Appendix: Proofs of Propositions and Lemmas

Appendix A. Proof of Lemma 1

Proof:

Before the shift, the profit of the generator j is $\hat{W}_{s-j} = \int_{\hat{s}_j^{-1}(0)}^{\hat{s}_j^{-1}(s_j(p^*))} \hat{s}_j(p)dp + \left(p^* - \hat{s}_j^{-1}\left(s_j(p^*)\right)\right) s_j(p^*)$.

After the shift, its profit $\hat{W}_{s-j}^{'} = \int_{\hat{s}_j^{-1}(0)}^{\hat{s}_j^{-1}\left(s_j^{'}(p^*+\Delta p)\right)} \hat{s}_j(p)dp + \left((p^* + \Delta p) - \hat{s}_j^{-1}\left(s_j^{'}(p^* + \Delta p)\right)\right) s_j^{'}(p^* + \Delta p)$. Therefore, the change in its profits is

$$
\begin{aligned}
\hat{W}_{s-j}^{'} - \hat{W}_{s-j} &= \Delta p s_j^{'}(p^* + \Delta p) - [\ \left(p^* - \hat{s}_j^{-1}\left(s_j(p^*)\right)\right)\left(s_j(p^*) - s_j^{'}(p^* + \Delta p)\right) \\
&\quad + \int_{\hat{s}_j^{-1}\left(s_j^{'}(p^*+\Delta p)\right)}^{\hat{s}_j^{-1}(s_j(p^*))} \left(\hat{s}_j(p) - s_j^{'}(p^* + \Delta p)\right) dp\ ] \\
&= \Delta W_1 - \Delta W_2
\end{aligned} \tag{1}
$$

where ΔW_1 and ΔW_2 are the increased and decreased profits after the shift of $s_j(p)$, respectively. From the meaning of "offset", when $\Delta p \to 0$, there is $s_j^{'}(p^* + \Delta p) \to s_j(p^*)$, $\hat{s}_j^{-1}\left(s_j^{'}(p^* + \Delta p)\right) \to \hat{s}_j^{-1}\left(s_j(p^*)\right)$, and $s_j(p^*) = \hat{s}_j\left(\hat{s}_j^{-1}\left(s_j(p^*)\right)\right)$. Therefore,

$$
\begin{aligned}
\hat{W}_{s-j}^{'} - \hat{W}_{s-j} &= \Delta p s_j^{'}(p^* + \Delta p) - \left(p^* - \hat{p}_j^*\right)\left(s_j(p^*) - s_j^{'}(p^* + \Delta p)\right) + o(\Delta p) \\
&= \Delta p\left(d_0 - s_{-j}(p^* + \Delta p)\right) - \left(p^* - \hat{p}_j^*\right)\left(s_{-j}(p^* + \Delta p) - s_{-j}(p^*)\right) + o(\Delta p)
\end{aligned} \tag{2}
$$

where $\hat{p}_j^* = \hat{s}_j^{-1}\left(s_j(p^*)\right)$, $o(\Delta p)$ represents second or higher order infinitesimal of Δp. Clearly, when $\hat{W}_{s-j}^{'} - \hat{W}_{s-j} > 0$, a rightward or downward shift of $s_j(p)$ for generator j would be advantageous, implying the possibility of exercising market power. Here, $\Delta p\left(d_0 - s_{-j}(p^* + \Delta p)\right) > \left(p^* - \hat{p}_j^*\right)\left(s_{-j}(p^* + \Delta p) - s_{-j}(p^*)\right) + o(\Delta p)$. So, let $\Delta p \to 0$, we can get $\frac{s_j(p^*)}{p^* - \hat{p}_j^*} > \left.\frac{ds_{-j}(p)}{dp}\right|_{p=p^*}$. Also, when $\hat{W}_{s-j}^{'} - \hat{W}_{s-j} < 0$, i.e., $\frac{s_j(p^*)}{p^* - \hat{p}_j^*} < \left.\frac{ds_{-j}(p)}{dp}\right|_{p=p^*}$, a leftward or upward shift of $s_j(p)$ for generator j would be advantageous, implying the possibility of mitigating market power. And while $\hat{W}_{s-j}^{'} - \hat{W}_{s-j} = 0$, i.e., $\frac{s_j(p^*)}{p^* - \hat{p}_j^*} = \left.\frac{ds_{-j}(p)}{dp}\right|_{p=p^*}$, not shifting $s_j(p)$ is advantageous for generator j. In summary, the market will reach an equilibrium when $\forall j \in \mathbf{N}$, $\hat{W}_{s-j}^{'} - \hat{W}_{s-j} = 0$ holds. ■

Appendix B. Proof of Lemma 2

Proof: From Lemma 1, when the bidding of SEM reaches an equilibrium, $\forall j \in \mathbf{N}$, $s_j(p^*) = \left(p^* - \hat{p}_j^*\right) \times \left.\frac{ds_{-j}(p)}{dp}\right|_{p=p^*}$. Write it to a matrix form as $\mathrm{S}(p^*) = \mathrm{P}(p^*)\mathrm{R}\tilde{\mathrm{S}}(p^*)$, where p^* is the ECP, the vector $\mathrm{S}(p^*) = \left[s_1(p^*),\ s_2(p^*), \cdots, s_n(p^*)\right]^{\mathrm{T}}$, $\tilde{\mathrm{S}}(p^*) = \left.\frac{d\mathrm{S}(p)}{dp}\right|_{p=p^*} = \left.\left[\frac{ds_1(p)}{dp}, \frac{ds_2(p)}{dp}, \cdots, \frac{ds_n(p)}{dp}\right]^{\mathrm{T}}\right|_{p=p^*}$, the diagonal matrix $\mathrm{P}(p^*) = diag\left[p^* - \hat{p}_j^*\right]_{n \times n}$, and the constant matrix $\mathrm{R} = \begin{bmatrix} 0 & 1 & \cdots & 1 \\ 1 & 0 & \cdots & 1 \\ \cdots & \cdots & \cdots & \cdots \\ 1 & 1 & \cdots & 0 \end{bmatrix}_{\mathrm{n \times n}}$. Clearly, given $p^* \in \left(p^0, p^{\max}\right]$, $\mathrm{P}(p^*)\mathrm{R}$ is an $n \times n$ matrix with rank n. Then,

$$
\tilde{\mathrm{S}}(p^*) = \mathrm{R}^{-1}\mathrm{P}^{-1}(p^*)\mathrm{S}(p^*) \tag{B.1}
$$

From Eq. (B.1), $\tilde{\mathrm{S}}(p^*)$ and $\mathrm{S}(p^*)$ can be mutually determined, that is, given $\tilde{\mathrm{S}}(p^*)$, $\mathrm{S}(p^*)$ can be uniquely determined based on Eq. (B.1). Next, we need to design $\tilde{\mathrm{S}}(p^*)$ and $\mathrm{S}(p^*)$ that satisfy $\tilde{\mathrm{S}}(p^*) > [0, 0, \cdots, 0]^{\mathrm{T}}$

and $\mathrm{S}(p^*) = [\hat{s}_1(\hat{p}_1^*), \hat{s}_2(\hat{p}_2^*), \cdots, \hat{s}_n(\hat{p}_n^*)]^{\mathrm{T}} > [0, 0, \cdots, 0]^{\mathrm{T}}$ with given $\hat{s}_i(p)\,(p \in [0, \hat{p}_i], i \in \mathrm{N})$ and d_0. To do so, write Eq. (B.1) in a component form as below:

$$\left.\frac{ds_j(p)}{dp}\right|_{p=p^*} = \frac{1}{n-1} \sum_{i \in \mathbf{N}, i \neq j} \frac{s_i(p^*)}{p^* - \hat{p}_i^*} - \frac{n-2}{n-1} \frac{s_j(p^*)}{p^* - \hat{p}_j^*} \tag{B.2}$$

When market competition reaches an equilibrium, there is $s(p^*) = \sum_{i \in \mathbf{N}} s_i(p^*) = d_0$ from Lemma 1. $\mathrm{S}(p^*)$ is designed under different scenarios as follows.

(1) When there are only two generators (named generator 1 and 2) in $\mathbf{N}$, we can always find $\hat{p}_1^* \in [0, \hat{p}_1]$ and $\hat{p}_2^* \in [0, \hat{p}_2]$ satisfying $\hat{s}_1(\hat{p}_1^*) > 0$, $\hat{s}_2(\hat{p}_2^*) > 0$ and $\hat{s}_1(\hat{p}_1^*) + \hat{s}_2(\hat{p}_2^*) = d_0$. Then, let $s_1(p^*) = \hat{s}_1(\hat{p}_1^*)$ and $s_2(p^*) = \hat{s}_2(\hat{p}_2^*)$, there is $\left.\frac{ds_1(p)}{dp}\right|_{p=p^*} = \frac{s_2(p^*)}{p^* - \hat{p}_2^*} > 0$ and $\left.\frac{ds_2(p)}{dp}\right|_{p=p^*} = \frac{s_1(p^*)}{p^* - \hat{p}_1^*} > 0$ based on Eq. (B.1). Clearly, both $s_1(p^*)$ and $s_2(p^*)$ satisfy the design requirements above. Therefore, p^* is an ECP from Lemma 1.

(2) When there are three and more generators in $\mathbf{N}$, the design method in (1) can barely guarantee $\tilde{\mathrm{S}}(p^*) > [0, 0, \cdots, 0]^{\mathrm{T}}$. Here, consider a special design of weighted average distribution for $s(p^*) = d_0$, i.e., $\forall j \in \mathbf{N}$, let $\frac{s_j(p^*)}{p^* - \hat{p}_j^*} = \mu$. Thereupon, solving the $n+1$ equations $\frac{s_j(p^*)}{p^* - \hat{p}_j^*} = \mu$ and $\sum_{i \in \mathbf{N}} s_i(p^*) = d_0$ consisting of $n+1$ variables yields $s_j(p^*) = \frac{(p^* - \hat{p}_j^*)d_0}{np^* - \sum_{i \in \mathbf{N}} \hat{p}_i^*} > 0$, and there is $\left.\frac{ds_j(p)}{dp}\right|_{p=p^*} = \frac{\mu}{n-1} > 0$ according to Eq. (B.2). Clearly, $\mathrm{S}(p^*) > [0, 0, \cdots, 0]^{\mathrm{T}}$ and $\tilde{\mathrm{S}}(p^*) > [0, 0, \cdots, 0]^{\mathrm{T}}$. Next, it needs to prove that the designed $s_j(p^*)$ can satisfy $s_j(p^*) = \hat{s}_j(\hat{p}_j^*)$, that is, $\hat{s}_j(\hat{p}_j^*) = \frac{(p^* - \hat{p}_j^*)d_0}{np^* - \sum_{i \in \mathbf{N}} \hat{p}_i^*}$ has a solution about $\hat{p}_j^*$.

Denoting $f_j(x) = \frac{(p^* - x)d_0}{np^* - x - \sum_{i \in \mathbf{N}, i \neq j} \hat{p}_i^*}$, there is $\frac{df_j(x)}{dx} < 0$ and $f_j(0) > 0$. From the assumptions of this paper, we have $\frac{\hat{s}_j(x)}{dx} \geq 0$ and $\hat{s}_j(0) \leq 0$, then $f_j(0) > \hat{s}_j(0)$. Next, discuss the solution of the equation $\hat{s}_j(x) = f_j(x)$.

(2.1) If $\forall j \in \mathbf{N}$, the equation $\hat{s}_j(x) = f_j(x)$ has solutions, then p^* is an ECP.

(2.2) If $\forall j \in \mathbf{N}$, the equation $\hat{s}_j(x) = f_j(x)$ has no solutions, which implies that the curves $\hat{s}_j(x)$ and $f_j(x)$ do not intersect in the interval $[0, \hat{p}_i]$, and there is $\hat{s}_i(\hat{p}_i) < f_i(\hat{p}_i)$ due to $f_i(0) > \hat{s}_i(0)$. Then, there is $\sum_{i \in \mathbf{N}} \hat{s}_i(\hat{p}_i) < \sum_{i \in \mathbf{N}} f_i(\hat{p}_i) = d_0$, which contradicts the assumption that "supply exceeds demand" of this paper. Therefore, there is some $i \in \mathbf{N}$ such that $\hat{s}_i(x) = f_i(x)$ has a solution.

Here, no solution to the equation $\hat{s}_i(x) = f_i(x)$ implies that $\hat{s}_i(\hat{p}_i) < s_i(p^*) < f_i(\hat{p}_i)$ reflecting the actual clearing supply for generator i at a given p^* should be $\hat{s}_i(\hat{p}_i)$ rather than $\hat{s}_i(\hat{p}_i^*)\,(= s_i(p^*))$, i.e., insufficient supply. Then, $\frac{\hat{s}_i(\hat{p}_i)}{p^* - \hat{p}_i^*} < \frac{s_i(p^*)}{p^* - \hat{p}_i^*} = \left.\frac{ds_{-i}(p)}{dp}\right|_{p=p^*}$, indicating that the supply function $s_i(p)$ will shift left (up) (as in Fig.2). However, since generator i has already reached the maximum supply $\hat{s}_i(\hat{p}_i)$, this deviation has no benefit. In other words, the rationally submitted supply function of generator i has no effect on the formation of the market equilibrium or the uniform clearing price p^*. Therefore, the above analysis can remove i from $\mathbf{N}$, and p^* is only related to the generator set $\hat{\mathbf{N}} = \mathbf{N}/\{i\}$ and the demand $d_0 - \hat{s}_i(\hat{p}_i) > 0$. Then, we can consider each equation until all similar generators are eliminated.

In particular, if there is only the unique generator k among the final $\hat{\mathbf{N}}$ obtained, then k has the full power to decide the uniform clearing price p^*. At this point, $p^* = p^{\max}$ is the most favorable for all generators, so the equilibrium clearing price is $p^{\max}$. Alternatively, choose any one of the removed generators to join $\hat{\mathbf{N}}$ to make $\hat{\mathbf{N}}$ have two elements. Then, we may follow the method in (1) to design $\mathrm{S}(p^*)$ such that p^* is an ECP.

In conclusion, Lemma 2 holds. ■

Appendix C. Proof of Proposition 1

Proof: Given the market demand d_0, the supply function $s_i(p)$ for each generator $i \in \mathbf{N}$, and the ECP p^* such that $s(p^*) = \sum_{i \in \mathbf{N}} s_i(p^*) = d_0$. If generator $j \in \mathbf{N}$ does not exercise its market power, then $p^* = \hat{p}_j^* = \hat{s}_j^{-1}(s_j(p^*))$ from Definition 1.

If generator j shifts $s_j(p)$ to the right or down by a small offset $\Delta p > 0$, then $\hat{W}'_{s-j} - \hat{W}_{s-j} = \Delta p \left(d_0 - s_{-j}\left(p^* + \Delta p\right)\right) + o\left(\Delta p\right) > 0$ from Eq. (2), indicating that $\exists s'_j(p) < s_j(p)$ may increase the profit of generator j. And the clearing price $p^{*'} > p^*$, the clearing supply $s_j\left(p^{*'}\right) < s_j\left(p^*\right)$ decided by $\left(s'_j(p), s_{-j}(p)\right)$, but the welfare $\hat{W}'_{s-j} > \hat{W}_{s-j}$. Thus, p^* is not the clearing price, and Proposition 1 holds. ■

Appendix D. Proof of Proposition 2

Proof: At the bidding equilibrium, $s\left(p^*\right) = d_0$ from Lemma 3. And Lemma 1 shows that $\forall j \in \mathbf{N}$, $\frac{s_j(p^*)}{p^* - \hat{p}_j^*} = \left.\frac{ds_{-j}(p)}{dp}\right|_{p=p^*}$. Accordingly, $\left.\frac{ds_{-j}(p)}{dp}\right|_{p=p^*} = \sum\limits_{i \in \mathbf{N}, i \neq j} \left.\frac{ds_i(p)}{dp}\right|_{p=p^*} \geq (n-1)\,\delta\left(p^*\right)$ and $\frac{s_j(p^*)}{p^* - \hat{p}_j^*} \leq \frac{d_0}{p^* - \max\limits_{i \in \mathbf{N}} \hat{p}_i^*}$. Therefore, $\frac{d_0}{p^* - \max\limits_{i \in \mathbf{N}} \hat{p}_i^*} \geq (n-1)\,\delta\left(p^*\right) \geq (n-1)\frac{d_0}{p^{\max} - \max\limits_{i \in \mathbf{N}} \hat{p}_i^*}$. Namely,

$$p^* \leq \frac{1}{n-1}\left(p^{\max} + (n-2)\max_{i \in \mathbf{N}} \hat{p}_i^*\right) \tag{D.1}$$

■

Appendix E. Proof of Lemma 4

Proof: Suppose the total ECS $s\left(p^*\right) < d_0$ when the market competition reaches an equilibrium and the ECP $p^* < p^{\max}$. Consider generator $j \in \mathbf{N}$ making the deviation s'_j as follows.

$$s'_j\left(p^*\right) = s_j\left(p^*\right) \quad \text{and} \quad s'_j\left(p^* + \varepsilon\right) = s_j\left(p^*\right) + \zeta \tag{E.1}$$

where $\varepsilon > 0$ and $\zeta > \frac{\varepsilon s(p^*)}{p^{\max} - (p^* + \varepsilon)} > 0$. Except for generator j, the supply functions submitted by the remaining $n-1$ generators keep unchanged. Here, ε is an infinitesimal. So, the total clearing supply $s'\left(p^{*'}\right)$ can be approximated as $s\left(p^*\right) + \zeta$, and the clearing price $p^{*'} = p^* + \varepsilon$. Therefore, the objective value of model C3 after deviation is

$$\begin{aligned} s'\left(p^{*'}\right)\left(p^{\max} - p^{*'}\right) &= \left(s\left(p^*\right) + \zeta\right)\left(p^{\max} - \left(p^* + \varepsilon\right)\right) \\ &> s\left(p^*\right)\left(p^{\max} - p^*\right) \end{aligned} \tag{E.2}$$

Besides, the profit of generator j after deviation is

$$\begin{aligned} \hat{W}'_{s-j} &= \int_{\hat{s}_j^{-1}(0)}^{\hat{s}_j^{-1}\left(s'_j(p^{*'})\right)} \hat{s}_j(p)dp + \left(p^{*'} - \hat{s}_j^{-1}\left(s'_j(p^{*'})\right)\right) s'_j(p^{*'}) \\ &\geq \int_{\hat{s}_j^{-1}(0)}^{\hat{s}_j^{-1}(s_j(p^*))} \hat{s}_j(p)dp + \left(p^{*'} - \hat{s}_j^{-1}\left(s'_j(p^{*'})\right)\right) s_j(p^{*'}) \\ &\qquad + \left(\hat{s}_j^{-1}\left(s'_j(p^{*'})\right) - \hat{s}_j^{-1}\left(s_j(p^*)\right)\right) s_j(p^*) \\ &= \int_{\hat{s}_j^{-1}(0)}^{\hat{s}_j^{-1}(s_j(p^*))} \hat{s}_j(p)dp - \hat{s}_j^{-1}\left(s_j(p^*)\right) s_j(p^*) + p^{*'} s_j\left(p^{*'}\right) \\ &> \int_{\hat{s}_j^{-1}(0)}^{\hat{s}_j^{-1}(s_j(p^*))} \hat{s}_j(p)dp + \left(p^* - \hat{s}_j^{-1}\left(s_j(p^*)\right)\right) s_j(p^*) \end{aligned} \tag{E.3}$$

In conclusion, combining Eq. (E.2) and Eq. (E.3), the above deviation is compatible with the interests of all sides, so that the total clearing supply will definitely increase, until $s\left(p^*\right) = d_0$. ■

Appendix F. Proof of Proposition 3

Proof: The optimality condition of the objective $W^{'}$ in model C3 yields $\frac{ds(p)}{dp}\left(p^{\max}-p\right)-s\left(p\right)=0$. So, at the ECP p^{*},

$$\left.\frac{ds\left(p\right)}{dp}\right|_{p=p^{*}}=\frac{s\left(p^{*}\right)}{p^{\max}-p^{*}} \tag{F.1}$$

From Lemma 1, when the market is at an equilibrium, $\forall j\in\mathbf{N}$,

$$s_{j}\left(p^{*}\right)=\left(p^{*}-\hat{p}_{j}^{*}\right)\left.\frac{ds_{-j}\left(p\right)}{dp}\right|_{p=p^{*}}\geq\left(p^{*}-\max_{i\in\mathbf{N}}\hat{p}_{i}^{*}\right)\left.\frac{ds_{-j}\left(p\right)}{dp}\right|_{p=p^{*}} \tag{F.2}$$

Summing the two sides of Eq. (F.2) with respect to j,

$$s\left(p^{*}\right)\geq\left(p^{*}-\max_{i\in\mathbf{N}}\hat{p}_{i}^{*}\right)\sum_{j\in\mathbf{N}}\left.\frac{ds_{-j}\left(p\right)}{dp}\right|_{p=p^{*}}=\left(n-1\right)\left(p^{*}-\max_{i\in\mathbf{N}}\hat{p}_{i}^{*}\right)\left.\frac{ds\left(p\right)}{dp}\right|_{p=p^{*}} \tag{F.3}$$

Therefore, substituting Eq. (F.1) into Eq. (F.3),

$$s\left(p^{*}\right)\geq\left(n-1\right)\left(p^{*}-\max_{i\in\mathbf{N}}\hat{p}_{i}^{*}\right)\frac{s\left(p^{*}\right)}{p^{\max}-p^{*}} \tag{F.4}$$

i.e., $p^{*}\leq\frac{1}{n}\left(p^{\max}+\left(n-1\right)\max_{i\in\mathbf{N}}\hat{p}_{i}^{*}\right)$. ■

Appendix G. Proof of Proposition 4

Proof: 1) For the MCP-C2 mechanism, Proposition 2 indicates that $p^{*}\leq\frac{1}{n-1}\left(p^{\max}+\left(n-2\right)\max_{i\in\mathbf{N}}\hat{p}_{i}^{*}\right)$ holds when $\delta\left(p\right)\geq\frac{d_{0}}{p^{\max}-\max_{i\in\mathbf{N}}\hat{p}_{i}^{*}}$. Hence, $p^{*}<p^{\max}$. Meanwhile, we have $\lim_{n\to\infty}p^{*}\leq\lim_{n\to\infty}\frac{1}{n-1}\left(p^{\max}+\left(n-2\right)\max_{i\in\mathbf{N}}\hat{p}_{i}^{*}\right)=\max_{i\in\mathbf{N}}\hat{p}_{i}^{*}$, and $p^{*}>p^{0}=\max_{i\in\mathbf{N}}\hat{p}_{i}^{*}$ from Lemma 2. Hence, $\lim_{n\to\infty}p^{*}=\max_{i\in\mathbf{N}}\hat{p}_{i}^{*}$. So, if $\hat{p}_{j}^{*}=\max_{i\in\mathbf{N}}\hat{p}_{i}^{*}$, then $\lim_{n\to\infty}\sigma_{j}=0$. If $\hat{p}_{j}^{*}<\max_{i\in\mathbf{N}}\hat{p}_{i}^{*}$, then $p_{j}<p^{*}$ implies $\sigma_{j}=0$ from Definition 2.

2) The analysis for the MCP-C3 mechanism is similar. ■