1. Ans:

P(16 or more correct) = P(16 correct) + P(17 correct) + P(18 correct) + P(19 correct) + P(20 correct)
[C(20, 16) * (1/4)^16 * (3/4)^4] + [C(20, 17) * (1/4)^17 * (3/4)^3] + [C(20, 18) * (1/4)^18 * (3/4)^2] + [C(20, 19) * (1/4)^19 * (3/4)^1] + [C(20, 20) * (1/4)^20 * (3/4)^0] ~= 0.0058

The answer is approximately **0.58%** #

2. Ans:

Expected number of matches = 4*P(The series end at 4 games) + 5*P(The series end at 5 games) + 6*P(The series end at 6 games) + 7*P(The series end at 7 games)

4* [(1/2)^4] + 5 * [C(4,3) * (1/2)^5 * 2] + 6 * [C(5,4) * (1/2)^6 * 2] + 7 * [C(7,4) * (1/2)^7 * 2]
= 4 * 0.125 + 5 * 0.25 + 6 * 0.3125 + 7 * 0.3125 = 5.8125

The answer is approximately **5.8125** #

3. Ans:

If use Poisson distribution to describe, let λ = (1/80) * 20 = 1/4.
We can now use the Poisson distribution to calculate the probability of witnessing the show during the 20-minute waiting period: P(X >= 1) = 1 - P(X = 0) = 1 - e^(-λ) * λ^0 / 0! = 1 - e^(-1/4) = 0.219

The answer is approximately **0.219** #

4. Ans:

a. P(T > t) = e^(-μ1*t), t = 10 => P(T > 10) = **e^(-μ1*10)** #
b. P(Mary finishes before John) = P(T2 < T - 10), T = 10 + Exp(μ1) = ∫[0,∞] (1 - e^(-μ2*t*)) * *μ1* * *e*^(-*μ1*t) dt = **μ2 / (μ1 + μ2)** #

5.Ans:

Let X be the delay time of the train and Y be the arrival time of the bus at John's office. We can use the joint probability distribution function (PDF) to calculate the probability of John being late for work:

P(X + Y > 30) = ∬ fX,Y(x,y) dxdy

Since X and Y are independent, the joint PDF can be expressed as the product of their marginal PDFs:

fX,Y(x,y) = fX(x) * fY(y)

fX(4) = 1/16, fX(6) = 1/8, fX(8) = 1/2, fX(10) = 1/4, fX(12) = 1/16

fY(y) = (1 / (sqrt(2π) σ)) * exp(-((y - μ)^2) / (2σ^2))

P(X + Y > 30) = ∬ fX(x) * fY(y) dx dy, where X + Y > 30

$= \int 4^\infty \int 30 - x^\infty (1/16) * (1 / (\mathrm{sqrt}(2\pi)\ 2)) * \exp(-((y - 500)^2) / 8)\ dy\ dx$

- $\int 6^\infty \int 30 - x^\infty (1/8) * (1 / (\mathrm{sqrt}(2\pi)\ 2)) * \exp(-((y - 500)^2) / 8)\ dy\ dx$
- $\int 8^\infty \int 30 - x^\infty (1/2) * (1 / (\mathrm{sqrt}(2\pi)\ 2)) * \exp(-((y - 500)^2) / 8)\ dy\ dx$
- $\int 10^\infty \int 30 - x^\infty (1/4) * (1 / (\mathrm{sqrt}(2\pi)\ 2)) * \exp(-((y - 500)^2) / 8)\ dy\ dx$
- $\int 12^\infty \int 30 - x^\infty (1/16) * (1 / (\mathrm{sqrt}(2\pi)\ 2)) * \exp(-((y - 500)^2) / 8)\ dy\ dx$

Using a numerical integration tool, we can evaluate this expression to get:

$P(X + Y > 30) \approx 0.0426$

The final answer is approximately **4.26%** #

6.Ans:

a. P(Cancer | X-ray positive) = 0.99 * 0.01 / (0.99 * 0.01 + 0.01 * 0.99) = 0.5 or 50% #
b. P(Cancer | X-ray positive and MRI positive) = 0.999 * 0.5 / (0.999 * 0.5 + 0.001 * 0.5) = **0.998 or 99.8%** #

7.Ans:

$Z = (\bar{x} - \mu) / (\sigma/\sqrt{n})$

a. $Z = (10 - 8.5) / (0.5) = 3$

Using a standard normal table or calculator, we can find that the probability of Z being less than 3 is approximately **0.9987** #

b. $Z1 = (7 - 8.5) / (0.5) = -3$, $Z2 = (10 - 8.5) / (0.5) = 3$

Z1 being less than -3 is approximately 0.0013
Z2 being less than 3 is approximately 0.9987

the average time waiting in line for these 49 customers is between 7 and 10 minutes is approximately 0.9987 - 0.0013 = **0.9974** #

c. $Z = (7.5 - 8.5) / (0.5) = -2$

Z being less than -2 is approximately **0.0228** #

8. Ans:

$E = np = 600 \times 0.4 = 240$

$\sigma = \mathrm{sqrt}(p(1-p)/n) = \mathrm{sqrt}(0.4 \times 0.6/600) = 0.0258$

$Z = (\hat{p} - p) / \sigma$

If the number of iPhone users is between 216 and 264, then the sample proportion is:

$\hat{p} = (216 + 264) / 600 = 0.8$

The corresponding *z*-score is:

Z = (0.8 - 0.4) / 0.0258 = 15.50

The probability of observing a z-score of 15.50 or greater (in either tail) under the null hypothesis is essentially zero. Therefore, **the type I error probability for this analysis procedure is essentially zero, assuming that p = 0.4 for real**.

9. Ans:

P(X = 0 (0 under-filled bottles)) * 75 = (24 choose 0) * (0.05)^0 * (0.95)^24 * 75 ≈ 44.38

P(X = 1 (1 under-filled bottle)) * 75 = (24 choose 1) * (0.05)^1 * (0.95)^23 * 75 ≈ 27.61

P(X = 2 (2 under-filled bottles)) * 75 = (24 choose 2) * (0.05)^2 * (0.95)^22 * 75 ≈ 7.33

P(X = 3 (3 under-filled bottles)) * 75 = (24 choose 3) * (0.05)^3 * (0.95)^21 * 75 ≈ 0.64

χ^2 = ((39 - 44.38)^2 / 44.38) + ((23 - 27.61)^2 / 27.61) + ((12 - 7.33)^2 / 7.33) + ((1 - 0.64)^2 / 0.64) ≈ 5.22

Refer to chi-squared distribution table, we find that p-value is approximately 0.157.
Because the p-value is greater than 0.05, we cannot reject the null hypothesis and **we can conclude that binomial distribution is a suitable model for this data.**

10. Ans:

(10).

$$\text{Expected Frequency} = \frac{\text{Total row} \times \text{Total column}}{\text{Overall Total}}$$

=)

Prob \ OR	A	B	C	Total	Expeted A	Epectes B	Expe C
A	24	11	10	45	15.75	13.5	15.75
B	7	13	5	25	8.75	7.5	8.75
C	4	6	20	30	10.5	9	10.5
Total	35	30	35	100			

$$ch2 = \frac{(24-15.75)^2}{15.75} + \frac{(11-13.5)^2}{13.5} + \frac{(10-15.75)^2}{15.75} + \frac{(7-8.75)^2}{8.75} + \frac{(13-7.5)^2}{7.5} + \frac{(5-8.75)^2}{8.75} + \frac{(4-10.5)^2}{10.5} + \frac{(6-9)^2}{9} + \frac{(20-10.5)^2}{10.5} \approx 17.333\ldots$$

0.01 and 4=(3-1)*(3-1) degrees of freedom to be 13.277
Since our test statistic (17.333) is greater than the critical value (13.277),
The null hypothesis was false and conclude that there is evidence of an association between the grades in two classes.

11. Ans:

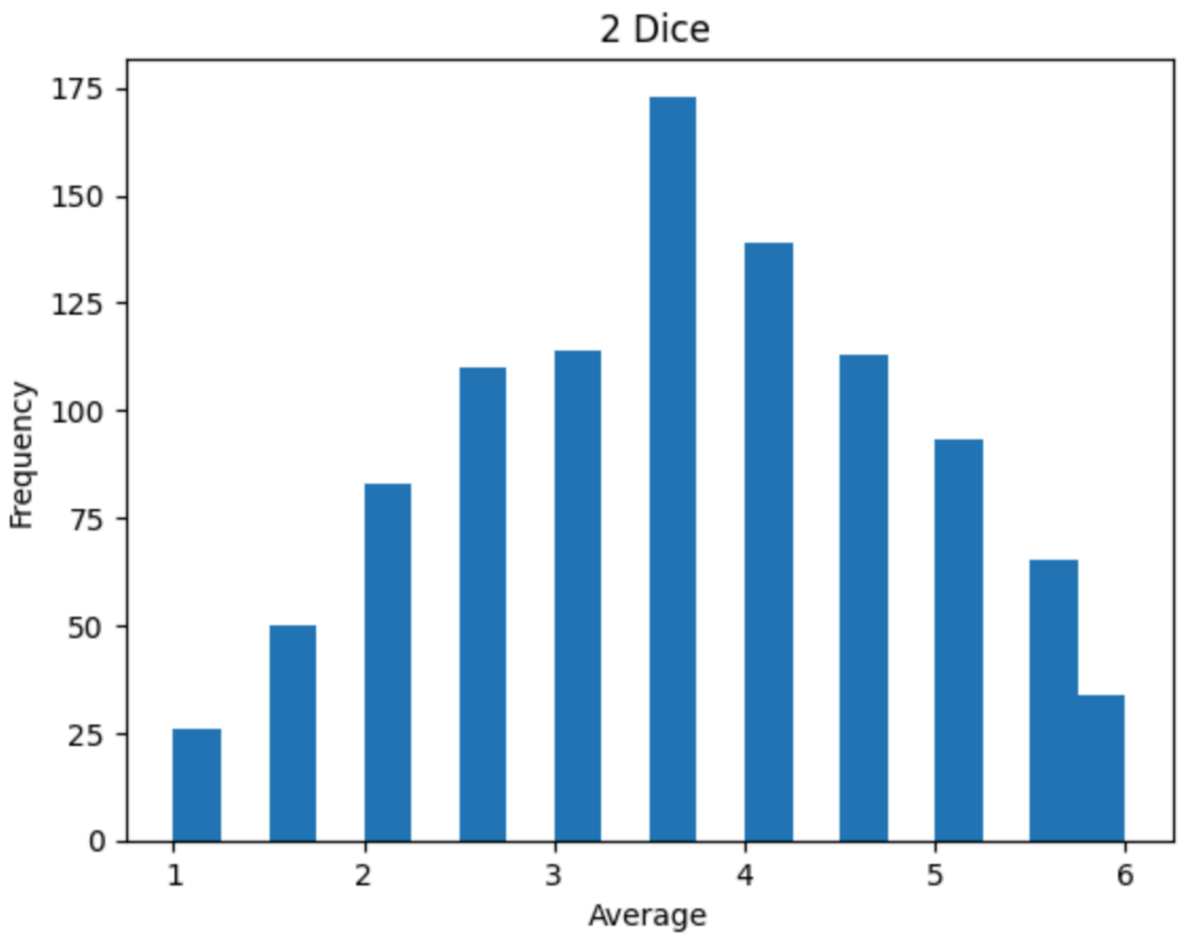
Please check the detail on ipynb files.

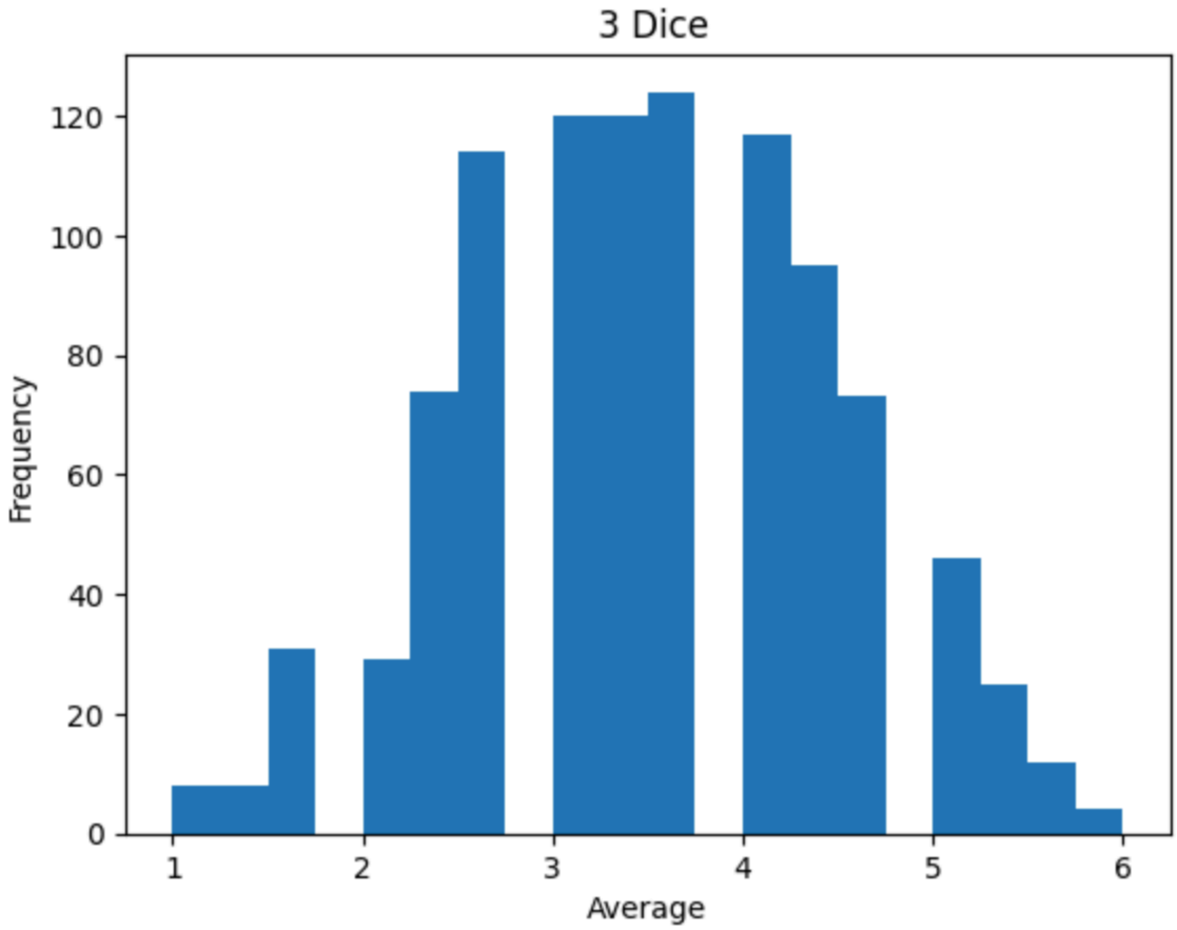
```
import random
import matplotlib.pyplot as plt

times = 1000
dice = [2, 3, 4, 5]

for n in dice:
    averages_list = []
    for i in range(times):
        rolls = [random.randint(1, 6) for j in range(n)]
        average = sum(rolls) / n
        averages_list.append(average)

    plt.hist(averages_list, bins=20)
    plt.title(f" {n} Dice")
    plt.xlabel("Average")
    plt.ylabel("Frequency")
    plt.show()
```




3 Dice
Frequency
Average
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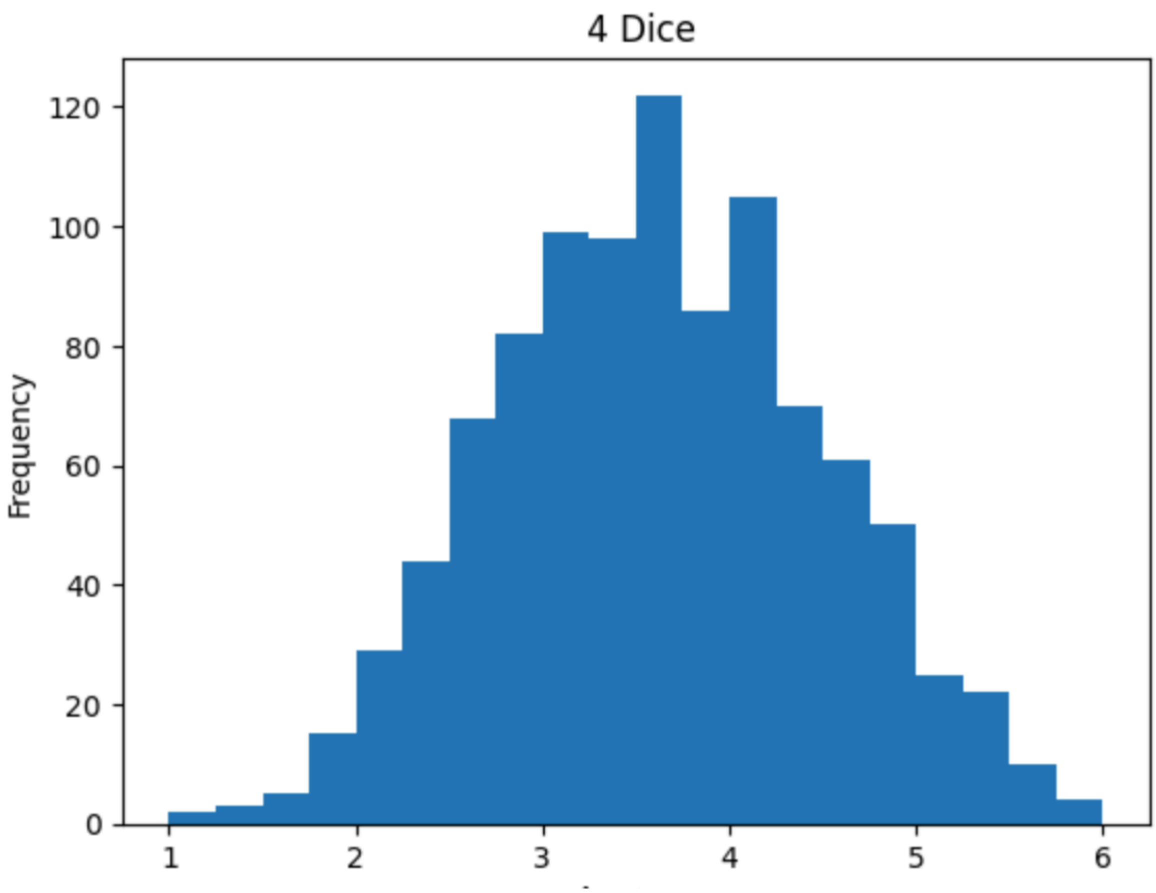
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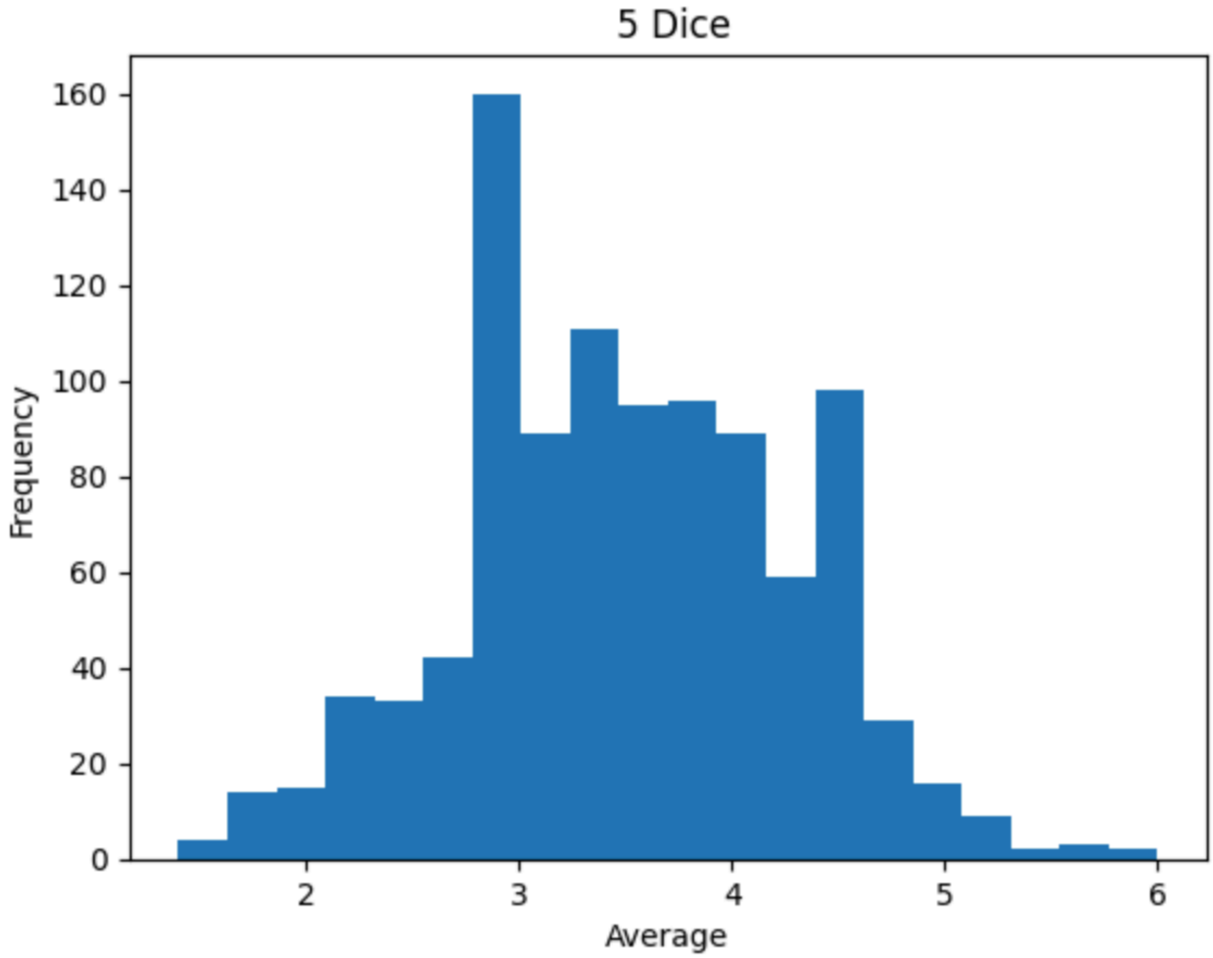

12. Ans:

Please check the detail code on ipynb files.

a. The probability that all the first 10 cards reach the same end is: 0.4916

b.

```
n_cards=52, face_steps=1, probability=0.4896
n_cards=52, face_steps=3, probability=0.5037
n_cards=52, face_steps=5, probability=0.4916
n_cards=52, face_steps=7, probability=0.4984
n_cards=52, face_steps=9, probability=0.5002
n_cards=104, face_steps=1, probability=0.502
n_cards=104, face_steps=3, probability=0.4976
n_cards=104, face_steps=5, probability=0.5028
n_cards=104, face_steps=7, probability=0.4999
n_cards=104, face_steps=9, probability=0.5036
```

We can observe that the probability that all the first 10 cards reach the same end decreases as the number of face steps increases and as the number of cards increases.

系所：工工所
姓名：許世佑
學號： R 10546001