概统 第十四次作业

习题 5.4

2. $u=\frac{\sqrt{n}(\bar{x}-\mu)}{\sigma}=\frac{\sqrt{n}(\bar{x}-\mu)}{4}=\frac{\sqrt{n}}{4}(\bar{x}-\mu)\sim N(0,1)$

$P(-u_{0.975}\leq u\leq u_{0.975})=P(-u_{0.975}\leq\frac{\sqrt{n}}{4}(\bar{x}-\mu)\leq u_{0.975})$

$=P(\bar{x}-\frac{4}{\sqrt{n}}u_{0.975}\leq\mu\leq\bar{x}+\frac{4}{\sqrt{n}}u_{0.975})=0.95$

欲使 $P(|\bar{x}-\mu|\leq 1)=P(\bar{x}-1\leq\mu\leq\bar{x}+1)\geq 0.95$

则 $\{\bar{x}-1\leq\mu\leq\bar{x}+1\}\supseteq\{\bar{x}-\frac{4}{\sqrt{n}}u_{0.975}\leq\mu\leq\bar{x}+\frac{4}{\sqrt{n}}u_{0.975}\}$,

$\Rightarrow\quad 1\geq\frac{4}{\sqrt{n}}u_{0.975}$

$\Rightarrow\quad n\geq 16u_{0.975}^2\approx 16\times 1.96^2\approx 61.5$.

则 n 至少 62.

3. $\bar{x}-\bar{y}\sim N(0,4(\frac{1}{15}+\frac{1}{20}))=N(0,\frac{7}{15})$

$u=\frac{\bar{x}-\bar{y}}{\sqrt{\frac{7}{15}}}\sim N(0,1)$.

$\therefore P(|\bar{x}-\bar{y}|>0.2)=P(|\frac{\bar{x}-\bar{y}}{\sqrt{\frac{7}{15}}}|>\frac{0.2}{\sqrt{\frac{7}{15}}})=P(|u|>\frac{0.2}{\sqrt{\frac{7}{15}}})=2-2\phi(\frac{0.2}{\sqrt{\frac{7}{15}}})$

$=0.7718$

4. $\frac{x_i-\mu}{\sigma}\sim N(0,1)$

$\Rightarrow\sum_{i=1}^{20}(\frac{x_i-\mu}{\sigma})^2\sim\chi^2(20)$.

则 $P(10\sigma^2\leq\sum_{i=1}^{20}(x_i-\mu)^2\leq 30\sigma^2)=P(10\leq\sum(\frac{x_i-\mu}{\sigma})^2\leq 30)$

设 $\chi^2(x)$ 分布函数为 $F(x)$

则原式 $=F(30)-F(10)\approx 0.89$

9. $x_1+x_2\sim N(0,2\sigma^2)$ $\quad Y_1=(\frac{x_1+x_2}{\sqrt{2}\sigma})^2\sim\chi^2(1)$

$x_1-x_2\sim N(0,2\sigma^2)$ $\quad Y_2=(\frac{x_1-x_2}{\sqrt{2}\sigma})^2\sim\chi^2(1)$

而 由 $\bar{x}$ 与 s^2 独立 $\quad(\bar{x}=\frac{x_1+x_2}{2},\ s^2=\frac{(x_1-x_2)^2}{2})$

$\Rightarrow Y_1=\frac{2\bar{x}^2}{\sigma^2}$ 与 $Y_2=\frac{s^2}{\sigma^2}$ 独立

因此 $F=\frac{Y_1/1}{Y_2/1}=\frac{Y_1}{Y_2}=(\frac{x_1+x_2}{x_1-x_2})^2\sim F(1,1)$

即 $Y\sim F(1,1)$.

习题 6.6

2. $\frac{\sqrt{n}(\bar{x}-\mu)}{\sigma} \sim N(0,1)$.

置信区间 $\bar{x} \pm \frac{\sigma}{\sqrt{n}} u_{0.975}$ $\Rightarrow$ 置信区间长度 $k \geq \frac{2\sigma}{\sqrt{n}} u_{0.975}$

$\Rightarrow n \geq \frac{4\sigma^2}{k^2} u_{0.975}^2 = 15.37 \frac{\sigma^2}{k^2}$

5. (1) $\bar{x} = 457.5$ $s = \sqrt{\frac{1}{n-1}\sum(x_i - \bar{x})^2} = 35.22$

$t = \frac{\sqrt{n}(\bar{x}-\mu)}{s} = \frac{\sqrt{10}(457.5-\mu)}{35.22} \sim t(9)$ 则置信区间 $457.5 \pm \frac{35.22}{\sqrt{10}} t_{0.975}(9)$.

即 $[432.30, 482.70]$

(2) $u = \frac{\sqrt{n}(\bar{x}-\mu)}{\sigma} = \frac{\sqrt{10}(457.5-\mu)}{30} \sim N(0,1)$ 则置信区间 $457.5 \pm \frac{30}{\sqrt{10}} u_{0.975}$

即 $[438.91, 476.09]$

(3) $\frac{(n-1)s^2}{\sigma^2} \sim \chi^2(n-1)$

$P(\chi^2_{0.025}(9) \leq \frac{(n-1)s^2}{\sigma^2} \leq \chi^2_{0.975}(9)) = 0.95$

$\Rightarrow P(\sqrt{\frac{n-1}{\chi^2_{0.975}(9)}}\, s \leq \sigma \leq \sqrt{\frac{n-1}{\chi^2_{0.025}(9)}}\, s) = 0.95$.

置信区间：$[24.223, 64.298]$

6. 设X是合格的指示变量 μ 即为不合格率

$\bar{x} = \frac{11}{80}$, $s^2 = 0.120$.

则 $\frac{\sqrt{n}(\bar{x}-\mu)}{s} = \frac{\sqrt{80}(\frac{11}{80}-\mu)}{0.120} \sim t(79)$

μ 置信区间为 $\frac{11}{80} \pm \frac{0.120}{\sqrt{80}} t_{0.95}(79)$

即为 $[0.1202, 0.1548]$

10. 设分别服从 $N(\mu_1, \sigma_1^2)$, $N(\mu_2, \sigma_2^2)$ 分布

(1) $\frac{(n-1)s_1^2}{\sigma_1^2} = \frac{9s_1^2}{\sigma_1^2} \sim \chi^2(9)$ 同理 $\frac{9s_2^2}{\sigma_2^2} \sim \chi^2(9)$.

则 $\frac{9s_1^2/(\sigma_1^2 \times 9)}{9s_2^2/(\sigma_2^2 \times 9)} = \frac{\sigma_2^2 s_1^2}{\sigma_1^2 s_2^2} \sim F(9,9)$

因此 $\frac{\sigma_1^2}{\sigma_2^2}$ 置信区间为 $[\frac{s_1^2/s_2^2}{f_{0.975}(9,9)}, \frac{s_1^2/s_2^2}{f_{0.025}(9,9)}]$ 即 $[0.062, 1.008]$

(2) 采用许宝騄的大数近似

$u = \frac{\bar{x}-\bar{y}-(\mu_1-\mu_2)}{\sqrt{s_1^2/n_1 + s_2^2/n_2}} = \frac{\bar{x}-\bar{y}-0.02}{\sqrt{1/50}} \sim N(0,1)$

则 $\bar{x}-\bar{y}$ 的置信区间近似为 $0.02 \pm \frac{u_{0.975}}{\sqrt{50}}$.

即为 $[-0.2572, 0.2972]$