概统 第六次作业

习题2.5

3. $x^2+Kx+1=0$ 有实根 $\Leftrightarrow \Delta=K^2-4\geq 0 \Leftrightarrow K\in(-\infty,-2]\cup[2,+\infty)$

考虑到 $\int_{-\infty}^{+\infty} p(x)dx = \int_1^6 p\,dx = 5p = 1 \Rightarrow p(x)=\begin{cases}\frac{1}{5}, & 1<x<6 \\ 0, & else\end{cases}$

设A为"$x^2+Kx+1=0$有实根"

$P(A) = \int_2^6 p(x)dx = \int_2^6 \frac{1}{5}dx = \frac{4}{5} = 0.8$

11. 设A为一次内他 等到服务

$P(A) = \int_0^{10} p(x)dx = \int_0^{10}\frac{1}{5}e^{-\frac{x}{5}}dx = -e^{-\frac{x}{5}}\Big|_0^{10} = 1-e^{-2}$

$P(Y<1)=P(Y=0)=P^5(\bar{A}) = (1-e^{-2})^5$

则 $P(Y\geq 1) = P(\overline{Y<1}) = 1-P(Y<1) = 1-(1-e^{-2})^5 \approx 0.5167$

19. 设学生分数为X

$P(X\geq 96) = 1-\phi(\frac{96-\mu}{\sigma}) = 1-\phi(\frac{24}{\sigma}) = 2.3\%$

经查表 $\phi(\frac{24}{\sigma})\approx 0.977$ 则 $\frac{24}{\sigma}\approx 2$.

$P(60\leq X\leq 84) = P(X\leq 84) - P(X\leq 60)$

$= \phi(\frac{84-\mu}{\sigma}) - \phi(\frac{60-\mu}{\sigma})$

$= \phi(\frac{12}{\sigma}) - \phi(-\frac{12}{\sigma}) = 2(\frac{12}{\sigma})-1$

$\approx 2\phi(1)-1 = 0.6826$

22. $X\sim N(\mu=20, \sigma^2=1600)$

$P(-30\leq x\leq 30) = \phi(\frac{30-\mu}{\sigma}) - \phi(\frac{-30-\mu}{\sigma})$

$= \phi(0.25)-\phi(-1.25) = -1+\phi(0.25)-\phi(1.25) = 0.4931$

设A为三次测量误差均大于30m

$P(A) = (1-P(-30\leq x\leq 30))^3 \approx 0.1302$

则 $p = P(\bar{A}) = 1-0.1302 = 0.8698$

23. 以min计, $\mu=240$, $\sigma=20$

(1) $P(X\geq 260) = 1-P(X\leq 260) = 1-\phi(\frac{260-\mu}{\sigma}) = 1-\phi(1) = 0.1587$

(2) $P(X\leq 250) = \phi(\frac{250-\mu}{\sigma}) = \phi(0.5) = 0.6915$

(3) $P(220\leq X\leq 260) = \phi(\frac{260-\mu}{\sigma}) - \phi(\frac{220-\mu}{\sigma})$

$= \phi(1)-\phi(-1) = 2\phi(1)-1 = 0.6826$

30. $E|X-\mu| = \int_{-\infty}^{+\infty}|x-\mu|\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx = 2\int_{\mu}^{+\infty}(x-\mu)\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx$

$= 2\sigma\int_0^{+\infty}\frac{x}{\sqrt{2\pi}}e^{-\frac{1}{2}x^2}dx = 2\sigma\int_0^{+\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}x^2}d(\frac{1}{2}x^2) = -2\sigma\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}x^2}\Big|_0^{+\infty} = 2\sigma\frac{1}{\sqrt{2\pi}} = \sigma\sqrt{\frac{2}{\pi}}$

习题2.6

1.

Y	0	1	4	9
P	$\frac{1}{5}$	$\frac{1}{6}+\frac{1}{15}$	$\frac{1}{5}$	$\frac{11}{30}$

$\Rightarrow$

Y	0	1	4	9
P	$\frac{1}{5}$	$\frac{7}{30}$	$\frac{1}{5}$	$\frac{11}{30}$

Z	0	1	2	3
P	$\frac{1}{5}$	$\frac{1}{6}+\frac{1}{15}$	$\frac{1}{5}$	$\frac{11}{30}$

$\Rightarrow$

Z	0	1	2	3
P	$\frac{1}{5}$	$\frac{7}{30}$	$\frac{1}{5}$	$\frac{11}{30}$

2. $F(X<0) = \int_{-\infty}^{0} p(x)\,dx = \int_{-\infty}^{0} \frac{2}{\pi}\frac{1}{e^x+e^{-x}}\,dx = \frac{1}{2}\int_{-\infty}^{+\infty}\frac{2}{\pi}\frac{1}{e^x+e^{-x}}\,dx = \frac{1}{2}\cdot 1 = \frac{1}{2}$

故 $P(g(X)=-1) = F(X<0) = \frac{1}{2}$

$P(g(X)=1) = 1-P(g(X)=-1) = 1-\frac{1}{2} = \frac{1}{2}$

$Y=g(X)$	-1	1
P	0.5	0.5

13.

$g(x)=e^x$ 单调连续，$h(y)=g^{-1}(y)=\ln y$

$p_Y(y) = p_X(h(y))|h'(y)| = \frac{1}{\sqrt{2\pi}\sigma y}e^{-\frac{(\ln y-\mu)^2}{2\sigma^2}} \quad (y>0)$

$$
\begin{aligned}
EY &= \int_{-\infty}^{+\infty} I_{y>0}\, p_Y(y)\, y\,dy = \int_0^{+\infty}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(\ln(y)-\mu)^2}{2\sigma^2}}dy \\
&= \int_0^{+\infty}\frac{y}{\sqrt{2\pi}\sigma}e^{-\frac{(\ln y-\mu)^2}{2\sigma^2}}d(\ln y) \\
&= \int_{-\infty}^{+\infty}\frac{e^x}{\sqrt{2\pi}\sigma}e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx \\
&= \int_{-\infty}^{+\infty}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-(\mu+\sigma^2))^2}{2\sigma^2}}dx \cdot e^{\frac{2\mu+\sigma^2}{2}} \\
&= \int_{-\infty}^{+\infty}\frac{1}{\sqrt{\pi}}e^{-x^2}dx\cdot e^{\mu+\frac{\sigma^2}{2}} = \frac{1}{\sqrt{\pi}}\times\sqrt{\pi}\cdot e^{\mu+\frac{\sigma^2}{2}} = e^{\mu+\frac{\sigma^2}{2}}
\end{aligned}
$$

$$
\begin{aligned}
EY^2 &= \int_{-\infty}^{+\infty} I_{y>0}\, p_Y(y)\, y^2\,dy = \int_0^{+\infty}\frac{y}{\sqrt{2\pi}\sigma}e^{-\frac{(\ln y-\mu)^2}{2\sigma^2}}dy \\
&= \int_0^{+\infty}\frac{y^2}{\sqrt{2\pi}\sigma}e^{-\frac{(\ln y-\mu)^2}{2\sigma^2}}d(\ln y) \\
&= \int_{-\infty}^{+\infty}\frac{e^{2x}}{\sqrt{2\pi}\sigma}e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx \\
&= \int_{-\infty}^{+\infty}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-(\mu+2\sigma^2))^2}{2\sigma^2}}dx\cdot e^{2\mu+2\sigma^2} \\
&= \int_{-\infty}^{+\infty}\frac{1}{\sqrt{\pi}}e^{-x^2}dx\cdot e^{2\mu+2\sigma^2} = e^{2\mu+2\sigma^2}
\end{aligned}
$$

则 $\mathrm{Var}\,Y = EY^2-(EY)^2 = e^{2\mu+2\sigma^2}-e^{2\mu+\sigma^2} = e^{2\mu+\sigma^2}(e^{\sigma^2}-1)$

即 $EY = e^{\mu+\frac{\sigma^2}{2}}$，$\mathrm{Var}\,Y = e^{2\mu+\sigma^2}(e^{\sigma^2}-1)$

习题2.7

3.(1) $\int_a^{x_{0.5}}\frac{1}{b-a}dx = \frac{x_{0.5}-a}{b-a} = 0.5 \Rightarrow x_{0.5} = \frac{a+b}{2}$

(2) $P(X\le x_{0.5}) = \phi\left(\frac{x_{0.5}-\mu}{\sigma}\right) = 0.5 \Rightarrow \frac{x_{0.5}-\mu}{\sigma} = \phi^{-1}(0.5) = 0 \Rightarrow x_{0.5} = \mu$

(3) $Y\sim Ln(\mu,\sigma^2)$ 令 $X=\ln Y$，则 $X\sim N(\mu,\sigma^2)$

$P(Y\le x_{0.5}) = P(X\le \ln x_{0.5}) = \phi\left(\frac{\ln x_{0.5}-\mu}{\sigma}\right) = 0.5 \Rightarrow \frac{\ln x_{0.5}-\mu}{\sigma} = \phi^{-1}(0.5) = 0$

$\Rightarrow \ln x_{0.5} = \mu$

$\Rightarrow x_{0.5} = e^{\mu}$