离散数学第七次作业

4. (4) $(\neg(\exists x)P(x)\vee(\forall y)Q(y))\rightarrow(\forall z)R(z)$

$=\neg(\neg(\exists x)P(x)\vee(\forall y)Q(y))\vee(\forall z)R(z)$

$=((\exists x)P(x)\wedge\neg(\forall y)Q(y))\vee(\forall z)R(z)$

$=((\exists x)P(x)\wedge(\exists y)\neg Q(y))\vee(\forall z)R(z)$

$=(\exists x)(P(x)\wedge(\exists y)\neg Q(y))\vee(\forall z)R(z)$

$=(\exists x)(\exists y)(P(x)\wedge\neg Q(y))\vee(\forall z)R(z)$

$=(\exists x)(\exists y)(\forall z)((P(x)\wedge\neg Q(y))\vee R(z))$

(8) $(\forall x)(P(x)\rightarrow Q(x))\rightarrow((\exists x)P(x)\rightarrow(\exists x)Q(x))$

$=\neg(\forall x)(\neg P(x)\vee Q(x))\vee(\neg(\exists x)P(x)\vee(\exists x)Q(x))$

$=(\exists x)\neg(\neg P(x)\vee Q(x))\vee((\forall x)\neg P(x)\vee(\exists x)Q(x))$

$=(\exists x)(P(x)\wedge\neg Q(x))\vee(\forall x)\neg P(x)\vee(\exists x)Q(x)$

$=(\exists x)((P(x)\wedge\neg Q(x))\vee Q(x))\vee(\forall x)\neg P(x)$

$=(\exists x)(P(x)\vee Q(x)\vee(\forall y)\neg P(y))$

$=(\exists x)(\forall y)(P(x)\vee Q(x)\vee\neg P(y))$

(9) $(\forall x)(P(x)\rightarrow(\exists y)Q(x,y))\vee(\forall z)R(z)$

$=(\forall x)(\neg P(x)\vee(\exists y)Q(x,y))\vee(\forall z)R(z)$

$=(\forall x)(\exists y)(\neg P(x)\vee Q(x,y))\vee(\forall z)R(z)$

$=(\forall x)(\exists y)(\forall z)(\neg P(x)\vee Q(x,y)\vee R(z))$

Skolem范式

$(\forall x)(\forall z)(\neg P(x)\vee Q(x,f(x))\vee R(z))$

(10) $(\exists y)(\forall x)(\forall z)(\exists u)(\forall v)P(x,y,z,u,v)$

Skolem范式

$(\forall x)(\forall z)(\forall v)P(x,a,z,f(x,z),v)$

5. (1) <u>归结法</u> $(\forall x)(P(x)\vee Q(x))$ 的子句集：$\{P(x)\vee Q(x)\}$

$(\forall x)(Q(x)\rightarrow\neg R(x))=(\forall x)(\neg Q(x)\vee\neg R(x))$ 的子句集 $\{\neg Q(x)\vee\neg R(x)\}$

$\neg(\exists x)(R(x)\rightarrow P(x))=(\forall x)\neg(\neg R(x)\vee P(x))$

$=(\forall x)(R(x)\wedge\neg P(x))$ 的子句集 $\{R(x),\neg P(x)\}$

<u>合公式子句集 $\{\neg Q(x)\vee\neg R(x), R(x), \neg P(x)\}$</u>

归结过程：

(1) $P(x)\vee Q(x)$

(2) $\neg Q(x)\vee\neg R(x)$

(3) $R(x)$

(4) $\neg P(x)$

(5) $Q(x)$ (1)(4)归结

(6) $\neg R(x)$ (2)(5)归结

(7) $\square$ (3)(7)归结

推理规则　前提　$(\forall x)(P(x)\vee Q(x))$，$(\forall x)(Q(x)\to\neg R(x))$

结论　$(\exists x)(R(x)\to P(x))$

(1) $(\forall x)(P(x)\vee Q(x))$　前提

(2) $P(c)\vee Q(c)$　全称量词消去

(3) $(\forall x)(Q(x)\to\neg R(x))$　前提

(4) $Q(c)\to\neg R(c)$　全称量词消去

(5) $\neg P(c)\to Q(c)$　(2)

(6) $\neg P(c)\to\neg R(c)$　(4)(5)三段论

(7) $R(c)\to P(c)$　(6)

(8) $(\exists x)(R(x)\to P(x))$　存在量词引入

(4) 设 $P(x)$：x是大学里的学生　$Q(x)$：x是本科生　$R(x)$：x是研究生

$S(x)$：x是高材生

只需证　$(\forall x)(P(x)\to(Q(x)\bar{\vee}R(x)))\wedge(\exists x)(P(x)\wedge S(x))$

$\wedge(\neg R(John)\wedge S(John))\Rightarrow P(John)\to Q(John)$

归结法　$(\forall x)(P(x)\to(Q(x)\bar{\vee}R(x)))=(\forall x)(\neg P(x)\vee(\neg Q(x)\wedge R(x))\vee(Q(x)\wedge\neg R(x)))$

$=(\forall x)((\neg P(x)\vee R(x)\vee Q(x))\wedge(\neg P(x)\vee\neg R(x)\vee\neg Q(x))$

的子句集：$\{\neg P(x)\vee R(x)\vee Q(x),\ \neg P(x)\vee\neg R(x)\vee\neg Q(x)\}$

$(\exists x)(P(x)\wedge S(x))$ 的子句集：$\{P(a),\ S(a)\}$

$(\neg R(John)\wedge S(John))$ 的子句集：$\{\neg R(John),\ S(John)\}$

$\neg(P(John)\to Q(John))=\neg(\neg P(John)\vee Q(John))=P(John)\wedge\neg Q(John)$

的子句集：$\{P(John),\ \neg Q(John)\}$

从而公式的子句集 $\{\neg P(x)\vee R(x)\vee Q(x),\ \neg P(x)\vee\neg R(x)\vee\neg Q(x),\ P(a),\ S(a),$

$\neg R(John),\ S(John),\ P(John),\ \neg Q(John)\}$

归结过程：(1) $\neg P(x)\vee R(x)\vee Q(x)$

(2) $\neg P(x)\vee\neg R(x)\vee\neg Q(x)$

(3) $S(a)$

(4) $\neg R(John)$

(5) $S(John)$

(6) $P(John)$

(7) $\neg Q(John)$

(8) $\neg P(John)\vee R(John)$　(1)(7)归结

(9) $\neg P(John)$　(4)(8)归结

(10) $\square$　(6)(9)归结

推理规则　前提　$(\forall x)(P(x)\to(Q(x)\bar{\vee}R(x)))$，$(\exists x)(P(x)\wedge S(x))$

$\neg R(John)\wedge S(John)$

结论　$P(John)\to Q(John)$

(1) $(\forall x)(P(x)\to(Q(x)\bar{\vee}R(x)))$　前提

(2) $P(John)\to(Q(John)\bar{\vee}R(John))$　全称量词消去

(3) $P(John)$　附加前提引入

(4) $Q(John)\bar{\vee}R(John)$　(2)(3)分离

(5) $\neg R(John)\wedge S(John)$　前提

(6) $\neg R(John)$　(5)

(7) $Q(John)$　(4)(6)

(8) $P(John)\to Q(John)$　条件证明规则