$termvar,\ x,\ y,\ z$			
$trmlabel,\ a,\ b,\ c$			
$typlabel,\ A,\ B,\ C$			
$varref,\ v$	$::=$		
	$\vert$	x	
$trm,\ t$	$::=$		
	$\vert$	v	
	$\vert$	val	
	$\vert$	$v.a$	
	$\vert$	$v_1\ v_2$	
	$\vert$	$\mathbf{let}\ x = t_1\ \mathbf{in}\ t_2$	bind x in t_2
	$\vert$	$[v/x]t$	M
val	$::=$		
	$\vert$	$\nu\,(x : T)\,defs$	bind x in $defs$
	$\vert$	$\lambda(x : T).t$	bind x in t
$defs$	$::=$		
	$\vert$	$\{\}$	
	$\vert$	$d \wedge defs$	
	$\vert$	$[v/x]defs$	M
$def,\ d$	$::=$		
	$\vert$	$\{a = t\}$	
	$\vert$	$\{A = T\}$	
$typ,\ T$	$::=$		
	$\vert$	$\forall\,(x : T_1)\,T_2$	bind x in T_2
	$\vert$	$\mu\,(x : T)$	bind x in T
	$\vert$	dec	
	$\vert$	$v.A$	
	$\vert$	$T_1 \wedge T_2$	
	$\vert$	$\top$	
	$\vert$	$\bot$	
	$\vert$	$[v/x]T$	M
dec	$::=$		
	$\vert$	$\{a : T\}$	
	$\vert$	$\{A : T_1..T_2\}$	
$terminals$	$::=$		
	$\vert$	μ	
	$\vert$	ν	
	$\vert$	λ	
	$\vert$	$\wedge$	
	$\vert$	$\top$	
	$\vert$	$\bot$	
	$\vert$	$\forall$	
	$\vert$	$\emptyset$	

	$\mid$	$\vdash$
	$\mid$	$\vdash_{\#}$
	$\mid$	$\vdash_{!}$
	$\mid$	$\vdash_{\#\#}$
	$\mid$	$\rightarrow$
	$\mid$	$\gg$
	$\mid$	$\notin$
	$\mid$	$\in$
	$\mid$	$,$
$ctx,\ E,\ F,\ \Gamma$	$::=$	
	$\mid$	$\emptyset$
	$\mid$	$(\Gamma, x : T)$
$stack,\ s$	$::=$	
	$\mid$	$\emptyset$
	$\mid$	$(s, x : t)$
$formula$	$::=$	
	$\mid$	$judgement$
	$\mid$	$\Gamma(x) = T$
	$\mid$	$\mathbf{uniq}\,\Gamma$
	$\mid$	$x \notin \Gamma$
	$\mid$	$a \notin \mathbf{labels}\ T$
	$\mid$	$A \notin \mathbf{labels}\ T$
	$\mid$	$s(x) = t$
	$\mid$	$\mathbf{uniq}\,s$
	$\mid$	$d \in defs$
	$\mid$	$T_1 = T_2$
$Typing$	$::=$	
	$\mid$	$\Gamma \vdash t : T$
	$\mid$	$\Gamma \vdash d : T$
	$\mid$	$\Gamma \vdash defs : T$
	$\mid$	$\Gamma \vdash T_1 <: T_2$
$Inert$	$::=$	
	$\mid$	$\mathbf{record}\ T$
	$\mid$	$\mathbf{inert}\ T$
	$\mid$	$\mathbf{inert}\,\Gamma$
$PreciseTyping$	$::=$	
	$\mid$	$\Gamma\vdash_{!} val : T$
	$\mid$	$\Gamma\vdash_{!} x : T_1 \gg T_2$
$TightTyping$	$::=$	
	$\mid$	$\Gamma\vdash_{\#} t : T$

$$
\begin{array}{lll}
 & | & \Gamma \vdash_{\#} T_1 <: T_2 \\
\\
\mathit{InvertibleTyping} & ::= & \\
 & | & \Gamma \vdash_{\#\#} x : T \\
 & | & \Gamma \vdash_{\#\#} \mathit{val} : T \\
\\
\mathit{OperationalSemantics} & ::= & \\
 & | & (s_1, t_1) \to (s_2, t_2) \\
\\
\mathit{judgement} & ::= & \\
 & | & \mathit{Typing} \\
 & | & \mathit{Inert} \\
 & | & \mathit{PreciseTyping} \\
 & | & \mathit{TightTyping} \\
 & | & \mathit{InvertibleTyping} \\
 & | & \mathit{OperationalSemantics} \\
\\
\mathit{user\_syntax} & ::= & \\
 & | & \mathit{termvar} \\
 & | & \mathit{trmlabel} \\
 & | & \mathit{typlabel} \\
 & | & \mathit{varref} \\
 & | & \mathit{trm} \\
 & | & \mathit{val} \\
 & | & \mathit{defs} \\
 & | & \mathit{def} \\
 & | & \mathit{typ} \\
 & | & \mathit{dec} \\
 & | & \mathit{terminals} \\
 & | & \mathit{ctx} \\
 & | & \mathit{stack} \\
 & | & \mathit{formula}
\end{array}
$$

$\boxed{\Gamma \vdash t : T}$

$$\frac{\Gamma(x) = T}{\Gamma \vdash x : T} \quad \textsc{ty\_var}$$

$$\frac{(\Gamma, x : T_1) \vdash t : T_2}{\Gamma \vdash \lambda(x : T_1).t : \forall\,(x : T_1)\,T_2} \quad \textsc{ty\_all\_intro}$$

$$\frac{\begin{array}{l}\Gamma \vdash x : \forall\,(z : T_1)\,T_2 \\ \Gamma \vdash y : T_1\end{array}}{\Gamma \vdash x\,y : [y/z]\,T_2} \quad \textsc{ty\_all\_elim}$$

$$\frac{\begin{array}{l}T_1 = T_2 \\ (\Gamma, x : T_1) \vdash \mathit{defs} : T_2\end{array}}{\Gamma \vdash \nu\,(x : T_1)\mathit{defs} : \mu\,(x : T_2)} \quad \textsc{ty\_new\_intro}$$

$$\frac{\Gamma \vdash x : \{a : T\}}{\Gamma \vdash x.a : T} \quad \textsc{ty\_new\_elim}$$

$$\frac{\begin{array}{l}\Gamma \vdash t_1 : T_1 \\ (\Gamma, x : T_1) \vdash t_2 : T_2\end{array}}{\Gamma \vdash \mathbf{let}\,x = t_1\,\mathbf{in}\,t_2 : T_2} \quad \textsc{ty\_let}$$

$$\frac{\Gamma \vdash x : T}{\Gamma \vdash x : \mu\,(z : T)} \quad \text{TY\_REC\_INTRO}$$

$$\frac{\Gamma \vdash x : \mu\,(z : T)}{\Gamma \vdash x : [x/z]\,T} \quad \text{TY\_REC\_ELIM}$$

$$\frac{\begin{array}{c}\Gamma \vdash x : T_1 \\ \Gamma \vdash x : T_2\end{array}}{\Gamma \vdash x : T_1 \wedge T_2} \quad \text{TY\_AND\_INTRO}$$

$$\frac{\begin{array}{c}\Gamma \vdash t : T_1 \\ \Gamma \vdash T_1 <: T_2\end{array}}{\Gamma \vdash t : T_2} \quad \text{TY\_SUB}$$

$\boxed{\Gamma \vdash d : T}$

$$\frac{\Gamma \vdash t : T}{\Gamma \vdash \{a = t\} : \{a : T\}} \quad \text{TY\_DEF\_TRM}$$

$$\frac{}{\Gamma \vdash \{A = T\} : \{A : T..T\}} \quad \text{TY\_DEF\_TYP}$$

$\boxed{\Gamma \vdash \mathit{defs} : T}$

$$\frac{\Gamma \vdash d : T}{\Gamma \vdash d \wedge \{\} : T} \quad \text{TY\_DEFS\_ONE}$$

$$\frac{\begin{array}{c}\Gamma \vdash d : T_1 \\ \Gamma \vdash \mathit{defs} : T_2\end{array}}{\Gamma \vdash d \wedge \mathit{defs} : T_1 \wedge T_2} \quad \text{TY\_DEFS\_CONS}$$

$\boxed{\Gamma \vdash T_1 <: T_2}$

$$\frac{}{\Gamma \vdash T <: \top} \quad \text{SUBTYP\_TOP}$$

$$\frac{}{\Gamma \vdash \bot <: T} \quad \text{SUBTYP\_BOT}$$

$$\frac{}{\Gamma \vdash T <: T} \quad \text{SUBTYP\_REFL}$$

$$\frac{\begin{array}{c}\Gamma \vdash T_1 <: T_2 \\ \Gamma \vdash T_2 <: T_3\end{array}}{\Gamma \vdash T_1 <: T_3} \quad \text{SUBTYP\_TRANS}$$

$$\frac{}{\Gamma \vdash T_1 \wedge T_2 <: T_1} \quad \text{SUBTYP\_AND11}$$

$$\frac{}{\Gamma \vdash T_1 \wedge T_2 <: T_2} \quad \text{SUBTYP\_AND12}$$

$$\frac{\begin{array}{c}\Gamma \vdash T_1 <: T_2 \\ \Gamma \vdash T_1 <: T_3\end{array}}{\Gamma \vdash T_1 <: T_2 \wedge T_3} \quad \text{SUBTYP\_AND2}$$

$$\frac{\Gamma \vdash T_1 <: T_2}{\Gamma \vdash \{a : T_1\} <: \{a : T_2\}} \quad \text{SUBTYP\_FLD}$$

$$\frac{\begin{array}{c}\Gamma \vdash T_1 <: T_2 \\ \Gamma \vdash T_3 <: T_4\end{array}}{\Gamma \vdash \{A : T_2..T_3\} <: \{A : T_1..T_4\}} \quad \text{SUBTYP\_TYP}$$

$$\frac{\Gamma \vdash x : \{A : T_1 .. T_2\}}{\Gamma \vdash x.A <: T_2} \quad \text{SUBTYP\_SEL1}$$

$$\frac{\Gamma \vdash x : \{A : T_1 .. T_2\}}{\Gamma \vdash T_1 <: x.A} \quad \text{SUBTYP\_SEL2}$$

$$\frac{\begin{array}{l}\Gamma \vdash T_3 <: T_1 \\ (\Gamma, x : T_1) \vdash T_2 <: T_4\end{array}}{\Gamma \vdash \forall\,(x : T_1)\, T_2 <: \forall\,(x : T_3)\, T_4} \quad \text{SUBTYP\_ALL}$$

$\boxed{\textbf{record}\ T}$

$$\frac{}{\textbf{record}\,\{a : T\}} \quad \text{RT\_ONE\_TRM}$$

$$\frac{}{\textbf{record}\,\{A : T..T\}} \quad \text{RT\_ONE\_TYP}$$

$$\frac{\begin{array}{c}\textbf{record}\ T_1 \\ a \notin \textbf{labels}\ T_1\end{array}}{\textbf{record}\ T_1 \wedge \{a : T_2\}} \quad \text{RT\_AND\_TRM}$$

$$\frac{\begin{array}{c}\textbf{record}\ T_1 \\ A \notin \textbf{labels}\ T_1\end{array}}{\textbf{record}\ T_1 \wedge \{A : T_2..T_2\}} \quad \text{RT\_AND\_TYP}$$

$\boxed{\textbf{inert}\ T}$

$$\frac{}{\textbf{inert}\,\forall\,(x : T_1)\, T_2} \quad \text{INERT\_TYP\_ALL}$$

$$\frac{\textbf{record}\ T}{\textbf{inert}\,\mu\,(x : T)} \quad \text{INERT\_TYP\_BND}$$

$\boxed{\textbf{inert}\,\Gamma}$

$$\frac{}{\textbf{inert}\,\emptyset} \quad \text{INERT\_EMPTY}$$

$$\frac{\begin{array}{c}\textbf{inert}\,\Gamma \\ \textbf{inert}\ T \\ x \notin \Gamma\end{array}}{\textbf{inert}\,(\Gamma, x : T)} \quad \text{INERT\_ALL}$$

$\boxed{\Gamma \vdash_! val : T}$

$$\frac{(\Gamma, x : T_1) \vdash t : T_2}{\Gamma \vdash_! \lambda(x : T_1).t : \forall\,(x : T_1)\, T_2} \quad \text{TY\_ALL\_INTRO\_P}$$

$$\frac{(\Gamma, x : T) \vdash defs : T}{\Gamma \vdash_! \nu\,(x : T)\, defs : \mu\,(x : T)} \quad \text{TY\_NEW\_INTRO\_P}$$

$\boxed{\Gamma \vdash_! x : T_1 \gg T_2}$

$$\frac{\Gamma(x) = T}{\Gamma \vdash_! x : T \gg T} \quad \text{PF\_BIND}$$

$$\frac{\Gamma \vdash_! x : T_1 \gg \mu\,(z : T_2)}{\Gamma \vdash_! x : T_1 \gg [x/z]\, T_2} \quad \text{PF\_OPEN}$$

$$\frac{\Gamma \vdash_{!} x : T_1 \gg T_2 \wedge T_3}{\Gamma \vdash_{!} x : T_1 \gg T_2} \quad \text{PF\_AND1}$$

$$\frac{\Gamma \vdash_{!} x : T_1 \gg T_2 \wedge T_3}{\Gamma \vdash_{!} x : T_1 \gg T_3} \quad \text{PF\_AND2}$$

$\boxed{\Gamma \vdash_{\#} t : T}$

$$\frac{\Gamma(x) = T}{\Gamma \vdash_{\#} x : T} \quad \text{TY\_VAR\_T}$$

$$\frac{(\Gamma, x : T_1) \vdash t : T_2}{\Gamma \vdash_{\#} \lambda(x : T_1).t : \forall\,(x : T_1)\,T_2} \quad \text{TY\_ALL\_INTRO\_T}$$

$$\frac{\begin{array}{l}\Gamma \vdash_{\#} x : \forall\,(z : T_1)\,T_2 \\ \Gamma \vdash_{\#} y : T_1\end{array}}{\Gamma \vdash_{\#} x\,y : [y/z]\,T_2} \quad \text{TY\_ALL\_ELIM\_T}$$

$$\frac{(\Gamma, x : T) \vdash \mathit{defs} : T}{\Gamma \vdash_{\#} \nu\,(x : T)\,\mathit{defs} : \mu\,(x : T)} \quad \text{TY\_NEW\_INTRO\_T}$$

$$\frac{\Gamma \vdash_{\#} x : \{a : T\}}{\Gamma \vdash_{\#} x.a : T} \quad \text{TY\_NEW\_ELIM\_T}$$

$$\frac{\begin{array}{l}\Gamma \vdash_{\#} t_1 : T_1 \\ (\Gamma, x : T) \vdash t_2 : T_2\end{array}}{\Gamma \vdash_{\#} \mathbf{let}\, x = t_1 \,\mathbf{in}\, t_2 : T_2} \quad \text{TY\_LET\_T}$$

$$\frac{\Gamma \vdash_{\#} x : T}{\Gamma \vdash_{\#} x : \mu\,(z : T)} \quad \text{TY\_REC\_INTRO\_T}$$

$$\frac{\Gamma \vdash_{\#} x : \mu\,(z : T)}{\Gamma \vdash_{\#} x : [x/z]\,T} \quad \text{TY\_REC\_ELIM\_T}$$

$$\frac{\begin{array}{l}\Gamma \vdash_{\#} x : T_1 \\ \Gamma \vdash_{\#} x : T_2\end{array}}{\Gamma \vdash_{\#} x : T_1 \wedge T_2} \quad \text{TY\_AND\_INTRO\_T}$$

$$\frac{\begin{array}{l}\Gamma \vdash_{\#} t : T_1 \\ \Gamma \vdash_{\#} T_1 <: T_2\end{array}}{\Gamma \vdash_{\#} t : T_2} \quad \text{TY\_SUB\_T}$$

$\boxed{\Gamma \vdash_{\#} T_1 <: T_2}$

$$\frac{}{\Gamma \vdash_{\#} T <: \top} \quad \text{SUBTYP\_TOP\_T}$$

$$\frac{}{\Gamma \vdash_{\#} \bot <: T} \quad \text{SUBTYP\_BOT\_T}$$

$$\frac{}{\Gamma \vdash_{\#} T <: T} \quad \text{SUBTYP\_REFL\_T}$$

$$\frac{\begin{array}{l}\Gamma \vdash_{\#} T_1 <: T_2 \\ \Gamma \vdash_{\#} T_2 <: T_3\end{array}}{\Gamma \vdash_{\#} T_1 <: T_3} \quad \text{SUBTYP\_TRANS\_T}$$

$$\frac{}{\Gamma \vdash_{\#} T_1 \wedge T_2 <: T_1} \quad \text{SUBTYP\_AND11\_T}$$

$$\frac{}{\Gamma \vdash_{\#} T_1 \wedge T_2 <: T_2} \quad \text{SUBTYP\_AND12\_T}$$

$$\frac{\Gamma \vdash_{\#} T_1 <: T_2 \qquad \Gamma \vdash_{\#} T_1 <: T_3}{\Gamma \vdash_{\#} T_1 <: T_2 \wedge T_3} \quad \text{SUBTYP\_AND2\_T}$$

$$\frac{\Gamma \vdash_{\#} T_1 <: T_2}{\Gamma \vdash_{\#} \{a : T_1\} <: \{a : T_2\}} \quad \text{SUBTYP\_FLD\_T}$$

$$\frac{\Gamma \vdash_{\#} T_1 <: T_2 \qquad \Gamma \vdash_{\#} T_3 <: T_4}{\Gamma \vdash_{\#} \{A : T_2..T_3\} <: \{A : T_1..T_4\}} \quad \text{SUBTYP\_TYP\_T}$$

$$\frac{\Gamma \vdash_{!} x : T_1 \gg \{A : T_2..T_2\}}{\Gamma \vdash_{\#} x.A <: T_2} \quad \text{SUBTYP\_SEL1\_T}$$

$$\frac{\Gamma \vdash_{!} x : T_1 \gg \{A : T_2..T_2\}}{\Gamma \vdash_{\#} T_2 <: x.A} \quad \text{SUBTYP\_SEL2\_T}$$

$$\frac{\Gamma \vdash_{\#} T_3 <: T_1 \qquad (\Gamma, x : T_1) \vdash T_2 <: T_4}{\Gamma \vdash_{\#} \forall (x : T_1)\, T_2 <: \forall (x : T_3)\, T_4} \quad \text{SUBTYP\_ALL\_T}$$

$\boxed{\Gamma \vdash_{\#\#} x : T}$

$$\frac{\Gamma \vdash_{!} x : T_1 \gg T_2}{\Gamma \vdash_{\#\#} x : T_2} \quad \text{TY\_PRECISE\_INV}$$

$$\frac{\Gamma \vdash_{\#\#} x : \{a : T_1\} \qquad \Gamma \vdash_{\#} T_1 <: T_2}{\Gamma \vdash_{\#\#} x : \{a : T_2\}} \quad \text{TY\_DEC\_TRM\_INV}$$

$$\frac{\Gamma \vdash_{\#\#} x : \{A : T_2..T_3\} \qquad \Gamma \vdash_{\#} T_1 <: T_2 \qquad \Gamma \vdash_{\#} T_3 <: T_4}{\Gamma \vdash_{\#\#} x : \{A : T_1..T_4\}} \quad \text{TY\_DEC\_TYP\_INV}$$

$$\frac{\Gamma \vdash_{\#\#} x : [x/z]\, T}{\Gamma \vdash_{\#\#} x : \mu\, (z : T)} \quad \text{TY\_BND\_INV}$$

$$\frac{\Gamma \vdash_{\#\#} x : \forall (z : T_2)\, T_3 \qquad \Gamma \vdash_{\#} T_1 <: T_2 \qquad (\Gamma, z : T_1) \vdash T_3 <: T_4}{\Gamma \vdash_{\#\#} x : \forall (z : T_1)\, T_4} \quad \text{TY\_ALL\_INV}$$

$$\frac{\Gamma \vdash_{\#\#} x : T_1 \qquad \Gamma \vdash_{\#\#} x : T_2}{\Gamma \vdash_{\#\#} x : T_1 \wedge T_2} \quad \text{TY\_AND\_INV}$$

$$\frac{\Gamma \vdash_{\#\#} x : T_1 \qquad \Gamma \vdash_{!} y : T_2 \gg \{A : T_1..T_1\}}{\Gamma \vdash_{\#\#} x : y.A} \quad \text{TY\_SEL\_INV}$$

$$\frac{\Gamma \vdash_{\#\#} x : T}{\Gamma \vdash_{\#\#} x : \top} \quad \text{TY\_TOP\_INV}$$

$\boxed{\Gamma \vdash_{\#\#} val : T}$

$$\frac{\Gamma \vdash_{!} val : T}{\Gamma \vdash_{\#\#} val : T} \quad \text{TY\_PRECISE\_INV\_V}$$

$$\frac{\begin{array}{l}\Gamma \vdash_{\#\#} val : \forall\,(z : T_2)\,T_3 \\ \Gamma \vdash_{\#} T_1 <: T_2 \\ (\Gamma, z : T_1) \vdash T_3 <: T_4\end{array}}{\Gamma \vdash_{\#\#} val : \forall\,(z : T_1)\,T_4} \quad \text{TY\_ALL\_INV\_V}$$

$$\frac{\begin{array}{c}\Gamma \vdash_{\#\#} val : T_1 \\ \Gamma \vdash_{\#\#} val : T_2\end{array}}{\Gamma \vdash_{\#\#} val : T_1 \wedge T_2} \quad \text{TY\_AND\_INV\_V}$$

$$\frac{\begin{array}{l}\Gamma \vdash_{\#\#} val : T_1 \\ \Gamma \vdash_{!} y : T_2 \gg \{A : T_1..T_1\}\end{array}}{\Gamma \vdash_{\#\#} val : y.A} \quad \text{TY\_SEL\_INV\_V}$$

$$\frac{\Gamma \vdash_{\#\#} val : T}{\Gamma \vdash_{\#\#} val : \top} \quad \text{TY\_TOP\_INV\_V}$$

$\boxed{(s_1, t_1) \to (s_2, t_2)}$

$$\frac{\begin{array}{c}s(x) = \nu\,(z : T)\,defs \\ \{a = t\} \in [x/z]\,defs\end{array}}{(s, x.a) \to (s, t)} \quad \text{RED\_SEL}$$

$$\frac{s(x) = \lambda(z : T_1).t}{(s, x\ y) \to (s, [y/z]t)} \quad \text{RED\_APP}$$

$$\frac{}{(s, \textbf{let}\ x = val\ \textbf{in}\ t) \to ((s, x : val), t)} \quad \text{RED\_LET\_VAL}$$

$$\frac{}{(s, \textbf{let}\ x = y\ \textbf{in}\ t) \to (s, [y/x]t)} \quad \text{RED\_LET\_VAR}$$

$$\frac{(s_1, t_1) \to (s_2, t_2)}{(s_1, \textbf{let}\ x = t_1\ \textbf{in}\ t_3) \to (s_2, \textbf{let}\ x = t_2\ \textbf{in}\ t_3)} \quad \text{RED\_LET\_TGT}$$

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Definition rules:        80 good    0 bad
Definition rule clauses: 177 good    0 bad
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