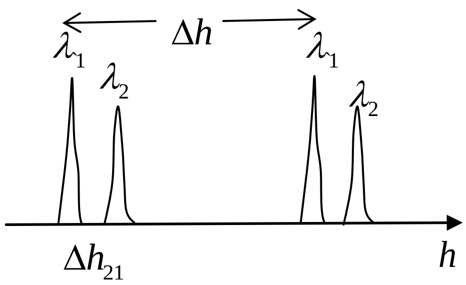
Homework1 of General Physics II

1. (Hecht, 9.40 with some variation)Given that the mirrors of a Fabry-Perot interferometer have amplitude reflection coefficient of *r*=0.8944, find:
   (a) The coefficient of finesse, F
   (b) The finesse
   (c) The contrast factor defined by: $C=(I_t/I_0)_{max}/(I_t/I_0)_{min}$
   (d) For 500 nm light, if the cavity length of the F-P is 1 cm, calculate the spectral FWHM (full width at half maximum) in wavelength and in frequency; and its free spectral range (in frequency)for this cavity; and its minimum resolvable wavelength difference around 500nm.
   (e) If I used the above F-P as an etalon, the input light is around 500nm with a frequency distribution width about 10GHz, please roughly estimate how much energy will pass through the etalon (Roughly estimate means you **do not** need to use Airy function, the exact solution of the output I, to do the calculation. You can not do this anyway because I do not give you the exact form of the frequency distribution of the input. Just estimate using the bandwidth)
2. Given a F-P type scanning spectrometer, the average length of cavity is h, as h varies (a small range on order of wavelength of light in use), different frequency (or wavelength) will appear in the output. However if the frequencies difference between the input light satisfy a certain relation (here I am referring to the frequency difference is much larger than the resolution limit), the F-P spectrometer cannot tell them apart, i.e. two different frequencies of light may appear at same h. 1) For the given F-P, if frequency υ_1 appears in output at certain h, What is the next higher frequency υ_2 appears at **same h**? (the answer would be no-surprisingly $\upsilon_2 - \upsilon_1$ equals one free spectral range, you need to prove this) 2) The result of 1) shows that the given F-P cannot tell the two frequency, but suppose you do want to tell them apart, what would you do?
3. Consider the interference of the Michelson interferometer as arising from two beams of equal energy flux density, so the interference pattern can be described by relation 4.1 (Pg 170, Zhao's book). $I(p)=2A^2[1+\cos\delta(P)]=4A^2\cos^2\frac{\delta(P)}{2}$. In analogy to what we have treated in the F-P case, derive the expression for the FWHM in phase domain, and what is its finesse?
4. This problem shows you how to use Fabry-Perot spectrometer (scanning h) to know the frequency differences of light:
   Consider a Fabry-Perot scanning spectrometer, with n=1, h=1.5cm; Finesse=100. The h can be increased by applying voltage to piezo material. All light input are at normal incident angle to the F-P mirror. For a reference light whose wavelength is known as $\lambda_1 = 600nm$, another light with unknown wavelength λ_2 (or frequency υ_2) which is very close to λ_1. As we

scan the F-P (change its h), on the oscilloscope (the horizontal axis is scanning time which in turn is proportional to the piezo-voltage, the voltage in turn is proportional to the length of F-P cavity, so we can say the horizontal is proportional to h; the vertical axis is intensity of light at the output) we observe the following picture:



Δh
λ_1
λ_2
λ_1
λ_2
Δh_{21}
h

The reason that you have multiple-sets is that they correspond to different interference order, say the 1st set corresponds to order m; the 2nd will be m+1 (if you scan h further, you will see more sets). By measuring the spacing of the peaks, we can determine the wavelength difference between λ_1, λ_2, and thus if we know λ_1, then λ_2 is known too. Here provides with parameters given above, and Δh_{21} is the spacing between peaks of λ_1, λ_2 within same set (same order of m); Δh is the spacing between the λ_1 peaks of adjacent order, and $\Delta h_{21} = 0.2\Delta h$, what is the wavelength of λ_2 and frequency υ_2?

5. (This problem is optional) In the discussion of multi-beam interference and Fabry-Perot, we assume the two reflecting surfaces are lossless, i.e. the total energy are either reflected or transmitted. In this case, the transmitted light can reach 100% I_0. However in real application for high finesse F-P, transmission seldom exceeds 20%, higher the finesse, lower the transmission. This is of cause due to the loss of light when interacting with the surface (such as absorption, scattering and diffraction loss etc). For a cavity formed by two parallel high reflecting surfaces, with $R=r^2=0.99$ (here R, r are the reflection for the ideal lossless case), but due to some reason (such as dirty mirror) a loss occurs. For each time the light hits the surface, 4% of its energy is lost. With such cavity, what is the maximum transmission of light, $(It/Io)_{max}$? (I give a hint at the end of document, you may choose ignore it)

   [Hint on problem 4: The important thing is what are the energy or the field distributions when light interacts with the surface each time. For the lossless surface, we have R+T=1, where $R=r^2$, $T=t^2$ if n is same on both sides of the thin reflecting film. (if n's are different, it will complicate the estimation a little but will not affect the basic idea). R, r, T, t are for lossless case. We can write the reflecting and transmitting field and work out the problem as in the class. For the surface where 4% energy is lost every time the light hits it, it is R"+T"+A=1. A is the loss (0.04 here), R" and T" are reflected and transmitted. We make an assumption that the loss is same for reflection and transmission, then $R''=a^2r^2$, $T''=a^2t^2$. You can find out the a and write out the reflected and transmitted field at each interaction between light and surface, the rest would be similar to the derivation I gave in the class; You may also read Hecht book where he discussed this issue in the F-P part, but I encourage you derive the result yourself]

6. A tungsten-filament lamp having a straight filament that is 0.1 mm in diameter is used as a source for a Young's interference experiment, the wavelength is ~590 nm. A double-slit aperture with 1mm between the slits has to be placed at least to what distance in order to observe the interference fringes? If we require the degree of coherence (the contrast) is more than 90%, what is the distance? Should the slits be oriented parallel to the lamp filament?

7. Using Michelson interferometer to observe an equal inclination fringes. The light source is a broad band thermal source, with the light centers about 500 nm, with a width of 100 nm. Using such light sources, how many interference fringes can you observe?

8. Given a Femto-second ($\Delta t = 10^{-15}$ s) laser pulse, (i.e. the duration of pulse in time, or temporal width) estimate its spectral width, i.e. the width in frequency domain. Given the central wavelength of the spectral distribution is 500 nm, what is the spectral width in wavelength (i.e. the approximate longest and shortest wavelengths such pulse contains)? Is it able to excite Sodium transition around 580 nm?