Homework 2

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June 3, 2019

CS389: Foundations of Data Science @ SJTU ACM Class, Zhiyuan College, SJTU
Prof. **John Hopcroft** Due Date: June 03, 2019

Exercise 3.12

(1)

$$||A_k||_F^2 = \sum_{i=1}^{k} Av_iv_i^T = \sum_{i=1}^{k} \sigma_i^2$$

(2)

$$||A_k||_2^2 = max|A_kx| = \sigma_1$$

(3)

$$||A - A_k||_F^2 = \sum_{i=k+1}^{n} Av_iv_i^T = \sum_{i=k+1}^{n} \sigma_i^2$$

(4)

From Lemma 3.8,

$$||A - A_k||_2^2 = \sigma_{k+1}$$

Exercise 3.13

$\because A$ is symmetric
$\therefore A^T = A, A = Q\Sigma Q^T$, eigen-decomp
$\therefore A^2 = A^T A = V\Sigma^2 V^T = V\Sigma V^T V\Sigma V^T$
$\therefore A = V\Sigma V^T$, V is unique.
Use the same rule, $A = U\Sigma U^T$
$\therefore U = V$
$\therefore u_i = v_i$

Exercise 3.16

(1)

Estimate [0.00390622, 0.99999237]

(2)

$$
\begin{aligned}
B &= \begin{pmatrix} 4 & 0 \\ 0 & 16 \end{pmatrix} \\
\Lambda &= \begin{pmatrix} 4 & 0 \\ 0 & 16 \end{pmatrix} \\
\therefore \Sigma &= \begin{pmatrix} 4 & 0 \\ 0 & 2 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \\
\therefore V &= \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \; U = \begin{pmatrix} -0.5 & -0.5 \\ -0.5 & -0.5 \\ 0.5 & -0.5 \\ 0.5 & 0.5 \end{pmatrix}
\end{aligned}
$$

(3)

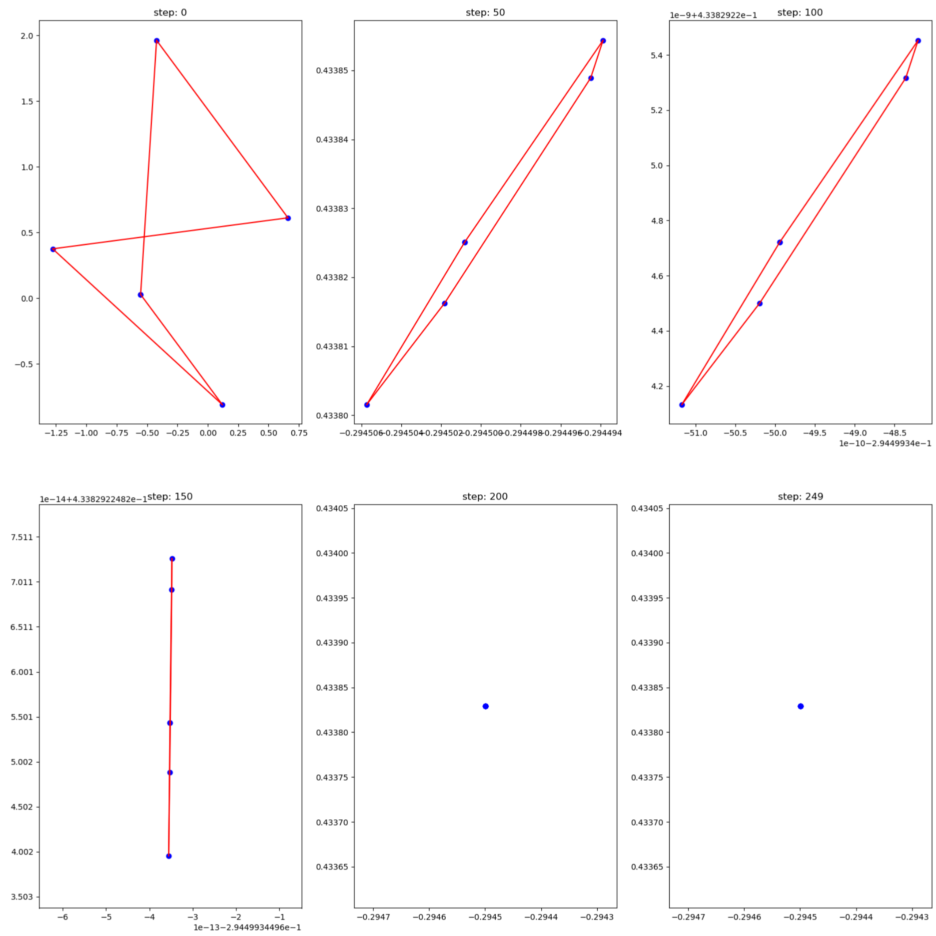
v_1 represents the likelhood of each resturant whether people like resturants. u_1 represents the likelihood of each people like resturants or dislikes. $\sigma_1 - \sigma_2$ represents the differ of customer of less like and most like.

Exercise 3.18

(1)

in next question.

(2)



It converges to a dot where places all dots.

(3)

It converges to a line where places all dots.

(4)

$$A = \begin{pmatrix} 0.5 & 0.5 & 0 & \cdots & 0 \\ 0 & 0.5 & 0.5 & \cdots & 0 \\ & & \cdots & & \\ 0.5 & 0 & \cdots & 0 & 0.5 \end{pmatrix}$$

(5)

For $n = 5$,

$$\sigma_1 = 1 \sigma_2 = 0.80901699 v_1 = (-0.44721360.195439510.60150096 - 0.371748030.51166727)^T$$

Normally,

$$\sigma_1 = 1 \sigma_2 < 1$$

The rate of converge is $\sigma_1 = 1$, so direction of v_1 doesn't change, but σ_i converge to 0, so finally converge to v_1.

(6)

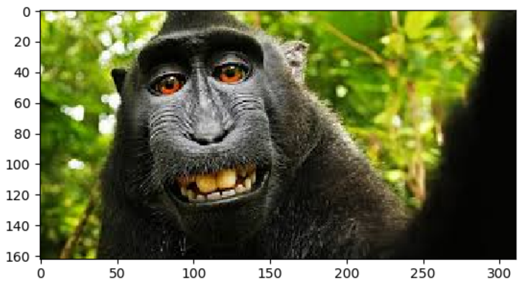
when n is odd, the dots will expands to a n-polygon.

Exercise 3.28

(1)

code

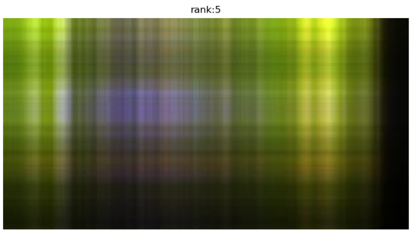
origin

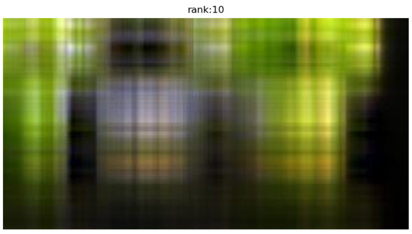


rank	F-norm
1	97.3
2	98.1
4	98.8
16	99.8

Table 1: Feature id

build

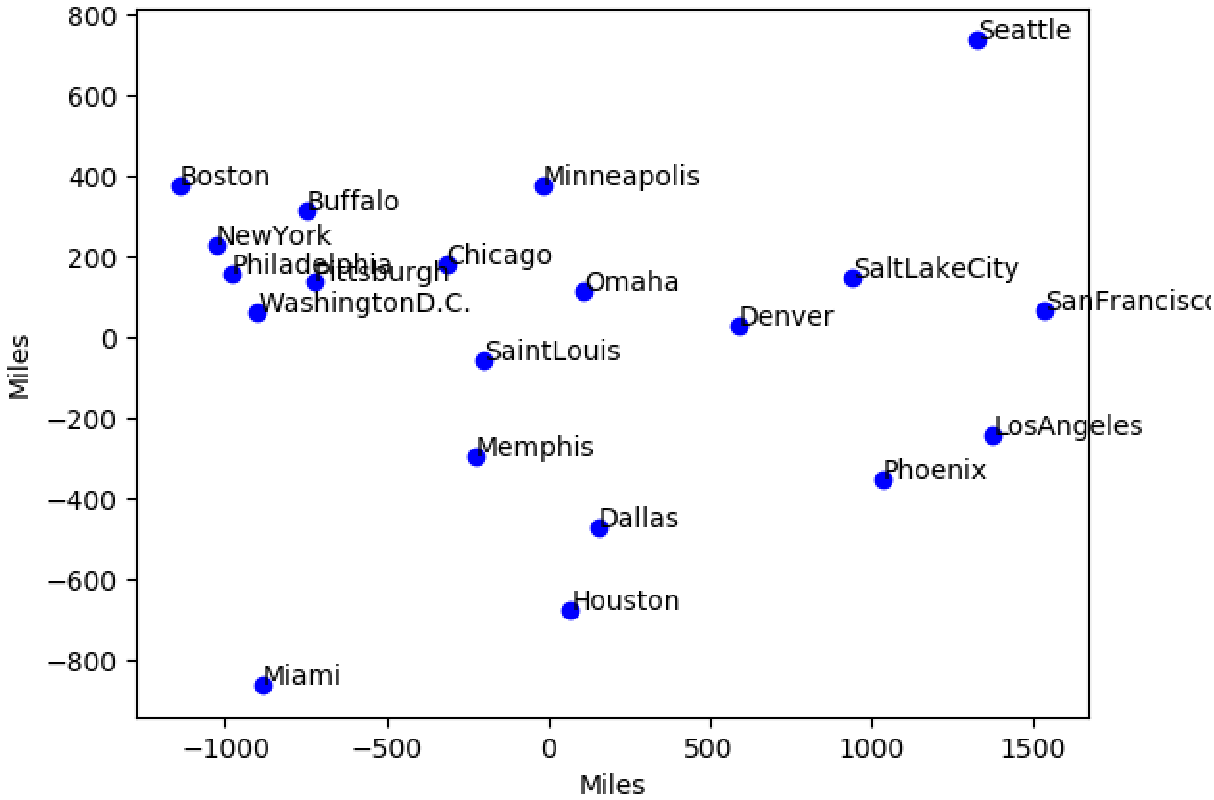




(2)

Exercise 3.32

(1)



(2)

We can do this.

It is a almost 4-dimensional space and needed to truncked to 3-dimensional. Determine how the quality of reconstruction varies when you reduce points to distances using different metrics.

```
rng default   % Set the seed for reproducibility
A = [normrnd(0,1,10,3) normrnd(0,0.1,10,1)];
B = randn(4,4);
X = A*B;
D = pdist(X,'euclidean'); % distance
Y = cmdscale(D);
maxerr2 = max(abs(pdist(X)-pdist(Y(:,1:2))))
maxerr3 = max(abs(pdist(X)-pdist(Y(:,1:3))))
maxerr4 = max(abs(pdist(X)-pdist(Y)))

D = pdist(X,'cityblock');
```

```
[Y,e] = cmdscale(D);
maxerr = max(abs(pdist(X)-pdist(Y)))
```