Homework 1
CS 259 @ SJTU
Prof. David Bindel
TA. Zhou Fan

Xun Gong(517020910141
ACM Class, Zhiyuan College, SJTU
Due Date: May 29, 2019
Submit Date: June 8, 2019

Problem 1: Constrained least squares

(1)

$$
\begin{aligned}
&\because \sum x_i = 1 \\
&\therefore c = (1 \cdots 1),\ cxc^T = 1 \\
&\mathcal{L} = ||Ax - b||^2 + \lambda(cxc^T - 1) \\
&\delta\mathcal{L} = \delta||Ax - b||^2 + \lambda c\delta xc^T + \delta\lambda(cx - 1) \\
&\delta\mathcal{L} = \delta x^T(2A^TAx - 2A^Tb - \lambda cxc^T) + \delta\lambda(cxc^T - 1) = 0
\end{aligned}
$$

$$
\therefore KKT \begin{cases} 2A^TAx - 2A^Tb - \lambda cxc^T = 0 \\ cx - 1 = 0 \end{cases}
$$

(2)

$$x = 1/2 * (A^TA)^{-1}(2A^Tb + \lambda c^T)$$

and $cx = 1$

$$\lambda = 2(R^TR)(1 - R^{-1}R^{-T}A^Tbc)$$

$$x = 0.5 * (R^{-1}R^{-T}(2A^Tb + 2R^TRc^T - 2A^Tbcc^T))$$

$$x = R^{-1}R^{-T}A^Tb + c^T - R^{-1}R^{-T}A^Tb$$

```
print('Hello world!')
```

```
% \begin{lstlisting}[style = python]
import numpy as np
from sympy import *
Q, R = np.linalg.qr(A)
c = [1]*n
X = R.I * (R.T).I * A.T * b + c.T -  R.I * (R.T).I * A.T * b
% \end{lstlisting}
```

Problem 2: Residual sensitivity

(1)

Equal to show $||r||\delta||r|| = r^T\delta r$
Equal to show $\delta(||r||^2) = 2r^T\delta r$

$$\begin{aligned}\delta(||r||^2) &= \delta(r^T r)\\ &= (\delta r^T)r + r^T \delta r\\ &= 2r^T \delta r\end{aligned}$$

(2)

Equal to show $||r||\delta||r|| = -r^T \delta Ax$
And from (1), $r^T \delta r = -r^T \delta Ax$
Equal to show $\delta r = -\delta Ax$ ()
And $r = b - Ax$
$\therefore \delta r = 0 - \delta Ax$ is equal to (*).
$\square$