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In[11]:= eq2 = L^2 * m (3 / 2 * (a'[t])^2 + (b'[t])^2 +
      (1 / 2) * (c'[t])^2 + 2 * a'[t] * b'[t] * Cos[b[t] - a[t]] +
      a'[t] * c'[t] * Cos[c[t] - a[t]] + b'[t] * c'[t] * Cos[c[t] - b[t]]) -
      m * L * g * (3 * (1 - Cos[a[t]]) + 2 * (1 - Cos[b[t]]) + (1 - Cos[c[t]]))
Out[11]= -g L m (1 + 3 (1 - Cos[a[t]]) + 2 (1 - Cos[b[t]]) - Cos[c[t]]) +
      L^2 m  $\left( \frac{3}{2} a'[t]^2 + 2 \cos[a[t] - b[t]] a'[t] b'[t] + b'[t]^2 + \right.$ 
       $\left. \cos[a[t] - c[t]] a'[t] c'[t] + \cos[b[t] - c[t]] b'[t] c'[t] + \frac{1}{2} c'[t]^2 \right)$ 

In[12]:= eaf2 = D[eq2, a[t]]
      ea2 = D[eq2, a'[t]]
Out[12]= -3 g L m Sin[a[t]] + L^2 m (-2 Sin[a[t] - b[t]] a'[t] b'[t] - Sin[a[t] - c[t]] a'[t] c'[t])
Out[13]= L^2 m (3 a'[t] + 2 Cos[a[t] - b[t]] b'[t] + Cos[a[t] - c[t]] c'[t])

In[14]:= ebf2 = D[eq2, b[t]]
      eb2 = D[eq2, b'[t]]
Out[14]= -2 g L m Sin[b[t]] + L^2 m (2 Sin[a[t] - b[t]] a'[t] b'[t] - Sin[b[t] - c[t]] b'[t] c'[t])
Out[15]= L^2 m (2 Cos[a[t] - b[t]] a'[t] + 2 b'[t] + Cos[b[t] - c[t]] c'[t])

In[16]:= ecf2 = D[eq2, c[t]]
      ec2 = D[eq2, c'[t]]
Out[16]= -g L m Sin[c[t]] + L^2 m (Sin[a[t] - c[t]] a'[t] c'[t] + Sin[b[t] - c[t]] b'[t] c'[t])
Out[17]= L^2 m (Cos[a[t] - c[t]] a'[t] + Cos[b[t] - c[t]] b'[t] + c'[t])

In[18]:= eat2 = D[ea2, t]
Out[18]= L^2 m (-2 Sin[a[t] - b[t]] (a'[t] - b'[t]) b'[t] - Sin[a[t] - c[t]] (a'[t] - c'[t]) c'[t] +
      3 a''[t] + 2 Cos[a[t] - b[t]] b''[t] + Cos[a[t] - c[t]] c''[t])

In[19]:= ebt2 = D[eb2, t]
Out[19]= L^2 m (-2 Sin[a[t] - b[t]] a'[t] (a'[t] - b'[t]) - Sin[b[t] - c[t]] (b'[t] - c'[t]) c'[t] +
      2 Cos[a[t] - b[t]] a''[t] + 2 b''[t] + Cos[b[t] - c[t]] c''[t])

In[20]:= ect2 = D[ec2, t]
Out[20]= L^2 m (-Sin[a[t] - c[t]] a'[t] (a'[t] - c'[t]) - Sin[b[t] - c[t]] b'[t] (b'[t] - c'[t]) +
      Cos[a[t] - c[t]] a''[t] + Cos[b[t] - c[t]] b''[t] + c''[t])

In[21]:= eat2
      eaf2
Out[21]= L^2 m (-2 Sin[a[t] - b[t]] (a'[t] - b'[t]) b'[t] - Sin[a[t] - c[t]] (a'[t] - c'[t]) c'[t] +
      3 a''[t] + 2 Cos[a[t] - b[t]] b''[t] + Cos[a[t] - c[t]] c''[t])
Out[22]= -3 g L m Sin[a[t]] + L^2 m (-2 Sin[a[t] - b[t]] a'[t] b'[t] - Sin[a[t] - c[t]] a'[t] c'[t])

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In[23]:= **Solve**[eat2 == eaf2, a''[t]]

$$\text{Out[23]} = \left\{ \left\{ a''[t] \rightarrow \frac{1}{3L} \left(-3g \sin[a[t]] - 2L \sin[a[t] - b[t]] b'[t]^2 - L \sin[a[t] - c[t]] c'[t]^2 - 2L \cos[a[t] - b[t]] b''[t] - L \cos[a[t] - c[t]] c''[t] \right) \right\} \right\}$$

y[0] = phi_1(t) and y[1] = dphi_1/dt

y[2] = phi_2(t) and y[3] = dphi_2/dt

y[4] = phi_3(t) and y[5] = dphi_3/dt

In[24]:= **Solve**[ebt2 == ebf2, b''[t]]

$$\text{Out[24]} = \left\{ \left\{ b''[t] \rightarrow \frac{1}{2L} \left(-2g \sin[b[t]] + 2L \sin[a[t] - b[t]] a'[t]^2 - L \sin[b[t] - c[t]] c'[t]^2 - 2L \cos[a[t] - b[t]] a''[t] - L \cos[b[t] - c[t]] c''[t] \right) \right\} \right\}$$

In[25]:= **Solve**[ect2 == ecf2, c''[t]]

$$\text{Out[25]} = \left\{ \left\{ c''[t] \rightarrow \frac{1}{L} \left(-g \sin[c[t]] + L \sin[a[t] - c[t]] a'[t]^2 + L \sin[b[t] - c[t]] b'[t]^2 - L \cos[a[t] - c[t]] a''[t] - L \cos[b[t] - c[t]] b''[t] \right) \right\} \right\}$$

$$\text{In[26]} := \text{finaleqa} = \frac{1}{3L} \left(-3g \sin[a[t]] - 2L \sin[a[t] - b[t]] b'[t]^2 - \right.$$

$$\left. L \sin[a[t] - c[t]] c'[t]^2 - 2L \cos[a[t] - b[t]] b''[t] - L \cos[a[t] - c[t]] c''[t] \right)$$

$$\text{finaleqb} = \frac{1}{2L} \left(-2g \sin[b[t]] + 2L \sin[a[t] - b[t]] a'[t]^2 - \right.$$

$$\left. L \sin[b[t] - c[t]] c'[t]^2 - 2L \cos[a[t] - b[t]] a''[t] - L \cos[b[t] - c[t]] c''[t] \right)$$

$$\text{finaleqc} = \frac{1}{L} \left(-g \sin[c[t]] + L \sin[a[t] - c[t]] a'[t]^2 + \right.$$

$$\left. L \sin[b[t] - c[t]] b'[t]^2 - L \cos[a[t] - c[t]] a''[t] - L \cos[b[t] - c[t]] b''[t] \right)$$

$$\text{Out[26]} = \frac{1}{3L} \left(-3g \sin[a[t]] - 2L \sin[a[t] - b[t]] b'[t]^2 - \right.$$

$$\left. L \sin[a[t] - c[t]] c'[t]^2 - 2L \cos[a[t] - b[t]] b''[t] - L \cos[a[t] - c[t]] c''[t] \right)$$

$$\text{Out[27]} = \frac{1}{2L} \left(-2g \sin[b[t]] + 2L \sin[a[t] - b[t]] a'[t]^2 - \right.$$

$$\left. L \sin[b[t] - c[t]] c'[t]^2 - 2L \cos[a[t] - b[t]] a''[t] - L \cos[b[t] - c[t]] c''[t] \right)$$

$$\text{Out[28]} = \frac{1}{L} \left(-g \sin[c[t]] + L \sin[a[t] - c[t]] a'[t]^2 + \right.$$

$$\left. L \sin[b[t] - c[t]] b'[t]^2 - L \cos[a[t] - c[t]] a''[t] - L \cos[b[t] - c[t]] b''[t] \right)$$

In[29]:= Solve[{finaleqa == a'[t], finaleqb == b'[t], finaleqc == c'[t]},
{a'[t], b'[t], c'[t]}]

Out[29]:= $\left\{ \left\{ a''[t] \rightarrow \right. \right.$

$$- \left(\left(\left(1 - \frac{1}{2} \cos[b[t] - c[t]]^2 \right) \left(\frac{g \sin[a[t]]}{L} + \frac{2}{3} \sin[a[t] - b[t]] b'[t]^2 + \frac{1}{3} \cos[a[t] - \right. \right. \right.$$

$$c[t]] \left(- \frac{g \sin[c[t]]}{L} + \sin[a[t] - c[t]] a'[t]^2 + \sin[b[t] - c[t]] b'[t]^2 \right) +$$

$$\frac{1}{3} \sin[a[t] - c[t]] c'[t]^2 \right) - \left(\frac{2}{3} \cos[a[t] - b[t]] - \right.$$

$$\frac{1}{3} \cos[a[t] - c[t]] \cos[b[t] - c[t]] \left(\frac{g \sin[b[t]]}{L} - \sin[a[t] - b[t]] a'[t]^2 + \right.$$

$$\frac{1}{2} \cos[b[t] - c[t]] \left(- \frac{g \sin[c[t]]}{L} + \sin[a[t] - c[t]] a'[t]^2 + \right.$$

$$\left. \left. \sin[b[t] - c[t]] b'[t]^2 \right) + \frac{1}{2} \sin[b[t] - c[t]] c'[t]^2 \right) \Bigg) /$$

$$\left(1 - \frac{2}{3} \cos[a[t] - b[t]]^2 - \frac{1}{3} \cos[a[t] - c[t]]^2 + \frac{2}{3} \cos[a[t] - b[t]] \right.$$

$$\left. \cos[a[t] - c[t]] \cos[b[t] - c[t]] - \frac{1}{2} \cos[b[t] - c[t]]^2 \right) \Bigg),$$

$$b''[t] \rightarrow - \left(\left(6 g \cos[a[t] - b[t]] \sin[a[t]] - 3 g \cos[a[t] - c[t]] \cos[b[t] - c[t]] \right. \right.$$

$$\sin[a[t]] - 6 g \sin[b[t]] + 2 g \cos[a[t] - c[t]]^2 \sin[b[t]] -$$

$$2 g \cos[a[t] - b[t]] \cos[a[t] - c[t]] \sin[c[t]] + 3 g \cos[b[t] - c[t]] \sin[c[t]] +$$

$$6 L \sin[a[t] - b[t]] a'[t]^2 - 2 L \cos[a[t] - c[t]]^2 \sin[a[t] - b[t]] a'[t]^2 +$$

$$2 L \cos[a[t] - b[t]] \cos[a[t] - c[t]] \sin[a[t] - c[t]] a'[t]^2 -$$

$$3 L \cos[b[t] - c[t]] \sin[a[t] - c[t]] a'[t]^2 +$$

$$4 L \cos[a[t] - b[t]] \sin[a[t] - b[t]] b'[t]^2 -$$

$$2 L \cos[a[t] - c[t]] \cos[b[t] - c[t]] \sin[a[t] - b[t]] b'[t]^2 +$$

$$2 L \cos[a[t] - b[t]] \cos[a[t] - c[t]] \sin[b[t] - c[t]] b'[t]^2 -$$

$$3 L \cos[b[t] - c[t]] \sin[b[t] - c[t]] b'[t]^2 +$$

$$2 L \cos[a[t] - b[t]] \sin[a[t] - c[t]] c'[t]^2 -$$

$$L \cos[a[t] - c[t]] \cos[b[t] - c[t]] \sin[a[t] - c[t]] c'[t]^2 -$$

$$3 L \sin[b[t] - c[t]] c'[t]^2 + L \cos[a[t] - c[t]]^2 \sin[b[t] - c[t]] c'[t]^2 \Bigg) /$$

$$\left(L \left(-6 + 4 \cos[a[t] - b[t]]^2 + 2 \cos[a[t] - c[t]]^2 - \right. \right.$$

$$\left. \left. 4 \cos[a[t] - b[t]] \cos[a[t] - c[t]] \cos[b[t] - c[t]] + 3 \cos[b[t] - c[t]]^2 \right) \right),$$

$$c''[t] \rightarrow - \left(\left(6 g \cos[a[t] - c[t]] \sin[a[t]] - 6 g \cos[a[t] - b[t]] \cos[b[t] - c[t]] \right. \right.$$

$$\sin[a[t]] - 4 g \cos[a[t] - b[t]] \cos[a[t] - c[t]] \sin[b[t]] +$$

$$6 g \cos[b[t] - c[t]] \sin[b[t]] - 6 g \sin[c[t]] + 4 g \cos[a[t] - b[t]]^2 \sin[c[t]] +$$

$$4 L \cos[a[t] - b[t]] \cos[a[t] - c[t]] \sin[a[t] - b[t]] a'[t]^2 -$$

$$6 L \cos[b[t] - c[t]] \sin[a[t] - b[t]] a'[t]^2 + 6 L \sin[a[t] - c[t]] a'[t]^2 -$$

$$4 L \cos[a[t] - b[t]]^2 \sin[a[t] - c[t]] a'[t]^2 +$$

$$4 L \cos[a[t] - c[t]] \sin[a[t] - b[t]] b'[t]^2 -$$

$$\frac{4 L \cos[a[t] - b[t]] \cos[b[t] - c[t]] \sin[a[t] - b[t]] b'[t]^2 + 6 L \sin[b[t] - c[t]] b'[t]^2 - 4 L \cos[a[t] - b[t]]^2 \sin[b[t] - c[t]] b'[t]^2 + 2 L \cos[a[t] - c[t]] \sin[a[t] - c[t]] c'[t]^2 - 2 L \cos[a[t] - b[t]] \cos[b[t] - c[t]] \sin[a[t] - c[t]] c'[t]^2 - 2 L \cos[a[t] - b[t]] \cos[a[t] - c[t]] \sin[b[t] - c[t]] c'[t]^2 + 3 L \cos[b[t] - c[t]] \sin[b[t] - c[t]] c'[t]^2}{L (-6 + 4 \cos[a[t] - b[t]]^2 + 2 \cos[a[t] - c[t]]^2 - 4 \cos[a[t] - b[t]] \cos[a[t] - c[t]] \cos[b[t] - c[t]] + 3 \cos[b[t] - c[t]]^2)}} \}}}$$

In[31]:= **Solve**[{x2 + y2 + z2 == x2, x2 + 2 * y2 + z2 == z2, x2 + 2 y2 + 2 c2 == x2}, {x2, y2, z2}]

Out[31]:= {{x2 -> 2 c2, y2 -> -c2, z2 -> c2}}

In[32]:= **Simplify**[

$$\begin{aligned} & - \left(\left(\left(1 - \frac{1}{2} \cos[b[t] - c[t]] \right)^2 \right) \left(\frac{g \sin[a[t]]}{L} + \frac{2}{3} \sin[a[t] - b[t]] b'[t]^2 + \frac{1}{3} \cos[a[t] - c[t]] \right. \right. \\ & \quad \left. \left(- \frac{g \sin[c[t]]}{L} + \sin[a[t] - c[t]] a'[t]^2 + \sin[b[t] - c[t]] b'[t]^2 \right) + \frac{1}{3} \right. \\ & \quad \left. \sin[a[t] - c[t]] c'[t]^2 \right) - \\ & \quad \left(\frac{2}{3} \cos[a[t] - b[t]] - \frac{1}{3} \cos[a[t] - c[t]] \cos[b[t] - c[t]] \right) \\ & \quad \left(\frac{g \sin[b[t]]}{L} - \sin[a[t] - b[t]] a'[t]^2 + \frac{1}{2} \cos[b[t] - c[t]] \right. \\ & \quad \left. \left(- \frac{g \sin[c[t]]}{L} + \sin[a[t] - c[t]] a'[t]^2 + \sin[b[t] - c[t]] b'[t]^2 \right) + \right. \\ & \quad \left. \frac{1}{2} \sin[b[t] - c[t]] c'[t]^2 \right) \Bigg) / \left(1 - \frac{2}{3} \cos[a[t] - b[t]]^2 - \frac{1}{3} \cos[a[t] - c[t]]^2 + \right. \\ & \quad \left. \frac{2}{3} \cos[a[t] - b[t]] \cos[a[t] - c[t]] \cos[b[t] - c[t]] - \frac{1}{2} \cos[b[t] - c[t]]^2 \right) \Bigg] \end{aligned}$$

Out[32]:= (10 g Sin[a[t]] + 4 g Sin[a[t] - 2 b[t]] -

$$\begin{aligned} & g \sin[a[t] + 2 b[t] - 2 c[t]] - g \sin[a[t] - 2 b[t] + 2 c[t]] + \\ & 4 L \sin[2 (a[t] - b[t])] a'[t]^2 + 8 L \sin[a[t] - b[t]] b'[t]^2 + \\ & 2 L \sin[a[t] - c[t]] c'[t]^2 + 2 L \sin[a[t] - 2 b[t] + c[t]] c'[t]^2) / \\ & (2 L (-5 + 2 \cos[2 (a[t] - b[t])] + \cos[2 (b[t] - c[t])])) \end{aligned}$$

$d_1(t) = a(t)$
 $d_2(t) = b(t)$
 $d_3(t) = c(t)$

$\leftarrow a''(t)$

In[39]:= Simplify[

$$\begin{aligned}
& - \left((6 g \cos[a[t] - b[t]] \sin[a[t]] - 3 g \cos[a[t] - c[t]] \cos[b[t] - c[t]] \sin[a[t]] - \right. \\
& \quad 6 g \sin[b[t]] + 2 g \cos[a[t] - c[t]]^2 \sin[b[t]] - \\
& \quad 2 g \cos[a[t] - b[t]] \cos[a[t] - c[t]] \sin[c[t]] + 3 g \cos[b[t] - c[t]] \sin[c[t]] + \\
& \quad 6 L \sin[a[t] - b[t]] a'[t]^2 - 2 L \cos[a[t] - c[t]]^2 \sin[a[t] - b[t]] a'[t]^2 + \\
& \quad 2 L \cos[a[t] - b[t]] \cos[a[t] - c[t]] \sin[a[t] - c[t]] a'[t]^2 - \\
& \quad 3 L \cos[b[t] - c[t]] \sin[a[t] - c[t]] a'[t]^2 + 4 L \cos[a[t] - b[t]] \sin[a[t] - b[t]] \\
& \quad b'[t]^2 - 2 L \cos[a[t] - c[t]] \cos[b[t] - c[t]] \sin[a[t] - b[t]] b'[t]^2 + \\
& \quad 2 L \cos[a[t] - b[t]] \cos[a[t] - c[t]] \sin[b[t] - c[t]] b'[t]^2 - \\
& \quad 3 L \cos[b[t] - c[t]] \sin[b[t] - c[t]] b'[t]^2 + 2 L \cos[a[t] - b[t]] \sin[a[t] - c[t]] \\
& \quad c'[t]^2 - L \cos[a[t] - c[t]] \cos[b[t] - c[t]] \sin[a[t] - c[t]] c'[t]^2 - \\
& \quad \left. 3 L \sin[b[t] - c[t]] c'[t]^2 + L \cos[a[t] - c[t]]^2 \sin[b[t] - c[t]] c'[t]^2 \right) / \\
& \left(L (-6 + 4 \cos[a[t] - b[t]]^2 + 2 \cos[a[t] - c[t]]^2 - \right. \\
& \quad \left. 4 \cos[a[t] - b[t]] \cos[a[t] - c[t]] \cos[b[t] - c[t]] + 3 \cos[b[t] - c[t]]^2) \right) \Big]
\end{aligned}$$

Out[39]=
$$\begin{aligned}
& (-7 g \sin[2 a[t] - b[t]] + 7 g \sin[b[t]] + g \sin[b[t] - 2 c[t]] + g \sin[2 a[t] + b[t] - 2 c[t]] + \\
& \quad 2 L (-7 \sin[a[t] - b[t]] + \sin[a[t] + b[t] - 2 c[t]]) a'[t]^2 + \\
& \quad 2 L (-2 \sin[2 (a[t] - b[t])] + \sin[2 (b[t] - c[t])]) b'[t]^2 - \\
& \quad 2 L \sin[2 a[t] - b[t] - c[t]] c'[t]^2 + 6 L \sin[b[t] - c[t]] c'[t]^2) / \\
& \quad (2 L (-5 + 2 \cos[2 (a[t] - b[t])] + \cos[2 (b[t] - c[t])]))
\end{aligned}$$

← $b''(t)$

In[40]:= Simplify[

$$\begin{aligned}
& - \left((6 g \cos[a[t] - c[t]] \sin[a[t]] - 6 g \cos[a[t] - b[t]] \cos[b[t] - c[t]] \sin[a[t]] - \right. \\
& \quad 4 g \cos[a[t] - b[t]] \cos[a[t] - c[t]] \sin[b[t]] + \\
& \quad 6 g \cos[b[t] - c[t]] \sin[b[t]] - 6 g \sin[c[t]] + 4 g \cos[a[t] - b[t]]^2 \sin[c[t]] + \\
& \quad 4 L \cos[a[t] - b[t]] \cos[a[t] - c[t]] \sin[a[t] - b[t]] a'[t]^2 - \\
& \quad 6 L \cos[b[t] - c[t]] \sin[a[t] - b[t]] a'[t]^2 + 6 L \sin[a[t] - c[t]] a'[t]^2 - \\
& \quad 4 L \cos[a[t] - b[t]]^2 \sin[a[t] - c[t]] a'[t]^2 + 4 L \cos[a[t] - c[t]] \sin[a[t] - b[t]] \\
& \quad b'[t]^2 - 4 L \cos[a[t] - b[t]] \cos[b[t] - c[t]] \sin[a[t] - b[t]] b'[t]^2 + \\
& \quad 6 L \sin[b[t] - c[t]] b'[t]^2 - 4 L \cos[a[t] - b[t]]^2 \sin[b[t] - c[t]] b'[t]^2 + \\
& \quad 2 L \cos[a[t] - c[t]] \sin[a[t] - c[t]] c'[t]^2 - \\
& \quad 2 L \cos[a[t] - b[t]] \cos[b[t] - c[t]] \sin[a[t] - c[t]] c'[t]^2 - \\
& \quad 2 L \cos[a[t] - b[t]] \cos[a[t] - c[t]] \sin[b[t] - c[t]] c'[t]^2 + \\
& \quad \left. 3 L \cos[b[t] - c[t]] \sin[b[t] - c[t]] c'[t]^2 \right) / \\
& \left(L (-6 + 4 \cos[a[t] - b[t]]^2 + 2 \cos[a[t] - c[t]]^2 - \right. \\
& \quad \left. 4 \cos[a[t] - b[t]] \cos[a[t] - c[t]] \cos[b[t] - c[t]] + 3 \cos[b[t] - c[t]]^2) \right) \Big]
\end{aligned}$$

Out[40]=
$$\begin{aligned}
& - \left((2 \sin[b[t] - c[t]] (g \cos[2 a[t] - b[t]] + g \cos[b[t]] + \right. \\
& \quad 2 L \cos[a[t] - b[t]] a'[t]^2 + 2 L b'[t]^2 + L \cos[b[t] - c[t]] c'[t]^2) \Big) / \\
& \quad (L (-5 + 2 \cos[2 (a[t] - b[t])] + \cos[2 (b[t] - c[t])]))
\end{aligned}$$

← $c''(t)$