Homework requirements

- Digital format (can be typeset or photos) is preferred
- Submit by next lecture
- Each homework 10 points; 1 point deducted for each day

Contact information

- TA:
  - 陈程：jackchen1990@gmail.com
- zlin@pku.edu.cn
- http://www.cis.pku.edu.cn/faculty/vision/zlin/zlin.htm

Homework (1)

1. Judge the properties of the following sets (openess, closeness, boundedness, compactness) and give their interiors, closures, and boundaries:

   a. $\mathcal{C}_1 = \emptyset$.

   b. $\mathcal{C}_2 = \mathbb{R}^n$.

   c. $\mathcal{C}_3 = \{x|0 \leq x < 1\} \cup \{x|2 \leq x \leq 3\} \cup \{x|4 < x \leq 5\}$.

   d. $\mathcal{C}_4 = \{(x, y)^T|x \geq 0, y > 0\}$.

   e. $\mathcal{C}_5 = \{k|k \in \mathbb{Z}\}$.

   f. $\mathcal{C}_6 = \{k^{-1}|k \in \mathbb{Z}\}$.

   g. $\mathcal{C}_7 = \{(1/k, \sin k)^T|k \in \mathbb{Z}\}$.

Homework (1)

2. For each of the following sequences, determine the rate of convergence and the rate constant.

a. $x_k = 2^{-k}$, for $k = 1, 2, \cdots$.

b. $x_k = 1 + 5 \times 10^{-2k}$, for $k = 1, 2, \cdots$.

c. $x_k = 2^{-2^k}$.

d. $x_k = 3^{-k^2}$.

e. $x_k = 1 - 2^{-2^k}$ for k odd, and $x = 1 + 2^{-k}$ for k even.

Homework (1)

3. Compute the gradient and the Hessian of the following functions (write in vector or matrix form, rather than in entries. Give details.):

   a. $f(\mathbf{x}) = \|\mathbf{x}\|_p$, $\mathbf{x} \neq \mathbf{0}$, $p \geq 2$.

   b. $f(\mathbf{x}) = (\mathbf{a}^T\mathbf{x})(\mathbf{b}^T\mathbf{x})$.

   c. $f(\mathbf{x}) = \frac{1}{2}\|\mathbf{A}\mathbf{x} - \mathbf{b}\|_2^2$.

Homework (1)

4. Find the dual norm of Mahalanobis norm: $\|\mathbf{x}\|_{\mathbf{M}} = \sqrt{\mathbf{x}^T\mathbf{M}\mathbf{x}}$, where $\mathbf{M}$ is a positive definite matrix.
5. Prove that the eigenvalues λ_i of $(\mathbf{A}+\mathbf{B})^{-1}\mathbf{A}$, where $\mathbf{A}$ is positive semidefinite and $\mathbf{B}$ is positive definite, satisfy $0 \leq \lambda_i < 1$.
6. Compute the condition number of the following matrix:

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 5 & 6 & 9 \end{bmatrix}.$$

7. Suppose $\mathbf{X} \in \mathbb{R}^{3\times 3}$, $\mathcal{A}(\mathbf{X}) = X_{11} + X_{12} - X_{31} + 2X_{33}$, find $\mathcal{A}^*$.