Homework 1
CS 259 @ SJTU
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Problem 1: Constrained least squares

(1)

$$\because \sum x_i = 1$$
$$\therefore c = (1 \cdots 1),\ cxc^T = 1$$
$$\mathcal{L} = ||Ax-b||^2 + \lambda(cxc^T - 1)$$
$$\delta\mathcal{L} = \delta||Ax-b||^2 + \lambda c\delta xc^T + \delta\lambda(cx-1)$$
$$\delta\mathcal{L} = \delta x^T(2A^TAx - 2A^Tb - \lambda cxc^T) + \delta\lambda(cxc^T - 1) = 0$$

$$\therefore KKT \begin{cases} 2A^TAx - 2A^Tb - \lambda cxc^T = 0 \\ cx - 1 = 0 \end{cases}$$

(2)

$$x = 1/2 * (A^TA)^{-1}(2A^Tb + \lambda c^T)$$

and $cx = 1$

$$\lambda = 2(R^TR)(1 - R^{-1}R^{-T}A^Tbc)$$
$$x = 0.5 * (R^{-1}R^{-T}(2A^Tb + 2R^TR - 2A^Tbc))$$

```
import numpy as np
from sympy import *
Q, R = np.linalg.qr(A)
X = 0.5*(R^{-1}R^{-T}(2A^Tb + 2R^TR - 2A^T b c))
```

Problem 2: Residual sensitivity

(1)

Equal to show $||r||\delta||r|| = r^T\delta r$
Equal to show $\delta(||r||^2) = 2r^T\delta r$

$$\begin{aligned}\delta(||r||^2) &= \delta(r^Tr) \\ &= (\delta r^T)r + r^T\delta r \\ &= 2r^T\delta r\end{aligned}$$

(2)

Equal to show $||r||\delta||r|| = -r^T\delta Ax$
And from (1), $r^T\delta r = -r^T\delta Ax$
Equal to show $\delta r = -\delta Ax$ ()
And $r = b - Ax$
$\therefore \delta r = 0 - \delta Ax$ is equal to (*).
$\square$