Measuring Stellar Elemental Abundance

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Motivation:

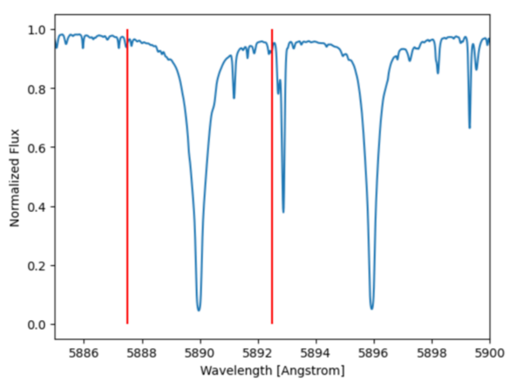
By measuring the elemental abundance of a star or planet, we can discover not only what is contained in their interior, but also what their lifetime was like. Also, we can gain a broad understanding of their age and way of formation. However, due to our inability to physically examine the center of any planetary or stellar body, let alone our own planet, there are many questions left unanswered regarding the core. While this is for research in the future, finding the stellar abundance using the curve of growth method gives us a basic understanding. The curve of growth measures the width of a spectral line. Hence, the specific element we'll be exploring for this project is Sodium (Na) in the sun, an element known to reside within our parent star. We'll collect data on how to find the number density of Na atoms in certain conditions, including when they're neutralized, ionized, and in the ground state. Also, finding the relative abundance of Na to H, the most abundant element in the sun, will give us an idea how much of it is present.

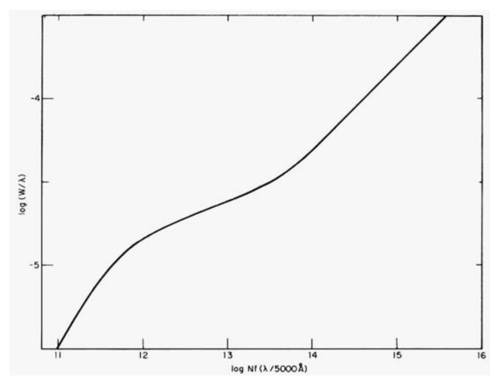
Methods:

To grasp a better understanding on the elemental abundance of Na in the Sun, we must accomplish four tasks: Find the number density of Na atoms in the ground state, excited state, neutral/ionized state, and all states combined. Finally, we find the relative abundance for Na to H, and make a comparison.

Task 1: Number Density of Ground State Sodium Using Equivalent Width

In order to get the number density of ground state Na, and ultimately analyze the abundance of Sodium in the Sun, we first must find it's equivalent width. The equivalent width allows us to roughly estimate the length of a spectral line, which is that of Sodium's in this case. Using the D-line transition of Sodium from the 3p to 3s energy levels, occurring at 5890 and 5896 Angstroms. On a Wavelength vs Normalized Flux graph, there are absorption lines at 5890 and 5896 Angstroms, which are the ground state and excited state respectively. So, we'll use the 5890 A line. For this project, the continuum intensity level is estimated to be +- 2.5 A. Using the graph below and assistance with *Python*, we can find the equivalent width. Oscillator strength f = 0.65. From there, we can use a curve of growth for the sun to get our wanted number density, shown below. By finding the solution for log(W/ λ) on the y-axis, we can estimate the location on the x-axis, and solve for N in log(Nf(λ/5000A)) to get the Number Density of ground state Na.





Task 2: Number Density of Excited State Sodium Using Boltzmann Equation

Now that we have the results for ground state, we can now look at the excited state for sodium, which has an absorption line at 5896 A. Using the Boltzmann equation, we can find our solution*:*

$$\frac{N_2}{N_1} = \frac{g_2}{g_1} \exp\left(-\frac{E_2 - E_1}{kT}\right)$$

In this formula, N2 is the excited state, N1 is the ground state, g is the number of degenerate energy states, E is energy at the state, k is the Boltzmann constant, and T is the temperature. The energy of the ground and excited states are -5.14 eV and -3.04 eV respectively. The g2/g1 ratio is 2, due to both having three degenerate states and a filled 2p orbital. T is 5780 K, the sun temperature.

Task 3: Number Density of Neutral & Ionized Na Atoms Using Saha Equation

The Saha Equation is provided below:

$$\frac{Na_{II}}{Na_I} = \frac{2kT}{P_e}\frac{Z_{II}}{Z_I}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} \exp\left(-\frac{\chi}{kT}\right)$$

Here, partition functions Z(II) = 1.0 and Z(I) = 2.4, electron pressure = m(e)kT = 1.0, the ionization energy X = 5.1 eV, and the Temperature T = 5780 K. The neutral state number density Na(I) = N1 + N2, which is the Number Density of Neutral Na atoms. Na(II) is the Number Density of Ionized Na atoms.

Task 4: Total Number Density of Na Atoms in Every State

Using the previous values, we can calculate the column density of Sodium, where the total number of sodium molecules in the sun's photosphere is shown below:

$$N_1 \times \left(1 + \frac{N_2}{N_1}\right) \times \left(1 + \frac{Na_{II}}{Na_I}\right)$$

The solution to this equation will give us our desired solution.

Final Task: Relative Abundance for Na to H

Finally, we compare our calculated abundance of sodium in the sun's photosphere to that of Hydrogen. The column density of hydrogen atoms is ~6.6 x 10^23. To get the solution, we must express the equation by a logarithm, where the base Hydrogen is set at 12:

$$\frac{N_{element}}{N_H} = 12 + log_{10}\left(\frac{N_{element}}{N_H}\right)$$

The solution to this formula allows us to find the number density ratio (N(Na) / N(H)) and the relative log abundance (Na/H).

Results:

In order to calculate the number density of Na atoms in the ground state, we needed to find the equivalent width of the Na spectrum. For our solution, we got 0.834 Angstroms, which is reasonable. Then, we used the curve of growth method. Due to it's fairly linear nature, similar to the equation y = x, if we can find the value for y, we can estimate a value for x. In this case, y = log(W/ λ), and using the equivalent width, got a solution of y = -3.85. Using this, we approximated the x-axis to have a value of x = 14.8. The equation for this is x = log(Nf(λ/5000A)), and finding N, we get a solution of 8.24 x 10^12 atoms/cm^2 for the number density of Na atoms in the ground state.

For the number density of neutral Na atoms, we used the Boltzmann equation to get our answer. Ultimately, we calculated a value of 2.43 x 10^12 atoms/cm^2. This value is about 3.39 times lower than the Na ground state. For the Na ionized state, we calculated a value of 2.136 x 10^18 atoms/cm^2.

Due to the ionized Na value being exponentially higher than the values for ground and excited state Na, it was the dominant factor when we calculated the total number density for Na atoms in all states. We calculated this number to be 2.137 x 10^18 atoms/cm^2, which is nearly similar to the ionized value. Our number density ratio of N(Na) / N(H) was 3.24 x 10^-6, and the relative log abundance (Na/H) was 6.51. These findings are very similar to what was discovered prior.

Conclusion:

The main goal of this research was to measure the stellar elemental abundance of Sodium using the curve of growth method. Using a lot of mathematics with finding the number density of Ana atoms in the ground, excited, neutral, and ionized states, we were able to find the total number density of Na. From here, we found the number density ratio and the relative log abundance of Sodium and Hydrogen. With Hydrogen being the most abundant element of our Sun and a relative log abundance (Na/H) of 6.51, there is much more Hydrogen in the Sun than there is of Sodium. However, a decent amount of Sodium is present, and that deserves to be recognized.

References:

Wang, J., Mar 29, 2023. *Project 3: Measuring Stellar Elemental Abundance*, URL: https://buckeyemailosu-my.sharepoint.com/personal/wang_12220_osu_edu/_layouts/15/onedrive.aspx?ga=1&id=%2Fpersonal%2Fwang%5F12220%5Fosu%5Fedu%2FDocuments%2FEarthScAstron%205205%2FASTRON5205%5F2023SP%2FCollaborative%5FProjects%2FProject%5F3%2FProject%5F03%5FMeasuring%5Fabundance%2Eipynb&parent=%2Fpersonal%2Fwang%5F12220%5Fosu%5Fedu%2FDocuments%2FEarthScAstron%205205%2FASTRON5205%5F2023SP%2FCollaborative%5FProjects%2FProject%5F3

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