Homework requirements

- Digital format (can be typeset or photos) is preferred
- Submit by next lecture
- Each homework 10 points; 1 point deducted for each day of delay

Contact information

- Course Email:
  - optinml2019@163.com
- TA:
  - 陈程： jackchen1990@gmail.com (submit your homework here)
- zlin@pku.edu.cn
- http://www.cis.pku.edu.cn/faculty/vision/zlin/zlin.htm

Homework (5)

1.

a) Solve the optimization problem

$$\min_{\mathbf{x}} f(x_1, x_2) := 2x_1 + 3x_2, \quad s.t. \quad \sqrt{x_1} + \sqrt{x_2} = 5,$$

using Lagrange multipliers.

b) Find all its KKT points. Do they all correspond to local minima?

Homework (5)

2. With $f(\mathbf{x}) := x_1^2 + x_2^2$ for $\mathbf{x} \in \mathbb{R}^2$ consider

$$(P)\begin{cases} \min\limits_{\mathbf{x}} f(\mathbf{x}) \\ -x_2 \leq 0 \\ x_1^3 - x_2 \leq 0 \\ x_1^3(x_2 - x_1^3) \leq 0 \end{cases}.$$

a) Find all its KKT points. Do they all correspond to local minima?

b) Check whether SCQ holds.

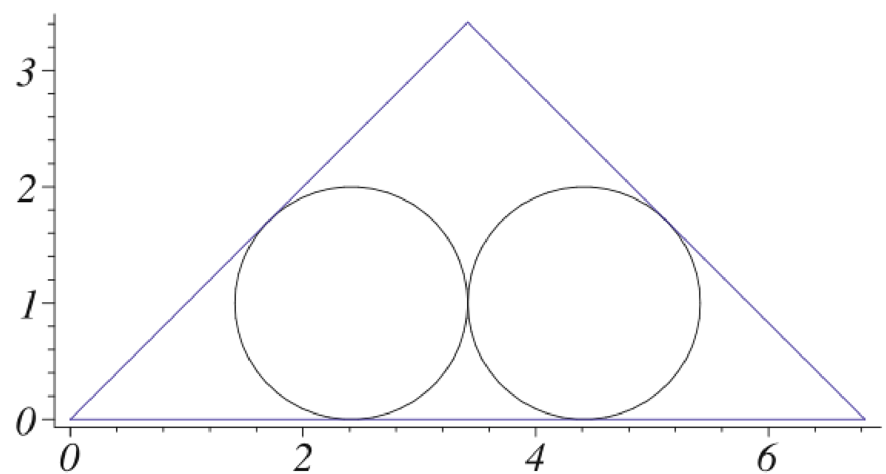
c) Find its dual function, with the domain specified.

Homework (5)

3. Determine a triangle with minimal area containing two disjoint disks with radius 1. Without loss of generalization, let $(0,0)$, $(x_1,0)$ and (x_2,x_3) with $x_1, x_3 \geq 0$ be the vertices of the triangle; (x_4,x_5) and (x_6,x_7) denote the centers of the disks.

   a) Formulate this problem as a minimization problem in terms of seven variables and nine constraints.

   b) $\mathbf{x}^* = (4+2\sqrt{2}, 2+\sqrt{2}, 2+\sqrt{2}, 1+\sqrt{2}, 1, 3+\sqrt{2}, 1)^T$ is a solution of this problem; calculate the corresponding Lagrange multipliers $\boldsymbol{\lambda}^*$, such that the KKT conditions are fulfilled.



Homework (5)

4. Find the point $\mathbf{x} \in \mathbb{R}^2$ that lies closest to the point $\mathbf{p} := (2,3)^T$ under the constraints $g_1(\mathbf{x}) := x_1 + x_2 \leq 0$ and $g_2(\mathbf{x}) := x_1^2 - 4 \leq 0$.

a) Verify that the problem is convex and fulfills SCQ.

b) Determine the KKT points by differentiating between three cases: none is active, exactly the first one is active, exactly the second one is active.

c) Find its dual function, with the domain specified.

Homework (5)

5. Given a support vector machine:

$$\min_{\mathbf{w},\beta} \frac{1}{2}\|\mathbf{w}\|^2,$$
$$s.t.\ y_i(\langle \mathbf{w}, \mathbf{x}_i \rangle + \beta) \geq 1, (i = 1, \cdots, m).$$

a) Check whether the problem fulfills SCQ. What does SCQ mean in this scenario?

b) Find its dual function, with the domain specified.

Homework (5)

6. Find the dual problem of the following problems and check whether the strong dualities hold:

a)

$$\min_x \; x^2 + 1$$
$$s.t. \; (x-2)(x-4) \le 0,$$

b)

$$\min_{\mathbf{x}} \; \mathbf{c}^T \mathbf{x}$$
$$s.t. \; f(\mathbf{x}) \le 0,$$

with $\mathbf{c} \neq \mathbf{0}$.

c) (Regularized Empirical Risk Minimization)

$$\min_{\mathbf{x},\mathbf{y}} F(\mathbf{x}) \equiv \frac{1}{n} \sum_{i=1}^{n} \phi_i(y_i) + \frac{\mu}{2} \|\mathbf{x}\|^2,$$
$$s.t. \; y_i = \mathbf{a}_i^T \mathbf{x}.$$