# RI3004A 3D Graphics Rendering

## Discussion 7 (Answers)

For **Lecture 10: Local Reflection Models**

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Please attempt the following questions before you go to your discussion class. Some of the questions may be quite open-ended and some may be even ambiguous. In those cases, you are encouraged to make your own (reasonable) assumptions.

(1) If the only light source is an area light source with radiance $L$, is it possible that some surfaces in the scene have outgoing radiance greater than $L$? If yes, in what situations? If not, why?

Yes, it is possible some surfaces may have outgoing radiance greater than $L$. This can happen when there is a large reflector or refractor in the scene that focuses the light from the light source onto a very small area, e.g. caustics.

(2) Suppose a very small surface reflects light in such a way that the <u>light power</u> given out in every direction is the same per unit solid angle. Describe any unusual appearance of the surface when viewed from some viewing directions. <u>Use radiance</u> to explain why.

The surface will look infinitely bright when the viewing direction is parallel to the surface. This is because the surface's projected area becomes zero, and when the power per unit solid angle is divided by this projected area to get the radiance, the radiance becomes infinity.

(3) Given a very small surface $S$, a viewer, looking at $S$ from a distance of $D$ away, can observe a radiance of $L$ in the direction $\omega$ from $S$. What would be the radiance observed by the viewer if he is looking at $S$ from a distance of $2D$ from the same direction as before?

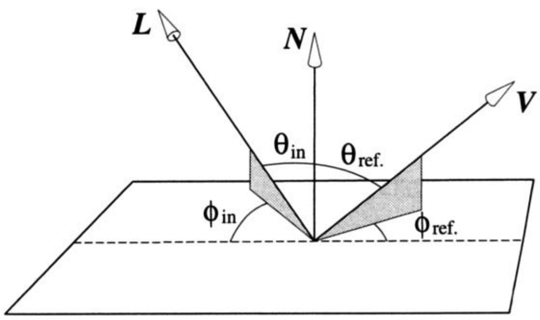
The radiance remains as $L$. Even though the light power received by the viewer has become 1/4 of the original (inverse-square law), the solid angle subtended by the viewer has also become 1/4 of the original.

(4) List two visual characteristics of <u>metal</u> that cannot be produced by the <u>Phong</u> reflection model.

* Fresnel effect.
* The color of the specular highlight changes with incident light direction.
* Off-specular highlight.

(5) Is the following function $f$ a valid BRDF? List two reasons why it is/is not valid. Refer to the figure for the meanings of the function parameters.

$$f(\theta_{in}, \varphi_{in}, \theta_{ref}, \varphi_{ref}) = \frac{1}{2} + \frac{1}{3}\cos^2\theta_{ref} - \frac{1}{6}\sin^3\theta_{in}$$




It is not a valid BRDF because

(i) The value of $f$ is always greater than $1/\pi$ for all parameters. This means that the total outgoing light energy in all directions is greater than the incoming, which is physically impossible.

(ii) The function $f$ does not follow the Helmholtz Reciprocity, that means $f(\theta_{in}, \varphi_{in}, \theta_{ref}, \varphi_{ref}) \neq f(\theta_{ref}, \varphi_{ref}, \theta_{in}, \varphi_{in})$ in general.

(6) A point light source $p$ is giving out light power $\Phi$ uniformly in all directions. Let $x$ be a surface point on a differential surface $s$ of area $dA$, which is directly illuminated by the point light. The following quantities are provided:

$r$ — the distance between $p$ and $x$;
$N$ — the unit surface normal vector at $x$;
$\Psi$ — the unit vector from $x$ to $p$;
$\Theta$ — the unit vector from $x$ to the sensor;
$f(x, \Psi\to\Theta)$ — the BRDF at $x$ for an incoming direction $\Psi$ and an out-going direction $\Theta$.

**(a)** Write an expression for the light power received by the differential surface $s$ from the point light. (Hint: the surface area of a sphere of radius $R$ is $4\pi R^2$.)

$$\frac{\Phi}{4\pi r^2} \cdot (N \bullet \Psi) \cdot dA$$

**(b)** Write an expression for the radiance reflected from $x$ in the direction $\Theta$, due to the direct illumination from the point light. Let $L(x\to\Theta)$ be this reflected radiance from $x$.

$$L(x \to \Theta) = f(x, \Psi \to \Theta) \cdot \frac{\Phi}{4\pi r^2} \cdot (N \bullet \Psi)$$

(7) The BRDF of a diffuse surface is a constant for all incident and outgoing directions. Show your workings to derive the BRDF (which is a constant $\rho$) of a *perfect* diffuse surface. A perfect diffuse surface reflects *all* incident light energy. Start your derivation from the following:

$$\frac{\text{total outgoing light power}}{\text{incident light power}} = \frac{\int_{\Omega} dA \cdot L_{\text{out}} \cdot \cos\theta_{\text{out}} \cdot d\omega_{\text{out}}}{dA \cdot L_{\text{in}} \cdot \cos\theta_{\text{in}} \cdot d\omega_{\text{in}}} = 1$$

where $dA$ is the differential surface area, and $\Omega$ denotes integration over the solid angle of the entire hemisphere.

$$\frac{\text{total outgoing light power}}{\text{incident light power}} = \frac{\int_{\Omega} dA \cdot L_{\text{out}} \cdot \cos\theta_{\text{out}} \cdot d\omega_{\text{out}}}{dA \cdot L_{\text{in}} \cdot \cos\theta_{\text{in}} \cdot d\omega_{\text{in}}} = 1$$

$$\Rightarrow \frac{dA \cdot L_{\text{out}} \int_{\Omega} \cos\theta_{\text{out}} \cdot d\omega_{\text{out}}}{dA \cdot L_{\text{in}} \cdot \cos\theta_{\text{in}} \cdot d\omega_{\text{in}}} = 1$$

$$\Rightarrow \frac{dA \cdot L_{\text{out}} \cdot \pi}{dA \cdot L_{\text{in}} \cdot \cos\theta_{\text{in}} \cdot d\omega_{\text{in}}} = 1$$

$$\Rightarrow \frac{L_{\text{out}}}{L_{\text{in}} \cdot \cos\theta_{\text{in}} \cdot d\omega_{\text{in}}} = \frac{1}{\pi}$$

$$\Rightarrow \rho = \frac{1}{\pi}$$

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