Week 5: Bayesian linear regression and introduction to Stan

11/02/2023

Introduction

Today we will be starting off using Stan, looking at the kid's test score data set (available in resources for the Gelman Hill textbook).

```
library(tidyverse)
library(rstan)
library(tidybayes)
library(here)
```

The data look like this:

```
kidiq <- read_rds("D:\\kidiq.RDS")

kidiq
```

```
# A tibble: 434 x 4
   kid_score mom_hs mom_iq mom_age
       <int>  <dbl>  <dbl>   <int>
 1        65      1  121.       27
 2        98      1   89.4      25
 3        85      1  115.       27
 4        83      1   99.4      25
 5       115      1   92.7      27
 6        98      0  108.       18
 7        69      1  139.       20
 8       106      1  125.       23
```

```
 9       102      1   81.6      24
10        95      1   95.1      19
# ... with 424 more rows
```

As well as the kid's test scores, we have a binary variable indicating whether or not the mother completed high school, the mother's IQ and age.

Descriptives

Question 1

Use plots or tables to show three interesting observations about the data. Remember:

- Explain what your graph/ tables show
- Choose a graph type that's appropriate to the data type

```r
library(skimr)
library(janitor)
library(ggplot2)
skim(kidiq)
```

Table 1: Data summary

Name	kidiq
Number of rows	434
Number of columns	4
Column type frequency:	
numeric	4
Group variables	None

Variable type: numeric

skim_variable	n_missing	complete_rate	mean	sd	p0	p25	p50	p75	p100	hist
kid_score	0	1	86.80	20.41	20.00	74.00	90.00	102.00	144.00	
mom_hs	0	1	0.79	0.41	0.00	1.00	1.00	1.00	1.00	
mom_iq	0	1	100.00	15.00	71.04	88.66	97.92	110.27	138.89	
mom_age	0	1	22.79	2.70	17.00	21.00	23.00	25.00	29.00	

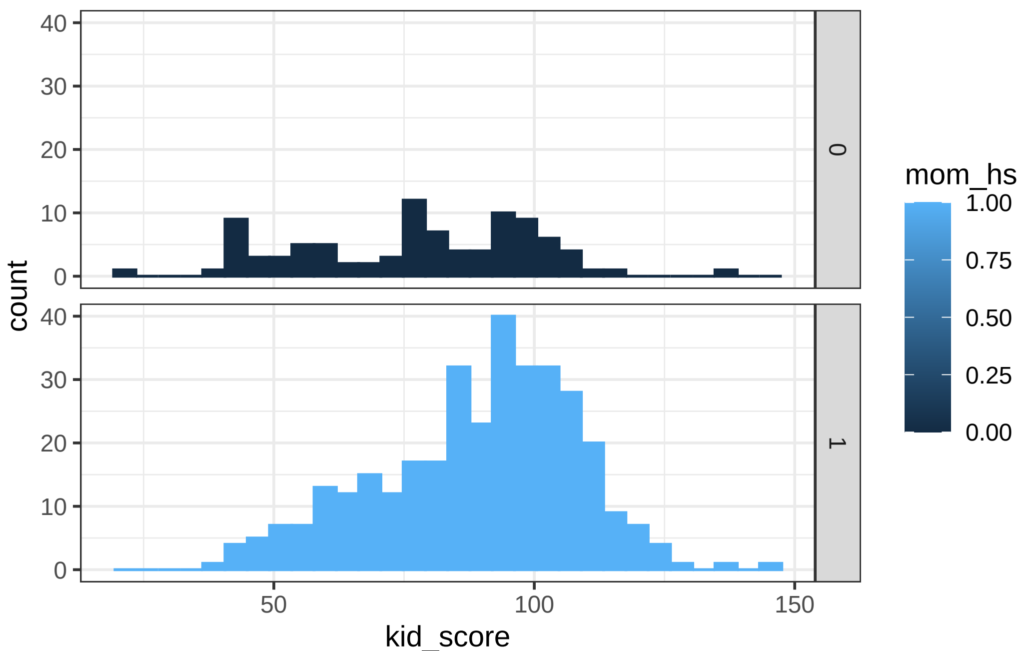
```r
kidiq |>get_dupes()
```

```
# A tibble: 2 x 5
  kid_score mom_hs mom_iq mom_age dupe_count
      <int>  <dbl>  <dbl>   <int>      <int>
1       104      1   125.      23          2
2       104      1   125.      23          2
```

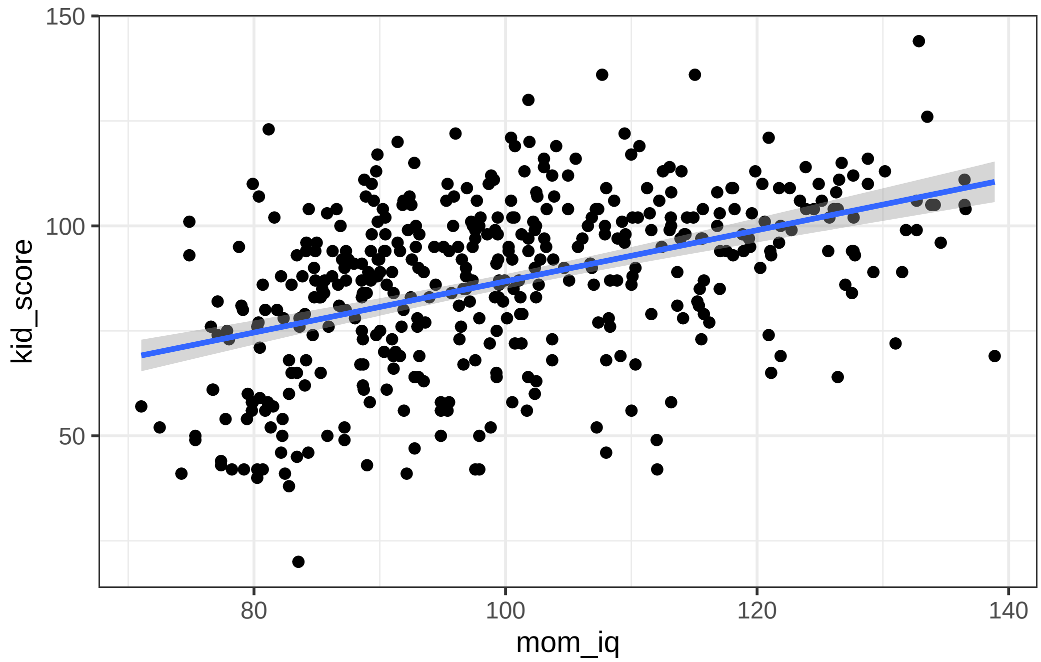
```r
kidiq1<-kidiq |> distinct()
summary(kidiq1$mom_age)
```

```
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max. 
  17.00   21.00   23.00   22.79   25.00   29.00 
```

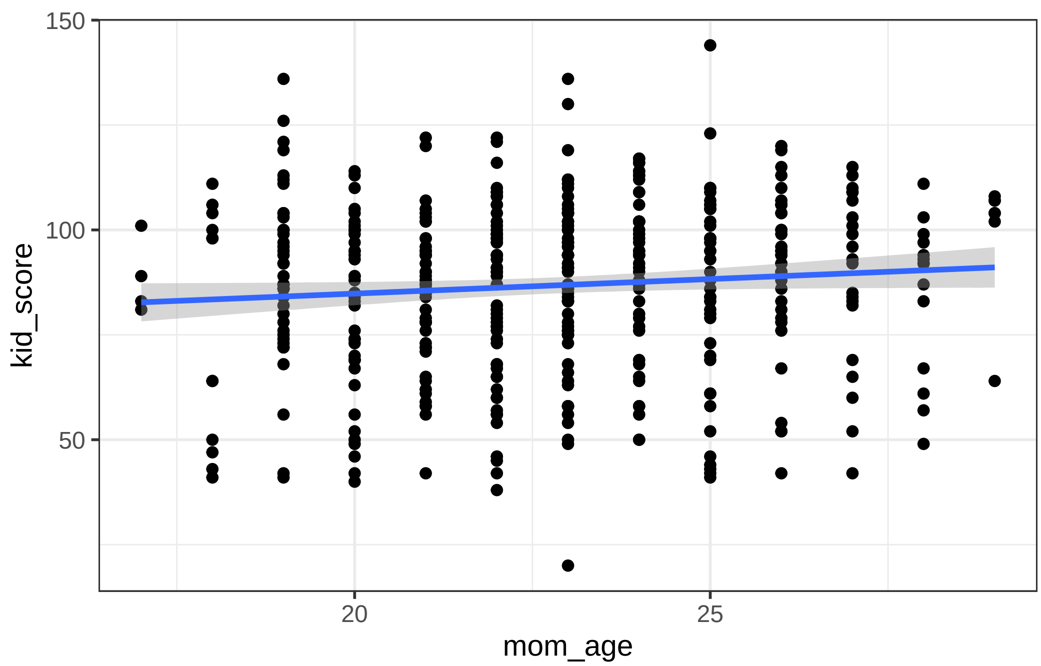
```r
kidiq1 |> ggplot(aes(x=kid_score,fill=mom_hs, color=mom_hs)) +geom_histogram( position="id
```




```r
kidiq1 |> ggplot(aes(x=mom_iq,y=kid_score))+geom_point()+theme_bw()+geom_smooth(method = "
```




```
kidiq1 |> ggplot(aes(x=mom_age,y=kid_score))+geom_point()+theme_bw()+geom_smooth(method =
```



From the above 3 graphs, the mother who completed high schools education could have kids with a higher test score.

The higher the mother's IQ, the higher her kids' test score. The measurement should conform to our prior knowledge from our basic gene genetics .

There are no any relationship between kid_score and their mother ages within 30-year-old.

A graph type that's appropriate to the data type is mom_iq VS kid_score plot.

Estimating mean, no covariates

In class we were trying to estimate the mean and standard deviation of the kid's test scores. The `kids2.stan` file contains a Stan model to do this. If you look at it, you will notice the first `data` chunk lists some inputs that we have to define: the outcome variable `y`, number of observations `N`, and the mean and standard deviation of the prior on `mu`. Let's define all these values in a `data` list.

```
y <- kidiq1$kid_score
mu0 <- 80
sigma0 <- 10

# named list to input for stan function
data <- list(y = y,
             N = length(y),
             mu0 = mu0,
             sigma0 = sigma0)
```

Now we can run the model:

```
library(tidyverse)
library(rstan)
library(tidybayes)
library(here)

fit <-rstan::stan(file =here::here("D:\\kids2.stan"),
            data = data,
            chains = 3,
            iter = 500)
```

```
SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 1).
```

```
Chain 1:
Chain 1: Gradient evaluation took 3.4e-05 seconds
Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.34 seconds.
Chain 1: Adjust your expectations accordingly!
Chain 1:
Chain 1:
Chain 1: Iteration:   1 / 500 [  0%]  (Warmup)
Chain 1: Iteration:  50 / 500 [ 10%]  (Warmup)
Chain 1: Iteration: 100 / 500 [ 20%]  (Warmup)
Chain 1: Iteration: 150 / 500 [ 30%]  (Warmup)
Chain 1: Iteration: 200 / 500 [ 40%]  (Warmup)
Chain 1: Iteration: 250 / 500 [ 50%]  (Warmup)
Chain 1: Iteration: 251 / 500 [ 50%]  (Sampling)
Chain 1: Iteration: 300 / 500 [ 60%]  (Sampling)
Chain 1: Iteration: 350 / 500 [ 70%]  (Sampling)
Chain 1: Iteration: 400 / 500 [ 80%]  (Sampling)
Chain 1: Iteration: 450 / 500 [ 90%]  (Sampling)
Chain 1: Iteration: 500 / 500 [100%]  (Sampling)
Chain 1:
Chain 1:  Elapsed Time: 0.015 seconds (Warm-up)
Chain 1:                0.006 seconds (Sampling)
Chain 1:                0.021 seconds (Total)
Chain 1:

SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 2).
Chain 2:
Chain 2: Gradient evaluation took 2.5e-05 seconds
Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.25 seconds.
Chain 2: Adjust your expectations accordingly!
Chain 2:
Chain 2:
Chain 2: Iteration:   1 / 500 [  0%]  (Warmup)
Chain 2: Iteration:  50 / 500 [ 10%]  (Warmup)
Chain 2: Iteration: 100 / 500 [ 20%]  (Warmup)
Chain 2: Iteration: 150 / 500 [ 30%]  (Warmup)
Chain 2: Iteration: 200 / 500 [ 40%]  (Warmup)
Chain 2: Iteration: 250 / 500 [ 50%]  (Warmup)
Chain 2: Iteration: 251 / 500 [ 50%]  (Sampling)
Chain 2: Iteration: 300 / 500 [ 60%]  (Sampling)
Chain 2: Iteration: 350 / 500 [ 70%]  (Sampling)
Chain 2: Iteration: 400 / 500 [ 80%]  (Sampling)
Chain 2: Iteration: 450 / 500 [ 90%]  (Sampling)
Chain 2: Iteration: 500 / 500 [100%]  (Sampling)
```

```
Chain 2:
Chain 2:  Elapsed Time: 0.013 seconds (Warm-up)
Chain 2:                0.005 seconds (Sampling)
Chain 2:                0.018 seconds (Total)
Chain 2:

SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 3).
Chain 3:
Chain 3: Gradient evaluation took 2.5e-05 seconds
Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.25 seconds.
Chain 3: Adjust your expectations accordingly!
Chain 3:
Chain 3:
Chain 3: Iteration:   1 / 500 [  0%]  (Warmup)
Chain 3: Iteration:  50 / 500 [ 10%]  (Warmup)
Chain 3: Iteration: 100 / 500 [ 20%]  (Warmup)
Chain 3: Iteration: 150 / 500 [ 30%]  (Warmup)
Chain 3: Iteration: 200 / 500 [ 40%]  (Warmup)
Chain 3: Iteration: 250 / 500 [ 50%]  (Warmup)
Chain 3: Iteration: 251 / 500 [ 50%]  (Sampling)
Chain 3: Iteration: 300 / 500 [ 60%]  (Sampling)
Chain 3: Iteration: 350 / 500 [ 70%]  (Sampling)
Chain 3: Iteration: 400 / 500 [ 80%]  (Sampling)
Chain 3: Iteration: 450 / 500 [ 90%]  (Sampling)
Chain 3: Iteration: 500 / 500 [100%]  (Sampling)
Chain 3:
Chain 3:  Elapsed Time: 0.014 seconds (Warm-up)
Chain 3:                0.006 seconds (Sampling)
Chain 3:                0.02 seconds (Total)
Chain 3:
```

Look at the summary

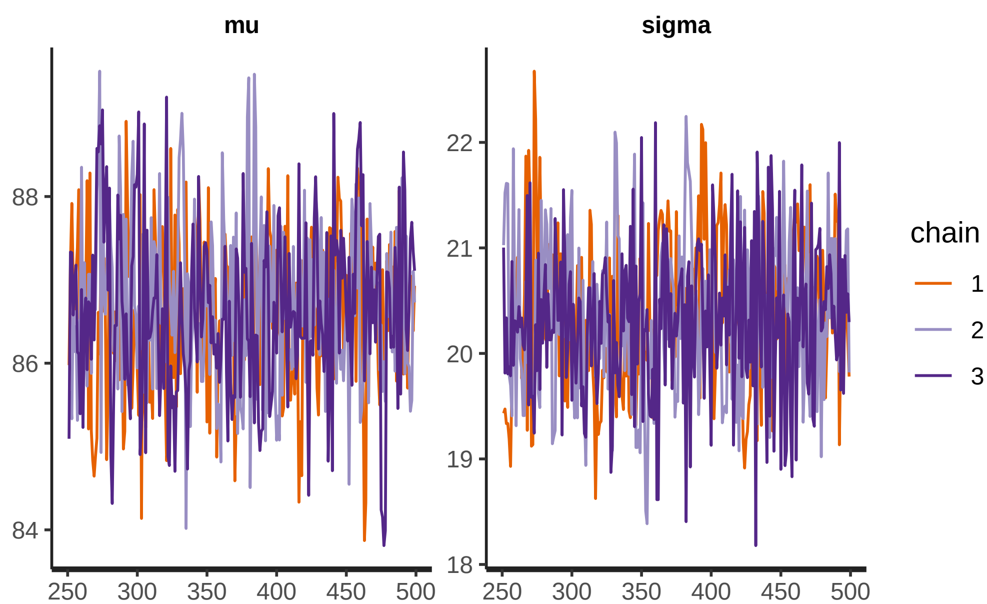
```
fit
```

```
Inference for Stan model: anon_model.
3 chains, each with iter=500; warmup=250; thin=1;
post-warmup draws per chain=250, total post-warmup draws=750.

          mean se_mean   sd     2.5%      25%      50%      75%    97.5% n_eff
mu       86.63    0.05 0.99    84.73    85.93    86.59    87.31    88.64   466
sigma    20.35    0.04 0.72    19.07    19.81    20.34    20.86    21.81   311
```

```
lp__  -1522.46    0.06 1.04 -1525.47 -1522.86 -1522.15 -1521.70 -1521.41   307
      Rhat
mu    1.00
sigma 1.01
lp__  1.00

Samples were drawn using NUTS(diag_e) at Sun Feb 12 22:14:11 2023.
For each parameter, n_eff is a crude measure of effective sample size,
and Rhat is the potential scale reduction factor on split chains (at
convergence, Rhat=1).
```

Traceplot

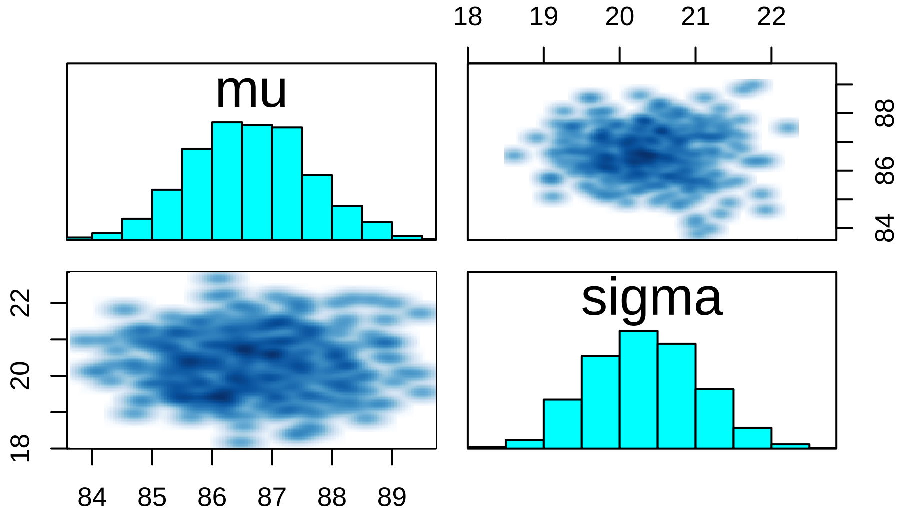
```
traceplot(fit)
```



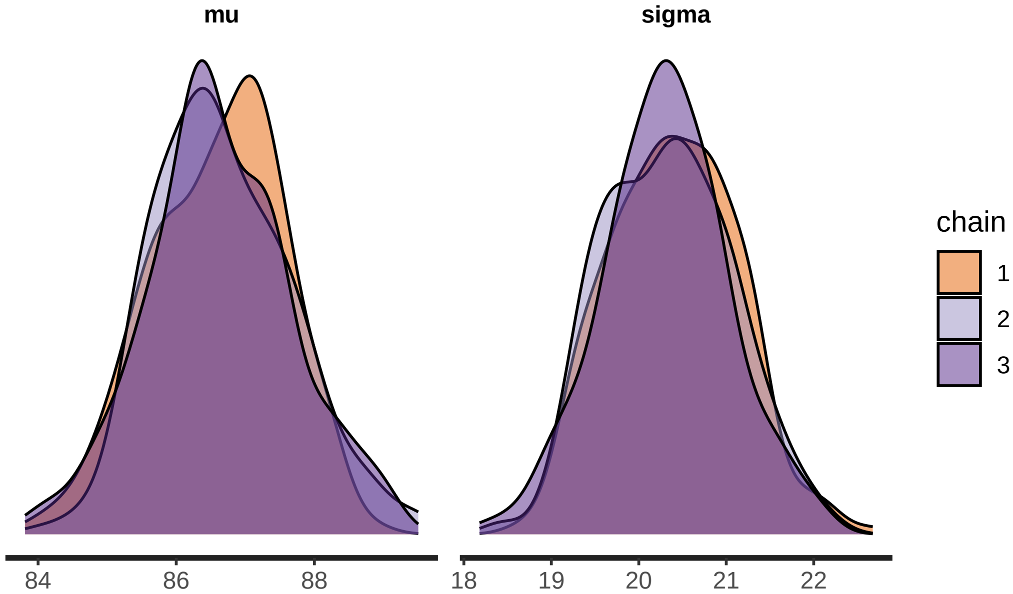
All looks fine.

```
library(tidyverse)
library(rstan)
library(tidybayes)
library(here)
```

```r
pairs(fit, pars = c("mu", "sigma"))
```




```r
stan_dens(fit, separate_chains = TRUE)
```



Understanding output

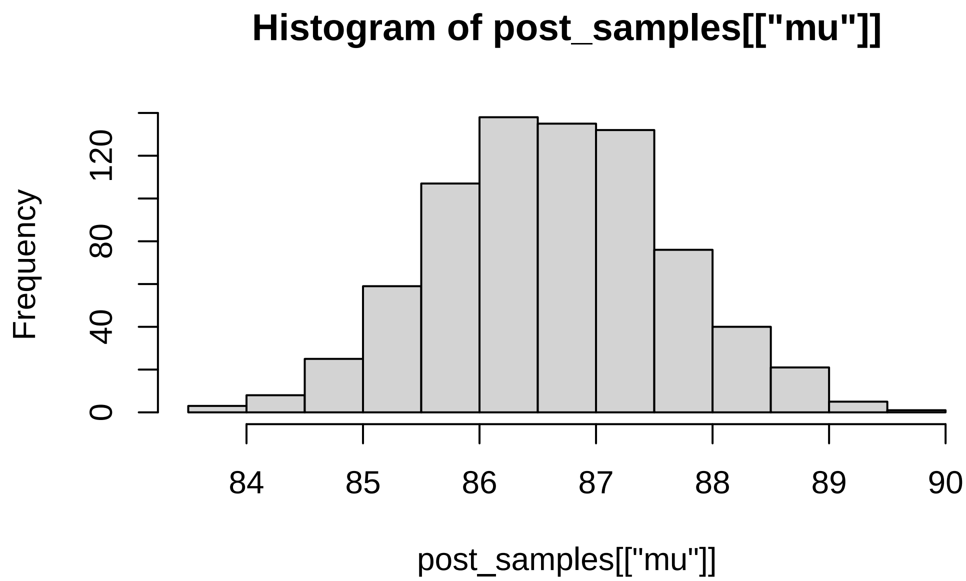
What does the model actually give us? A number of samples from the posteriors. To see this, we can use `extract` to get the samples.

```
post_samples <- extract(fit)
head(post_samples[["mu"]])
```

```
[1] 87.15583 86.11437 86.51366 86.90869 87.55650 86.64331
```

This is a list, and in this case, each element of the list has 4000 samples. E.g. quickly plot a histogram of mu

```
hist(post_samples[["mu"]])
```




```
median(post_samples[["mu"]])
```

```
[1] 86.58619
```

```
# 95% bayesian credible interval
quantile(post_samples[["mu"]], 0.025)
```

```
   2.5%
84.7284
```

```
quantile(post_samples[["mu"]], 0.975)
```

```
   97.5%
88.64016
```

Plot estimates

There are a bunch of packages, built-in functions that let you plot the estimates from the model, and I encourage you to explore these options (particularly in `bayesplot`, which we will most likely be using later on). I like using the `tidybayes` package, which allows us to easily get the posterior samples in a tidy format (e.g. using gather draws to get in long format). Once we have that, it's easy to just pipe and do ggplots as usual.

Get the posterior samples for mu and sigma in long format:

```
dsamples <- fit  |>
  gather_draws(mu, sigma) # gather = long format
dsamples
```

```
# A tibble: 1,500 x 5
# Groups:   .variable [2]
   .chain .iteration .draw .variable .value
    <int>      <int> <int> <chr>      <dbl>
 1      1          1     1 mu          86.0
 2      1          2     2 mu          87.2
 3      1          3     3 mu          87.9
 4      1          4     4 mu          87.0
 5      1          5     5 mu          86.6
 6      1          6     6 mu          87.1
 7      1          7     7 mu          87.4
 8      1          8     8 mu          88.1
 9      1          9     9 mu          86.0
10      1         10    10 mu          86.7
# ... with 1,490 more rows
```

```
# wide format
fit  |>  spread_draws(mu, sigma)
```

```
# A tibble: 750 x 5
   .chain .iteration .draw    mu sigma
    <int>      <int> <int> <dbl> <dbl>
 1      1          1     1  86.0  19.4
 2      1          2     2  87.2  19.5
 3      1          3     3  87.9  19.3
 4      1          4     4  87.0  19.3
 5      1          5     5  86.6  19.2
```

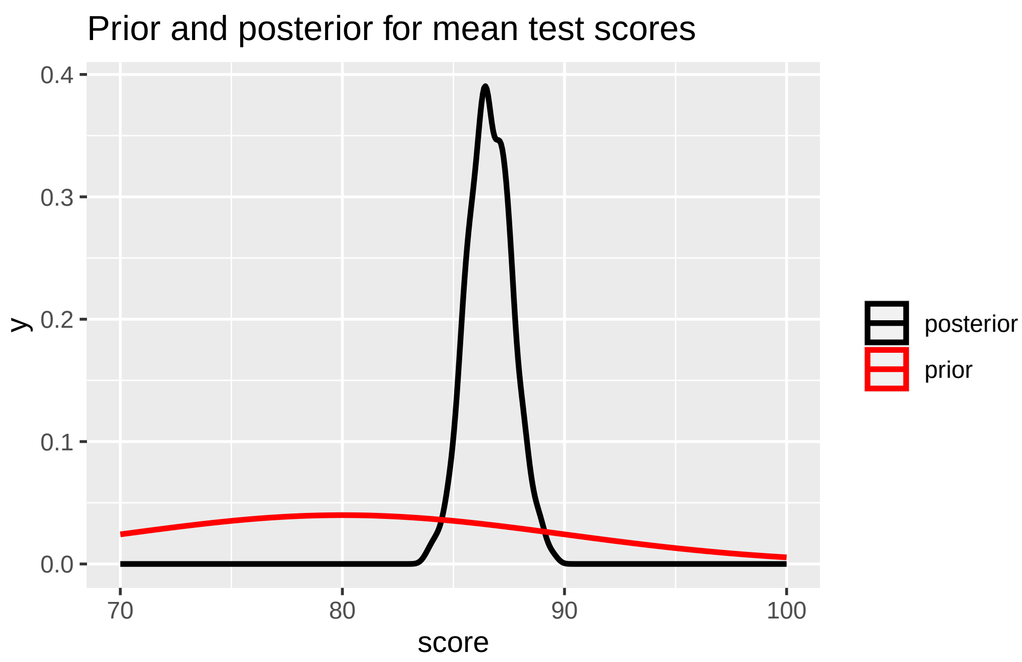
```
 6      1          6     6  87.1  18.9
 7      1          7     7  87.4  19.8
 8      1          8     8  88.1  20.5
 9      1          9     9  86.0  20.0
10      1         10    10  86.7  20.5
# ... with 740 more rows
```

```r
# quickly calculate the quantiles using

dsamples |>
  median_qi(.width = 0.8)
```

```
# A tibble: 2 x 7
  .variable .value .lower .upper .width .point .interval
  <chr>      <dbl>  <dbl>  <dbl>  <dbl> <chr>  <chr>
1 mu          86.6   85.4   87.9    0.8 median qi
2 sigma       20.3   19.4   21.3    0.8 median qi
```

Let's plot the density of the posterior samples for mu and add in the prior distribution

```r
dsamples |>
  filter(.variable == "mu") |>
  ggplot(aes(.value, color = "posterior")) + geom_density(size = 1) +
  xlim(c(70, 100)) +
  stat_function(fun = dnorm,
        args = list(mean = mu0,
                    sd = sigma0),
        aes(colour = 'prior'), size = 1) +
  scale_color_manual(name = "", values = c("prior" = "red", "posterior" = "black")) +
  ggtitle("Prior and posterior for mean test scores") +
  xlab("score")
```



Question 2

Change the prior to be much more informative (by changing the standard deviation to be 0.1). Rerun the model. Do the estimates change? Plot the prior and posterior densities.

```
sigma0=0.1
data1 <- list(y = y,
              N = length(y),
              mu0 = mu0,
              sigma0 = sigma0)

mod1 <- rstan::stan(file = here::here("D:\\kids2.stan"),
             data = data1,
             chains = 3,
             iter = 2000)
```

```
SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 1).
Chain 1:
Chain 1: Gradient evaluation took 6e-06 seconds
Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.06 seconds.
Chain 1: Adjust your expectations accordingly!
```

```
Chain 1:
Chain 1:
Chain 1: Iteration:    1 / 2000 [  0%]  (Warmup)
Chain 1: Iteration:  200 / 2000 [ 10%]  (Warmup)
Chain 1: Iteration:  400 / 2000 [ 20%]  (Warmup)
Chain 1: Iteration:  600 / 2000 [ 30%]  (Warmup)
Chain 1: Iteration:  800 / 2000 [ 40%]  (Warmup)
Chain 1: Iteration: 1000 / 2000 [ 50%]  (Warmup)
Chain 1: Iteration: 1001 / 2000 [ 50%]  (Sampling)
Chain 1: Iteration: 1200 / 2000 [ 60%]  (Sampling)
Chain 1: Iteration: 1400 / 2000 [ 70%]  (Sampling)
Chain 1: Iteration: 1600 / 2000 [ 80%]  (Sampling)
Chain 1: Iteration: 1800 / 2000 [ 90%]  (Sampling)
Chain 1: Iteration: 2000 / 2000 [100%]  (Sampling)
Chain 1:
Chain 1:  Elapsed Time: 0.02 seconds (Warm-up)
Chain 1:                0.025 seconds (Sampling)
Chain 1:                0.045 seconds (Total)
Chain 1:

SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 2).
Chain 2:
Chain 2: Gradient evaluation took 6e-06 seconds
Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.06 seconds.
Chain 2: Adjust your expectations accordingly!
Chain 2:
Chain 2:
Chain 2: Iteration:    1 / 2000 [  0%]  (Warmup)
Chain 2: Iteration:  200 / 2000 [ 10%]  (Warmup)
Chain 2: Iteration:  400 / 2000 [ 20%]  (Warmup)
Chain 2: Iteration:  600 / 2000 [ 30%]  (Warmup)
Chain 2: Iteration:  800 / 2000 [ 40%]  (Warmup)
Chain 2: Iteration: 1000 / 2000 [ 50%]  (Warmup)
Chain 2: Iteration: 1001 / 2000 [ 50%]  (Sampling)
Chain 2: Iteration: 1200 / 2000 [ 60%]  (Sampling)
Chain 2: Iteration: 1400 / 2000 [ 70%]  (Sampling)
Chain 2: Iteration: 1600 / 2000 [ 80%]  (Sampling)
Chain 2: Iteration: 1800 / 2000 [ 90%]  (Sampling)
Chain 2: Iteration: 2000 / 2000 [100%]  (Sampling)
Chain 2:
Chain 2:  Elapsed Time: 0.02 seconds (Warm-up)
Chain 2:                0.026 seconds (Sampling)
Chain 2:                0.046 seconds (Total)
```

```
Chain 2:

SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 3).
Chain 3:
Chain 3: Gradient evaluation took 6e-06 seconds
Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.06 seconds.
Chain 3: Adjust your expectations accordingly!
Chain 3:
Chain 3:
Chain 3: Iteration:    1 / 2000 [  0%]  (Warmup)
Chain 3: Iteration:  200 / 2000 [ 10%]  (Warmup)
Chain 3: Iteration:  400 / 2000 [ 20%]  (Warmup)
Chain 3: Iteration:  600 / 2000 [ 30%]  (Warmup)
Chain 3: Iteration:  800 / 2000 [ 40%]  (Warmup)
Chain 3: Iteration: 1000 / 2000 [ 50%]  (Warmup)
Chain 3: Iteration: 1001 / 2000 [ 50%]  (Sampling)
Chain 3: Iteration: 1200 / 2000 [ 60%]  (Sampling)
Chain 3: Iteration: 1400 / 2000 [ 70%]  (Sampling)
Chain 3: Iteration: 1600 / 2000 [ 80%]  (Sampling)
Chain 3: Iteration: 1800 / 2000 [ 90%]  (Sampling)
Chain 3: Iteration: 2000 / 2000 [100%]  (Sampling)
Chain 3:
Chain 3:  Elapsed Time: 0.022 seconds (Warm-up)
Chain 3:                0.019 seconds (Sampling)
Chain 3:                0.041 seconds (Total)
Chain 3:
```

```r
summary(mod1)
```

```
$summary
             mean     se_mean         sd        2.5%         25%         50%
mu       80.06216 0.001937312 0.09947123    79.86371    79.99739    80.06364
sigma    21.40077 0.014183704 0.70201467    20.06621    20.90404    21.39176
lp__  -1544.65277 0.027131992 0.96158825 -1547.21623 -1545.05073 -1544.33607
              75%       97.5%    n_eff     Rhat
mu       80.12864    80.25633 2636.306 1.000073
sigma    21.87984    22.82255 2449.701 1.000209
lp__  -1543.96760 -1543.70302 1256.073 1.000742

$c_summary
, , chains = chain:1
```

```
         stats
parameter        mean         sd        2.5%         25%         50%
    mu       80.05835 0.09869074    79.87102    79.99299    80.05851
    sigma    21.43740 0.69528072    20.14373    20.91850    21.46733
    lp__  -1544.63545 0.94515360 -1547.26692 -1544.98569 -1544.31687
         stats
parameter         75%       97.5%
    mu       80.12794    80.24235
    sigma    21.91645    22.86516
    lp__  -1543.97586 -1543.70661

, , chains = chain:2

         stats
parameter        mean         sd        2.5%         25%         50%
    mu       80.06635 0.09809149    79.86563    80.00540    80.06631
    sigma    21.37475 0.67935841    20.12428    20.88985    21.36401
    lp__  -1544.60933 0.91581538 -1546.83700 -1545.00566 -1544.30385
         stats
parameter         75%       97.5%
    mu       80.12743    80.25700
    sigma    21.83152    22.75306
    lp__  -1543.95407 -1543.69870

, , chains = chain:3

         stats
parameter        mean        sd        2.5%         25%         50%         75%
    mu       80.06179 0.1015347    79.85886    79.99307    80.06471    80.13097
    sigma    21.39014 0.7296460    20.00992    20.89736    21.37080    21.88096
    lp__  -1544.71353 1.0187681 -1547.26130 -1545.16000 -1544.40152 -1543.96902
         stats
parameter       97.5%
    mu       80.25658
    sigma    22.82241
    lp__  -1543.70324
```

mu was greatly changed from 86.67 to the current value 80.06593,but sigma was smally altered from 20.40 to the current value 21.37953.

```r
mod1samples <- mod1  |>
  gather_draws(mu, sigma) # gather = long format
mod1samples
```

```
# A tibble: 6,000 x 5
# Groups:   .variable [2]
   .chain .iteration .draw .variable .value
    <int>      <int> <int> <chr>      <dbl>
 1      1          1     1 mu          80.1
 2      1          2     2 mu          80.0
 3      1          3     3 mu          80.0
 4      1          4     4 mu          80.1
 5      1          5     5 mu          80.2
 6      1          6     6 mu          80.0
 7      1          7     7 mu          80.0
 8      1          8     8 mu          80.1
 9      1          9     9 mu          79.9
10      1         10    10 mu          79.9
# ... with 5,990 more rows
```

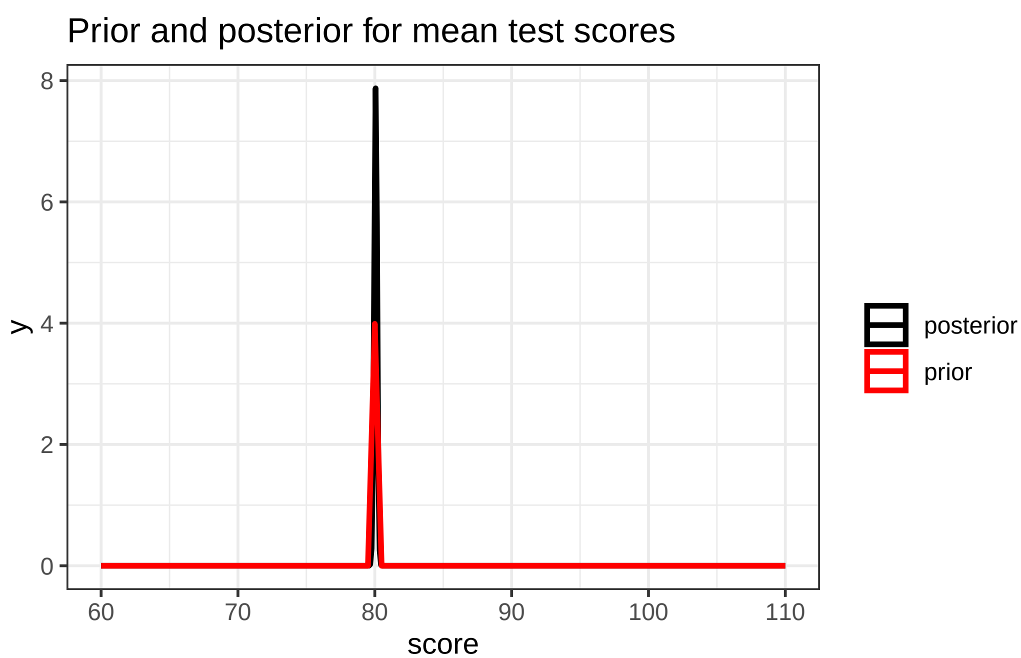
```r
# wide format
mod1  |>  spread_draws(mu, sigma)
```

```
# A tibble: 3,000 x 5
   .chain .iteration .draw    mu sigma
    <int>      <int> <int> <dbl> <dbl>
 1      1          1     1  80.1  21.8
 2      1          2     2  80.0  21.7
 3      1          3     3  80.0  22.0
 4      1          4     4  80.1  22.0
 5      1          5     5  80.2  21.5
 6      1          6     6  80.0  21.7
 7      1          7     7  80.0  22.2
 8      1          8     8  80.1  23.1
 9      1          9     9  79.9  21.6
10      1         10    10  79.9  20.6
# ... with 2,990 more rows
```

```r
# quickly calculate the quantiles using

mod1samples |>
  median_qi(.width = 0.8)
```

```
# A tibble: 2 x 7
  .variable .value .lower .upper .width .point .interval
  <chr>      <dbl>  <dbl>  <dbl>  <dbl> <chr>  <chr>
1 mu          80.1   79.9   80.2    0.8 median qi
2 sigma       21.4   20.5   22.3    0.8 median qi
```

```r
mod1samples |>
  filter(.variable == "mu") |>
  ggplot(aes(.value, color = "posterior")) + geom_density(size = 1) +
  xlim(c(60, 110)) +
  stat_function(fun = dnorm,
        args = list(mean = mu0,
                    sd = sigma0),
        aes(colour = 'prior'), size = 1) +
  scale_color_manual(name = "", values = c("prior" = "red", "posterior" = "black")) +
  ggtitle("Prior and posterior for mean test scores") + xlab("score") + theme_bw()
```



Adding covariates

Now let's see how kid's test scores are related to mother's education. We want to run the simple linear regression

$$Score = \alpha + \beta X$$

where $X = 1$ if the mother finished high school and zero otherwise.

`kid3.stan` has the stan model to do this. Notice now we have some inputs related to the design matrix X and the number of covariates (in this case, it's just 1).

Let's get the data we need and run the model.

```
X <- as.matrix(kidiq1$mom_hs, ncol = 1) # force this to be a matrix
K <- 1

data <- list(y = y, N = length(y),
             X =X, K = K)
fit2 <- rstan::stan(file = here::here("D:\\kids3.stan"),
            data = data,
            iter = 1000)
```

```
SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 1).
Chain 1:
Chain 1: Gradient evaluation took 6.2e-05 seconds
Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.62 seconds.
Chain 1: Adjust your expectations accordingly!
Chain 1:
Chain 1:
Chain 1: Iteration:   1 / 1000 [  0%]  (Warmup)
Chain 1: Iteration: 100 / 1000 [ 10%]  (Warmup)
Chain 1: Iteration: 200 / 1000 [ 20%]  (Warmup)
Chain 1: Iteration: 300 / 1000 [ 30%]  (Warmup)
Chain 1: Iteration: 400 / 1000 [ 40%]  (Warmup)
Chain 1: Iteration: 500 / 1000 [ 50%]  (Warmup)
Chain 1: Iteration: 501 / 1000 [ 50%]  (Sampling)
Chain 1: Iteration: 600 / 1000 [ 60%]  (Sampling)
Chain 1: Iteration: 700 / 1000 [ 70%]  (Sampling)
Chain 1: Iteration: 800 / 1000 [ 80%]  (Sampling)
Chain 1: Iteration: 900 / 1000 [ 90%]  (Sampling)
Chain 1: Iteration: 1000 / 1000 [100%]  (Sampling)
```

```
Chain 1:
Chain 1:  Elapsed Time: 0.105 seconds (Warm-up)
Chain 1:                0.072 seconds (Sampling)
Chain 1:                0.177 seconds (Total)
Chain 1:

SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 2).
Chain 2:
Chain 2: Gradient evaluation took 1.9e-05 seconds
Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.19 seconds.
Chain 2: Adjust your expectations accordingly!
Chain 2:
Chain 2:
Chain 2: Iteration:   1 / 1000 [  0%]  (Warmup)
Chain 2: Iteration: 100 / 1000 [ 10%]  (Warmup)
Chain 2: Iteration: 200 / 1000 [ 20%]  (Warmup)
Chain 2: Iteration: 300 / 1000 [ 30%]  (Warmup)
Chain 2: Iteration: 400 / 1000 [ 40%]  (Warmup)
Chain 2: Iteration: 500 / 1000 [ 50%]  (Warmup)
Chain 2: Iteration: 501 / 1000 [ 50%]  (Sampling)
Chain 2: Iteration: 600 / 1000 [ 60%]  (Sampling)
Chain 2: Iteration: 700 / 1000 [ 70%]  (Sampling)
Chain 2: Iteration: 800 / 1000 [ 80%]  (Sampling)
Chain 2: Iteration: 900 / 1000 [ 90%]  (Sampling)
Chain 2: Iteration: 1000 / 1000 [100%]  (Sampling)
Chain 2:
Chain 2:  Elapsed Time: 0.154 seconds (Warm-up)
Chain 2:                0.073 seconds (Sampling)
Chain 2:                0.227 seconds (Total)
Chain 2:

SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 3).
Chain 3:
Chain 3: Gradient evaluation took 1.8e-05 seconds
Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.18 seconds.
Chain 3: Adjust your expectations accordingly!
Chain 3:
Chain 3:
Chain 3: Iteration:   1 / 1000 [  0%]  (Warmup)
Chain 3: Iteration: 100 / 1000 [ 10%]  (Warmup)
Chain 3: Iteration: 200 / 1000 [ 20%]  (Warmup)
Chain 3: Iteration: 300 / 1000 [ 30%]  (Warmup)
Chain 3: Iteration: 400 / 1000 [ 40%]  (Warmup)
```

```
Chain 3: Iteration: 500 / 1000 [ 50%]  (Warmup)
Chain 3: Iteration: 501 / 1000 [ 50%]  (Sampling)
Chain 3: Iteration: 600 / 1000 [ 60%]  (Sampling)
Chain 3: Iteration: 700 / 1000 [ 70%]  (Sampling)
Chain 3: Iteration: 800 / 1000 [ 80%]  (Sampling)
Chain 3: Iteration: 900 / 1000 [ 90%]  (Sampling)
Chain 3: Iteration: 1000 / 1000 [100%]  (Sampling)
Chain 3:
Chain 3:  Elapsed Time: 0.147 seconds (Warm-up)
Chain 3:                0.079 seconds (Sampling)
Chain 3:                0.226 seconds (Total)
Chain 3:

SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 4).
Chain 4:
Chain 4: Gradient evaluation took 1.9e-05 seconds
Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.19 seconds.
Chain 4: Adjust your expectations accordingly!
Chain 4:
Chain 4:
Chain 4: Iteration:   1 / 1000 [  0%]  (Warmup)
Chain 4: Iteration: 100 / 1000 [ 10%]  (Warmup)
Chain 4: Iteration: 200 / 1000 [ 20%]  (Warmup)
Chain 4: Iteration: 300 / 1000 [ 30%]  (Warmup)
Chain 4: Iteration: 400 / 1000 [ 40%]  (Warmup)
Chain 4: Iteration: 500 / 1000 [ 50%]  (Warmup)
Chain 4: Iteration: 501 / 1000 [ 50%]  (Sampling)
Chain 4: Iteration: 600 / 1000 [ 60%]  (Sampling)
Chain 4: Iteration: 700 / 1000 [ 70%]  (Sampling)
Chain 4: Iteration: 800 / 1000 [ 80%]  (Sampling)
Chain 4: Iteration: 900 / 1000 [ 90%]  (Sampling)
Chain 4: Iteration: 1000 / 1000 [100%]  (Sampling)
Chain 4:
Chain 4:  Elapsed Time: 0.13 seconds (Warm-up)
Chain 4:                0.07 seconds (Sampling)
Chain 4:                0.2 seconds (Total)
Chain 4:
```

Question 3

a) Confirm that the estimates of the intercept and slope are comparable to results from `lm()`

```r
library(skimr)
library(janitor)

mod2<-lm(kid_score~mom_hs,data=kidiq1)
summary(fit2)$summary[c("alpha", "beta[1]"),]
```

```
            mean     se_mean       sd      2.5%       25%      50%      75%
alpha   78.02925 0.07519286 1.992863 74.167491 76.677353 78.02799 79.41924
beta[1] 11.13083 0.08497624 2.245152  6.781314  9.536871 11.12153 12.69320
           97.5%    n_eff      Rhat
alpha   81.88483 702.4278 1.002997
beta[1] 15.44220 698.0663 1.004600
```

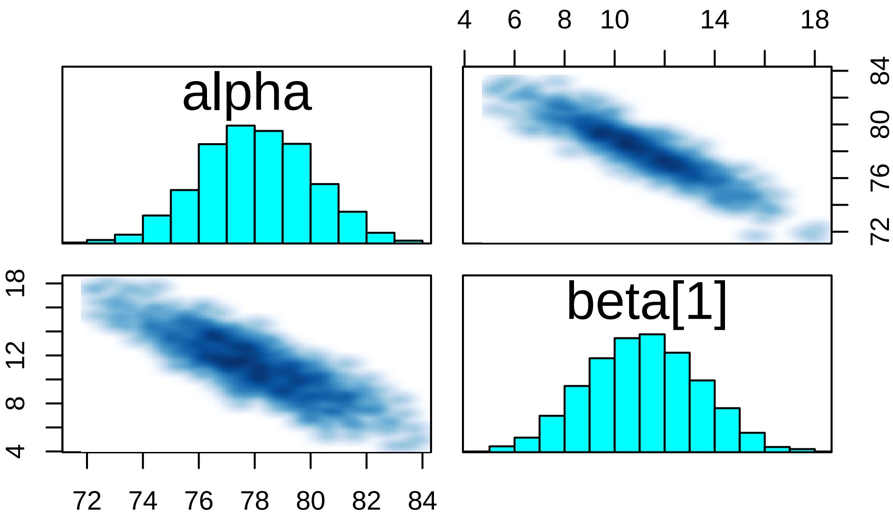
```r
summary(mod2)
```

```

Call:
lm(formula = kid_score ~ mom_hs, data = kidiq1)

Residuals:
    Min      1Q  Median      3Q     Max 
-57.548 -13.276   2.724  14.724  58.452 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)   77.548      2.060  37.651  < 2e-16 ***
mom_hs        11.728      2.324   5.046 6.67e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 19.86 on 431 degrees of freedom
Multiple R-squared:  0.05578,	Adjusted R-squared:  0.05358 
F-statistic: 25.46 on 1 and 431 DF,  p-value: 6.675e-07
```

So the STAN results are mean=77.98760 and beta[1]=11.14227,while the LM results are mean=77.548 and beta[1]=11.728.Both are almost same.

b) Do a `pairs` plot to investigate the joint sample distributions of the slope and intercept. Comment briefly on what you see. Is this potentially a problem?

```
library(tidyverse)
library(rstan)
library(tidybayes)
library(here)

pairs(fit2, pars = c("alpha", "beta"))
```

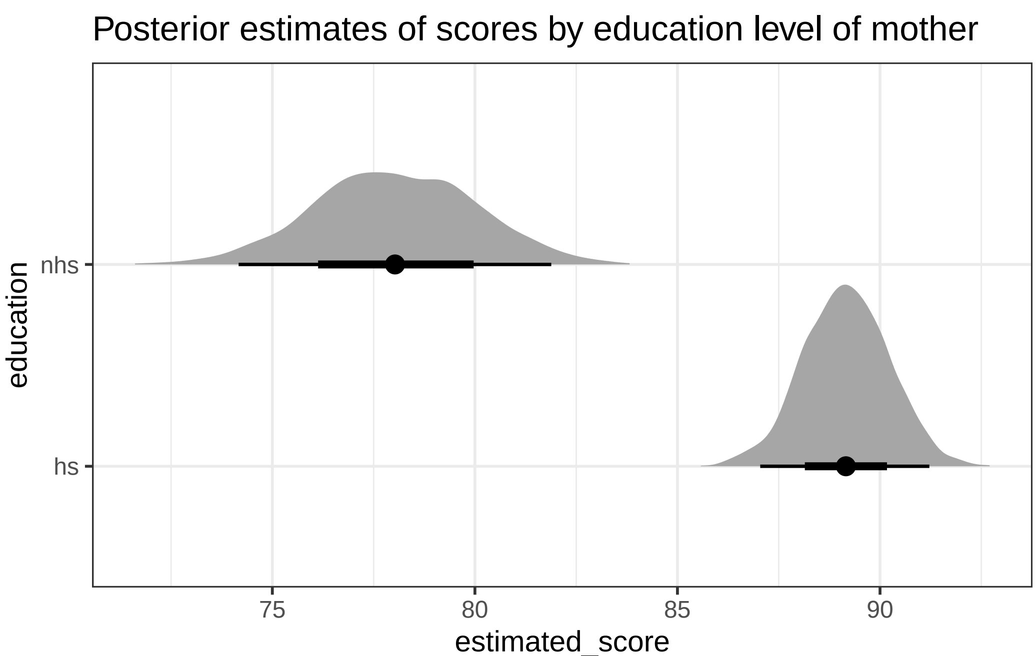


In the fit2, its intercept(alpha) has a wide distribution which means a good sampling but a little hard to compute the intercept and beta(slope) has a narrower distribution which means a bad sampling but easily to compute the slope.Thus,this is potentially a problem.

Plotting results

It might be nice to plot the posterior samples of the estimates for the non-high-school and high-school mothered kids. Here's some code that does this: notice the `beta[condition]` syntax. Also notice I'm using `spread_draws`, because it's easier to calculate the estimated effects in wide format

```r
fit2 |>
  spread_draws(alpha, beta[k], sigma) |>
     mutate(nhs = alpha, # no high school is just the intercept
          hs = alpha + beta) |>
  select(nhs, hs) |>
  pivot_longer(nhs:hs, names_to = "education", values_to = "estimated_score") |>
  ggplot(aes(y = education, x = estimated_score)) +
  stat_halfeye() +
  theme_bw() +
  ggtitle("Posterior estimates of scores by education level of mother")
```



Question 4

Add in mother's IQ as a covariate and rerun the model. Please mean center the covariate before putting it into the model. Interpret the coefficient on the (centered) mum's IQ.

```r
y <- kidiq1$kid_score
mu0 <- 80
sigma0 <- 10
# named list to input for stan function
```

```r
data <- list(y = y,
             N = length(y),
             mu0 = mu0,
             sigma0 = sigma0)

X <-  cbind(as.matrix(kidiq1$mom_hs),as.matrix(kidiq1$mom_iq)) # force this to be a matrix
K <- 2

data <- list(y = y, N = length(y),
             X =X, K = K)
mod3 <- stan(file = here::here("D:\\kids3.stan"),
            data = data,
            iter = 1000)
```

```
SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 1).
Chain 1:
Chain 1: Gradient evaluation took 2.2e-05 seconds
Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.22 seconds.
Chain 1: Adjust your expectations accordingly!
Chain 1:
Chain 1:
Chain 1: Iteration:   1 / 1000 [  0%]  (Warmup)
Chain 1: Iteration: 100 / 1000 [ 10%]  (Warmup)
Chain 1: Iteration: 200 / 1000 [ 20%]  (Warmup)
Chain 1: Iteration: 300 / 1000 [ 30%]  (Warmup)
Chain 1: Iteration: 400 / 1000 [ 40%]  (Warmup)
Chain 1: Iteration: 500 / 1000 [ 50%]  (Warmup)
Chain 1: Iteration: 501 / 1000 [ 50%]  (Sampling)
Chain 1: Iteration: 600 / 1000 [ 60%]  (Sampling)
Chain 1: Iteration: 700 / 1000 [ 70%]  (Sampling)
Chain 1: Iteration: 800 / 1000 [ 80%]  (Sampling)
Chain 1: Iteration: 900 / 1000 [ 90%]  (Sampling)
Chain 1: Iteration: 1000 / 1000 [100%]  (Sampling)
Chain 1:
Chain 1:  Elapsed Time: 0.667 seconds (Warm-up)
Chain 1:                0.353 seconds (Sampling)
Chain 1:                1.02 seconds (Total)
Chain 1:

SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 2).
Chain 2:
```

```
Chain 2: Gradient evaluation took 2e-05 seconds
Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.2 seconds.
Chain 2: Adjust your expectations accordingly!
Chain 2:
Chain 2:
Chain 2: Iteration:   1 / 1000 [  0%]  (Warmup)
Chain 2: Iteration: 100 / 1000 [ 10%]  (Warmup)
Chain 2: Iteration: 200 / 1000 [ 20%]  (Warmup)
Chain 2: Iteration: 300 / 1000 [ 30%]  (Warmup)
Chain 2: Iteration: 400 / 1000 [ 40%]  (Warmup)
Chain 2: Iteration: 500 / 1000 [ 50%]  (Warmup)
Chain 2: Iteration: 501 / 1000 [ 50%]  (Sampling)
Chain 2: Iteration: 600 / 1000 [ 60%]  (Sampling)
Chain 2: Iteration: 700 / 1000 [ 70%]  (Sampling)
Chain 2: Iteration: 800 / 1000 [ 80%]  (Sampling)
Chain 2: Iteration: 900 / 1000 [ 90%]  (Sampling)
Chain 2: Iteration: 1000 / 1000 [100%]  (Sampling)
Chain 2:
Chain 2:  Elapsed Time: 0.597 seconds (Warm-up)
Chain 2:                0.261 seconds (Sampling)
Chain 2:                0.858 seconds (Total)
Chain 2:

SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 3).
Chain 3:
Chain 3: Gradient evaluation took 5.1e-05 seconds
Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.51 seconds.
Chain 3: Adjust your expectations accordingly!
Chain 3:
Chain 3:
Chain 3: Iteration:   1 / 1000 [  0%]  (Warmup)
Chain 3: Iteration: 100 / 1000 [ 10%]  (Warmup)
Chain 3: Iteration: 200 / 1000 [ 20%]  (Warmup)
Chain 3: Iteration: 300 / 1000 [ 30%]  (Warmup)
Chain 3: Iteration: 400 / 1000 [ 40%]  (Warmup)
Chain 3: Iteration: 500 / 1000 [ 50%]  (Warmup)
Chain 3: Iteration: 501 / 1000 [ 50%]  (Sampling)
Chain 3: Iteration: 600 / 1000 [ 60%]  (Sampling)
Chain 3: Iteration: 700 / 1000 [ 70%]  (Sampling)
Chain 3: Iteration: 800 / 1000 [ 80%]  (Sampling)
Chain 3: Iteration: 900 / 1000 [ 90%]  (Sampling)
Chain 3: Iteration: 1000 / 1000 [100%]  (Sampling)
Chain 3:
```

```
Chain 3:  Elapsed Time: 0.722 seconds (Warm-up)
Chain 3:                0.344 seconds (Sampling)
Chain 3:                1.066 seconds (Total)
Chain 3:

SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 4).
Chain 4:
Chain 4: Gradient evaluation took 2.1e-05 seconds
Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.21 seconds.
Chain 4: Adjust your expectations accordingly!
Chain 4:
Chain 4:
Chain 4: Iteration:   1 / 1000 [  0%]  (Warmup)
Chain 4: Iteration: 100 / 1000 [ 10%]  (Warmup)
Chain 4: Iteration: 200 / 1000 [ 20%]  (Warmup)
Chain 4: Iteration: 300 / 1000 [ 30%]  (Warmup)
Chain 4: Iteration: 400 / 1000 [ 40%]  (Warmup)
Chain 4: Iteration: 500 / 1000 [ 50%]  (Warmup)
Chain 4: Iteration: 501 / 1000 [ 50%]  (Sampling)
Chain 4: Iteration: 600 / 1000 [ 60%]  (Sampling)
Chain 4: Iteration: 700 / 1000 [ 70%]  (Sampling)
Chain 4: Iteration: 800 / 1000 [ 80%]  (Sampling)
Chain 4: Iteration: 900 / 1000 [ 90%]  (Sampling)
Chain 4: Iteration: 1000 / 1000 [100%]  (Sampling)
Chain 4:
Chain 4:  Elapsed Time: 0.453 seconds (Warm-up)
Chain 4:                0.251 seconds (Sampling)
Chain 4:                0.704 seconds (Total)
Chain 4:
```

```r
summary(mod3)$summary[c("alpha", "beta[1]","beta[2]"),]
```

```
             mean     se_mean         sd       2.5%        25%        50%
alpha   25.6975836 0.208242903 5.86510972 14.0316833 21.7878733 25.5614176
beta[1]  5.6572935 0.064420318 2.23181585  1.3753124  4.0955391  5.6021092
beta[2]  0.5666353 0.002263409 0.05921372  0.4521845  0.5264855  0.5646773
              75%      97.5%     n_eff     Rhat
alpha   29.7646074 37.4141685  793.2533 1.003381
beta[1]  7.2522955 10.0395390 1200.2480 1.000318
beta[2]  0.6066058  0.6869141  684.4135 1.003825
```

```r
y <- kidiq1$kid_score
mu0 <- 80
sigma0 <- 10
# named list to input for stan function
data <- list(y = y,
             N = length(y),
             mu0 = mu0,
             sigma0 = sigma0)

X <-  cbind(as.matrix(kidiq1$mom_hs),as.matrix(kidiq1$mom_iq - mean(kidiq1$mom_iq))) # for
K <- 2

data <- list(y = y, N = length(y),
             X =X, K = K)
mod4 <- stan(file = here::here("D:\\kids3.stan"),
            data = data,
            iter = 1000)
```

```
SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 1).
Chain 1:
Chain 1: Gradient evaluation took 1.9e-05 seconds
Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.19 seconds.
Chain 1: Adjust your expectations accordingly!
Chain 1:
Chain 1:
Chain 1: Iteration:   1 / 1000 [  0%]  (Warmup)
Chain 1: Iteration: 100 / 1000 [ 10%]  (Warmup)
Chain 1: Iteration: 200 / 1000 [ 20%]  (Warmup)
Chain 1: Iteration: 300 / 1000 [ 30%]  (Warmup)
Chain 1: Iteration: 400 / 1000 [ 40%]  (Warmup)
Chain 1: Iteration: 500 / 1000 [ 50%]  (Warmup)
Chain 1: Iteration: 501 / 1000 [ 50%]  (Sampling)
Chain 1: Iteration: 600 / 1000 [ 60%]  (Sampling)
Chain 1: Iteration: 700 / 1000 [ 70%]  (Sampling)
Chain 1: Iteration: 800 / 1000 [ 80%]  (Sampling)
Chain 1: Iteration: 900 / 1000 [ 90%]  (Sampling)
Chain 1: Iteration: 1000 / 1000 [100%]  (Sampling)
Chain 1:
Chain 1:  Elapsed Time: 0.118 seconds (Warm-up)
Chain 1:                0.086 seconds (Sampling)
Chain 1:                0.204 seconds (Total)
```

```
Chain 1:

SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 2).
Chain 2:
Chain 2: Gradient evaluation took 1.8e-05 seconds
Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.18 seconds.
Chain 2: Adjust your expectations accordingly!
Chain 2:
Chain 2:
Chain 2: Iteration:   1 / 1000 [  0%]  (Warmup)
Chain 2: Iteration: 100 / 1000 [ 10%]  (Warmup)
Chain 2: Iteration: 200 / 1000 [ 20%]  (Warmup)
Chain 2: Iteration: 300 / 1000 [ 30%]  (Warmup)
Chain 2: Iteration: 400 / 1000 [ 40%]  (Warmup)
Chain 2: Iteration: 500 / 1000 [ 50%]  (Warmup)
Chain 2: Iteration: 501 / 1000 [ 50%]  (Sampling)
Chain 2: Iteration: 600 / 1000 [ 60%]  (Sampling)
Chain 2: Iteration: 700 / 1000 [ 70%]  (Sampling)
Chain 2: Iteration: 800 / 1000 [ 80%]  (Sampling)
Chain 2: Iteration: 900 / 1000 [ 90%]  (Sampling)
Chain 2: Iteration: 1000 / 1000 [100%]  (Sampling)
Chain 2:
Chain 2:  Elapsed Time: 0.134 seconds (Warm-up)
Chain 2:                0.08 seconds (Sampling)
Chain 2:                0.214 seconds (Total)
Chain 2:

SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 3).
Chain 3:
Chain 3: Gradient evaluation took 2e-05 seconds
Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.2 seconds.
Chain 3: Adjust your expectations accordingly!
Chain 3:
Chain 3:
Chain 3: Iteration:   1 / 1000 [  0%]  (Warmup)
Chain 3: Iteration: 100 / 1000 [ 10%]  (Warmup)
Chain 3: Iteration: 200 / 1000 [ 20%]  (Warmup)
Chain 3: Iteration: 300 / 1000 [ 30%]  (Warmup)
Chain 3: Iteration: 400 / 1000 [ 40%]  (Warmup)
Chain 3: Iteration: 500 / 1000 [ 50%]  (Warmup)
Chain 3: Iteration: 501 / 1000 [ 50%]  (Sampling)
Chain 3: Iteration: 600 / 1000 [ 60%]  (Sampling)
Chain 3: Iteration: 700 / 1000 [ 70%]  (Sampling)
```

```
Chain 3: Iteration: 800 / 1000 [ 80%]  (Sampling)
Chain 3: Iteration: 900 / 1000 [ 90%]  (Sampling)
Chain 3: Iteration: 1000 / 1000 [100%]  (Sampling)
Chain 3:
Chain 3:  Elapsed Time: 0.132 seconds (Warm-up)
Chain 3:                0.091 seconds (Sampling)
Chain 3:                0.223 seconds (Total)
Chain 3:

SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 4).
Chain 4:
Chain 4: Gradient evaluation took 1.8e-05 seconds
Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.18 seconds.
Chain 4: Adjust your expectations accordingly!
Chain 4:
Chain 4:
Chain 4: Iteration:   1 / 1000 [  0%]  (Warmup)
Chain 4: Iteration: 100 / 1000 [ 10%]  (Warmup)
Chain 4: Iteration: 200 / 1000 [ 20%]  (Warmup)
Chain 4: Iteration: 300 / 1000 [ 30%]  (Warmup)
Chain 4: Iteration: 400 / 1000 [ 40%]  (Warmup)
Chain 4: Iteration: 500 / 1000 [ 50%]  (Warmup)
Chain 4: Iteration: 501 / 1000 [ 50%]  (Sampling)
Chain 4: Iteration: 600 / 1000 [ 60%]  (Sampling)
Chain 4: Iteration: 700 / 1000 [ 70%]  (Sampling)
Chain 4: Iteration: 800 / 1000 [ 80%]  (Sampling)
Chain 4: Iteration: 900 / 1000 [ 90%]  (Sampling)
Chain 4: Iteration: 1000 / 1000 [100%]  (Sampling)
Chain 4:
Chain 4:  Elapsed Time: 0.157 seconds (Warm-up)
Chain 4:                0.078 seconds (Sampling)
Chain 4:                0.235 seconds (Total)
Chain 4:
```

```r
summary(mod4)$summary[c("alpha", "beta[1]","beta[2]"),]
```

```
              mean     se_mean         sd       2.5%        25%        50%
alpha   82.3429047 0.060796615 1.90243158 78.5968477 81.0579477 82.3344845
beta[1]  5.6523603 0.068658586 2.14749637  1.6096660  4.1641036  5.6513650
beta[2]  0.5661089 0.001905995 0.06415049  0.4394008  0.5212819  0.5670989
               75%      97.5%     n_eff      Rhat
```

```
alpha   83.6477500 86.0149840  979.1727 1.001370
beta[1]  7.1027257  9.9117289  978.3070 1.002040
beta[2]  0.6084374  0.6893613 1132.8085 1.002166
```

```r
mean(kidiq1$kid_score)
```

```
[1] 86.75751
```

Here the alpha(intercept) means that when the mom_iq was in the average of all mom_iqs and moms did not complete their high school education, the kids' test score actually was what should be(82.217).The centered intercept should be totally different from the non-centered data before(25.9479477).

Here the beta[1] is an estimator that shows a positive relationship between the kids' test score and moms' education level.It means that when the moms completed their high school education her kids' test score also increased by 5.6837998 scores corresponding to the moms' education variation.

Here the beta[2] is an estimator that shows a positive relationship between the kids' test score and moms' IQ.It means that when the moms' IQ increased or reduced by one unit and their kids' test score also increased or reduced by 0.5656852 scores corresponding to the moms' IQ variation.

Question 5

Confirm the results from Stan agree with `lm()`

```r
library(tidyverse)
library(stringr)
library(dplyr)
library(janitor)


summary(mod4)$summary[c("alpha", "beta[1]","beta[2]"),]
```

```
              mean      se_mean         sd       2.5%        25%        50%
alpha   82.3429047 0.060796615 1.90243158 78.5968477 81.0579477 82.3344845
beta[1]  5.6523603 0.068658586 2.14749637  1.6096660  4.1641036  5.6513650
beta[2]  0.5661089 0.001905995 0.06415049  0.4394008  0.5212819  0.5670989
               75%      97.5%      n_eff      Rhat
```

```
alpha   83.6477500 86.0149840  979.1727 1.001370
beta[1]  7.1027257  9.9117289  978.3070 1.002040
beta[2]  0.6084374  0.6893613 1132.8085 1.002166
```

```
kidiq2<- kidiq1 |> mutate(mom_iq=mom_iq-mean(mom_iq))
kidiq2<-as.data.frame(kidiq2)

mod5<- lm(kid_score~mom_hs+mom_iq,data=kidiq2)
summary(mod5)$coeff
```

```
              Estimate Std. Error   t value      Pr(>|t|)
(Intercept) 82.0859652 1.94540681 42.194756 5.838177e-155
mom_hs       5.9493443 2.21435766  2.686713  7.495551e-03
mom_iq       0.5633781 0.06081298  9.264110  9.493682e-19
```

The 3 estimators in both models are almost same.

Question 6

Plot the posterior estimates of scores by education of mother for mothers who have an IQ of 110.

```
library(plyr)
library(dplyr)
library(tidyverse)
mean_mom_iq<-mean(kidiq1$mom_iq)
mean_mom_iq
```

```
[1] 99.94338
```

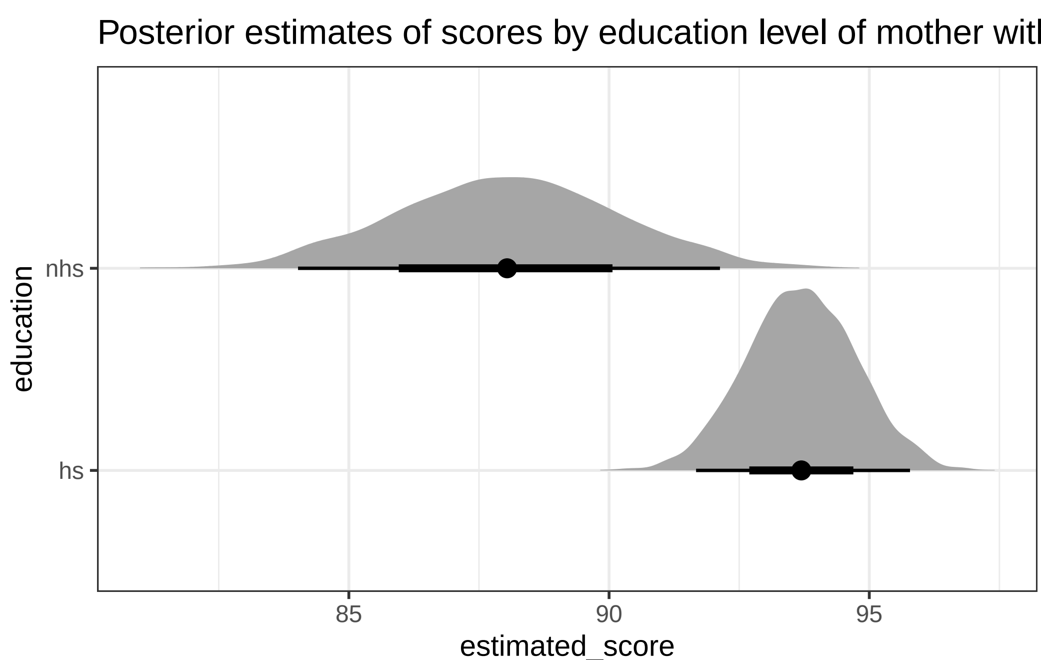
```
post_mod4_samples <- extract(mod4)
length(post_mod4_samples)
```

```
[1] 4
```

```
dim(post_mod4_samples[["beta"]])
```

```
[1] 2000    2
```

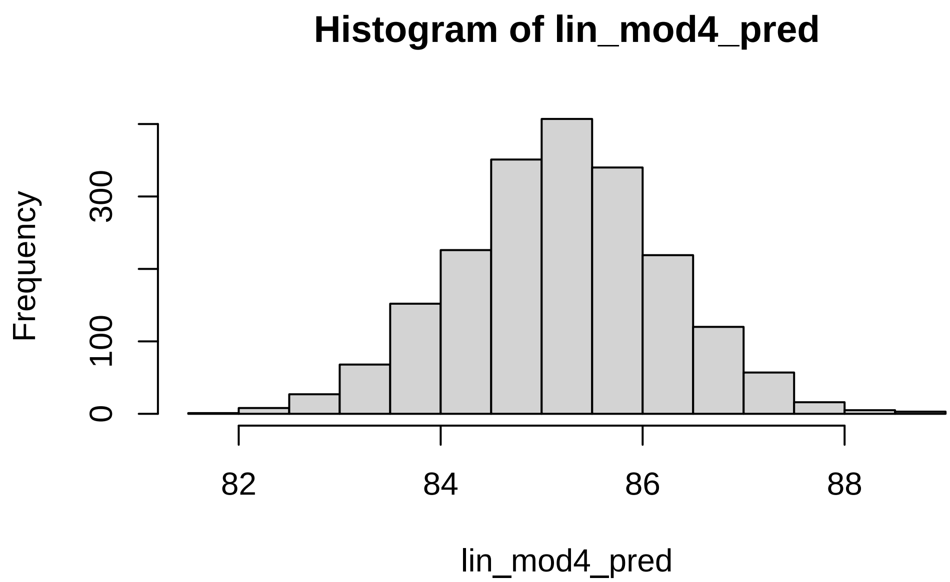
```
x_new1 <- 110-mean_mom_iq

mod4 |>
  spread_draws(alpha, beta[k]) |> pivot_wider( names_from = "k", values_from = "beta")|>
  dplyr::rename(beta1="1",beta2="2")|>
     mutate(nhs = alpha+beta2*x_new1, # no high school is just the intercept
          hs = alpha + beta1+beta2*x_new1) |>
  select(nhs, hs) |>
  pivot_longer(nhs:hs, names_to = "education", values_to = "estimated_score") |>
  #plyr::ddply("education", summarise, grp.mean=mean(estimated_score)) |>
  ggplot(aes(y = education, x = estimated_score)) +
  stat_halfeye()+
  theme_bw() +
  ggtitle("Posterior estimates of scores by education level of mother with IQ=110")
```



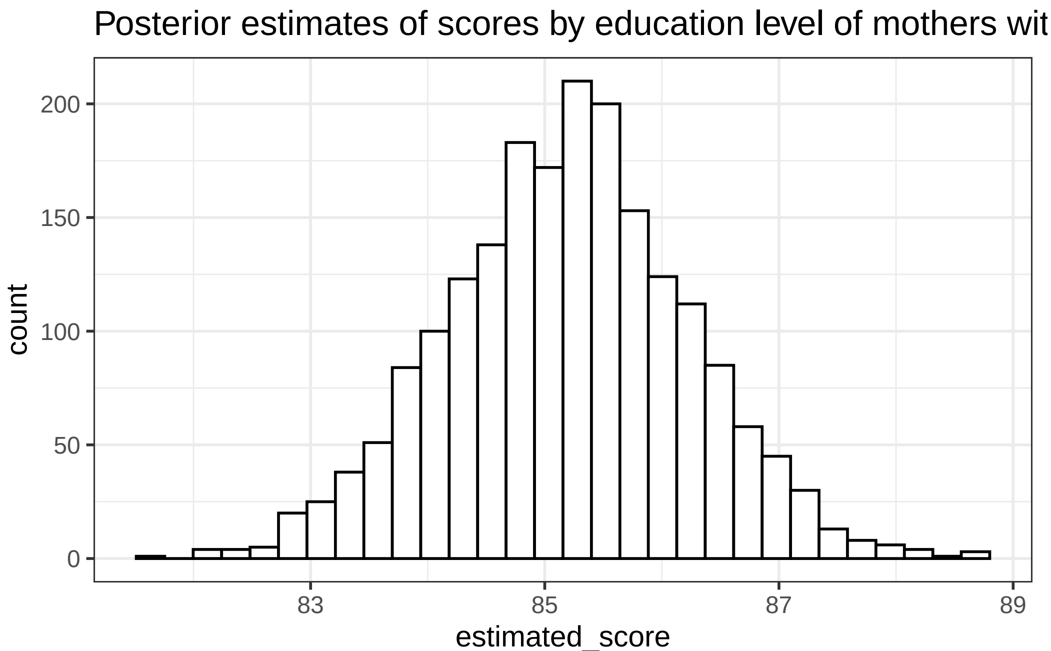
Question 7

Generate and plot (as a histogram) samples from the posterior predictive distribution for a new kid with a mother who graduated high school and has an IQ of 95.

```
library(ggplot2)
mod4_alpha <- post_mod4_samples[["alpha"]]
mod4_beta1 <- post_mod4_samples[["beta"]][,1]
mod4_beta2 <- post_mod4_samples[["beta"]][,2]
x_new2 <-95-mean_mom_iq
lin_mod4_pred <- mod4_alpha + mod4_beta1*1+mod4_beta2*x_new2
hist(lin_mod4_pred)
```




```
as.data.frame(lin_mod4_pred) |> ggplot(aes(x=lin_mod4_pred))+geom_histogram(color="black",
```


Posterior estimates of scores by education level of mothers wit
count
200
150
100
50
0
83
85
87
89
estimated_score

===