MATH 22 Section 010
Summary I

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Disclaimer: This is a short summary of the content we have covered in MATH 22. I strongly suggest students to at least thoroughly work through all questions mentioned in the Canvas Exam Review Page. This document was created by me and only serves for a quick review purpose.

1 Linear Function y=mx+b

1.1 Basic Properties

(a) $y = mx + b$, where m and b are constants.

(b) Rate of change is the slope m.

(c) Difference quotient is the rate of change m.

(d) x-intercept is $(\frac{-b}{m}, 0)$. The x-coordinate of the x-intercept is the zero of f.

(e) y-intercept is $(0, b)$.

1.2 Rate of Change

The rate of change of f between x_1 and x_2 is

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

1.3 Difference Quotient

Suppose we have a function $f(x)$, where f is any function. The difference quotient of $f(x)$ is the rate of change of f from x to $x + h$, ie:

$$\frac{f(x+h) - f(x)}{h}$$

If f is a linear function $f(x) = mx + b$, then the difference quotient

$$\frac{f(x+h) - f(x)}{h} = \frac{m(x+h) + b - mx - b}{h} = m$$

If f is not a linear function, then the difference quotient depends on x and h.

2 Polynomials

2.1 Definitions

Definition 2.1 (Polynomials)**.** *A function f is a polynomial if it is in form of*

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

*where $0, 1, \cdots, n-1, n$ are all **nonnegative** integers.*

- Degree of f is n.
- The leading coefficient is a_n.

Remark 2.2. *$f(x) = x^{-1} = \frac{1}{x}$ is not a polynomial. This is because -1 is a negative integer.*

2.2 Extrema

Definition 2.3 (Global/Absolute Maximum and Minimum)**.** *The largest or smallest value of $f(x)$ in the whole domain of f.*

Definition 2.4 (Local/Relative Maximum and Minimum)**.** *The largest or smallest value of $f(x)$ in a small open interval of f.*

Remark 2.5. *In polynomials, the turning points are the local maximum or minimum.*

2.3 Even and Odd Functions

2.3.1 Even Functions

A function is an even function if

$$f(-x) = f(x)$$

for all x in the domain of f. The graph of f is symmetric with respect to **y-axis**.

2.3.2 Odd Functions

A function is an odd function if

$$f(-x) = -f(x)$$

for all x in the domain of f. The graph of f is symmetric with respect to **origin**.

Remark 2.6. *To check whether a function f is even, odd or niether, just calculate $f(-x)$ and compare it with $f(x)$.*

3 Graph of a Polynomial

3.1 Basics

Definition 3.1 (Turning Point)**.** *A turning point (x, y) is the point of f where the graph turns from increasing to decreasing (Local Maximum) or from decreasing to increasing(Local Minimum).*

Theorem 3.2. *If a function is a polynomial with degree n, then it has* ***at most*** *$n-1$ turning points and n x-intercepts.*

3.2 End Behaviors of the Graph

End behaviors only depend on the **highest order** of the term in the polynomial. That is, if a polynomial $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$, then it only depends on $a_n x^n$.

- If $a_n > 0$, then $f(x) \to \infty$ as $x \to \infty$
- If n is odd, then $f(x)$ goes to the opposite direction as $x \to -\infty$ compared to $x \to \infty$
- If n is even, then $f(x)$ goes to the same direction as $x \to \pm\infty$

Remark 3.3. *If you get confused, think about the graphs of polynomials such as x^2, x^3 and x^4.*

3.3 Minimal Degree of a Polynomial

From **Theorem 3.2**, we can determine degree of a polynomial should be at least larger than a number. Use the end behavior of the graph to determine whether the degree should be even or odd. For instance, if the degree is at least 4, and is an odd number, then the minimum of the degree is 5.

4 Piecewise-Defined Polynomial

4.1 Basics

A function f is a piecewise-defined polynomial if it is defined to be a polynomial on some intervals. For instance,

$$f(x) = \begin{cases} x^3 & \text{if } x < 1 \\ x^2 - 1 & \text{if } x \geq 1 \end{cases}$$

- We sketch the graph based on the polynomial on each interval.
- To find $f(k)$, we first determine k is in which interval of the domain of f. Then plug k into the corresponding polynomial.
- f is continuous if there is no jump point in the graph.

4.2 Solve f(x)=c

To solve $f(x) = c$, we discuss all the cases of the polynomials and determine whether the solutions are valid (in the corresponding interval) or not.
Example: Let

$$f(x) = \begin{cases} x^3 & \text{if } x < 1 \\ x^2 - 1 & \text{if } x \geq 1 \end{cases}$$

Solve f(x)=1.
Solution:
We first look at the case $x < 1$: Then $f(x) = x^3$. We have

$$x^3 = 1 \Longrightarrow x = 1$$

But $x = 1$ is not in the region of $(-\infty, 1)$, so it is not a solution.
Then we look at the case $x \geq 1$: Then $f(x) = x^2 - 1$. We have

$$x^2 - 1 = 1 \Longrightarrow x^2 = 2 \Longrightarrow x = \pm\sqrt{2}$$

But $-\sqrt{2}$ is not in the region of $[1, \infty)$, so $\sqrt{2}$ is the only solution. Then we can conclude that the solution for $f(x) = 1$ is $x = \sqrt{2}$.

5 Division of Polynomials

5.1 Long Division

The process of doing the long divisions is omitted. Please see your own notes.

5.2 Synthetic Division

It is **only** useful when the divisor is in form of $x - k$.

Remark 5.1. *One may use synthetic division when the divisor is* $(ax + b)$*; however, I do not recommend. You need to do additional multiplication or division afterward.*

5.3 Remainder Theorem

If $f(x)$ is a polynomial and the divisor is $(x - k)$, then the remainder is $f(k)$.