

## 6.10

Assume  $k = \prod_i^n p_i^{\alpha_i}$

Construct vector  $x = (x_i)_i$  which is  $k$ -size

Here then we can pick  $a_i, b_i$  from  $p_i^{\alpha_i}$

And  $x_i = a_j * i + b_j \bmod p_j^{\alpha_j}$  for every  $j \in [n]$

Using the chinese remainder theorem,  $x_i = \sum_j^n (a_j * i + b_j) * \frac{k}{p_j^{\alpha_j}} * (\frac{p_j^{\alpha_j}}{k}) \bmod p_j^{\alpha_j} \bmod k$

Since  $0 \leq x_i \leq k-1$ ,  $x_i$  can be determined by above equation.

And  $k^2$  choose all  $a_1, b_1, a_2, b_2 \dots$ , it is next step to generate  $k^2$ .

Next, prove for  $x_i \neq x_j$  and for all  $0 \leq w, z \leq k-1$ , there is unique choice of  $a_i, b_i$  for all  $i$ , which  $x_i = w$ ,  $x_j = z$ .

Let  $w_j = w \bmod p_j^{\alpha_j}$ , and  $z_j = z \bmod p_j^{\alpha_j}$ , where  $w_j = a_j * m + b_j$  and  $z_j = a_j * n + b_j$ .

Then  $a_j$  and  $b_j$  can be unique.

Finally, for a certain  $i$  and  $j$ , each vector generated  $a_i, b_i$  for all  $i$ , which has relation to  $w, z$  in No.  $i$  position with No.  $j$  coordinates.

So,  $k^2$  where each pair of these values appears only once. \*\*\*\*