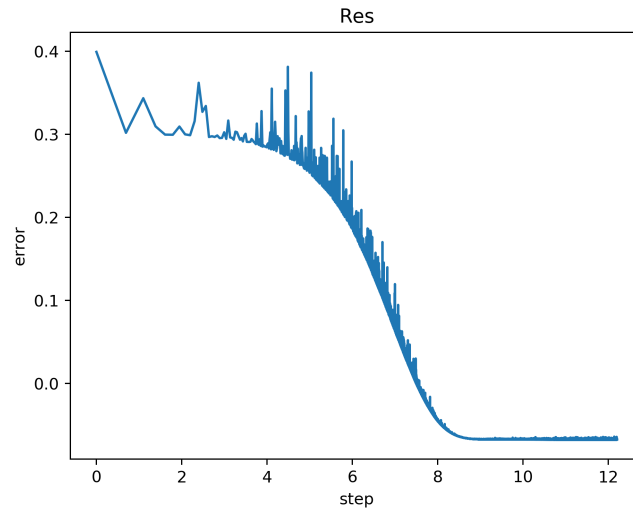


Problem 1: Stochastic gradient descent

(1)



Code is in attachment. 

(2)

Rule: Stochastic gradient descent is converge to 0.

Problem 2: Least 2p-norm regression

(1)

$$f(x) = \begin{pmatrix} \frac{1}{\sqrt{p}} r_1^p \\ \vdots \\ \frac{1}{\sqrt{p}} r_n^p \end{pmatrix}, \quad r_i = (b - Ax)_i = b_i - A_{i,:}x$$

$$J(x) = -\sqrt{p} \begin{pmatrix} r_1^{p-1} a_{1,1} & \cdots & r_1^{p-1} a_{1,n} \\ \vdots & \ddots & \vdots \\ r_n^{p-1} a_{n,1} & \cdots & r_n^{p-1} a_{n,n} \end{pmatrix}$$

(2)

Equal to show Gauss-Newton Step $\equiv \min ||D^k(Ap^k - r^k)||^2$

$$\begin{aligned} \mathcal{L} &= ||D^k(Ap^k - b + Ax)||^2 \\ \delta \mathcal{L} &= \delta(D^k(Ap^k - b + Ax))^T (D^k(Ap^k - b + Ax)) \\ \delta \mathcal{L} &= 2\delta x^T D^2(A^T Ax - A^T b + A^T Ap) \\ \therefore \delta \mathcal{L} &= 0 \\ \therefore 0 &= A^T Ax - A^T b + A^T Ap \\ p &= -(A^T A)^{-1}(A^T Ax - A^T b) \end{aligned}$$

What is interesting, $p = -[f'^T f']^{-1} \delta \phi$
 \therefore These 2 question has same solution.

□