Lecture 4 - Randomization and Complexity

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- Two goals this lecture:
  1. Focus on experimental works and their relevance to theory.
  2. Introduce you to some recent and ongoing debates and research areas.

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- Two goals this lecture:
  1. Focus on experimental works and their relevance to theory.
  2. Introduce you to some recent and ongoing debates and research areas.
- We will do this by delving into three topics:
  - Randomization in choice
  - Complexity and cognition

Randomization in Choice

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- Stochastic choice has a long history in economics, some models:
  - Random EU, Logit models
  - Drift Diffusion models
  - Rational Inattention
  - Ambiguity and hedging

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  - Random EU, Logit models
  - Drift Diffusion models
  - Rational Inattention
  - Ambiguity and hedging
- Recently, Agranov and Ortoleva (2017 JPE) shows conclusive evidence in favor of what they call “deliberate randomization".

Introduction and Motivation

- Individuals often exhibit **stochastic choice**: making different decisions when faced with the same set of options repeatedly.
- Possible explanations include:
    1. Random utility: preferences change over time.
    2. Bounded rationality: decision errors occur despite stable preferences.
    3. Deliberate randomization: individuals intentionally diversify decisions.
- This paper investigates the origins of stochastic choice using novel experimental designs.

Experimental Design

- Subjects chose between lotteries with outcomes based on a four-sided die.
- Two main treatments:
  1. **Distant repetitions:** Questions repeated apart, with no advance notice.
  2. **In-a-row repetitions:** Questions repeated consecutively, with prior notice.
- Additional features:
  - Option to pay for a randomizing coin flip.
  - Measurement of risk attitudes and violations of expected utility (Allais paradox).

Some Models of Stochastic Choice

- Gul and Pesendorfer (2006 ECMA) Random Expected Utility:

$$\rho(A)(p) = \mu(\{u \in U : u(p) > u(q) \text{ for all } q \in A\})$$

- The probability that lottery p is chosen from a set A is the probability that it is the best lottery for a randomly chosen utility $u \in U$ with distribution μ.

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- The following axioms + continuity* are sufficient and necessary for REU representation:
  - Regularity: $p \in A \subseteq B$ then $\rho(p, A) \geq \rho(p, B)$.
  - Extremeness: $\rho(ext(A), A) = 1$, $ext(A)$ is the set of extreme points of A (Draw ICs and set of lottery to illustrate).
  - Linearty: $\rho(p, A) = \rho(\alpha p + (1-\alpha)q, \alpha A + (1-\alpha)q)$ (Idea: vNM applied utility by utility.

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- Random behavior interpret as time-variant stochastic utility:
  - Experimental prediction: stochastic in distant repetition, but not in-a-row repetition.

Some Models of Stochastic Choice

- Drift Diffusion: choosing between $u(p)$ and $u(q)$ which are perceived with noise:

$$X(t) = X(t-1) + \alpha[u(p) - u(q)] + \epsilon(t)$$

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- Drift Diffusion: choosing between $u(p)$ and $u(q)$ which are perceived with noise:

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- Random behavior interpret as noisy perception
  - Experimental prediction 1: stochastic in distant repetition, but not in-a-row repetition.
  - Experimental prediction 2: stochastic behavior increasing between options with closer values.

Some Models of Stochastic Choice

- Deliberate Randomization, Cerreia-Violgio et al. (2016).
- Agent can randomize between lotteries to hedge uncertainty in utility evaluation (think back to Cautious EU).

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- Deliberate Randomization, Cerreia-Violgio et al. (2016).
- Agent can randomize between lotteries to hedge uncertainty in utility evaluation (think back to Cautious EU).
- Random behavior interpret deliberate preference
  - Experimental prediction 1: stochastic in distant repetition, and in-a-row repetition.

TABLE 1
LIST OF QUESTIONS ASKED

Question	Lottery 1 (1)	Lottery 1 (2)	Lottery 1 (3)	Lottery 1 (4)	Lottery 2 (1)	Lottery 2 (2)	Lottery 2 (3)	Lottery 2 (4)	Difference in Expected Value
FOSD1	98	98	98	98	103	103	103	103	5
FOSD2	17	17	18	18	17	17	17	17	.5
FOSD3	10	20	30	30	70	100	120	190	97.5
EASY1	23	23	30	30	5	5	5	31	15
EASY2	12	14	16	96	85	85	85	85	50.5
EASY3	100	100	100	100	20	20	20	101	59.75
HARD1	38	38	38	77	16	16	94	94	7.25
HARD2	10	10	90	90	32	45	45	56	5.5
HARD3	6	84	105	200	54	60	117	135	7.25
HARD4	13	30	51	81	19	32	38	86	0

Main Findings

- **Widespread stochastic choice:**
  90% in distant repetitions, 71% in in-a-row repetitions.
- **Hard questions drive behavior:** Stochastic choice occurs primarily when options are similarly appealing ("hard questions").
- **Deliberate randomization:** 79% of participants intentionally diversified decisions.
- Some participants (29%) opted to pay for randomization (costly coin flip).

Theoretical Implications

- Random utility and bounded rationality models (e.g., REU* and DDM*):
  - Explain some patterns but fail to account for consistent stochastic choice in both treatments.
- Models of deliberate randomization (e.g., cautious stochastic choice):
  - Successfully predict stochastic behavior in hard questions.
  - Link stochastic choice with Allais-type violations of expected utility.

Conclusions and Robustness Checks

- **Main conclusion:** Deliberate randomization is the primary driver of stochastic choice.
- Robustness:
  - Replicated findings with high stakes, time preferences, and social preferences.
  - Correlation between stochastic choice and deliberate hedging/diversification.

Some Recent Works: Randomization

- **Randomization as Deliberate Preference:**
  - Cerreia-Vioglio et al. (2019, AER): Models deliberately stochastic behavior.
  - Agranov et al. (2023, EJ): Randomization is stable across environments and task types.
  - Agranov and Ortoleva (2023, Restat): Large ranges of randomization observed in people.
  - Toussaert (2025, WP): Intentional violations of stochastic dominance for surprise effects.
- **Ambiguity, Hedging, and Randomization:**
  - Ke and Zhang (2020, ECMA): Theory of ambiguity aversion and hedging.
  - Baillon et al. (2022, ECMA): Randomization reveals ambiguity aversion.
- **Incomplete Preferences and Randomization:**
  - Nielsen and Rigotti (2024, WP): Direct elicitation of incomplete preferences.
  - Halevy et al. (2024, WP): Simple randomization patterns challenge complete preference models.

Three Works

- **Randomization as Deliberate Preference:**
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- **Ambiguity, Hedging, and Randomization:**
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Toussaert 2024

- Stochastic dominance is a normative rule in decision theory, often linked to rational choice.
- Violations are typically rare and attributed to cognitive errors or complex gambles.
- This paper explores stochastic dominance violations due to anticipatory utility.
    - Example: Sacrificing a guaranteed holiday destination for the excitement of a surprise lottery.
- Objective: Investigate the experimental conditions and theoretical implications of such violations.

Toussaert 2024

- Participants ranked 10 European holiday destinations and provided valuations for each.

- Presented with binary decision problems:
  - Option A: A sure trip to a specific destination.
  - Option B: A "surprise lottery" over two or more destinations.

- Key variations:
  - Number of destinations in the lottery.
  - Rankings of destinations within the lottery.
  - Probability distribution over destinations.

- Additional tasks:
  - Participants designed their preferred lottery and chose when to resolve uncertainty.
  - Monetary equivalents of trips were used to study the role of experiential value.

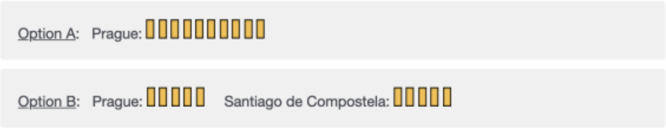
Toussaert 2024

Table 2: Structure of the survey

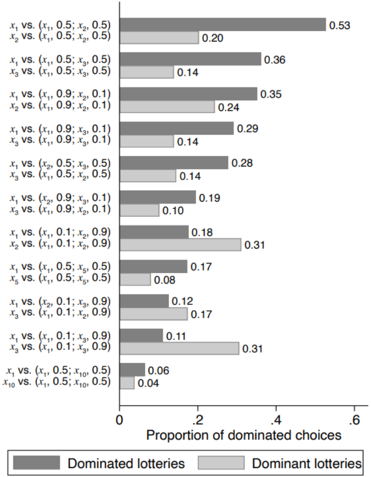
Part 1	**Elicitation of preferences $\succeq$ over holiday trips**
	▷ Ordinal ranking of 10 destinations, $x_1, x_2, ..., x_{10}$
	▷ Valuation $v_k \in [0, 500]$ for each destination x_k
Part 2	**Choices in 45 binary decision problems (DPs)**
	A: $(x_i, 1)$ vs. B: $(x_j, p_j; x_k, p_k; ...; x_l, p_l)$
Part 3	**Design of favorite lottery**
	▷ Selection of support and probability distribution
	▷ Selection of date at which to resolve the uncertainty
	▷ [If chose a positive delay] Valuation of delay
Part 4	**Preferences for a "wildcard trip"**
	▷ Valuation of a trip to an unknown destination
	▷ Questions to understand motives
Part 5	**Preferences over monetary gambles**
	3 DPs with each destination replaced by its valuation
	Valuation of one risky and one ambiguous monetary bet
	Questions about travel history and preferences
End	Selection of choice problem

Toussaert 2024

Figure 2: Sample decision problem



Toussaert 2024



Toussaert 2024

- Stochastic dominance violations occur in favor of randomization in 20% of decisions.
  - Respondents sacrifice an average of £46 (11% of trip value) for surprise.
- Violations occur more frequently for lotteries with higher entropy (more uncertainty).
- Preference for delay: 74% of participants who prefer randomization choose to delay the resolution of uncertainty.
- Violations largely disappear for monetary lotteries, emphasizing the role of experiential goods.

Toussaert 2024

- Standard models of expected and non-expected utility fail to explain observed violations:
    - Deterministic models typically satisfy stochastic dominance.
    - Violations tied to high entropy and preference for delay suggest a need for new models.
- Practical insights:
    - Goods with surprise elements can increase consumer welfare and motivation.
    - Implications for designing incentive structures and marketing strategies.
- Call for models that incorporate anticipatory utility and multidimensional choice objects.

Baillon, Halevy and Li 2022

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  - Classical: pay all or random incentive scheme (RIS).

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- This paper investigates the interaction between ambiguity, RIS and randomization.

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- If subjects are sophisticated, can use the RIS as a hedge and ambiguity averse subject would choose *as if* ambiguity neutral and randomize.

Baillon, Halevy and Li 2022

Red

(circle a or b)

a) €10 if a **red** chip is drawn from Bag K.

b) €10.20 if a **red** chip is drawn from Bag U.

Blue

(circle a or b)

a) €10 if a **blue** chip is drawn from Bag K.

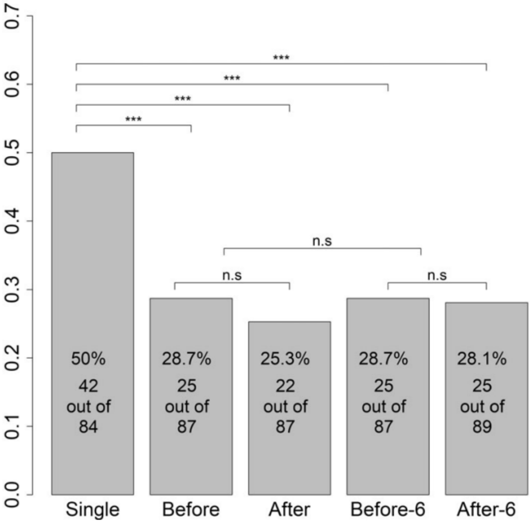
b) €10.20 if a **blue** chip is drawn from Bag U.

Baillon, Halevy and Li 2022

- Subjects faced two bags:
  - **Bag K (Known):** One red and one blue chip.
  - **Bag U (Unknown):** Unknown composition of red and blue chips.

- Tasks:
  - Choose between bets on Bag K and Bag U with slightly higher payouts for Bag U.
  - Treatments varied the timing of randomization and uncertainty resolution:
    - **Single:** No random incentives.
    - **Before:** Randomization before choice and uncertainty resolution.
    - **After:** Randomization after uncertainty resolution.

Baillon, Halevy and Li 2022



Baillon, Halevy and Li 2022

- Ambiguity aversion observed in **Single** treatment (50% chose Bag K).
- Lower ambiguity aversion in **Before** (29%) and **After** (25%) treatments.
- Evidence of integration:
  - Half of ambiguity-averse subjects integrated decisions, reducing ambiguity via RIS.
- Suggests that RIS distorts measurement of ambiguity aversion.

Baillon, Halevy and Li 2022

- If RIS selects one choice randomly for payment, ambiguity-averse individuals can hedge ambiguity:
  - Ambiguity is neutralized by the objective randomization device.
- Implication: Integrated decisions under RIS mimic ambiguity neutrality.
- Experimental evidence:
  - Subjects using integration showed less ambiguity aversion, supporting Raiffa's argument.

Halevy, Walker-Jones and Zrill 2023

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Halevy, Walker-Jones and Zrill 2023

- Question: What does it mean to have incomplete preferences - in a choice theoretic sense? And does it imply randomization?
- Challenge 1: If you only observe choices - can never distinguish preference for randomization vs incomplete preference $\Rightarrow$ need a richer dataset to identify incompleteness.
- New question: Is there a richer types of observables (in addition to choice) that is also natural enough that identifies complete vs incomplete behavior?

Halevy, Walker-Jones and Zrill 2023

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Halevy, Walker-Jones and Zrill 2023

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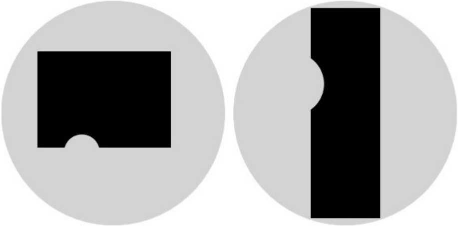
- Question: Is there a set of sufficient weak axioms such that if we reject them+completeness, on this new dataset, then it gives a good justification for questioning completeness?

Experiment

DM is tasked with determining which shape is larger: L or R



1. Would you rather bet on L or R or randomize?
2. What is the chance that the Right shape is larger?
   - Subjects get a lower bound l_R and upper bound u_R for this chance.
3. What is the chance that the Left shape is larger?
   - Subjects get a lower bound l_L and upper bound u_L for this chance.

A bet on a shape wins the \$m prize iff the shape is larger

The Acts L and R (Betting on the Left or Right Shape)

The Acts L and R

s	$L(s)$	$R(s)$
λ	100	0
ρ	0	100

The task is then:

- Choose L or R or randomize.
- Give $p(\lambda)$ the probability equivalent of L
- Give $p(\rho)$ the probability equivalent of R

Predictions: Subjective Expected Utility (SEU)

States of the world $S = \{\lambda, \rho\}$ (L or R is bigger)

Probabilities of the states $p(\lambda)$ and $p(\rho)$ such that $p(\lambda) + p(\rho) = 1$, and:

$$L \succeq R \iff p(\lambda) \geq p(\rho)$$

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$$L \succeq R \iff p(\lambda) \geq p(\rho)$$

- Only degenerate intervals — lower bound equals the upper bound — should be reported
- Should only randomize over L and R if 50-50

Prediction: Maxmin Expected Utility

There is a closed and convex set $\mathcal{C} \subseteq \Delta(S)$ such that

$$L \succeq R \iff min_{p \in \mathcal{C}} p(\lambda) \geq min_{p \in \mathcal{C}} p(\rho)$$

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- Only degenerate intervals should be reported
- Intervals should sum up to less than or equal to 100
- May randomize over L and R even if not 50-50 (ambiguity aversion)

Axioms

Sample of axioms they consider (in addition to completeness):

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Axioms

Sample of axioms they consider (in addition to completeness):

- Monotonicity
  - Higher constant act is strictly preferred
- Convexity
  - $\forall f, g \succeq h$, $\forall \alpha, \alpha f + (1-\alpha)gh$.
- Weak Certainty Independence
  - If x and y are constant acts, $\alpha \in (0, 1)$:
    $\alpha f + (1-\alpha)x \succeq \alpha g + (1-\alpha)x \Rightarrow \alpha f + (1-\alpha)y \succeq \alpha g + (1-\alpha)y$
- Statewise Transitivity:
  - $\forall\ f, g, h$, if h statewise dom. f, $f \succeq g$ then $h \succ g$.

Identification

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If 1) the DM's preference satisfies completeness, monotonicity, convexity and weak certainty independence and 2) the DM isolate problems, then $u_L + u_R \leq 1$.

Identification

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Theorem 2

If 1) the DM's preference satisfies completeness, monotonicity, convexity and weak certainty independence and 2) the DM isolate problems, then $u_L + u_R \leq 1$.

Theorem 5

If 1) the DM assigns weight α to $f \in \{L, R\}$ when asked how they would like to randomize over L and R, 2) their preferences satisfy completeness and statewise transitivity, and 3) the DM isolate problems then if $\alpha = 1$ it must be that $l_f \geq \min(u_f, \frac{1}{2})$ while if $\alpha = \frac{3}{4}$ it must be that $l_f \geq \frac{1}{2}$.

Figure 1: Testable implications - Theorems 1 and 2

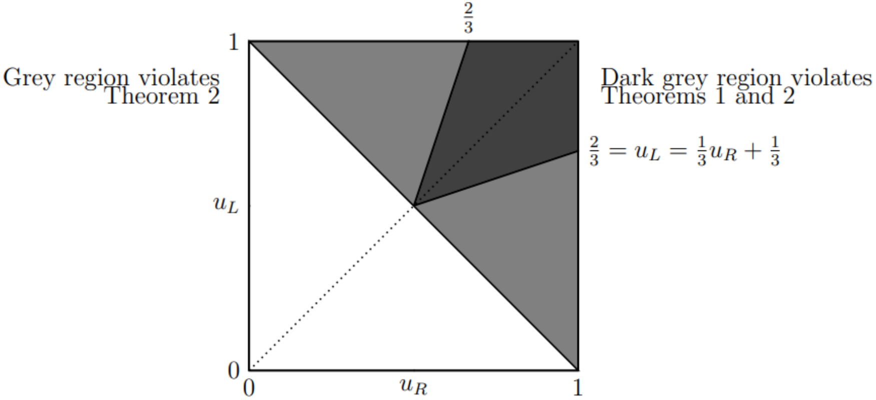
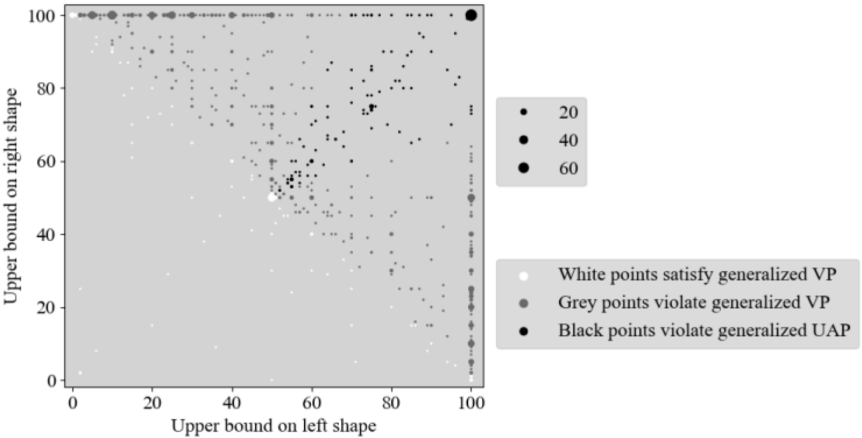


Table 2: Aggregate Distribution of Behavior

	Percent of rounds
Consistent with SEU	53%
Non-degenerate interval(s) of PE but inconsistent with generalized VP	37%
Non-degenerate interval(s) of PE consistent with generalized VP	4%
Two degenerate intervals that sum to more than 100	1%

Classification of consistency with the generalized VP model is based on Theorem 2 – considering responses in the two probability equivalent questions, and Theorem 7, Theorem 8, Theorem 9, and Theorem 10 – considering also the response in the randomization question.



Point Size Indicates Number of Observations

Table 4: Likelihood of Randomization (LOR) Conditional on PEs responses

LOR over L and R in a round	63%
LOR if two different degenerate PEs	13%
LOR if two identical degenerate PEs	98%
LOR if a non-degenerate PE interval	91%
LOR if PEs violate Theorem 1	95%
LOR if PEs violate Theorem 2	92%

Results: Behavioral Findings

- **Key Observations:**
  - 53% of choices consistent with Subjective Expected Utility (SEU).
  - 37% of choices inconsistent with generalized models of complete preferences (e.g., VP, Smooth Ambiguity).
- **Evidence of Incomplete Preferences:**
  - Non-degenerate intervals in 41% of rounds.
  - Inconsistent intervals linked to deliberate randomization.
- **Dimensionality Effect:**
  - Tasks involving rectangles (higher dimensionality) showed more violations of standard models compared to squares.

Complexity and Cognition

Complexity and Cognition

- A new perspective on choice behavior (past 10 years)
- Main line of thinking:
    - Many behavioral anomalies are driven by complexity reasons.
    - Subjects face difficulties in choice (cognitive uncertainty) and use heuristics that end up resembling to behavioral anomalies.

Literature

- Noisy Cognition can lead to non-EU behavior:
  - Theory: Steiner and Stewart (2016 AER), Khaw and Woodford (2021 Restud)
  - Experiment: Enke and Graeber (2023 QJE), Frydman and Jin (2022 QJE)

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  - Experiment: Enke and Graeber (2023 QJE), Frydman and Jin (2022 QJE)
- Complexity of objects can lead to NEU behavior:
  - Theory: Camara (2022 WP), Puri (2022 WP), Mononen (2024 WP), Hu (2024 WP)
  - Experiment: Oprea (2024 AER), de Clippel, Moscariello, Ortoleva and Rozen (2024 WP)

Three Papers

- We will cover three papers:
  - Steiner and Stewart (2016 AER): Perceiving Prospects Properly
  - Enke and Graeber (2023 QJE): Cognitive Uncertainty
  - Oprea (2024 AER): Decisions under Risk are Decisions under Complexity

Steiner and Stewart (2016)

- Result: If objects are perceived with noise, then optimal to bias and distort perceived probabilities.

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- Two Stage Model:
  - First: observes parameters of decision problem and encoded using a perception strategy.
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- Two Stage Model:
  - First: observes parameters of decision problem and encoded using a perception strategy.
  - Second: choose an action based on noisy values of parameters.
- Intuition: Suppose noise is mean-zero, then it makes a larger difference when

Steiner and Stewart (2016)

- The DM picks between s for sure and a lottery paying $\ell = (r_1, p; r_2, 1-p)$.

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- Finally she chooses the lottery if $qr_1 + (1-q)r_2 > s$.
- Payoff is:

$$f(\ell, q; s) = \begin{cases} pr_1 + (1-p)r_2 & \text{if } qr_1 + (1-q)r_2 > s, \\ s & \text{otherwise.} \end{cases}$$

Steiner and Stewart (2016)

- Nature chooses a *perception strategy* $m(p; s)$, where $m(\cdot; s) : [0, 1] \to [\underline{m}, \overline{m}]$, to maximize, for each p, the agent's expected payoff given the distribution over lotteries. An optimal perception strategy $m^*(p; s)$ satisfies

$$m^*(p; s) \in \arg \max_{m \in [\underline{m}, \overline{m}]} \mathbb{E}\big[f((p, r_1, r_2), c(m, \varepsilon))\big] \tag{1}$$

for each p, where the expectation is over the noise ε and the rewards (r_1, r_2).

Steiner and Stewart (2016)

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  for each p, where the expectation is over the noise ε and the rewards (r_1,r_2).

- We will consider a simple case $r_1,r_2\sim\mathcal{N}(0,1)$, $q=m+\varepsilon$ where $\varepsilon\in\{-\sigma,\sigma\}$ with equal probability.

Steiner and Stewart (2016)

Theorem 1: The optimal perception strategy $m^*(p; s)$ is nondecreasing in p. Furthermore, if $s > 3^{1/4}$, then the agent overstates small probabilities and understates large probabilities; that is, for all $p \in [\sigma, 1 - \sigma] \setminus \{1/2\}$,

$$|m^*(p; s) - 1/2| < |p - 1/2|.$$

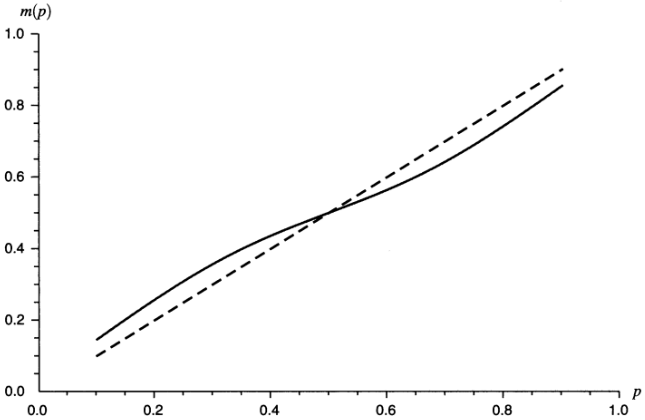

FIGURE 2. THE OPTIMAL PERCEPTION STRATEGY $m^*(p;s)$ (*Solid Curve*) FOR OPPORTUNITY COST $s = 2$ RELATIVE TO THE UNBIASED STRATEGY $m(p;s) \equiv p$ (*Dashed Line*)

Intuition

- Suppose you change perception slightly - if the expected lottery value and s are far apart then it doesn't change choice and hence doesn't impact outcomes.

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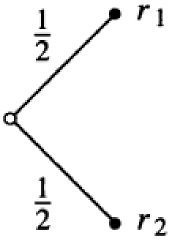
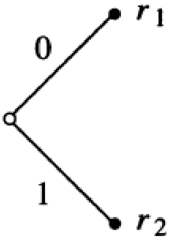
- So they make a difference when the two options are *perceived to be a tie*: $qr_1 + (1-q)r_2 = s$. Therefore the optimal strategy conditions on ties occuring.
- For what kinds of ps are ties more likely to occur?

Intuition





- First lottery $v \sim \mathcal{N}(0, \frac{1}{2})$ since the value is $(r_1 + r_2)/2)$.
- Second lottery $v \sim \mathcal{N}(0, 1)$

Intuition

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- Second lottery $v \sim \mathcal{N}(0, 1)$
- So if s is high, then much more likely to tie with the second lottery.

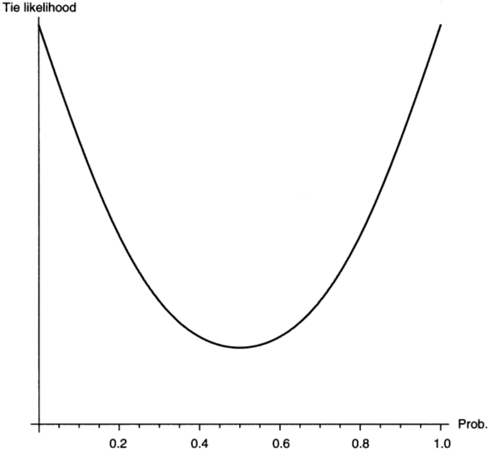

FIGURE 3. THE LIKELIHOOD OF A TIE BETWEEN A LOTTERY AND THE ALTERNATIVE AS A FUNCTION OF THE PERCEIVED PROBABILITY FOR $s = 2$

Optimal Distortion and Naïve Perception Strategy

Naïve Perception Strategy: Assume $m(p) \equiv p$. This strategy leads to *unbiased perception* of the expected lottery reward:

$$\mathbb{E}[r(p + \varepsilon) - r(p)] = 0,$$

where $r(p) = pr_1 + (1 - p)r_2$, and the expectation is over the noise ε.

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Conditional Bias:

- Although the perception is unbiased unconditionally, it becomes *biased conditional* on the agent perceiving a tie between the lottery and the alternative.
- When the opportunity cost s is high, one can show:

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Intuition: This is just like the winner's curse

- The naïve perception strategy tends to *overvalue the lottery.*

Enke and Graeber 2023

- They posit subjects face *cognitive uncertainty* in choice tasks, i.e., do not know if they chose correctly.

Enke and Graeber 2023

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Enke and Graeber 2023

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- They collect choices + a measure of choice confidence, study if probability weighting is more common if confidence is low.
- They do this in many domains: risk, ambiguity, belief updating and financial investment.

Enke and Graeber 2023

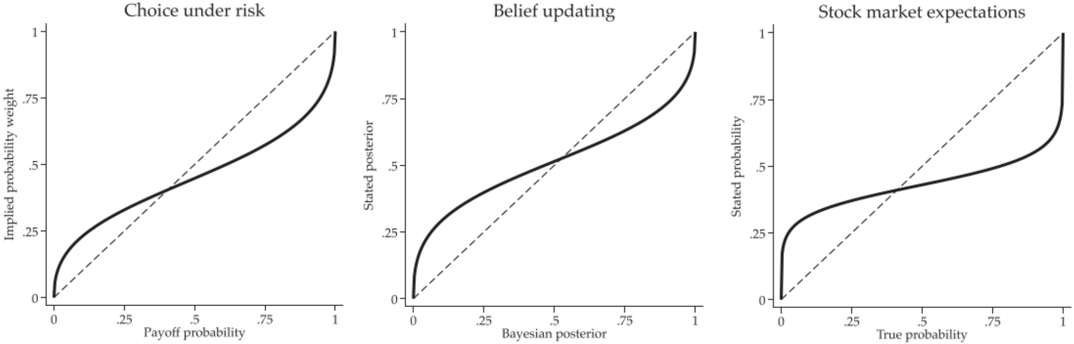


FIGURE I

Decisions as Functions of Objective Probabilities

Enke and Graeber 2023

Example

With probability **35% : Get $ 18**
With probability **65% : Get $0**

Which certain payment is worth as much to you as this lottery?

Getting $ ______ with certainty is worth as much to me as this lottery.

Enke and Graeber 2023

Decision screen 2: Your certainty about your decision

When you make your decision, you may feel **uncertain about which certain payment is worth as much to you as the lottery**. On decision screen 2, we will ask you to select a button to indicate **how certain** you are that you value the lottery within +/- $0.50 of the amount you entered on the previous screen.

Example

Suppose that on the first decision screen you indicated that you value a 5% chance of getting $18 as much as receiving $5.50 with certainty. Your second decision screen would look like this:

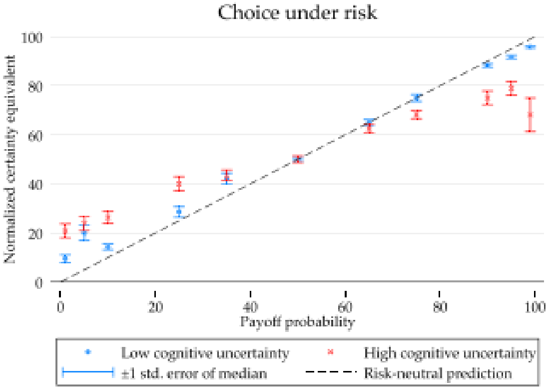
With probability **5% : Get $18**
With probability **95% : Get $0**

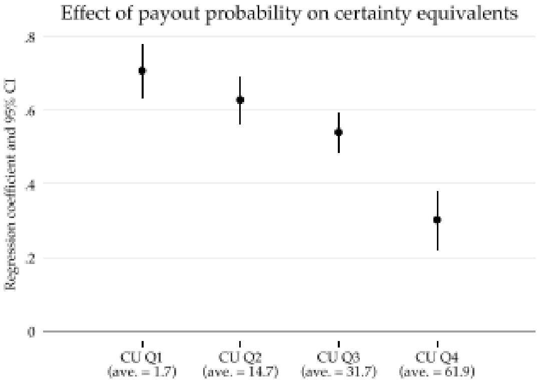
How certain are you that you actually value this lottery somewhere between getting $5.00 and $6.00?

0%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	100%

very uncertain **completely certain**

Enke and Graeber 2023





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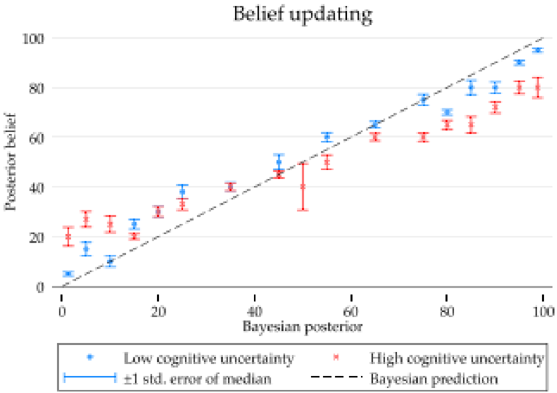

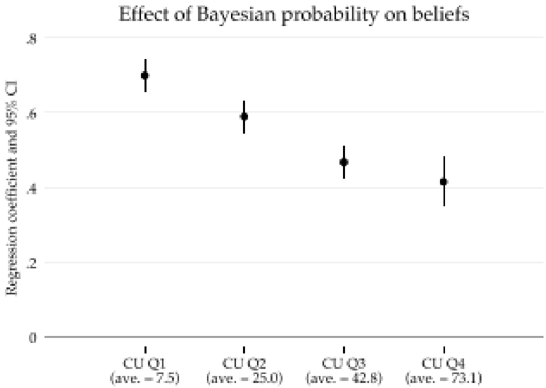

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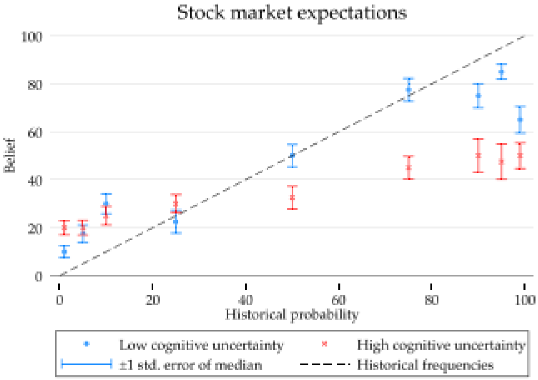

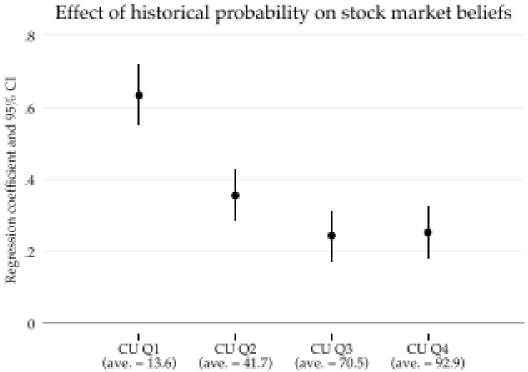

Oprea 2024

- Oprea focuses on the fourtold pattern of risk:

	Gains	*Losses*
Low probability	CE($100, .05) = $14 *Risk seeking*	CE(–$100, .05) = –$8 *Risk aversion*
High probability	CE($100, .95) = $78 *Risk aversion*	CE(–$100, .95) = –$84 *Risk seeking*

CE(x, p) is the certainty equivalent of the prospect that pays $x with probability p.
Source: Adapted from Tversky & Kahneman, 1992.

- Coined by Kahneman as: "the most distinctive implication of prospect theory"

Oprea 2024

- Consider you have 100 boxes with dollar amounts in them.
  - If you are paid randomly one of them - this is a lottery.

Oprea 2024

- Consider you have 100 boxes with dollar amounts in them.
  - If you are paid randomly one of them - this is a lottery.
  - If you are paid the average - this is what Oprea (2024) calls a simplicity equivalent.
- The idea: if fourfold pattern is only due to risk then it should only occur when you elicit the value of the lotteries and not when subjects are asked about the average.
- If instead it occurs in both, then the complexity of aggregation drives the pattern.

Oprea 2024

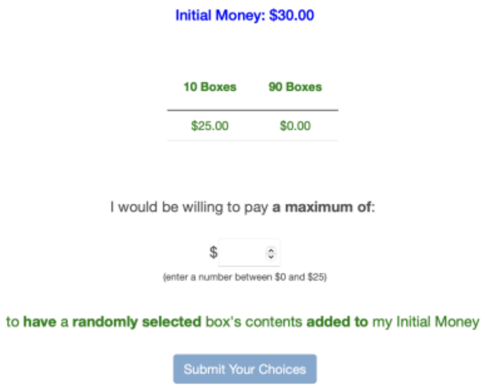

Figure 18: Screenshot from a lottery task (task G10) under BDM. *Notes: In mirror tasks, the screen is identical except for the text at the bottom which instead reads "...to have the average of these boxes' contents added to my Initial money.'*

Oprea 2024

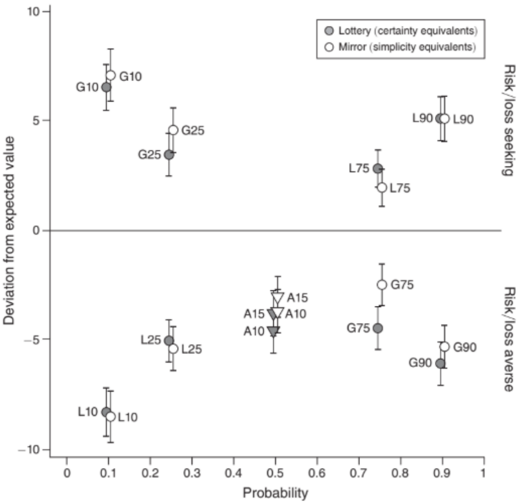

Figure 1. Mean Deviations from Expected Value in Lotteries (Solid Gray Dots) and Mirrors (Hollow Dots)

Oprea 2024

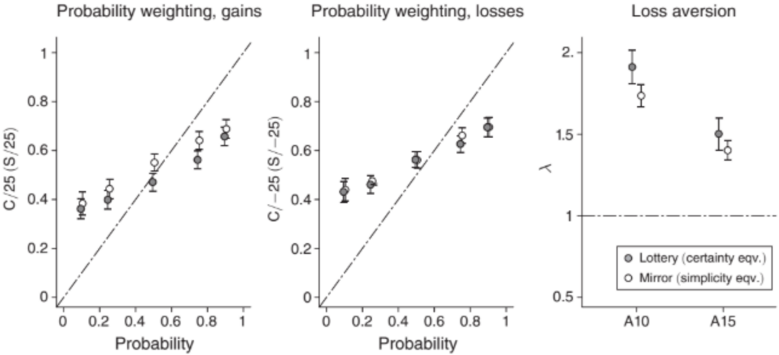

Figure 2. Naïve Visualization of the Probability Weighting Functions (Left Two Panels) and the Loss Aversion Parameter, λ

Oprea 2024

- Oprea 2024 is titled "Decisions under Risk are Decisions under Complexity"
- Oprea 2024: "These findings suggest that much of the behavior motivating our most important behavioral theorist of risk derive from complexity-driven mistakes rather than true risk preferences."

Oprea 2024

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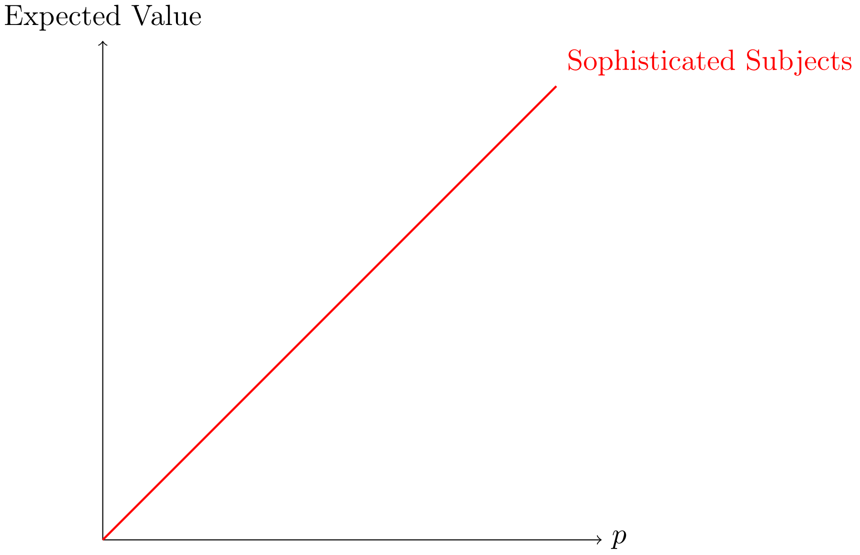
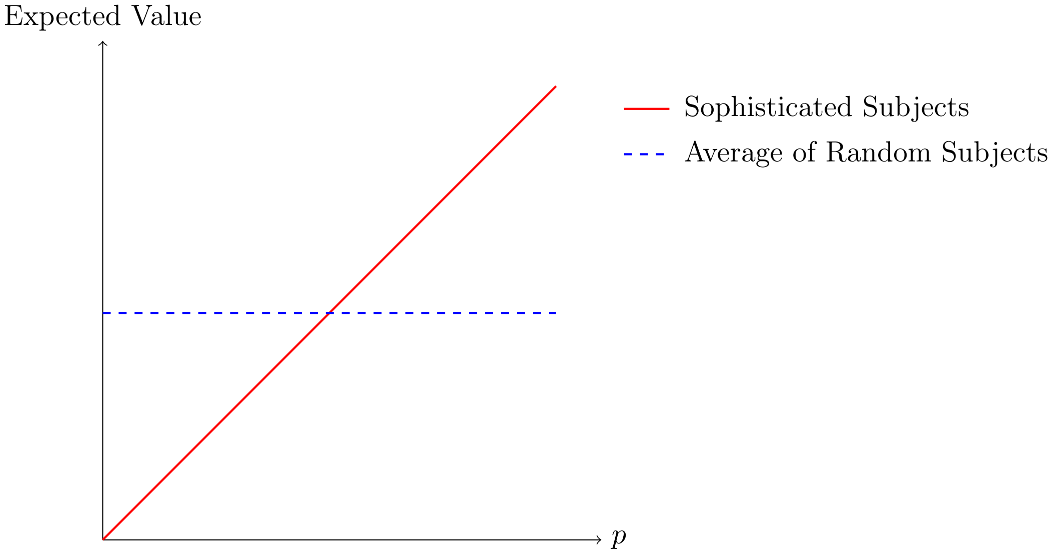
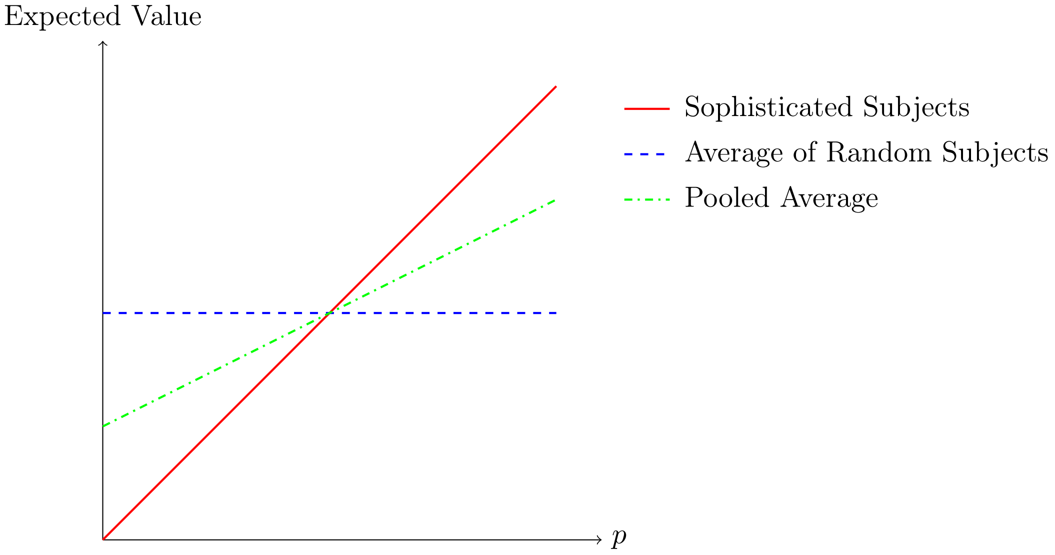
- Oprea 2024: “These findings suggest that much of the behavior motivating our most important behavioral theorist of risk derive from complexity-driven mistakes rather than true risk preferences."
- Clearly very controversial, let’s look at a recent comment paper.

Banki et.al (2025 Feb 6)

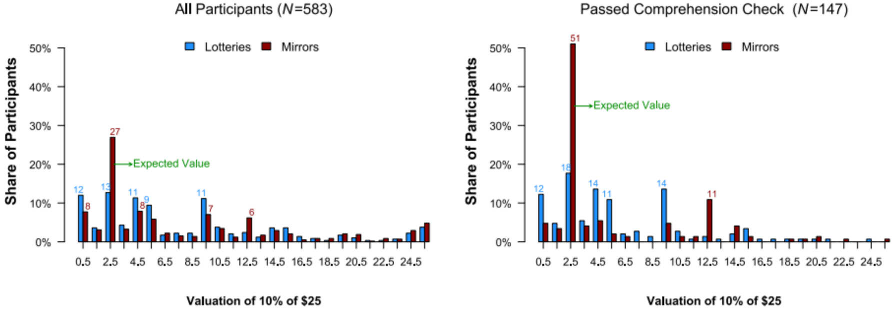
- Banki et.al: The equivalence between lotteries and mirrors is driven mostly by confused subjects.
- “This equivalence, however, was driven by the 75% of subjects who erred on comprehension questions. The remaining 25% of subjects largely valued mirrors at their expected value and lotteries in line with prospect theory."


Expected Value
Sophisticated Subjects
p


Expected Value
Sophisticated Subjects
Average of Random Subjects
p

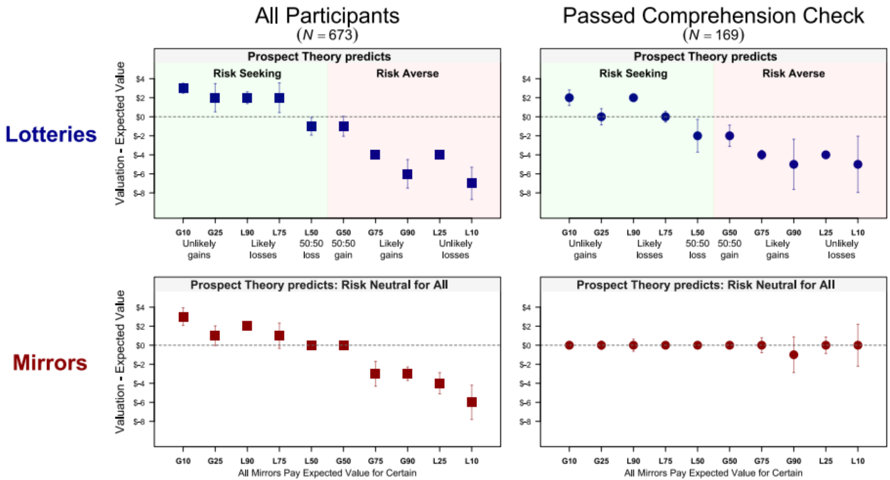

Expected Value
Sophisticated Subjects
Average of Random Subjects
Pooled Average
p

Banki 2025



(a) All Participants (b) Error-free

Banki 2025



All Participants
($N = 673$)
Passed Comprehension Check
($N = 169$)
Lotteries
Mirrors
Prospect Theory predicts
Risk Seeking
Risk Averse
Prospect Theory predicts: Risk Neutral for All
Valuation - Expected Value
$4
$2
$0
$-2
$-4
$-6
$-8
G10
G25
L90
L75
L50
G50
G75
G90
L25
L10
Unlikely gains
Likely losses
50:50 loss
50:50 gain
Likely gains
Unlikely losses
All Mirrors Pay Expected Value for Certain