Investigating the Mass, Radius and Density of the Exoplanet GJ 436 b

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Motivation:

When Galileo first peered into the sky with his telescope, people began to pose one particular question: why? Today, hundreds of years later, this question still stands. Why do we study planets and galaxies in the universe when they are so inconceivably far away from our own home? Generally speaking, the study of planets near and far from our own solar system serves the purpose of aiding us in understanding our own little galactic neighborhood. By observing the behavior and composition of such cosmic entities, we can gain a stronger grasp of the physics that governs the planet we call home and everything in it. Three major features of planets that astronomers typically analyze include the mass, radius, and density. Exoplanets are no exception to this rule, nor is GJ 436 b (Host Star: Gliese 436 (M-type/red-dwarf)). GJ 436 b in particular exhibits some strange behavior, and astronomers typically study such strange bodies, comparing them to some reference planets such as Earth, in order to determine whether we truly do understand the physics that governs the universe.

Methods:

In order to determine the mass, radius and density of GJ 436 b we are required to investigate its signal from two specific methodologies of exoplanet detection. In accordance with present day instrumentation, the radial velocity and transit methods are our best chance at getting accurate measurements of GJ 436b’s mass and radius, from which we can derive its density.

Another important variable that we will be considering is the errors of mass and radius in our calculations, which we will be determining using the propagation of uncertainty.

The radial velocity technique is used to measure the value “K” (in meters per second), which is the shift in light that the star produces when being gravitationally perturbed by an orbiting planet. We will use python to graph and fit the signal of the host star, as well as use the equation:

$$K = \frac{RV_{max} - RV_{min}}{2}$$

to determine the signal. With this we are able to derive it the mass of the planet from:

$$m_p = \frac{K}{\sin i} \cdot m_* \cdot \sqrt{\frac{a}{Gm_*}}$$

where mp and m* are the masses of the planet and star, a is the planet's semi-major axis, G is the gravitational constant and sin(i) measures the inclination of the system relative to the observer's perspective. Using the propagation of uncertainties we can find the errors of our calculations of mass:

$$\sigma_K = \frac{\sqrt{\sigma^2_{RV_{max}} + \sigma^2_{RV_{min}}}}{2}, \qquad \sigma_{m_p} = \frac{\sigma_K}{\sin i} \cdot m_* \cdot \sqrt{\frac{a}{Gm_*}}$$

The next step is to investigate the change in stellar flux “f” when an orbiting planet travels in front of our view of the star. With this measurement we can use the relation of stellar and planetary radius in order to determine GJ 436b’s radius, again using propagation of uncertainty:

$$R_p = \sqrt{f} \cdot R_s, \quad \sigma_{R_p} = \frac{1}{2} \cdot \frac{\sigma_f}{f} \cdot R_p$$

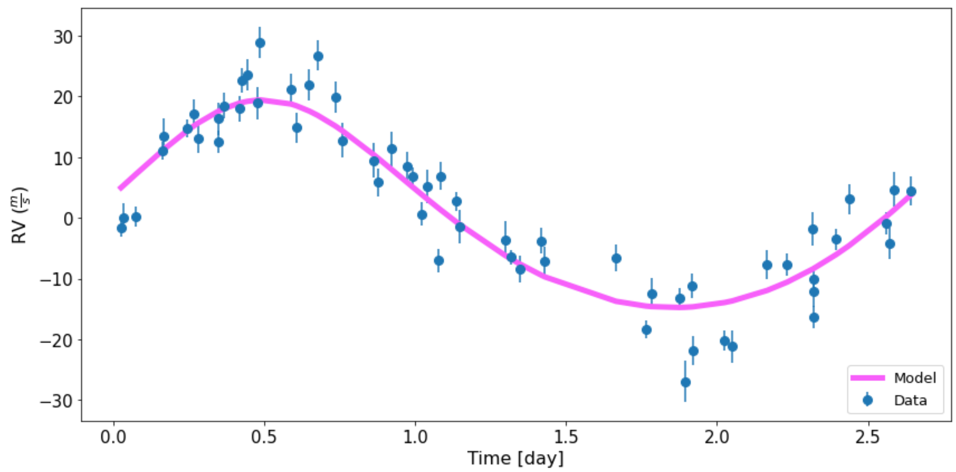
Once we find these measurements, all that remains is to simply plug our values into the equation for density, once again accounting for errors:

$$\rho = \frac{m}{(4/3)\cdot\pi R_p^3} \quad , \quad \sigma_\rho = \rho \cdot \sqrt{(\frac{\sigma_{m_p}}{m_p})^2 + (\frac{\sigma_{R_p}}{R_p})^2 + (\frac{\sigma_{R_p}}{R_p})^2 + (\frac{\sigma_{R_p}}{R_p})^2}$$

and we can find the density for GJ 436b.

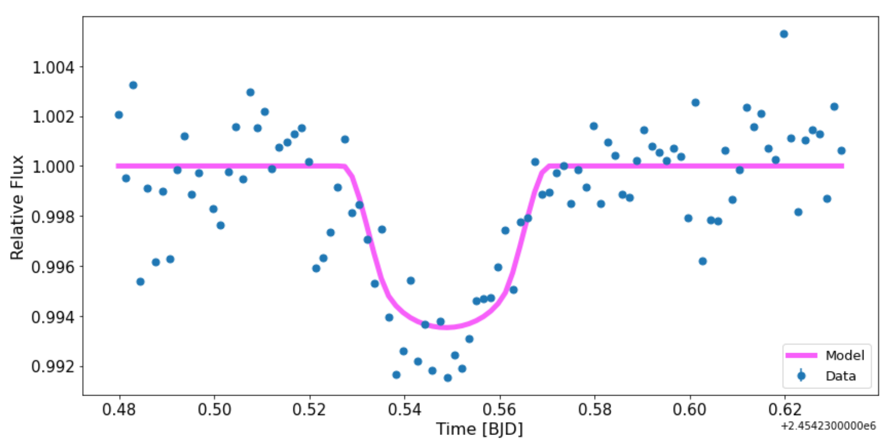
Results:

Plotting and fitting the RV signal of Gliese 436 we find:



which shows K in meters per second and the time in day. The graph shows one period of the signal, and using the amplitude we find the mean value of K to be 16.955 +- 0.113 m/s. Plugging this into the mass relation we find a mass for GJ 436b to be 21.517 +- 0.143 Earth mass.

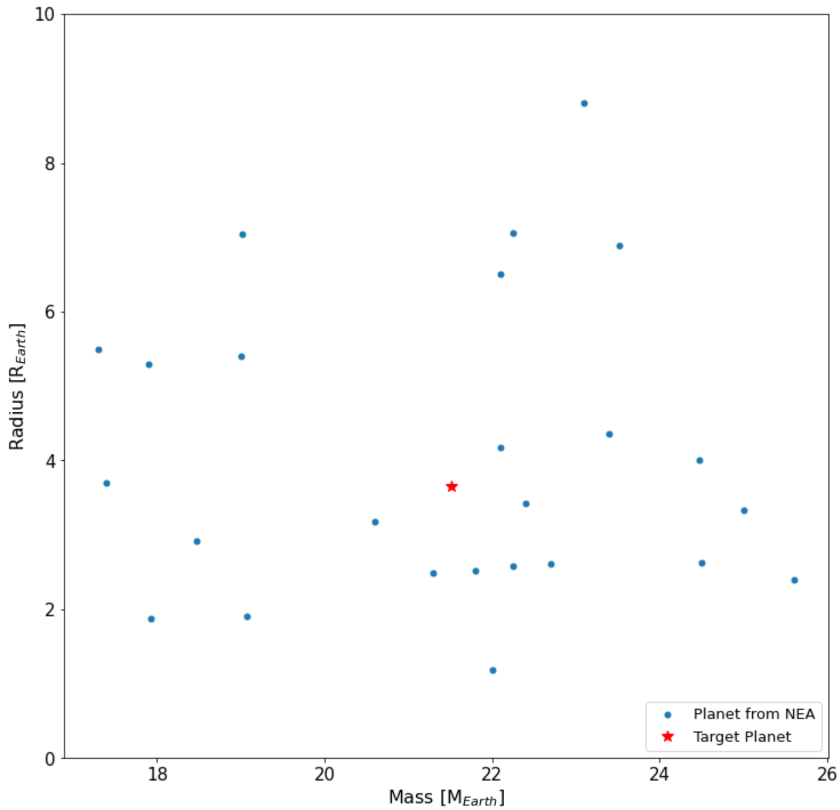
Plotting the Relative flux of the host star we find:



And we see the output of the star and the noticeable dip in flux as the planet transits and blocks some of the light. Using this value we can find the radius of GJ 436b to be 3.649 +- 0.008 Earth Radii. Using this value and our calculated mass we can plug them into the density formula to find it to be 2434.003 +- 18.385 kg/m^3, which is roughly half the density of Earth.

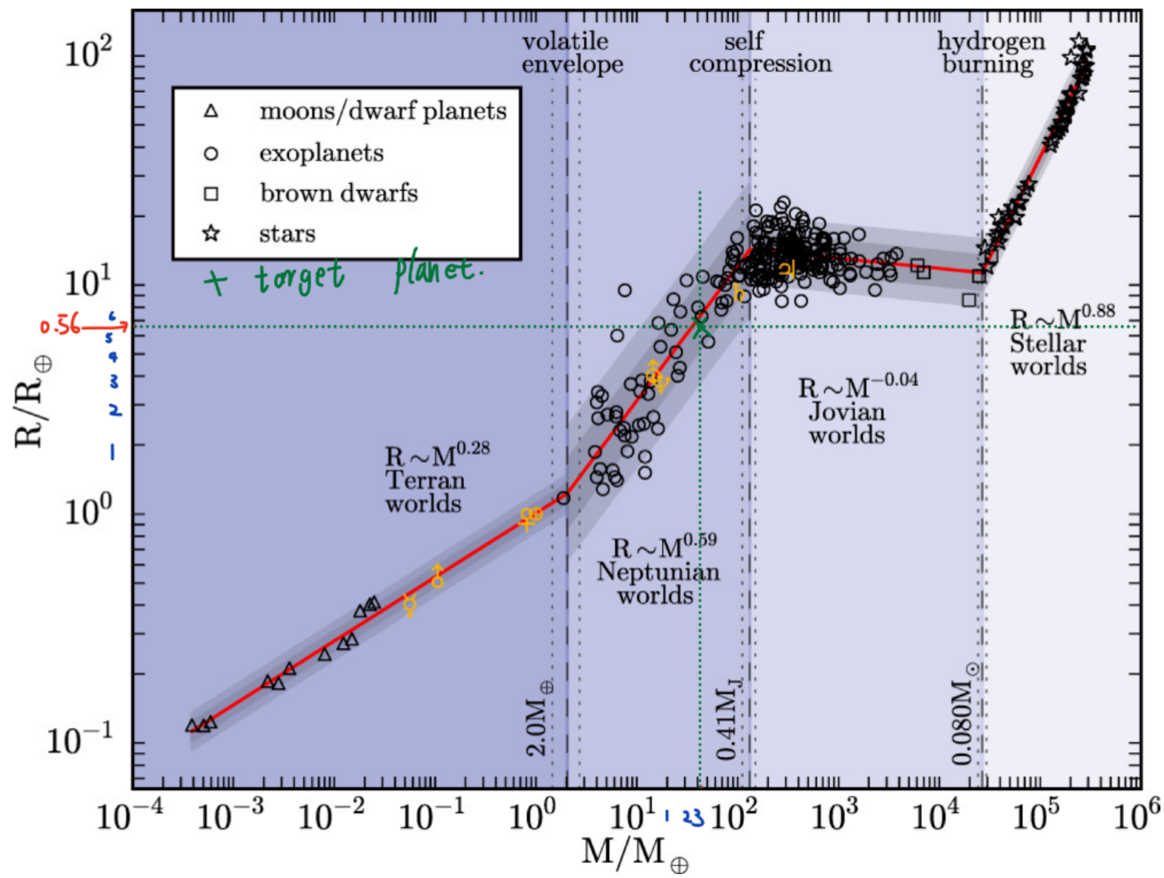
Comparisons:

Now we will see how our planet relates to other planets found in Nasa's Exoplanet Archive:



and we see that GJ 436b fits nicely amongst other planets. Another reference that we can use to check our data is to cross reference it from the paper "Probabilistic Forecasting of the Masses and Radii of Other Worlds" by Jingjing Chen and David Kipping where we are given Mass and

Radius relationships between exoplanets:



and we find that our measurements GJ 436b correlate with Chen and Kipping's relation of mass and radius.

Conclusion:

The main goal of this project was to identify an exoplanet, obtain its radial velocity and transit data, and use this information to find the mass, radius, and by extension, density of the planet. Radial velocity and transit data were the chosen methods of measuring the aforementioned planetary characteristics due to the fact that our current technology finds these methods the most accurate. After plotting the RV signal of Gliese 436 and using mass relations, we found that the mass of GJ 436 b would be approximately 21.517 +- 0.143 Earth mass. After plotting the relative flux, we were able to calculate the radius to be 3.649 +- 0.008 Earth Radii. As for the density, we used the relationship between mass and radius to calculate it to be about half the density of Earth, standing at 2434.003 +- 18.385 kg/m^3. Once we compared these results to the NASA Exoplanet Archive, we discovered that it blends in quite well. Additionally, we concluded it to be Neptune-sized. As previously mentioned, this information regarding GJ 436 b may seem arbitrary right now, but such data could very well contribute towards deepening our understanding of fundamental physical concepts. With such a strange planet at the center of this conversation, this data could even challenge some of those ideas and lead to future breakthroughs.

Contributions:

For Project 2, Huihao Zhang worked on the computational aspect, while Connor Michael created the powerpoint presentation and delivered the oral presentation. Connor McKiernan and Farah Abdulrahman completed the written report.

References:

Chen, J., & Kipping, D. (2016). Probabilistic forecasting of the masses and radii of other worlds. *The Astrophysical Journal*, *834*(1), 17. https://doi.org/10.3847/1538-4357/834/1/17

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