Homework 15 for GP1

1. KK 12.4 （12.12）
2. KK 12.6

   12.6 A rod of proper length l_0 oriented parallel to the x axis moves with speed u along the x axis in S. What is the length measured by an observer in S'?

   Ans. $l = l_0[(c^2 - v^2)(c^2 - u^2)]^{\frac{1}{2}}/(c^2 - uv)$

3. A train with proper length L, runs at velocity v_1 w.r.t. the ground, a person runs from back of the train to the front at v_2 w.r.t. the train. How much time this process takes as viewed by a person on the ground. (You may use LT or velocity transform)
4. ***(Optional**: I provide the answer but you do not need to answer this)* The same train above and this time the person walks with very low velocity u (u is relative to the train) from back to front. The person's clock is synchronized with that of train's at the back, and since the speed u is very small so that the time-dilation between the two clock's (the train and the person's) can be neglected, so when the person reaches the front, his clock will agree with the clock at the front of the train. Now considering observing the above events on the ground frame: for fixed time on the ground, the clocks on the train would appear reading differently. This is exactly like the figure I showed in notes, pg 430, but with ground clock's all read one value while the train's are not. You should be able to verify my next statement: the train's clock at the back will read Lv_1/c^2 more than the front (as viewed by the ground observer). So in view of the above statements, the ground observer will conclude that the time elapsed by the person's clock during the process (the person walks slowly from back to front of the train)must be Lv_1/c^2 less than that by the front clock. Please prove that the ground observer's conclusion is correct.
5. In my notes I derived formula of gamma factor in 1-D motion case, i.e.

   $\gamma_{u'} = \gamma_u \gamma_v (1 - \beta_u \beta_v)$ or $\gamma_u = \gamma_{u'} \gamma_v (1 + \beta_{u'} \beta_v)$. Now extend this to 2-D motion case, use transformation between velocities to prove that:

   $$\gamma_u = \gamma_{u'} \gamma_v (1 + \beta_{u'_x} \beta_v)$$

   Here, v is the velocity between frames and is along x direction. In frame S', the particle is moving with velocity (u'_x, u'_y), and $\gamma_u \equiv (1 - u^2/c^2)^{-1/2}$, where $u^2 = u_x^2 + u_y^2$
6. **(Optional)** Show that with the relativity momentum formula, if the momentum is conserved in the frame of ground then it is conserved in the frame travelling with A, in the example of elastic collision at the beginning of my notes section 13.1.
7. What is the velocity of the particle with rest mass m if its kinetic energy equals to the rest energy? What is the velocity if the kinetic energy is n times that of rest energy?
8. KK 13.1 （13.1）
9. KK 13.3 （13.3）
10. KK 13.4 （13.4）
11. KK 13.5 （13.5）