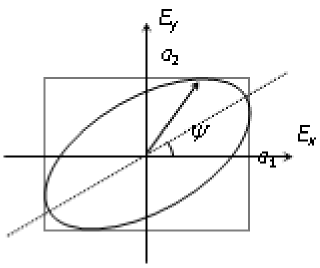
Homework for General Physics II set 6

1. Hecht's 8.1
2. Hecht's 8.6
3. Hecht's 8.8
4. Hecht's 8.19
5. For realistic linear polarizers, when the polarization of light is **parallel** with the transmission axis, not 100% will pass, let's denote T_1 for the percentage of light passing the polarizer in this case (polarization is parallel with transmission axis); when the polarization of light is **perpendicular** to the transmission axis, light is not completely blocked and let's denote T_2 percentage of light that will pass. If we have natural light input, after passing through a pair of realistic linear polarizers whose main transmission axis (T1 along this direction) has an angle of θ. Prove that $T_\theta = \frac{1}{2}(T_1^2 + T_2^2)\cos^2\theta + T_1 T_2 \sin^2\theta$, T is the transmission ratio of **energy** (I_{output}/I_{input}). T1, T2 as described at the beginning. (For the ideal polarizers, where T_1=1, T_2=0, results above agree with the Malus law. In this course if not specifically stated, we will treat linear polarizer as ideal for simplicity)
6. Hecht's 8.32. (Note the definition of the V degree of polarization is given by formula 8-29,8-30 in his book, section 8.6.1; and also you need Fresnel equations upon reflection, section 4.6.2 and 4.6.3 in his book)
7. Hecht's 8.34
8. For a polarized light whose Jones vector is $\begin{bmatrix} 2 \\ -i \end{bmatrix}$, what is polarization state (including the rotation)? If an ideal linear polarizer is placed horizontally, the polarized light given above illuminate the linear polarizer at normal angle, what is the output energy percentage? If the linear polarizer is placed with +30° (c.c.w) with respect to the horizontal direction, what is the output energy percentage?
9. A linear polarizer whose transmission axis is along direction ψ with respect to the horizontal direction:

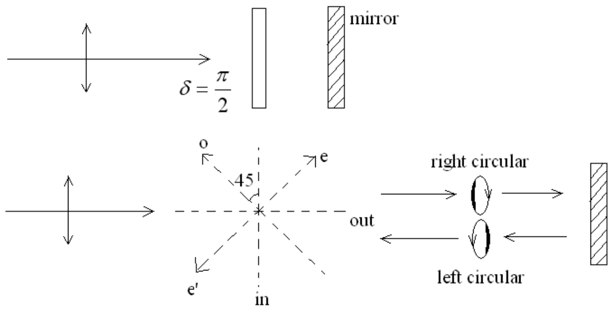


The elliptical polarized light with $a_1 = A, a_2 = B$ and phase $\Delta\phi = \varepsilon$, i.e. in Jones vector: $\begin{bmatrix} A \\ Be^{i\varepsilon} \end{bmatrix}$, when this light is passing the linear polarizer (dotted line in the figure), 1) what is the formula for the output intensity(I did this part in lecture)? 2) Then show that when the transmission axis is along the long principal axis of the ellipse, the outputs intensity is extreme. (the relation between principal axis direction and A, B, phase difference is given in Hecht's: pg. 328, formula 8-15: $\tan 2\alpha = \dfrac{2AB\cos\varepsilon}{A^2 - B^2}$)

10. A polarization state in the H-V base is: $\begin{bmatrix} 3 \\ 2i \end{bmatrix}$, 1) find out its expression in the $|R>, |L>$ base (right and left circular polarization), i.e. finding the expansion coefficients(projection) of the polarization state in terms of circular polarization. 2) what is the expression in another $|e_1>, |e_2>$ base, where $|e_1> = \frac{1}{\sqrt{5}}\begin{bmatrix} 2 \\ -i \end{bmatrix}, |e_2> = \frac{1}{\sqrt{5}}\begin{bmatrix} 1 \\ 2i \end{bmatrix}$ are two orthonormal elliptical base vectors, whose expression in H-V base is provided above.

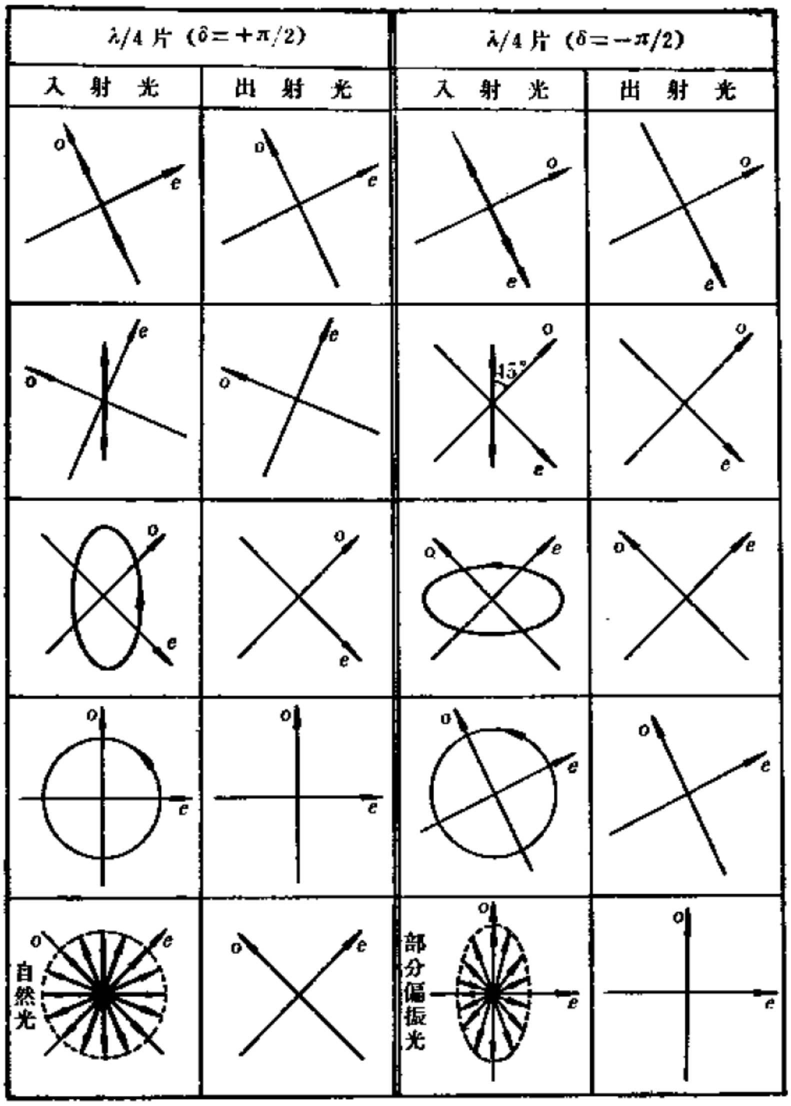
11. Using the Jones vector for a polarized light and Jones matrix for the reflection (in my note), show that if the incident light is a right-circular polarized light, at normal incidence angle (i=0) at an interface between (n_i, n_t), n_i is the refractive index of the incident media; n_t is the refractive index of the transmitted media. the polarization state of the reflected light will be left-circular polarized. Does it matter whether it is internal or external reflection? You may need the Fresnel Equations for reflection coefficients (pg. 80 of my note or pg. 114 in Hecht's and pg. 247 in Zhao's): at normal incident angle: $r_s = \dfrac{n_t - n_i}{n_t + n_i}, r_p = \dfrac{n_i - n_t}{n_t + n_i}$ s,p are two perpendicular directions defined in the notes and books)

12. Use the Jones vector and matrix, work the example given in the lecture. The input is a linear polarized light, after a quarter waveplate whose fast axis (o axis) is shown in the figure and a reflecting surface, the output is a linear light but orthogonal to the input. (verify the polarization state after each stage, for the reflecting surface use the results of above question)



13. Hecht, problem 8.41. (Understand how the half-plate 'flipped' polarization is the simplest way to solve the problem, while the matrix form here are too complicated)

14. Question 3, pg199-200, Zhao's book, vol. 2. Find the polarization state for the output given the input (1st and 3rd column) for the two quarter wave plate. (Note: In Zhao's problem, the δ is $\delta_{oe} = \varphi_o - \varphi_e$ which is $-\delta_{eo}$)



15. Please find out the matrix form of Right circular polarizer under H-V basis, where H, V stands for horizontal and vertical linear polarized light. A Right circular polarizer is for any input the output will be right-circular polarized. (I included the solution in the supplement on matrix representation, you may try all the 4 methods: Similarity transform; Operation on base vectors; matrix elements and projector)
16. For an optical element whose matrix form in H-V base is:

    $\hat{O} = \frac{1}{5}\begin{bmatrix} 4 & -2i \\ 2i & 1 \end{bmatrix}$, find out its eigenvalues and associated eigenvectors (normalized); can you tell the action of this O on any polarization input?

Other Recommended problems (no need to hand-in) in Zhao's book: Questions 2,6, problems 4,8,9 on pg 180-182; Question 2, problems 1,5 on pg 186-187; Hecht's: 8.43, 8.69, 8.71