概率论与数理统计 第五次作业

习题2.3

2. 设X是取出合格品前已取出的不合格品件数

$P(X=0)=\frac{8}{10}=\frac{4}{5}$

$P(X=1)=\frac{2}{10}\times\frac{8}{9}=\frac{8}{45}$

$P(X=2)=\frac{2}{10}\times\frac{1}{9}=\frac{1}{45}$

则 $EX=\frac{4}{5}\times0+\frac{8}{45}\times1+\frac{1}{45}\times2=\frac{2}{9}$

$EX^2=\frac{4}{5}\times0^2+\frac{8}{45}\times1^2+\frac{1}{45}\times2^2=\frac{4}{15}$

$Var X=EX^2-(EX)^2=\frac{4}{15}-\frac{4}{81}=\frac{88}{405}\approx0.217$

5.

$$p(x)=F'(x)=\begin{cases}\frac{e^x}{2}, & x<0\\ 0, & 0\le x<1\\ \frac{1}{4}e^{-\frac{1}{2}(x-1)}, & x\ge1\end{cases}$$

$$EX=\int_{-\infty}^{+\infty}xp(x)dx=\int_{-\infty}^{0}x\frac{e^x}{2}dx+\int_{1}^{+\infty}x\frac{1}{4}e^{-\frac{1}{2}(x-1)}dx$$

$$=\left[x\frac{e^x}{2}\Big|_{-\infty}^{0}-\int_{-\infty}^{0}\frac{e^x}{2}dx\right]+\left[-\frac{1}{2}xe^{-\frac{1}{2}(x-1)}\Big|_{1}^{+\infty}+\frac{1}{2}\int_{1}^{+\infty}e^{-\frac{1}{2}(x-1)}dx\right]$$

$$=\left[-\frac{1}{2}+0\right]+\left[\frac{1}{2}+1\right]=1$$

$$EX^2=\int_{-\infty}^{+\infty}x^2p(x)dx=\int_{-\infty}^{0}x^2\frac{e^x}{2}dx+\int_{1}^{+\infty}x^2\frac{1}{4}e^{-\frac{1}{2}(x-1)}dx$$

$$=\left[x^2\frac{e^x}{2}\Big|_{-\infty}^{0}-2\int_{-\infty}^{0}x\frac{e^x}{2}dx\right]+\left[-\frac{1}{2}x^2e^{-\frac{1}{2}(x-1)}\Big|_{1}^{+\infty}+\int_{1}^{+\infty}xe^{-\frac{1}{2}(x-1)}dx\right]$$

$$=\left[0-2(-\frac{1}{2})\right]+\left[\frac{1}{2}+6\right]=\frac{15}{2}$$

$$Var X=EX^2-(EX)^2=\frac{15}{2}-1^2=\frac{13}{2}=6.5$$

13.
pf: 设X分布函数为$p(t)$

$$P(X\geq x)=\int_x^{+\infty}p(t)dt \leq \int_x^{+\infty}\frac{e^{at}}{e^{ax}}p(t)dt \leq \frac{1}{e^{ax}}\int_{-\infty}^{+\infty}e^{at}p(t)dt=\frac{E(e^{aX})}{e^{ax}} \quad \square$$

14. 设X是每升白细胞数，则$EX=7.3\times10^9$，A为X在$5.2\times10^9\sim9.4\times10^9$

$$P(|X-EX|\leq\varepsilon)\geq 1-\frac{Var X}{\varepsilon^2}$$

取$\varepsilon=2.1\times10^9$ 则

$$P(|X-EX|\leq\varepsilon)=P(A)\geq 1-\frac{Var X}{\varepsilon^2}=1-\frac{(0.7\times10^9)^2}{(2.1\times10^9)^2}=\frac{8}{9}$$

习题2.4

4. 设没来的人数为X

$$p=P(X=0)+P(X=1)=(0.8)^{52}+52\times(0.8)^{51}\times0.2=1.28\times10^{-4}$$

7. 设X为不合格品件数，A为拒收

(1) $P(\bar{A})=P(X=0)+P(X=1)=(0.98)^{40}+40\times(0.98)^{39}\times0.02\approx0.8095$

$P(A)=1-P(\bar{A})\approx0.1905$

(2) X分布可近似为$\lambda=0.02\times40=0.8$的泊松分布

$P(\bar{A})=P(X=0)+P(X=1)=\lambda^0e^{-0.8}+\lambda^1e^{-0.8}\approx0.8088$

$P(A)=1-P(\bar{A})\approx0.1912$

9. 设B_0、B_1…为超市来0，1…名顾客的事件，显然$\bigcup_{n=0}^{\infty}B_n=\Omega$且$B_k$构成$\Omega$的分割

且$P(B_k)=P(X=k)=\frac{\lambda^k}{k!}e^{-\lambda}$

设Y为购买商品的顾客数，则

$$P(Y=l)=\sum_{k=0}^{\infty}P(Y=l|B_k)P(B_k)=\sum_{k=l}^{\infty}P(Y=l|B_k)P(X=l)$$
$$=\sum_{k=l}^{\infty}\left[\binom{k}{l}p^l(1-p)^{k-l}\right]\frac{\lambda^k}{k!}e^{-\lambda}$$
$$=\sum_{k=l}^{\infty}\frac{1}{l!(k-l)!}p^l(1-p)^{k-l}\lambda^ke^{-\lambda}$$
$$=\frac{p^l\lambda^le^{-\lambda}}{l!}\sum_{k=l}^{\infty}\frac{(1-p)^{k-l}\lambda^{k-l}}{(k-l)!}$$
$$=\frac{1}{l!}p^l\lambda^le^{-\lambda}e^{\lambda(1-p)}=\frac{1}{l!}(p\lambda)^le^{-\lambda p}$$

即购物人数Y服从参数为λp的泊松分布

12. 设摸球次数为X，$X\sim G(\frac{m}{m+n})$

则 $EX=\frac{1}{p}=\frac{m+n}{m}$.

取出黑球的期望个数 $E(X-1)=EX-1=\frac{n}{m}$.