 ）