Homework 1
CS 259 @ SJTU
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Due Date: May 29, 2019
Submit Date: May 30, 2019

Problem 1: Constrained least squares

(1)

$$
\begin{aligned}
&\because \sum x_i = 1 \\
&\therefore c = (1 \cdots 1),\ cxc^T = 1 \\
&\mathcal{L} = ||Ax - b||^2 + \lambda(cxc^T - 1) \\
&\delta\mathcal{L} = \delta||Ax - b||^2 + \lambda c\delta xc^T + \delta\lambda(cx - 1) \\
&\delta\mathcal{L} = \delta x^T(2A^TAx - 2A^Tb - \lambda cxc^T) + \delta\lambda(cxc^T - 1) = 0
\end{aligned}
$$

$$
\therefore KKT \begin{cases} 2A^TAx - 2A^Tb - \lambda cxc^T = 0 \\ cx - 1 = 0 \end{cases}
$$

(2)

$$x = 1/2 * (A^TA)^{-1}(2A^Tb + \lambda c^T)$$

and $cx = 1$

$$
\begin{aligned}
\lambda &= 2(R^TR)(1 - R^{-1}R^{-T}A^Tbc) \\
x &= 0.5 * (R^{-1}R^{-T}(2A^Tb + 2R^TRc^T - 2A^Tbcc^T)) \\
x &= R^{-1}R^{-T}A^Tb + c^T - R^{-1}R^{-T}A^Tb
\end{aligned}
$$

```python
import numpy as np
from sympy import *
Q, R = np.linalg.qr(A)
c = [1]*n
X = R.I * (R.T).I * A.T * b + c.T - R.I * (R.T).I * A.T * b
```

Problem 2: Residual sensitivity

(1)

Equal to show $||r||\delta||r|| = r^T\delta r$
Equal to show $\delta(||r||^2) = 2r^T\delta r$

$$
\begin{aligned}
\delta(||r||^2) &= \delta(r^T r) \\
&= (\delta r^T) r + r^T \delta r \\
&= 2 r^T \delta r
\end{aligned}
$$

(2)

Equal to show $||r||\delta||r|| = -r^T \delta A x$
And from (1), $r^T \delta r = -r^T \delta A x$
Equal to show $\delta r = -\delta A x$ ()
And $r = b - Ax$
$\therefore \delta r = 0 - \delta A x$ is equal to (*).
$\square$