Work-06-10

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4-6

Because 2^{100} is too much, we'd like to use a random walk to try this whole sampling. This means we only need to sample from p like $p = E(p|p_{others})$ to reduce computing.

4-7

From row -> column.

-	00	01	02	10	11	12	20	21	22
00	1/2	1/4	-	1/4	-	-	-	-	-
01	1/8	3/8	1/4	-	1/4	-	-	-	-
02	-	1/8	5/8	-	-	1/4	-	-	-
10	1/8	-	-	1/2	1/4	-	1/8	-	-
11	-	1/8	-	1/8	3/8	1/8	-	1/8	-
12	-	-	1/8	-	1/8	5/8	-	-	1/8
20	-	-	-	1/4	-	-	1/2	1/4	-
21	-	-	-	-	1/4	-	1/8	3/8	1/4
22	-	-	-	-	-	1/4	-	1/8	5/8

4-22-1

Let size be n and m,

$\pi_x = \frac{1}{m+n}, p_{xy} = 1/n$

Mixing time is equal to $O(\frac{ln(1/\pi_{min})}{\Phi^2\epsilon^3})$

Let S be the subset of V, and $|S| \leq (n+m)/2$

$\therefore \Phi = min(\Phi(S)) = min\frac{\sum\sum \pi_x p_{xy}}{\pi(S)}$

choose m vertices from clique-2, then we know $\Phi \leq 1/(mn)$

$\therefore \Phi = \frac{1}{mn}$

$\therefore \text{mix-time} = O(\frac{ln(m+n)m^2n^2}{\epsilon^3})$

4-23-2

Random select for $\pi_x = 1/n$ and $p_{yx} = p_{xy} = 1/4$ for x adjcent to y.

$\therefore \Phi = min(\Phi(S)) = min \frac{\sum\sum \pi_x p_{xy}}{\pi(S)}$

where this S has edges of $2 + \frac{|S|(n-|S|)}{n}$

$\therefore \Phi = \frac{|S|(n-|S|)+2n}{n|S|}$

And $|S| = n/2$

$\therefore$ mix-time $= O(\frac{4n^2 ln(n)}{\epsilon^3 (n+8)^2})$

4-26

(1)

No boundary means $\sum_y p_{xy} = 1$ And $g_x = \sum_y g_y p_{xy}$ let g_x be the max of all g_i then $g_x = \sum_y g_y p_{xy} \le g_x \sum_y p_{xy} = g_x $ therefore, $g_x = g_y$ for any g_y, g_x therefore, $g_a = g_b$ for any a, b in graph that's $g_x = Constant$

(2)

$$To\ prove\ g_x = \sum_y g_y p_{xy}$$

$$g_x = \frac{q_x}{d_x} = \frac{\sum_y q_y p_{xy}}{d_x} \because \text{edges are equally likely}$$
$$\therefore g_x = \frac{\sum_y q_y p_{xy}}{d_y} = \sum_y g_y p_{xy}$$

□

(3)

Use the conlusion of (1) and (2), because it is undirected graph(2), no boundary(1)

Therefore $\frac{q_x}{d_x}$ is the harmonic function, which equals to constant C

$\therefore q_x = C \cdot d_x$

$\because \sum q_x = 1$ according to (1)

$\therefore C \sum_x d_x = 1$

For a specific graph, d_x is unique, then C is unique.

(4)

The stationary prob is $q = (q_x)$, x is the vertice, $q_x = \frac{d_x}{\sum_i d_i}$