Homework requirements

- Digital format (can be typeset or photos) is preferred
- Submit this homework and ALL past homeworks by 23:59 of next Wednesday (July 24th).
- Each homework 10 points; 1 point deducted for each day of delay

Contact information

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Homework (6)

1. [Equality constrained entropy maximumization.] We consider the equality constrained entropy maximization problem

$$\min_{\mathbf{x}} f(\mathbf{x}) = \sum_{i=1}^{n} x_i \log x_i, \quad s.t. \quad \mathbf{A}\mathbf{x} = \mathbf{b}.$$

Use the following methods to solve it at $p = 100$ and $n = 500$ (you may generate $\mathbf{A}$ and $\mathbf{b}$ randomly):

- Direct projected gradient, with inexact line search.
- Dual approach.

Write a report to describe your settings and compare their performance (numerical accuracy vs. iteration number). Codes should also be handed in.

Homework (6)

2. Consider the problem

$$\min_{\mathbf{x}} \ \frac{1}{2}\|\mathbf{x}\|_2, \quad s.t. \quad \mathbf{A}\mathbf{x} = \mathbf{b}, \tag{1}$$

where $\mathbf{A} \in \mathbb{R}^{m\times n}, \mathbf{b} \in \mathbb{R}^m, m \le n$, and $\operatorname{rank} \mathbf{A} = m$.

Randomly generate $\mathbf{A} \in \mathbb{R}^{200\times 300}$ and $\mathbf{b} \in \mathbb{R}^{200}$ and solve problem (1) numerically by the penalty method. Hand in your code and report.

3. Consider:

$$\min_{\mathbf{x}\in\mathbb{R}^N} \|\mathbf{y} - \mathbf{D}\mathbf{x}\|_2^2, \quad s.t. \quad \|\mathbf{x}\|_? \le 1,$$

where $\|\mathbf{x}\|_?$ can be either the ℓ_1 norm or the ℓ_∞ norm. Randomly generate $\mathbf{D} \in \mathbb{R}^{200\times 300}$ and $\mathbf{y} \in \mathbb{R}^{200}$ and use Frank-Wolfe algorithm to solve it, for both ℓ_1 norm and ℓ_∞ norm. Further compare F-W algorithm with the projected gradient descent in convergence speed (objective function value vs. iteration number). Hand in your code and report.

Homework (6)

4. Use LADMPSAP to solve a graph construction problem:

$$\min_{\mathbf{Z},\mathbf{E}} ||\mathbf{Z}||_* + \lambda||\mathbf{E}||_{2,1}, \quad \text{s.t.} \quad \mathbf{D} = \mathbf{D}\mathbf{Z} + \mathbf{E}, \mathbf{Z}^T\mathbf{1} = \mathbf{1}, \mathbf{Z} \geq \mathbf{0}, \tag{1}$$

where $\mathbf{1}$ is an all-one vector. Randomly generate $\mathbf{D} \in \mathbb{R}^{200\times 300}$. Hand in your code and report.

Homework (6)

5. Use block coordinate descent to solve the low-rank matrix completion problem:

$$\min_{\mathbf{U},\mathbf{V},\mathbf{A}} \frac{1}{2}\|\mathbf{U}\mathbf{V}^T - \mathbf{A}\|_F^2, \quad s.t. \quad \mathcal{P}_\Omega(\mathbf{A}) = \mathcal{P}_\Omega(\mathbf{D}),$$

where $\mathcal{P}_\Omega(\cdot)$ is an operator that extracts entries of a matrix whose indices are in Ω and sets the remaining entries zeros.

Randomly generate $\mathbf{D} = \mathbf{U}_0\mathbf{V}_0^T$ and Ω, where $\mathbf{U}_0 \in \mathbb{R}^{200\times 5}$, $\mathbf{V}_0 \in \mathbb{R}^{300\times 5}$ and $|\Omega| = 0.1 \times 200 \times 300$. Hand in your code and report showing your settings and the difference $\|\mathbf{A}^* - \mathbf{D}\|_2$ where $\mathbf{A}^*$ is the optimal solution.