HW 14 for General Physics II

1. (Adapted from Griffiths P 5.1, for the general treatment of two-particle system, where electron in H atom belongs)

For the potential only depends on relative position between the two particles: V(r), $\vec{r} \equiv \vec{r}_1 - \vec{r}_2$, and the mass are m_1, m_2 respectively; $\vec{r}_1 = (x_1, y_1, z_1)$ for position of particle 1; and $\vec{r}_2 = (x_2, y_2, z_2)$ for particle 2. The S-equation in terms of $(x_1, y_1, z_1), (x_2, y_2, z_2)$ is:

$-\frac{\hbar^2}{2m_1}\nabla_1^2\psi - \frac{\hbar^2}{2m_2}\nabla_2^2\psi + V(r)\psi = E\psi$; where:

$\nabla_1^2 = \nabla_1 \cdot \nabla_1 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial y_1^2} + \frac{\partial^2}{\partial z_1^2}$ and $\nabla_2^2 = \nabla_2 \cdot \nabla_2 = \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial y_2^2} + \frac{\partial^2}{\partial z_2^2}$

In this case the S-equation can be separated into center of mass and reduced mass part, with CM defined: $\vec{R} \equiv \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$, and reduced mass: $\mu \equiv \frac{m_1 m_2}{m_1 + m_2}$.

(a) Show that $\vec{r}_1 = \vec{R} + (\mu/m_1)\vec{r}, \vec{r}_2 = \vec{R} - (\mu/m_2)\vec{r}$ and $\nabla_1 = (\mu/m_2)\nabla_R + \nabla_r, \nabla_2 = (\mu/m_1)\nabla_R - \nabla_r$

(b) Show that the time-independent S-equation then becomes:

$$-\frac{\hbar^2}{2(m_1 + m_2)}\nabla_R^2\psi - \frac{\hbar^2}{2\mu}\nabla_r^2\psi + V(r)\psi = E\psi$$

(c) Separate the variables, letting $\psi(\vec{R}, \vec{r}) = \psi_R(\vec{R})\psi_r(\vec{r})$. The S-equation will be separated into two parts. One is a free particle with mass M=m_1+m_2 and energy E_R; the other is particle with reduced mass in central field with energy E_r, with E_{total}=E_R+E_r. Like in classical mechanics, the M part is translation of the whole system and we seldom focus on it. The relative motion represented by $\psi_r(\vec{r})$ and E_r are what we concerned in such problems (as we did in H atom)

2. Griffiths P 4.10

Work out the **radial** wave functions R_{30}, R_{31} and R_{32} for Hydrogen atom, using the recursion formula and don't bother to normalize them.

3. Combined Griffiths P 4.13 and P 4.14 and more

(a) Find $<r>$ and $<r^2>$ for an electron in the ground state of hydrogen. Express the answer in terms of Bohr radius.

(b) What is the most probable value of r, in the ground state of hydrogen? Hint: First you must find the probability density that the electron would be found between r and r+dr.

c) For the ground state 1S orbit (same ψ_{100} as above),we are trying to calculate the radius of the sphere R, of which the electron in 1S orbit has 90% probability found inside this sphere. Express R in terms of Bohr radius. (This is the sphere I draw in my PPT of the 1S, and the size of it give us a rough idea to the distance between H atoms in ground state in order to have significant interaction, such as forming H2 molecule).

During the calculation you may need software (or graphical tool) to get the numerical value. I recommend https://www.wolframalpha.com/ ;you can do the numerical computation online there. (That is a famous website for numerical computation and really easy to use; and also you can access it in Mainland, at least for now)

d) For the 2S orbit (ψ_{200}, the 1st excited state), find the position of local extremes along the radial r; i.e. finding the local extremes of P(r), the probability density along r. (be warned P(r) is not same as $R_{nl}(r)^2$, and also you may need wolframalpha). Express the location of extremes in terms of Bohr radius. (you shall find 3 local extremes, corresponding to local max.; local mini. And another local max.)

4. Griffiths P 4.55. (modified, no addition of angular momentum part)

The electron in a hydrogen atom occupies the combined spin and position state:

$$R_{21}(\sqrt{1/3}Y_1^0\alpha_+ + \sqrt{2/3}Y_1^1\alpha_-)$$

(a) If you measure the orbital angular momentum squared L^2, what values might you get, and what is the probability of each?

(b) Same for the z component of orbital angular momentum L_z

(c) Same for the spin angular momentum S^2.

(d) Same for the z component of spin S_z.

(e) If you measure the position of the particle, what is the probability density for finding it at (r,θ,ϕ)

(f) If you measure both the S_z and the distance from the origin (these are commute operators and thus compatible observables), what is the probability density for finding the particle with spin up and at radius r?

5. Consider the hydrogen like carbon ion C^{5+}, calculate its "Bohr" radius and energy E_n, and what is its transition relation (energy difference) from n_1 state to n_2?

6. (Griffiths' 5.2, and you may use result from problem 1)

In view of P5.1 (problem 1 in our case), we can correct for the motion of the nucleus in hydrogen by simply replacing the electron mass with reduced mass. (m_e=0.51100MeV, m_p~m_n=938.27MeV)

(a) Find (to two significant digits) the percent error in the binding energy of hydrogen introduced by use of m instead of μ.

(b) Find the separation in wavelength between the Balmer lines (n=3 to n=2 transitions) for hydrogen and deuterium (proton+neutron for nucleus).

(c) Finding the binding energy of positronium (in which the proton is replaced by positron, same mass as electron but opposite charge)

(d) Suppose you want to confirm the existence of muonic hydrogen, in which the electron is replaced by muon (same –e charge, but 206.77 times heavier). What wavelength would you look for the Lyman transition (n=2 to n=1)?