Homework 16 for GP1

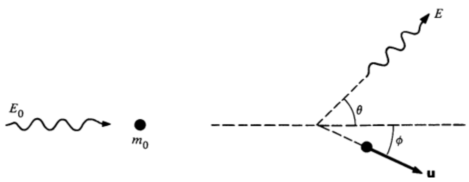
1. KK 13.9 （13.10 of 2nd edition）
2. KK 13.11 （13.12 in 2nd , but with a little difference, so I copy the 13.11 of the original）

   13.11 A photon of energy E_0 collides with a free particle of mass m_0 at rest. If the scattered photon flies off at angle θ, what is the scattering angle of the particle, ϕ?

   Ans. $\cot\phi = (1 + E_0/m_0c^2)\tan(\theta/2)$

3. Compton Scattering: The initial photon with wavelength λ_0 collides with a stationary free electron m_0, show that the scattered photon will have a wavelength shift (Compton effect):

   $\lambda = \lambda_0 + \frac{h}{m_0 c}(1-\cos\theta)$ (the KK has solution and I may also derive it in lecture, but you should try yourself anyway)



4. A particle of mass m and energy E collide with an identical stationary particle. What is the threshold energy (the minimum value of E) for a final state that containing N particles with mass m?
5. For a photon collide with a particle m head on and create one final new particle M. If the total energy of the system is E , how should it divided between the photon and mass m so that the final particle has largest M?

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6. To create Higgs boson (whose rest energy is over 100GeV), we collide a proton with an antiproton (their rest energy is 1 GeV).How much energy is required to create Higgs if:
   (a) A moving proton to collide with a stationary antiproton.
   (b) The proton and antiproton have equal and opposite momentum (that is the energy needed at least in the LHC (Large Hardron Collider)).
7. The 4-vector of frequency-wave vector: Considering a general plane wave form of $\sin(\omega t - \vec{k}\cdot\vec{r}) = \sin(\omega t - k_x x - k_y y - k_z z)$, the phase part should be invariant upon LT, otherwise the peak or valley of the wave observed in one frame would not be in another. This means $(\omega / c, \vec{k})\cdot(ct, \vec{r})$, the scalar product in 4-vector form is invariant upon LT, so we can construct a 4-vector $(\omega / c, \vec{k})$:
   (a) Using the transform property of 4-vector, prove the Doppler effect for a **light** wave whose k is in x-y plane: $\vec{k} = |k|(\cos\theta, \sin\theta, 0)$, find relation between ω', ω.
   (b) Prove that for the **light** wave, the E/ω is a constant independent of frames. (just forget about any quantum relations you know and only use special relativity to prove

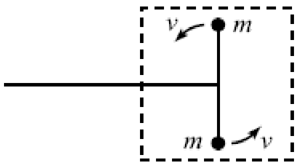
this)

8. For a number of material particles (rest mass m_i>0), each has (E_i,P_i) is S frame:
   (a) Prove that we can find one frame in which the total momentum P=0. (Hint: 1) You can prove that the energy-momentum 4-vector is time-like for one material particle, and for two particles, the total-energy-momentum can also be proved time-like: so for any number particles the total energy-momentum is also time-like. 2) Minkowski diagram can be used to easily show that you can find a frame in which total momentum is zero)
   (b) Prove the zero-momentum frame moves relative to S by: $\beta = \sum \vec{p}_i c \big/ \sum E_i$
   (c) Use the conclusion in this problem to show that it is impossible for a single photon to create a positron + electron pair.
   (d) If someone claims that he could create two identical particles each with mass m out of a single particle whose M<2m, because he can make arbitrary large energy by accelerating the M, is the claim correct or wrong and why?
9. KK 14.1 （14.1）
10. KK 14.3 （14.3）
11. KK 14.4 （14.4）
12. KK 14.6 (14.6)

The next four problems are about force, acceleration and equation of motion in SR, which is the material in chap. 14 of my note, and not covered in the lecture, so will not be required in the exam. Treat them as optional for your own interest.

13. For a particle with charge q moves in an uniform magnetic field B, and the initial velocity is perpendicular to the B, the force is Lorentz force=q|v||B| perpendicular to the motion.

    Show that the motion is a circular motion with radius: $r = |P|/q|B|$ even in the relativistic domain, where |P| is the value of momentum.
14. A particle m is moving initially with momentum (3-momentum)P_0 along x direction, a constant force with magnitude F_0 is applied along x direction. Find the velocity of the m at later time, and also the trajectory x(t). (You can leave any integral as it is)
15. Considering a “black box” (the black box itself is a massless “veil of ignorance”)inside which a dumbbell like object (one diatomic molecule) is rotating as shown in the figure:



   (a) What is the rest mass M of the black box as viewed by someone knowing nothing inside?
   (b) Suppose the black box is initially at stationary, and you apply a force on the horizontal stick (the stick can be treated as massless), convince that by using F=dP/dt=Ma (which is correct when the black box at low velocity), the inertial mass M is indeed the one in a).
16. Prove that from the relation (14-18) in the notes, the dynamic relation between 4-force and 4-acceleration, you can get the relation as (14-11). (My notes on pg 537 (all-English version) mistaken (14-18) with (14-17))

This is a long homework because it is the last one for this course and you **do not** need to hand them in.