离散数学 第九周作业

13. (3) $A\cup C=\{0,1,2,3,6,7,8,9,12,15,18\}$

$B-A\cup C=\{4,5\}$

14. (1) $\cup A_1=\{3,4,\{3\},\{4\}\}$

(2) $\cap A_2=\{3\}$

15. $P(\phi)=\{\phi\}$

$PP(\phi)=P(\{\phi\})=\{\phi,\{\phi\}\}$

$PPP(\phi)=P(\{\phi,\{\phi\}\})=\{\phi,\{\phi\},\{\{\phi\}\},\{\phi,\{\phi\}\}\}$

$\cup A_1=\{\phi,\{\phi\},\{\{\phi\}\},\{\phi,\{\phi\}\}\}$

$\cap A_2=\{\phi\}$

16. (1) $P(A)=P(\{\{\phi\},\{\{\phi\}\}\})=\{\phi,\{\{\phi\}\},\{\{\{\phi\}\}\},\{\{\phi\},\{\{\phi\}\}\}\}$

$\cup P(A)=\{\{\phi\},\{\{\phi\}\}\}$

(2) $\cup A=\{\phi,\{\phi\}\}$

$P(\cup A)=P(\{\phi,\{\phi\}\})=\{\phi,\{\phi\},\{\{\phi\}\},\{\phi,\{\phi\}\}\}$

17. (5) 求证：$C\subseteq A\wedge C\subseteq B \Leftrightarrow C\subseteq A\cap B$

证明：$C\subseteq A\wedge C\subseteq B$

$\Leftrightarrow (\forall x)(x\in C\to x\in A)\wedge(\forall x)(x\in C\to x\in B)$

$\Leftrightarrow (\forall x)((x\in C\to x\in A)\wedge(x\in C\to x\in B))$

$\Leftrightarrow (\forall x)(x\in C\to x\in(A\cap B))$

$\Leftrightarrow C\subseteq A\cap B$

(6) 求证：$A\cap B=\phi\Leftrightarrow A\subseteq -B\Leftrightarrow B\subseteq -A$

证明：先证 $A\cap B=\phi\Leftrightarrow A\subseteq -B$：

$A\cap B=\phi$

$\Leftrightarrow \neg(\exists x)(x\in A\cap B)$

$\Leftrightarrow (\forall x)\neg(x\in A\cap B)$

$\Leftrightarrow (\forall x)\neg(x\in A\wedge x\in B)$

$\Leftrightarrow (\forall x)(\neg(x\in A)\vee\neg(x\in B))$

$\Leftrightarrow (\forall x)(\neg(x\in A)\vee x\in -B)$

$\Leftrightarrow (\forall x)(x\in A\to x\in -B)$

$\Leftrightarrow A\subseteq -B$

同理：$A\cap B=\phi\Leftrightarrow B\subseteq -A$

故 $A\cap B=\phi\Leftrightarrow A\subseteq -B\Leftrightarrow B\subseteq -A$ 得证

18. (3) $A=B$

证：$A=A\cup(A\cap B)=A\cup(A\cup B)=A\cup B=B\cup(A\cup B)=B\cup(A\cap B)=B$

(4) $B\subseteq A$

证：$\forall x$，$x\in B\Rightarrow x\in B\vee x\in A\Rightarrow x\in(B\cup A)$

$\Rightarrow x\in(A\cup(B-A))\Rightarrow x\in(A\cap(B\cup -B)\cup(B-A))$

$\Rightarrow x\in((A\cap B)\cup(A\cap -B)\cup(B-A))$

$\Rightarrow x\in((A\cap B)\cup(A\oplus B))$

$\Rightarrow x\in((A\cap B)\cup A)$

$\Rightarrow x\in A$

故 $B\subseteq A$

19.(3) 充要条件 $A \subseteq B \cup C$

证: $\forall x$ $x \in A \Rightarrow x \in A \cap A$

$\Rightarrow x \in (A \cap (B \cup -B)) \cap (A \cap (C \cup -C))$

$\Rightarrow x \in ((A-B) \cup (A \cap B)) \cap ((A-C) \cup (A \cap C))$

$\Rightarrow x \in ((A-B) \cap (A-C)) \cup ((A-B) \cap (A \cap C))$

$\cup ((A \cap B) \cap (A-C)) \cup ((A \cap B) \cap (A \cap C))$

$\Rightarrow x \in \phi \cup ((A-B) \cap A \cap C) \cup (A \cap B \cap (A-C))$

$\cup (A \cap B \cap A \cap C)$

$\Rightarrow x \in C \cup B \cup B$

$\Rightarrow x \in B \cup C$

(4) 充要条件 $A \cap B = A \cap C$

证: $A \cap B = A \cap C$

$\Leftrightarrow (\forall x)(x \in A \cap B \leftrightarrow x \in A \cap C)$

$\Leftrightarrow (\forall x)((\neg(x \in A \cap B) \vee (x \in A \cap C)) \wedge (\neg(x \in A \cap C) \vee (x \in A \cap B)))$

$\Leftrightarrow (\forall x)(x \in -(A \cap B) \cup (A \cap C) \wedge x \in -(A \cap C) \cup (A \cap B))$

$\Leftrightarrow (\forall x)(x \in -A \cup -B \cup (A \cap C) \wedge x \in -A \cup -C \cup (A \cap B))$

$\Leftrightarrow (\forall x)(x \in -A \cup -B \cup C \wedge x \in -A \cup -C \cup B)$

而 $(A-B) \oplus (A-C) = \phi$

$\Leftrightarrow \neg(\exists x)(x \in ((A-B)-(A-C)) \cup ((A-C)-(A-B)))$

$\Leftrightarrow (\forall x)(x \notin ((A \cap -B) \cap (-A \cup C)) \cup ((A \cap -C) \cap (-A \cup B)))$

$\Leftrightarrow (\forall x)(x \notin (A \cap -B \cap C) \cup (A \cap -C \cap B))$

$\Leftrightarrow (\forall x)(x \in -((A \cap -B \cap C) \cup (A \cap -C \cap B)))$

$\Leftrightarrow (\forall x)(x \in -(A \cap -B \cap C) \cap -(A \cap -C \cap B))$

$\Leftrightarrow (\forall x)(x \in (-A \cup B \cup -C) \cap (-A \cup C \cup -B))$

$\Leftrightarrow (\forall x)(x \in -A \cup B \cup C \wedge x \in -A \cup -C \cup B)$

故 $A \cap B = A \cap C \Leftrightarrow (A-B) \oplus (A-C) = \phi$

26. (1) 充要条件 $A = \phi \vee B = \phi$

充分性 $A = \phi \Rightarrow A \times B = \phi$

$B = \phi \Rightarrow A \times B = \phi$

故 $A = \phi \vee B = \phi \Rightarrow A \times B = \phi$

必要性 $|A \times B| = |\phi| = 0$

$\Rightarrow |A| \cdot |B| = 0$

$\Rightarrow |A| = 0 \vee |B| = 0$

$\Rightarrow A = \phi \vee B = \phi$

(2) $A = \phi$ 时 $A \times A = A = \phi$

$A \neq \phi$ 时 $|A \times A| = |A| \cdot |A| = |A| \Rightarrow |A| = 1$

而若 $A = \{a\}$ $A \times A = \langle a, a \rangle = \{\{a\}\} \neq A = \{a\}$.

故 $A \neq \phi$ 时 $A \times A \neq A$.

综上: $A = \phi$ 时 $A \times A = A$ 成立.

28. 设 A_i 为能被 $i \in \mathbb{Z}_+$ 整除的 1~250 之间的数的集合

$|A_2| = 125$ $|A_3| = 83$ $|A_5| = 50$

$|A_2 \cap A_3| = 41$ $|A_2 \cap A_5| = 25$ $|A_3 \cap A_5| = 16$

$|A_2 \cap A_3 \cap A_5| = 8$

$|A_2 \cup A_3 \cup A_5| = |A_2| + |A_3| + |A_5| - |A_2 \cap A_3| - |A_2 \cap A_5| - |A_3 \cap A_5| + |A_2 \cap A_3 \cap A_5|$

$= 125 + 83 + 50 - 41 - 25 - 16 + 8 = 184$

故 1 至 250 之间能被 2, 3 或 5 整除的数有 184 个