Data Analytics Method HW#5

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Q1 (a)

Refer to the previous homework, we can see the result below.

```python
from PIL import Image
import numpy as np
from numpy import array
from tkinter import _flatten
from sklearn.linear_model import Lasso
from sklearn.linear_model import Ridge
import math
import matplotlib.pyplot as plt
```

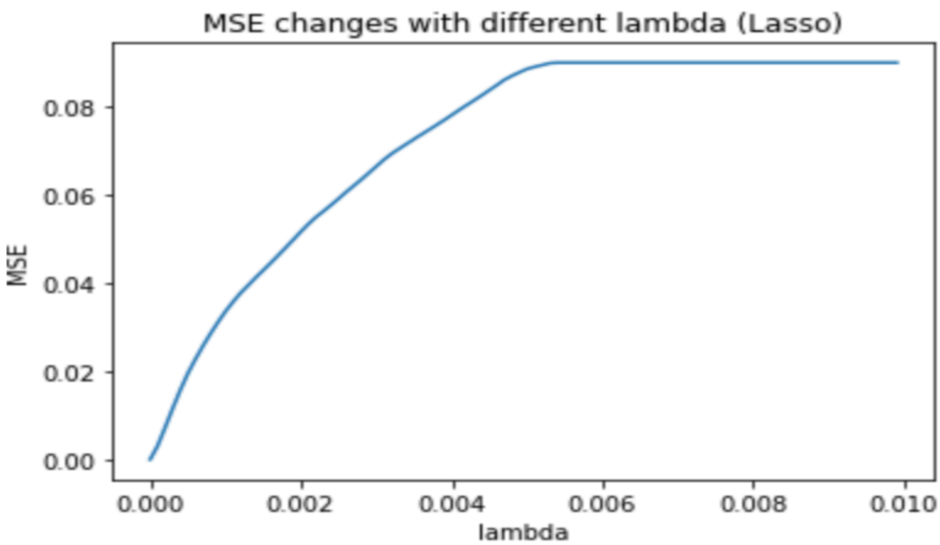
✓ 0.0s Python

```python
X = np.zeros((400, 2576))
for j in range(0, 40):
    for i in range(0, 10):
        image = Image.open(r"/Users/4yo/Desktop/NTU_Class/Data_Analyze_Method/ORL_Faces/%s_%s.png" %(j+1, i+1))
        image_array = array(image)
        X[i+j*10] = image_array.flatten()
        y = [10*[0],10*[1],10*[1],10*[1],10*[1],10*[1],10*[1],10*[0],10*[1],10*[0]
         ,10*[1],10*[1],10*[1],10*[1],10*[1],10*[1],10*[1],10*[1],10*[1],10*[1]
         ,10*[1],10*[1],10*[1],10*[1],10*[1],10*[1],10*[1],10*[1],10*[1],10*[1]
         ,10*[1],10*[0],10*[1],10*[1],10*[1],10*[1],10*[1],10*[1],10*[1],10*[1]]
y = list(_flatten(y))
```

✓ 0.2s Python

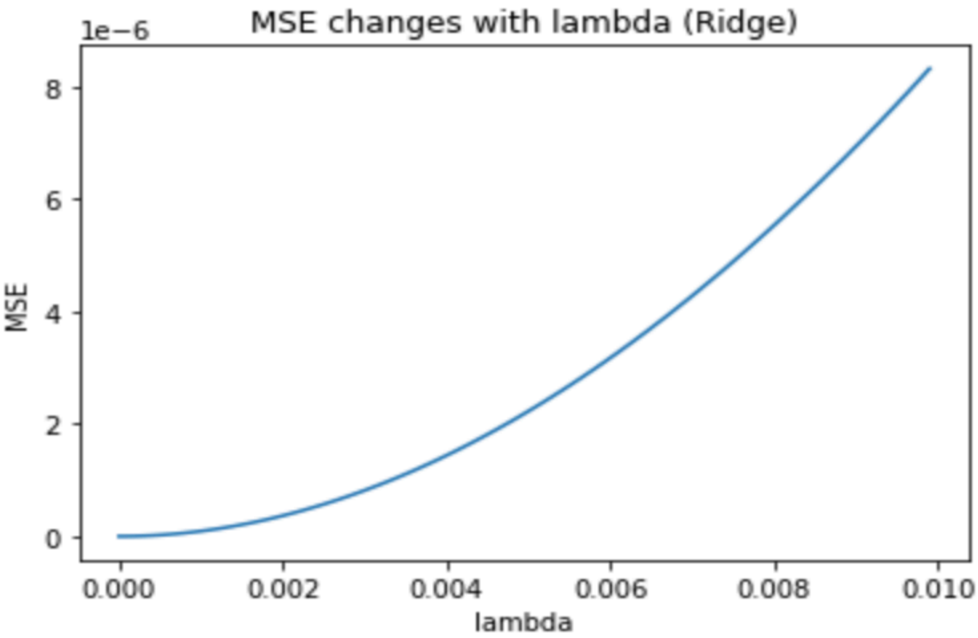
```python
mse = []
x_axis = []
for i in range(0,100,1):
    lasso = Lasso(alpha = i/10000,normalize = True)
    lasso.fit(X, y)
    mse.append(np.mean((lasso.predict(X) - y) ** 2))
    x_axis.append(i/10000)
plt.xlabel("lambda")
plt.ylabel("MSE")
plt.title("MSE changes with different lambda (Lasso)")
plt.plot(x_axis,mse)
```

✓ 4.0s Python



```python
mse = []
x_axis = []
for i in range(0,100,1):
    ridge = Ridge(alpha = i/10000,normalize = True)
    ridge.fit(X, y)
    mse.append(np.mean((ridge.predict(X) - y) ** 2))
    x_axis.append(i/10000)
plt.xlabel("lambda")
plt.ylabel("MSE")
plt.title("MSE changes with lambda (Ridge)")
plt.plot(x_axis,mse)
```

✓ 1.9s Python



Q1 (b)

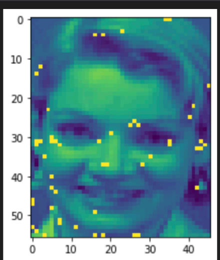
```python
lasso = Lasso(alpha = 0.001,normalize = True)
lasso.fit(X, y)
n = np.sum(lasso.coef_ != 0)
print('The total pixels were chosen from Lasso Regression:' + str(n))

important_pixels = []
for i in range(len(lasso.coef_)):
    if lasso.coef_[i] != 0:
        important_pixels.append(i)
```

✓ 0.1s Python

```
The total pixels were chosen from Lasso Regression：52
```

```python
image = Image.open(r"/Users/4yo/Desktop/NTU_Class/Data_Analyze_Method/ORL_Faces/1_1.png")
img_array = np.array(image)
#print(len(important_pixels),"important pixels at")
for i in range(0, len(important_pixels)): #math.floor()
    col = math.floor(important_pixels[i]/46)
    row = important_pixels[i]-46*col
    img_array[int(col)][int(row)]=255
plt.imshow(img_array, interpolation='nearest')
plt.show()
```

✓ 0.1s Python



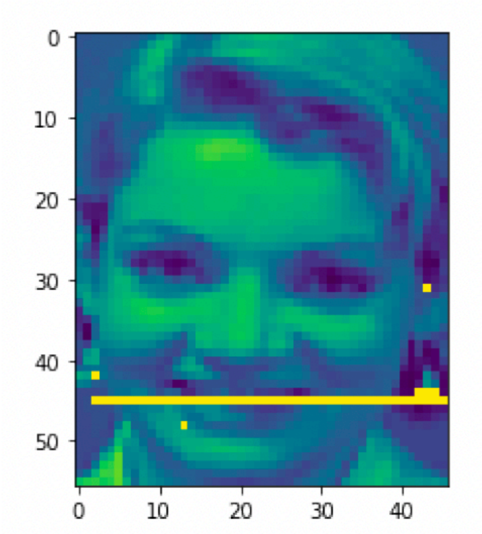


Figure1

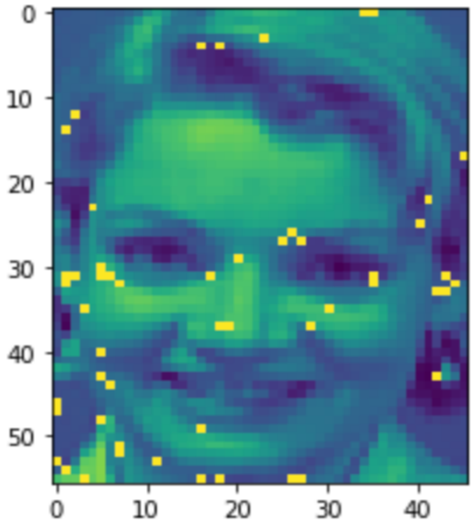


Figure2

The Figure1 is presented the chosen pixels from stepwise regression from HW#2, and the Figure2 is presented the chosen pixels from lasso regression. We can easily observe that the chosen points in figure2 is more scatter than figure 1 which is a simply line.

Q2 (a)

```
import pandas as pd
import numpy as np
from scipy.optimize import minimize
```
✓ 0.0s Python

```
# Read the CSV file
data = pd.read_csv('/Users/4yo/Desktop/NTU_Class/Data_Analyze_Method/transportation_table.csv')

# Define the objective function to minimize
def objective(beta, data):
    K = data['Capital']
    L = data['Labor']
    V = data['Value_Added']
    epsilon = np.exp(np.random.normal(size=len(data)))
    log_V_hat = np.log(beta[0]) + beta[1]*np.log(K) + beta[2]*np.log(L) + np.log(epsilon)
    return np.sum((np.log(V) - log_V_hat)**2)

# Minimize the objective function to estimate beta
beta0 = np.array([10000, 0.5, 0.5])
result = minimize(objective, beta0, args=(data,), method='Nelder-Mead')

# Print the estimated parameters
print('Estimates under the unconstrained model:')
print('beta1 =', result.x[1])
print('beta2 =', result.x[2])
```
✓ 0.9s Python

```
Estimates under the unconstrained model:
beta1 = 0.48739662520549115
beta2 = 0.45689725685502514
```

Estimates under the unconstrained model:

beta1 = 0.48739662520549115

beta2 = 0.45689725685502514

Q2 (b)

```
Q2 (b)

# Define the objective function with the Cobb-Douglas constraint
def objective_cd(beta, data):
    K = data['Capital']
    L = data['Labor']
    V = data['Value_Added']
    epsilon = np.exp(np.random.normal(size=len(data)))
    log_V_hat = np.log(beta[0]) + beta[1]*np.log(K) + (1-beta[1])*np.log(L) + np.log(epsilon)
    return np.sum((np.log(V) - log_V_hat)**2)

# Minimize the objective function with the Cobb-Douglas constraint to estimate beta
beta0 = np.array([10000, 0.5])
bounds = ((None, None), (0, 1))
result = minimize(objective_cd, beta0, args=(data,), method='L-BFGS-B', bounds=bounds)

# Print the estimated parameters under the Cobb-Douglas constraint
print('Estimates under the Cobb-Douglas constraint:')
print('beta1 =', result.x[1])
print('beta2 =', 1-result.x[1])

✓ 0.2s                                                                    Python

Estimates under the Cobb-Douglas constraint:
beta1 = 0.5000000000000004
beta2 = 0.49999999999999956

/Users/4yo/Library/Python/3.6/lib/python/site-packages/ipykernel_launcher.py:7: RuntimeWarning: invalid value encountered in log
  import sys
```

Estimates under the Cobb-Douglas constraint:

beta1 = 0.5000000000000004

beta2 = 0.49999999999999956

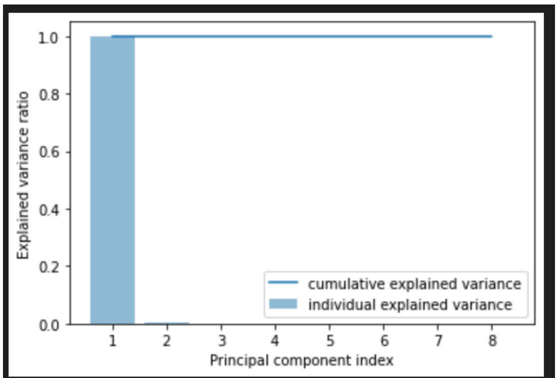
Q3 (a)

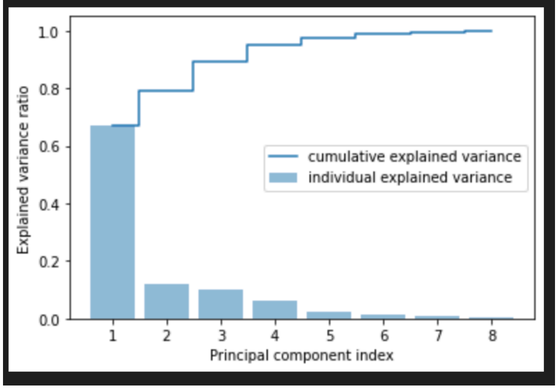
```
import numpy as np

def pca(X, isCorrMX=False):
    """
    Performs PCA on the input data matrix X.

    Parameters:
    X: array-like, shape (n_samples, n_features)
        The input data matrix, where n_samples is the number of samples and
        n_features is the number of features.

    isCorrMX: bool, default False
        If True, use the correlation matrix to perform PCA. Otherwise, use the
        covariance matrix.

    Returns:
    loadings: array-like, shape (n_features, n_components)
        The loading matrix, where n_components is the number of principal
        components.

    eigenvalues: array-like, shape (n_components,)
        The eigenvalue vector of the principal components.

    scores: array-like, shape (n_samples, n_components)
        The score matrix, i.e. the matrix of principal components.

    cum_var: array-like, shape (n_components,)
        The cumulative variance explained by each principal component.
    """
    # center the data
    X_centered = X - np.mean(X, axis=0)

    # compute the covariance/correlation matrix
    if isCorrMX:
        cov_matrix = np.corrcoef(X_centered, rowvar=False)
    else:
        cov_matrix = np.cov(X_centered, rowvar=False)

    # perform eigendecomposition
    eigenvalues, eigenvectors = np.linalg.eig(cov_matrix)
```

```python
    # sort eigenvalues and eigenvectors in decreasing order
    idx = np.argsort(eigenvalues)[::-1]
    eigenvalues = eigenvalues[idx]
    eigenvectors = eigenvectors[:, idx]

    # compute the loading matrix
    loadings = eigenvectors

    # compute the score matrix
    scores = np.dot(X_centered, loadings)

    # compute the cumulative variance explained
    cum_var = np.cumsum(eigenvalues) / np.sum(eigenvalues)
    var_exp = eigenvalues / np.sum(eigenvalues)
    cum_var_exp = np.cumsum(var_exp)

        # plot the scree plot
    import matplotlib.pyplot as plt
    plt.bar(range(1, len(eigenvalues)+1), var_exp, alpha=0.5, align='center',
            label='individual explained variance')
    plt.step(range(1, len(eigenvalues)+1), cum_var_exp, where='mid',
             label='cumulative explained variance')
    plt.ylabel('Explained variance ratio')
    plt.xlabel('Principal component index')
    plt.legend(loc='best')
    plt.show()

    return loadings, eigenvalues, scores, cum_var
```

```python
import pandas as pd

# load AutoMPG dataset
auto_mpg = pd.read_csv("DA_Demo.csv")

# extract features and labels
X = auto_mpg.iloc[:, :-1].values

# perform PCA with covariance matrix
loadings_cov, eigenvalues_cov, scores_cov, cum_var_cov = pca(X, isCorrMX=False)

# perform PCA with correlation matrix
loadings_corr, eigenvalues_corr, scores_corr, cum_var_corr = pca(X, isCorrMX=True)
```

✓ 1.4s Python





```python
# print results
print("\nisCorrMX=False: \n")
print("Loading matrix with covariance matrix:\n", loadings_cov)
print("Eigenvalues with covariance matrix:\n", eigenvalues_cov)
print("Score matrix with covariance matrix:\n", scores_cov)
print("Cumulative variance explained with covariance matrix:\n", cum_var_cov)
```

✓ 0.0s Python

```
Output exceeds the size limit. Open the full output data in a text editor

isCorrMX=False:

Loading matrix with covariance matrix:
 [[-7.59590581e-03 -1.75785097e-02 -4.19212264e-02  8.31000630e-01
   5.49535507e-01  5.61451547e-02 -3.97118180e-02  2.43984110e-02]
 [ 1.79257460e-03  1.33223878e-02 -7.28065126e-03 -4.31502411e-03
  -9.17751826e-03  1.53912563e-02  3.75218753e-01  9.26626917e-01]
 [ 1.14338202e-01  9.45572572e-01 -3.03873075e-01  9.22239231e-03
  -3.41321063e-03 -1.06909276e-02  9.19061696e-04 -1.63889805e-02]
 [ 3.89660894e-02  2.98328518e-01  9.47540130e-01  6.34424404e-02
  -5.73169239e-03 -8.63249865e-02 -8.51000586e-03  8.19890724e-03]
 [ 9.92644743e-01 -1.20867105e-01 -2.65874637e-03  3.53993832e-03
   3.93041220e-03  3.52931159e-03 -1.85727735e-04 -1.31438408e-04]
 [-1.35281211e-03 -3.48299933e-02 -7.68839939e-02 -2.08463123e-02
   1.25527681e-01 -9.88134411e-01  9.09879495e-03  1.37739900e-02]
 [-1.33689886e-03 -2.39497354e-02 -4.41269251e-02  5.51778482e-01
  -8.24191632e-01 -1.12006207e-01  3.03888666e-02 -1.60382361e-02]
 [-5.51527155e-04 -3.24675463e-03  1.23716014e-02  2.00685150e-02
   5.30801776e-02  8.77898223e-03  9.25502182e-01 -3.74145010e-01]]
Eigenvalues with covariance matrix:
 [7.32193919e+05 1.51443435e+03 2.61673181e+02 2.32569592e+01
 5.54405192e+00 2.85720860e+00 3.60437639e-01 2.58474909e-01]
Score matrix with covariance matrix:
 [[ 5.36470833e+02  5.08481249e+01 -1.06818979e+01 ...  2.34743018e+00
...
  -9.98056150e-01  7.82258105e-02]]
Cumulative variance explained with covariance matrix:
 [0.99753627 0.99959952 0.99995603 0.99998771 0.99999526 0.99999916
 0.99999965 1.        ]
```

```python
print("\nisCorrMX=True \n")
print("Loading matrix with correlation matrix:\n", loadings_corr)
print("Eigenvalues with correlation matrix:\n", eigenvalues_corr)
print("Score matrix with correlation matrix:\n", scores_corr)
print("Cumulative variance explained with correlation matrix:\n", cum_var_corr)
```

✓ 0.1s Python

```
Output exceeds the size limit. Open the full output data in a text editor

isCorrMX=True

Loading matrix with correlation matrix:
 [[-0.38586239  0.07663269  0.29228579  0.09998251 -0.74036644 -0.38735165
   0.19588516  0.1151321 ]
 [ 0.4023885   0.13842878  0.07223935 -0.21603551 -0.48261485  0.53092548
  -0.27878265  0.41774679]
 [ 0.41644435  0.12632499  0.07423622 -0.13581398 -0.30331627  0.00699705
   0.08422855 -0.82916553]
 [ 0.40183594 -0.11148007  0.23605571 -0.11971643  0.08426839 -0.6667096
  -0.53504996  0.13477548]
 [ 0.40157579  0.21102    -0.00089399 -0.32246785  0.13127292 -0.23585961
   0.72202073  0.30991105]
 [-0.2647309   0.41690206 -0.63943514 -0.49280794 -0.09773197 -0.20293343
  -0.22891382 -0.03518826]
 [-0.21386777  0.6904632   0.5871892  -0.10601968  0.30134385  0.11002592
  -0.12501506 -0.0542884 ]
 [-0.27786815 -0.50150064  0.30732382 -0.74328281  0.04739508  0.12086663
   0.0345266  -0.07951102]]
Eigenvalues with correlation matrix:
 [5.3758723  0.94366326 0.81164365 0.48615594 0.18282657 0.11432193
 0.05354682 0.03196954]
Score matrix with correlation matrix:
 [[ 2.74036781e+02  1.17077465e+02  1.10811235e+01 ... -1.36951193e+02
...
  -1.80076486e+02 -2.04894437e+01]]
Cumulative variance explained with correlation matrix:
 [0.67198404 0.78994195 0.8913974  0.95216689 0.97502021 0.98931046
 0.99600381 1.        ]
```

Q3 (b)

In this case, PCA is not scale-invariant. It is affected by the scale of the variables in the dataset. When the variables are measured on different scales, the variables with larger scales tend to dominate the principal components.

Q4 (a)

```
import numpy as np
import matplotlib.pyplot as plt
from PIL import Image
from numpy import array
from tkinter import _flatten
```

✓ 0.1s Python

```
X = np.zeros((400, 2576))
for j in range(0, 40):
    for i in range(0, 10):
        image = Image.open(r"/Users/4yo/Desktop/NTU_Class/Data_Analyze_Method/ORL_Faces/%s_%s.png" %(j+1, i+1))
        image_array = array(image)
        X[i+j*10] = image_array.flatten()
```

✓ 0.2s Python

```
def pca(dataMat, use_cov, topNfeat=20, ):
    meanVals = np.mean(dataMat, axis=0)
    meanRemoved = dataMat - meanVals
    if use_cov == True:
        covMat = np.cov(meanRemoved, rowvar=0)
    else:
        covMat = np.corrcoef(meanRemoved, rowvar=0)
    eigVal, eigVect = np.linalg.eig(np.mat(covMat))
    meanRemoved_score_matrix = meanRemoved * eigVect
    original_matrix = (meanRemoved_score_matrix * eigVect.T) + meanVals
    return meanRemoved_score_matrix, original_matrix, eigVal, eigVect
```

✓ 0.0s Python

```
def analyse_data(dataMat):
    Printed = 0
    meanVals = np.mean(dataMat, axis=0)
    meanRemoved = dataMat-meanVals
    covMat = np.cov(meanRemoved, rowvar=0)
    eigvals, eigVects = np.linalg.eig(np.mat(covMat))
    eigValInd = np.argsort(eigvals)

    topNfeat = 2576
    eigValInd = eigValInd[:-(topNfeat+1):-1]
    cov_all_score = complex(sum(eigvals)).real
    sum_cov_score = 0
    for i in range(0, len(eigValInd)):
        line_cov_score = complex(eigvals[eigValInd[i]]).real
        sum_cov_score += line_cov_score
        if  60 > (sum_cov_score/cov_all_score*100).real > 50 and Printed == 0:
            print('Principal components：%s, Variance percentage：%s%%, Cumulated percentage：%s%%' % (format(i+1, '2.0f'), format(line_cov_score
            Printed = 1
        elif 70 > (sum_cov_score/cov_all_score*100).real > 60 and Printed == 1:
            print('Principal components：%s, Variance percentage：%s%%, Cumulated percentage：%s%%' % (format(i+1, '2.0f'), format(line_cov_score
            Printed = 2
        elif 80 > (sum_cov_score/cov_all_score*100).real > 70 and Printed == 2:
            print('Principal components：%s, Variance percentage：%s%%, Cumulated percentage：%s%%' % (format(i+1, '2.0f'), format(line_cov_score
            Printed = 3
        elif 90 > (sum_cov_score/cov_all_score*100).real > 80 and Printed == 3:
            print('Principal components：%s, Variance percentage：%s%%, Cumulated percentage：%s%%' % (format(i+1, '2.0f'), format(line_cov_score
            Printed = 4
        elif (sum_cov_score/cov_all_score*100).real > 90 and Printed == 4:
            print('Principal components：%s, Variance percentage：%s%%, Cumulated percentage：%s%%' % (format(i+1, '2.0f'), format(line_cov_score
            Printed = 5
```

✓ 0.0s Python

```
#let X^T be a 2576x400 data matrix (X^T=2576x400)
meanRemoved_score_matrix, original_matrix, eigVals, eigVects = pca(X.T, True)
analyse_data(meanRemoved_score_matrix)
```

✓ 0.3s Python

```
Principal components： 2, Variance percentage：10.55%, Cumulated percentage：57.0%
Principal components： 3, Variance percentage：4.57%, Cumulated percentage：61.5%
Principal components： 6, Variance percentage：2.07%, Cumulated percentage：70.5%
Principal components：15, Variance percentage：0.66%, Cumulated percentage：80.2%
Principal components：47, Variance percentage：0.17%, Cumulated percentage：90.1%
```

Q4 (b)

```python
first_PC = X.T@eigVects
first_PC=first_PC.T[0]
first_PC_array=first_PC.reshape(56,46).real

min_first_PC_array = np.min(first_PC_array)
range_first_PC_array = np.max(first_PC_array) - np.min(first_PC_array)
for i, j in enumerate(first_PC_array):
    first_PC_array[i] = 255 * ((j - min_first_PC_array) / range_first_PC_array)

imgplot = plt.imshow(first_PC_array, cmap='gray', vmin=0, vmax=255)
plt.show()
```

✓ 1.0s Python