标题标题标题标题 Title 标题标题标题 长标题长标题长标题长标题

副标题副标题 Subtitle 副标题副标题

Htallone[1]　　作者 X[2]

[1] 北京航空航天大学 宇航学院

[2]xxxxxxx

2018年6月9日

目录

引言：伴随法

伴随法，又称为伴随仿真技术, is a useful computerized tool for the analysis of *linear time-varying systems*.

With the adjoint method, *error budgets* or sensitivities of the LTV system due to all disturbance input terms can be automatically generated.

In this paper, new interpretations of the adjoint method are achieved by using the *adjoint definition equation* in state-space form.

北京航空航天大学
BEIHANG UNIVERSITY

引言：伴随概念

Suppose $\boldsymbol{G} : \mathcal{U} \mapsto \mathcal{Y}$ is a linear system, $\mathcal{U}$ and $\mathcal{Y}$ are Hilbert spaces. The adjoint of $\boldsymbol{G}$ is the linear system $\boldsymbol{G}^* : \mathcal{Y} \mapsto \mathcal{U}$ such that

$$\langle \boldsymbol{G}u, y\rangle_{\mathcal{Y}} = \langle u, \boldsymbol{G}^*y\rangle_{\mathcal{U}} \quad \forall u \in \mathcal{U}, \ \forall y \in \mathcal{Y}, \tag{1}$$

where $\langle \cdot, \cdot \rangle$ denotes the *inner product* defined by

$$\langle f, g\rangle = \int_{t_0}^{t_f} g^{\mathrm{T}}(t)f(t)\mathrm{d}t, \quad f, g \in \mathcal{U} \text{ or } \mathcal{Y} \tag{2}$$

$$\langle \begin{bmatrix} f_0 \\ f \end{bmatrix}, \begin{bmatrix} g_0 \\ g \end{bmatrix}\rangle = g_0^{\mathrm{T}}f_0 + \int_{t_0}^{t_f} g^{\mathrm{T}}(t)f(t)\mathrm{d}t, \quad \begin{bmatrix} f_0 \\ f \end{bmatrix}, \begin{bmatrix} g_0 \\ g \end{bmatrix} \in \mathbb{R}^n \oplus \mathcal{U} \text{ or } \mathbb{R}^n \oplus \mathcal{Y} \tag{3}$$

北京航空航天大学
BEIHANG UNIVERSITY

伴随定义方程

$$p^{\mathrm{T}}(t_{\mathrm{f}})x(t_{\mathrm{f}}) - p^{\mathrm{T}}(0)x(0) = \int_0^{t_{\mathrm{f}}} -r^{\mathrm{T}}(t)y(t) + u^{\mathrm{T}}(t)q(t)\mathrm{d}t. \tag{4}$$

线性系统

$$\boldsymbol{G}: \mathbb{R}^n \oplus \mathcal{U} \mapsto \mathbb{R}^n \oplus \mathcal{Y}; \begin{bmatrix} x_0 \\ u \end{bmatrix} \mapsto \begin{bmatrix} x_{\mathrm{f}} \\ y \end{bmatrix} \tag{5}$$

伴随系统

$$\boldsymbol{G}^*: \mathbb{R}^n \oplus \mathcal{Y} \mapsto \mathbb{R}^n \oplus \mathcal{U}; \begin{bmatrix} p_{\mathrm{f}} \\ r \end{bmatrix} \mapsto \begin{bmatrix} p_0 \\ q \end{bmatrix} \tag{6}$$

$$\left\langle \boldsymbol{G} \begin{bmatrix} x_0 \\ u \end{bmatrix}, \begin{bmatrix} p_{\mathrm{f}} \\ r \end{bmatrix} \right\rangle_{\mathbb{R}^n \oplus \mathcal{Y}} = \left\langle \begin{bmatrix} x_0 \\ u \end{bmatrix}, \boldsymbol{G}^* \begin{bmatrix} p_{\mathrm{f}} \\ r \end{bmatrix} \right\rangle_{\mathbb{R}^n \oplus \mathcal{U}} \quad \forall \begin{bmatrix} x_0 \\ u \end{bmatrix} \in \mathbb{R}^n \oplus \mathcal{U}, \ \forall \begin{bmatrix} p_{\mathrm{f}} \\ r \end{bmatrix} \in \mathbb{R}^n \oplus \mathcal{Y}, \tag{7}$$

$$\left\langle \begin{bmatrix} f_0 \\ f \end{bmatrix}, \begin{bmatrix} g_0 \\ g \end{bmatrix} \right\rangle = g_0^{\mathrm{T}} f_0 + \int_{t_0}^{t_{\mathrm{f}}} g^{\mathrm{T}}(t)f(t)\mathrm{d}t. \tag{8}$$
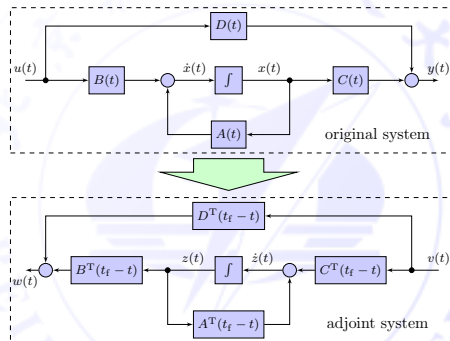
北京航空航天大学
BEIHANG UNIVERSITY

伴随系统总结

线性系统

$$\left.\begin{array}{l}\dot{x}(t) = A(t)x(t) + B(t)u(t), \\ y(t) = C(t)x(t) + D(t)u(t)\end{array}\right\}$$

伴随系统

$$\left.\begin{array}{l}\dot{z}(t) = A^{\mathrm{T}}(t_{\mathrm{f}} - t)z(t) + C^{\mathrm{T}}(t_{\mathrm{f}} - t)v(t), \\ w(t) = B^{\mathrm{T}}(t_{\mathrm{f}} - t)z(t) + D^{\mathrm{T}}(t_{\mathrm{f}} - t)v(t)\end{array}\right\}$$

$$z^{\mathrm{T}}(0)x(t_{\mathrm{f}}) - z^{\mathrm{T}}(t_{\mathrm{f}})x(0) = \int_0^{t_{\mathrm{f}}} -v^{\mathrm{T}}(t_{\mathrm{f}} - t)y(t) + u^{\mathrm{T}}(t)w(t_{\mathrm{f}} - t)\mathbf{d}t \qquad (9)$$

北京航空航天大学
BEIHANG UNIVERSITY

结　论

- The interpretation of adjoint simulation was achieved in a new and universal way, by using the adjoint definition equation in state-space form;

- The adjoint technique in covariance analysis was also derived;

- The adjoint concept is fundamental in mathematics, and naturally arises or is widely used in optimization, optimal control, stability analysis, navigation problems, circuit analysis, etc.

北京航空航天大学
BEIHANG UNIVERSITY

Thank You!

Questions?

Htallone
Email: Htallone@buaa.edu.cn

附　　录

北京航空航天大学
BEIHANG UNIVERSITY

Basic Definitions and Properties of LTV

Consider the linear system

$$\boldsymbol{G} : \mathcal{U} \mapsto \mathcal{Y}$$
$$: u \mapsto y = \boldsymbol{G}u, \tag{10}$$

described by state-space equations

$$\dot{x}(t) = A(t)x(t) + B(t)u(t), \quad x(t_0) = x_0 \in \mathbb{R}^n$$
$$y(t) = C(t)x(t) + D(t)u(t). \tag{11}$$

We are primarily concerned with the linear system $\boldsymbol{G}$ in the finite-horizon case. In (10), $\mathcal{U}$ and $\mathcal{Y}$ are finite-horizon Lebesgue 2-spaces $\mathcal{L}_2[t_0, t_f]$, and $u \in \mathcal{U}$, $y \in \mathcal{Y}$. In (11), $u(t) \in \mathbb{R}^m$ is the input vector, $x(t) \in \mathbb{R}^n$ is the state vector, $y(t) \in \mathbb{R}^p$ is the output vector; $A(t), B(t), C(t)$ and $D(t)$ are continuous real matrix valued functions of time with appropriate dimensions.

Let $\Phi(t, \tau)$ be the *state transition matrix* associated with system (11), which has the following properties

$$\frac{\mathrm{d}}{\mathrm{d}t}\Phi(t, \tau) = A(t)\Phi(t, \tau), \ \Phi(\tau, \tau) = I, \tag{12a}$$

$$\Phi^{-1}(t, \tau) = \Phi(\tau, t), \tag{12b}$$

in which I denotes the identity matrix. Then the solution to (11) is

$$x(t) = \Phi(t, t_0)x(t_0) + \int_{t_0}^{t} \Phi(t, \tau)B(\tau)u(\tau)\mathrm{d}\tau, \tag{13a}$$

$$y(t) = C(t)\Phi(t, t_0)x(t_0) + \int_{t_0}^{t} C(t)\Phi(t, \tau)B(\tau)u(\tau)\mathrm{d}\tau + D(t)u(t). \tag{13b}$$

In (13b), the first term is known as the *zero-input response*, the other terms the *zero-state response*. The zero-state response may be represented by the integral operator

$$y(t) = \int^{t_f} H(t, \tau)u(\tau)\mathrm{d}\tau, \tag{14}$$
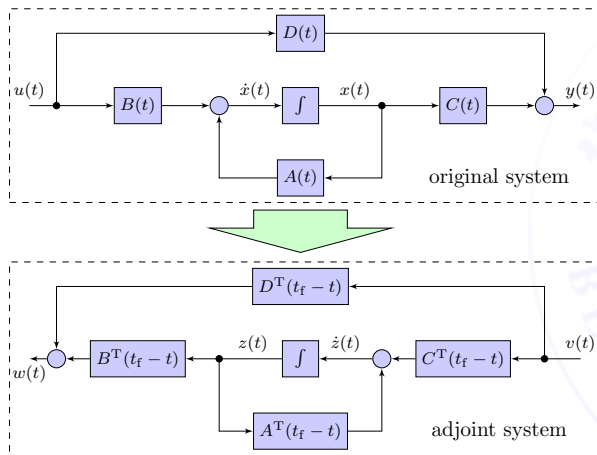
Construction Rules for Adjoint System

图 1: Construction of adjoint system from original system

The block diagrams of the orignal system and the adjoint system are shown in Figure. 1:, which can be used to explain the general construction rules [Laning; Zarchan; Yanushevsky] of an adjoint system from the orignal system:

1. Replace t by $t_f - t$ in the arguments of all variable coefficients where t_f is the final time.

2. Reverse all signal flow, redefine branch points (•) as sum points ($\bigcirc$), and vice versa.