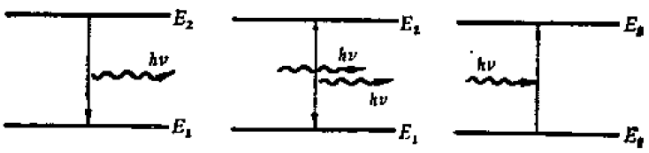
Homework for General Physics II, set7

(Materials covered can be found in Arthur Beiser's *Concept of Modern Physics* (pdf on internet), Chap.2, and Chap 4-1 to 4-5)

1. Problem 13.8, Hecht's
2. Problem 13.10, Hecht's（same as Zhao's book, problem 5,pg275， I include the derivation in my pdf Slide used in class）. Also estimate the Planck constant from the Stenfan-Boltzman constant σ= 5.67033 x 10^{-8}W/m^{2}K^4.
3. In blackbody radiation, I used $r(T,\upsilon)$ representing spectrum density of radiation power, where $r(T,\upsilon)|d\upsilon|$ is the radiation energy emitted by a unit area per unit time, at temperature T and within $d\upsilon$ around frequency υ; We also use wavelength λ as variable, $r'(T,\lambda)$, where $r(T,\lambda)|d\lambda|$ is the radiation energy emitted by a unit area per unit time, at temperature T and within $d\lambda$ around λ. Prove that: $r'(T,\lambda)=\dfrac{c}{\lambda^2}r(T,\dfrac{c}{\lambda})$
4. This is a long problem which will derive the Planck formula of the spectrum energy density of the field $u_T(\nu)$, the energy per volume at certain T for frequency ν.

   4-1. Einstein' model of emission and absorption (this part is background information):



(a) 自发辐射　(b) 受激辐射　(c) 受激吸收

(Zhao's fig. 3-4,Vol.2)

The ideal picture of two-level system is shown above, with 1 the ground and 2 the excited state. The 3 processes are spontaneous emission (a), stimulated emission (b) and stimulated absorption (c). The electrons make transitions between these energy states and photons being emitted or absorbed. $E_2-E_1=h\nu$. The stimulated transitions will depend on the spectrum density of the field while the spontaneous is not. The spontaneous emission rate is: (a) $K_{Spon.}=AN_2$, where N2 is the population (number of electrons at excited state) of state 2. A is a constant called spontaneous emission coefficient. The rate for stimulated emission (b) $K_{Sti-emis}=B_{2-1}u_T(\nu)N_2$, B_{2-1} is the stimulated emission coefficient. (c) $K_{Abs}=B_{1-2}u_T(\nu)N_1$, B_{1-2} is the absorption coefficient. At thermal equilibrium, the population at levels 1 and 2 will not change over time, and this can be written as:

$$\frac{dN_2}{dt} = -AN_2 - B_{2-1}u_T(\nu)N_2 + B_{1-2}u_T(\nu)N_1 = 0 \quad (1)$$

With Boltzmann distribution: $\frac{N_1}{N_2} = \frac{e^{-E_1/k_BT}}{e^{-E_2/k_BT}} = e^{h\nu/k_BT}$.

The relation should hold at all T and $u_T(\nu)$. At very high T, and very large $u_T(\nu)$, the spontaneous contribution can be neglected, and also N1/N2 approaches 1; so the $B_{2-1} = B_{1-2} = B$. Thus the general formula (1) can be simplified as:

$$\frac{dN_2}{dt} = -AN_2 - Bu_T(\nu)N_2 + Bu_T(\nu)N_1 = 0 \quad (2)$$

Question:

Using the above information to derive the expression of $u_T(\nu)$ in terms of A, B (and T, h…);

Then evaluate the A/B(which should be constant for all T) value by the Rayleigh-Jeans relation on $u_T(\nu)$ (10-14 in the note) which we know is correct at high T low ν, i.e. $h\nu / k_BT << 1$. You will get the Planck's formula.

4-2 From Bose-Einstein statistic: (result on average number of n is given without proof)

The average number of photons in a state with energy $E = h\nu$ at temperature T is given by: $n = (e^{E/k_BT} - 1)^{-1}$, and then we also know the spectrum density of modes (see notes pg 396) is given by: $g(\nu) = \frac{8\pi}{c^3}\nu^2$

Using above information to get the Planck formula of $u_T(\nu)$

5. The sodium (Na, 23g/mole, density=0.97g/cm^3)'s outer electron has work function of 2.36eV. We shine it with a light of 10^{-8}W/m^2. Assume you only know classical theory, use it to calculate how much time needed to produce a photoelectron with kinetic energy of 1 eV? (Hint: You will need a couple simplified assumptions here. You want first estimate how many sodium atom per area. This can be done by assume one layer of atoms will absorb all the light. From the density, atomic weight and Avagadro number, you know the atoms per volume, and assume atoms are small cubes, you can estimate how many atoms in one layer. Assume the photo energy will all be absorbed and converted to outer shell electron's energy with no other loss, then you can get the answer, which would be on the order of 10 years)
6. In a photoelectric experiment (Milligan's experiment)it is found that a stopping potential of 1.00eV is needed to stop all the electrons when the incident light of wavelength 260nm is used, and 2.30V is needed for the light of wavelength 207nm. From these data, estimate the Planck's constant and the work function of the metal.
7. If we are using 500nm visible light in the Compton experiment with free electrons (assume the initial speed of electrons is 0), at the 90 degree scattering angle, what is the ***shift*** of

wavelength of the light at this angle? What is the ***energy loss*** of the scattered light comparing with the input (in percentage comparing of the input)? If the input light is 0.05nm X-ray, please answer the same questions above. Can you think of an analogy in classical mechanics behaves at least qualitatively like this? (Hint: collisions between hard bodies)

8. A photon with 40keV scatters from a free electron at rest. What is the maximum energy the electron can obtain? (Hint: The calculation is easy, do not try to use differentiation)
9. Bohr' s H atom: Below you need to find formula for orbit energy E，angular momentum L and radius of electron R in hydrogen (H) atom, using Bohr's orbit model and de Broglie matter wave hypothesis:

   Bohr proposed electron like a particle moves around nuclei in a **circular orbit** subject to Coulomb force. It has orbit radius R and energy E and angular momentum L to be determined. Using de Broglie matter wave, the longest wavelength can stably exist on the orbit with radius R must satisfy: $n\lambda = 2\pi R_n \quad n = 1, 2, 3...$ (A standing wave on a ring), and you may also know his famous relation between momentum and wavelength: $p = \frac{h}{\lambda} \quad h = 6.63 \times 10^{-34}$ (SI unit). The Coulomb potential energy in hydrogen atom between electron an proton is: $U = -\frac{C}{r}$, $C = \frac{e^2}{4\pi\varepsilon_0} = 2.3 \times 10^{-28}$ (SI unit). Mass of electron is $m_e = 9.11 \times 10^{-31} kg$ (the reduced mass can be treated equal to this since proton is 1800 time massive).

   1) Using the provided information, write the formula for possible (different n) L, R an E of the electron in H atom. (Express L in h or $\hbar \equiv h / 2\pi$); and find out their minimum values (that is when n=1), express E in eV.

   2) Ritz-Rydberg formula for transition frequency, when the electron changes from an initial orbit n_i to a final orbit n_f (so called orbit quantum number), it emits out (or absorbs) light which satisfy Planck formula: $h\upsilon = -\Delta E = E_i - E_f$ (Conservation of energy: $\Delta E = E_f - E_i, \Delta E + h\upsilon = 0)$, express the frequency of light with physical constant and orbit quantum numbers.

10。AB's problem 4-9:

Notes: in question a), the velocity is the one estimated with classical mechanics, if electron moves in the 1st circular orbit (n=1), the velocity is v_1.

The **fine structure constant** is defined as $\alpha = e^2/2\epsilon_0 hc$. This quantity got its name because it first appeared in a theory by the German physicist Arnold Sommerfeld that tried to explain the fine structure in spectral lines (multiple lines close together instead of single lines) by assuming that elliptical as well as circular orbits are possible in the Bohr model. Sommerfeld's approach was on the wrong track, but α has nevertheless turned out to be a useful quantity in atomic physics. (*a*) Show that $\alpha = v_1/c$, where v_1 is the velocity of the electron in the ground state of the Bohr atom. (*b*) Show that the value of α is very close to 1/137 and is a pure number with no dimensions. Because the magnetic behavior of a moving charge depends on its velocity, the small value of α is representative of the relative magnitudes of the magnetic and electric aspects of electron behavior in an atom. (*c*) Show that $\alpha a_0 = \lambda_C/2\pi$, where a_0 is the radius of the ground-state Bohr orbit and λ_C is the Compton wavelength of the electron.

Recommended problems in Arthur Beiser's book (Shorthand for AB's henceforth): 2-9, 2-13, 2-17, 2-19, 2-27, 2-35. 4-12