

# Homework requirements

- Digital format (can be typeset or photos) is preferred
- Submit by next lecture
- Each homework 10 points; 1 point deducted for each day of delay

# Contact information

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# Homework (4)

1. Use Moreau decomposition to find the proximal mapping of

(a)  $\|\mathbf{x}\|_1$ .

(b)  $\|\mathbf{X}\|_*$ .

2. Use Moreau decomposition to prove that  $\mathbf{x} = P_L(\mathbf{x}) + P_{L^\perp}(\mathbf{x})$ , where  $L$  is a subspace and  $L^\perp$  is its orthogonal complement.

3. Consider the following problem of minimizing the Rosenbrock function:

$$\min_{\mathbf{x}} f(\mathbf{x}) = \sum_{i=2}^n [100(x_i - x_{i-1}^2)^2 + (1 - x_{i-1})^2].$$

For value of  $n = 50$  and starting from  $\mathbf{x}_0 = [-1.2, 1.0, -1.2, 1.0, \dots]^T$ , solve this problem using the following algorithms:

# Homework (4)

- 1) Gradient descent with exact line search;
- 2) Gradient descent with backtracking line search;
- 3) Steepest descent in  $\ell_1$  norm;
- 4) Steepest descent in  $\ell_2$  norm;
- 5) Damped Newton's method;
- 6) Levenberg-Marquardt method;
- 7) Gauss-Newton method;
- 8) Conjugate gradient method, using the Hestenes-Stiefel, Polak-Ribiere, and Fletcher-Reeves formula, respectively.

Plot the logarithm residual objective function value  $\log_{10}(f - f^*)$  (obviously  $f^* = 0$ ) versus the iteration number. Hand in your codes and a report showing how the parameters (including stopping criteria) are chosen and the figures.