Homework 1
CS 259 @ SJTU
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Problem 1: Constrained least squares

(1)

$$\begin{aligned}
&\because \sum x_i = 1 \\
&\therefore ||x||^2 = 1 \\
&\mathcal{L} = ||Ax-b||^2 + \lambda(||x||^2 - 1) \\
&\delta\mathcal{L} = \delta||Ax-b||^2 + \lambda * \delta(||x||^2 - 1) \\
&\delta\mathcal{L} = 2 * \delta x^T(A^TAx - A^Tb - \lambda x) = 0
\end{aligned}$$

$$\therefore KKT \begin{cases} x^* = (A^TA - \lambda)^{-1}A^Tb \\ ||x^*|| = 1 \end{cases}$$

(2)

```
import numpy as np
from sympy import *
q, r = np.linalg.qr(A)
p = Symbol('p')
X = (A.T*A - p).I * A.T * b
solve(np.norm(X) - 1)
```

Problem 2: Residual sensitivity

(1)

Equal to show $||r||\delta||r|| = r^T\delta r$
Equal to show $\delta(||r||^2) = 2r^T\delta r$

$$\begin{aligned}
\delta(||r||^2) &= \delta(r^Tr) \\
&= (\delta r^T)r + r^T\delta r \\
&= 2r^T\delta r
\end{aligned}$$

(2)

Equal to show $||r||\delta||r|| = -r^T\delta Ax$
And from (1), $r^T\delta r = -r^T\delta Ax$
Equal to show $\delta r = -\delta Ax$ ()

And $r = b - Ax$
$\therefore \delta r = 0 - \delta A x$ is equal to (*).

$\square$