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Faculty of Information Technology

Department of Big Data Analytics

Assignment I

Course: Data Mining and Information Retrieval

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ASSIGNMENT

Exercise 2.4, Question 1 Solution

For each of parts (a) through (d), indicate whether we would generally expect the performance of a **flexible** statistical learning method to be **better** or **worse** than an **inflexible** method. Justify your answer.

(a) The sample size n is extremely large, and the number of predictors p is small.

Answer: A flexible method would perform **better**.

Justification: With a large sample size and a small number of predictors, flexible methods can effectively learn complex relationships without overfitting. The low variance due to a large n allows the model to take advantage of its flexibility.

(b) The number of predictors p is extremely large, and the number of observations n is small.

Answer: A flexible method would perform **worse**.

Justification: In high-dimensional settings where $p \gg n$, flexible models tend to overfit, capturing noise instead of the true signal. Inflexible methods are more regularized and better suited to such situations.

(c) The relationship between the predictors and the response is highly non-linear.

Answer: A flexible method would perform **better**.

Justification: Flexible models can adapt to non-linear structures in the data, capturing complex patterns. Inflexible models, such as linear regression, would have high bias and fail to represent the non-linear relationship accurately.

(d) The variance of the error terms, i.e., $\sigma^2 = \mathrm{Var}(\varepsilon)$, is extremely high.

Answer: A flexible method would perform **worse**.

Justification: When the noise variance is high, flexible methods may overfit to the noise, resulting in high prediction variance. Inflexible methods tend to smooth over the noise, leading to more stable predictions.

Exercise 2.4, Question 10 Solution

(a) Dataset Loading

The Boston dataset was loaded successfully.

(b) Data Dimensions and Structure

- Number of observations (rows): 506
- Number of predictors (columns): 13
- Each row represents a suburb in the Boston metropolitan area.

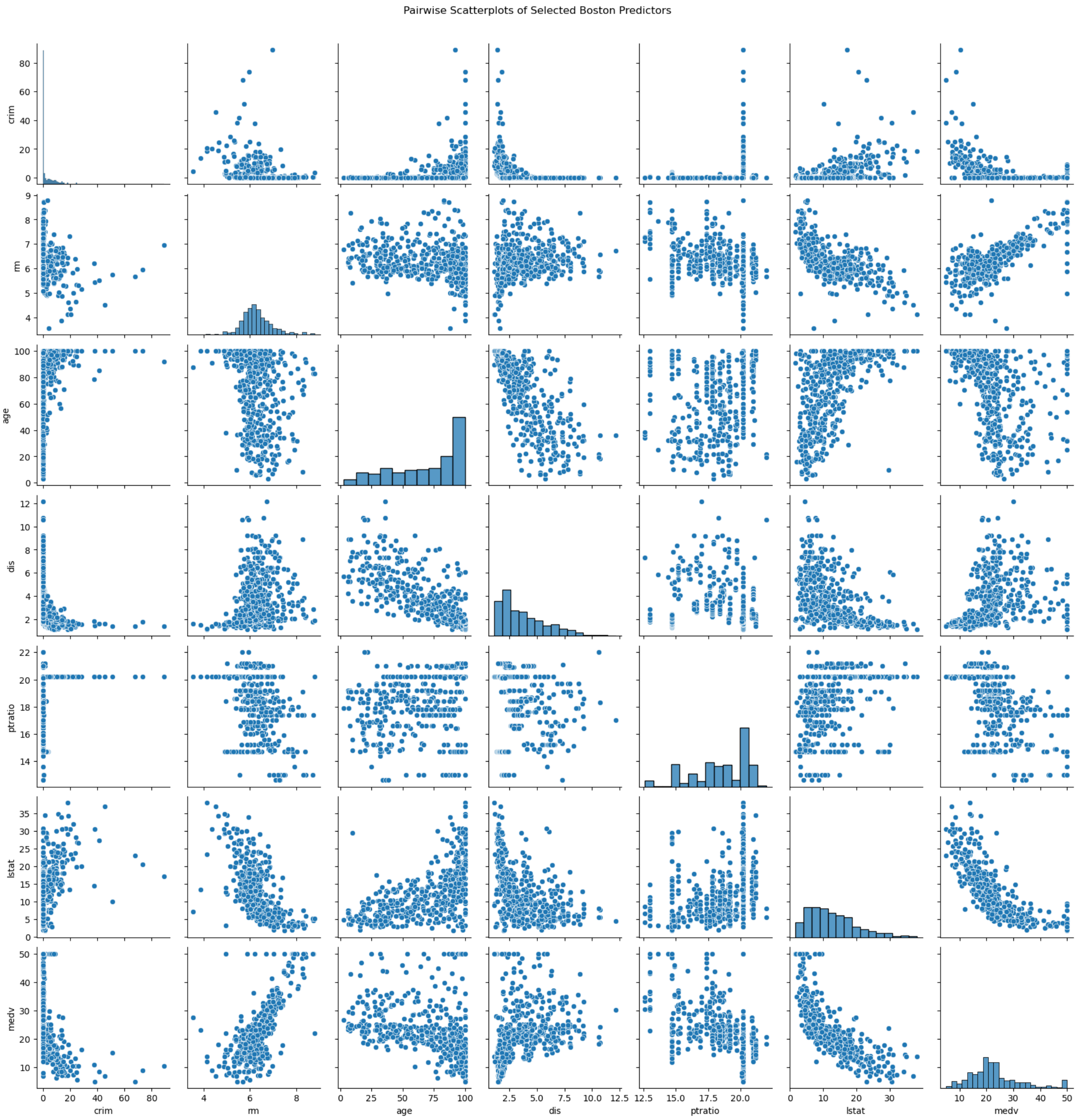
- Each column represents a different predictor variable (e.g., crime rate, tax rate).

Data Types:

```
float64 (10), int64 (3)
```

(c) Pairwise Scatterplots

A subset of pairwise scatterplots was generated for selected predictors to visualize their relationships:



- **medv vs. lstat:** Strong negative non-linear relationship.
- **medv vs. rm:** Positive linear trend; more rooms ⇒ higher home values.
- **dis vs. nox:** Negative correlation; farther suburbs have lower pollution.
- **crim:** Highly skewed, with few high-value outliers. Positively correlated with `rad`, `tax`, `lstat`; negatively with `medv`, `dis`.

(d) Predictors Correlated with Crime Rate (`crim`)

- **Strong Positive Correlations:** `rad (0.63)`, `tax (0.58)`, `lstat (0.46)`, `nox (0.42)`, `indus (0.41)`
- **Strong Negative Correlations:** `medv (-0.39)`, `dis (-0.38)`, `zn (-0.20)`

(e) Suburbs with High `crim`, `tax`, and `ptratio`

Top 5 Suburbs with Highest Crime Rate:

```
Index   crim     tax   ptratio   medv
380     88.9762  666   20.2      10.4
418     73.5341  666   20.2       8.8
405     67.9208  666   20.2       5.0
410     51.1358  666   20.2      15.0
414     45.7461  666   20.2       7.0
```

Top 5 Suburbs with Highest Tax Rate:

```
Index   tax     medv
492     711     20.1
491     711     13.6
490     711      8.1
489     711      7.0
488     711     15.2
```

Top 5 Suburbs with Highest Pupil-Teacher Ratio:

```
Index   ptratio  medv
354     22.0     18.2
355     22.0     20.6
135     21.2     18.1
127     21.2     16.2
136     21.2     17.4
```

Ranges for Predictors:

```
crim:    0.006 - 88.976
tax:     187 - 711
ptratio: 12.6 - 22.0
medv:    5.0 - 50.0 (truncated at upper bound)
```

(f) Suburbs Bounding the Charles River

Number of suburbs bordering the Charles River (`chas` = 1): **35**

(g) Median Pupil-Teacher Ratio

Median pupil-teacher ratio: **19.05**

(h) Suburb with Lowest Median Value

- Index: 398
- Median value (`medv`): $5,000
- High `crim`, `lstat`, `tax`, `age`, and `rad`
- Low `rm`, `dis`, `zn`, and `chas`
- Reflects socio-economic distress and possibly environmental pollution.

(i) Suburbs with Large Houses

- `rm` ¿ 7: 64 suburbs
- `rm` ¿ 8: 13 suburbs
- These suburbs have:
  - Very high `medv` (often capped at 50.0)
  - Very low `crim`, `lstat`, `nox`
  - Indicate affluent, clean, and safe neighborhoods

Example (Index 267):

```
rm = 8.297, medv = 50.0, crim = 0.57834, lstat = 7.44, nox = 0.575
```

Exercise 3.7, Question 1 Solution

The p-values in Table 3.4 test the null hypotheses that each advertising medium—TV, radio, and newspaper—has no effect on sales. Specifically:

- H_0^{TV} : TV advertising budget has no impact on sales.
- H_0^{radio} : Radio advertising budget has no impact on sales.
- $H_0^{\text{newspaper}}$: Newspaper advertising budget has no impact on sales.

Based on the multiple linear regression output below:

Predictor	**Coefficient**	**Std. Error**	**t-Statistic**	**p-value**
Intercept	2.939	0.3119	9.42	¡ 0.0001
TV	0.046	0.0014	32.81	¡ 0.0001
Radio	0.189	0.0086	21.89	¡ 0.0001
Newspaper	-0.001	0.0059	-0.18	0.8599

Table 1: Regression results for predicting Sales using TV, Radio, and Newspaper advertising budgets.

Conclusion in terms of Sales and Media Spend:

- **TV Advertising:** There is strong evidence that increasing TV advertising spend is associated with higher sales. The relationship is statistically significant with a very low p-value (< 0.0001).
- **Radio Advertising:** Similarly, radio advertising expenditure shows a significant positive association with sales. Increasing the radio budget is likely to increase sales, as indicated by the small p-value (< 0.0001).
- **Newspaper Advertising:** There is no statistical evidence that spending on newspaper advertising affects sales. The high p-value (0.8599) suggests that changes in newspaper ad budgets are not meaningfully associated with changes in sales, after accounting for TV and radio effects.

Exercise 3.7, Question 6 Solution

We aim to show that the least squares regression line from simple linear regression always passes through the point $(\bar{x}, \bar{y})$.

Least Squares Regression Line (Equation 3.4)

The least squares regression line is defined as:

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$$

where

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}, \quad \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

Proof that the Line Passes Through $(\bar{x}, \bar{y})$

Substitute $x = \bar{x}$ into the regression equation:

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 \bar{x}$$

Now replace $\hat{\beta}_0$ with $\bar{y} - \hat{\beta}_1 \bar{x}$:

$$\hat{y} = (\bar{y} - \hat{\beta}_1 \bar{x}) + \hat{\beta}_1 \bar{x} = \bar{y}$$

Thus, when $x = \bar{x}$, the predicted value $\hat{y} = \bar{y}$. Therefore, the least squares regression line always passes through the point $(\bar{x}, \bar{y})$.

Exercise 3.7, Question 11 Solution

(a) Regression of y onto x (without intercept)

- Coefficient estimate: $\hat{\beta}_{yx} = 1.9762$
- Standard error: ≈ 0.117
- **t-statistic**: ≈ 16.898
- p-value: < 0.0001
- R^2 (uncentered): 0.743

(b) Regression of x onto y (without intercept)

- Coefficient estimate: $\hat{\beta}_{xy} = 0.3757$
- Standard error: ≈ 0.022
- **t-statistic**: ≈ 16.898
- p-value: < 0.0001
- R^2 (uncentered): 0.743

(c) Relationship between (a) and (b)

- The t-statistics are **identical** in both regressions: $t \approx 16.898$.
- The slope coefficients are not reciprocals, due to differences in variance between x and y.
- This symmetry in t-statistics is expected in regressions **without intercept**.

(d) Algebraic Form of the t-Statistic

In a simple linear regression without intercept, the slope estimate is:

$$\hat{\beta} = \frac{\sum x_i y_i}{\sum x_i^2}$$

The standard error of $\hat{\beta}$ is:

$$SE(\hat{\beta}) = \sqrt{\frac{\sum (y_i - x_i\hat{\beta})^2}{(n-1)\sum x_i^2}}$$

The t-statistic for testing $H_0 : \beta = 0$ is:

$$t = \frac{\hat{\beta}}{SE(\hat{\beta})}$$

An alternative algebraic form of the t-statistic is:

$$t = \frac{\sqrt{n-1}\sum_{i=1}^n x_i y_i}{\sqrt{\left(\sum_{i=1}^n x_i^2\right)\left(\sum_{i=1}^n y_i^2\right) - \left(\sum_{i=1}^n x_i y_i\right)^2}}$$

This expression shows the symmetry between x and y.

(e) Equality of t-Statistics

From the expression above, it is clear that the t-statistic depends only on the sums:

$$\sum x_i y_i, \quad \sum x_i^2, \quad \sum y_i^2$$

This symmetry implies that the t-statistic for the regression of y on x is the same as for the regression of x on y, when both are performed **without an intercept**. Our computations confirm this: both regressions yielded $t \approx 16.898$.

(f) Regression with Intercept

When an intercept is included, the symmetry breaks. The regression line no longer passes through the origin, and the standard error and estimated variance change. As a result:

- The t-statistics for y on x and x on y **differ**.
- The R^2 and coefficient values also generally change.

Summary Table

Case	**Slope ($\hat{\beta}$)**	**t-statistic**	**Same t?**
$y \sim x$ (no intercept)	1.9762	16.898	Yes
$x \sim y$ (no intercept)	0.3757	16.898	Yes
$y \sim x$ (with intercept)	≈ 2.0	$\neq 16.898$	No
$x \sim y$ (with intercept)	≈ 0.5	$\neq 16.898$	No