

<i>termvar</i> , x, y, z		
<i>trmlabel</i> , a, b, c		
<i>typlabel</i> , A, B, C		
<i>varref</i> , v	$::=$	
		x
<i>trm</i> , t	$::=$	
		v
		val
		$v.a$
		$v_1 v_2$
		let $x = t_1$ in t_2 bind x in t_2
		$[v/x]t$ M
<i>val</i>	$::=$	
		$\nu(x : T)defs$ bind x in $defs$
		$\lambda(x : T).t$ bind x in t
<i>defs</i>	$::=$	
		$\{\}$
		$d \wedge defs$
		$[v/x]defs$ M
<i>def</i> , d	$::=$	
		$\{a = t\}$
		$\{A = T\}$
<i>typ</i> , T	$::=$	
		$\forall(x : T_1) T_2$ bind x in T_2
		$\mu(x : T)$ bind x in T
		dec
		$v.A$
		$T_1 \wedge T_2$
		\top
		\perp
		$[v/x]T$ M
<i>dec</i>	$::=$	
		$\{a : T\}$
		$\{A : T_1..T_2\}$
<i>terminals</i>	$::=$	
		μ
		ν
		λ
		\wedge
		\top
		\perp
		\forall
		\emptyset

		\vdash
		$\vdash_{\#}$
		$\vdash_{!}$
		$\vdash_{\#\#}$
		\rightarrow
		\gg
		\notin
		\in
		$,$
ctx, E, F, Γ	$::=$	
		\emptyset
		$(\Gamma, x : T)$
$stack, s$	$::=$	
		\emptyset
		$(s, x : t)$
$formula$	$::=$	
		<i>judgement</i>
		$\Gamma(x) = T$
		uniq Γ
		$x \notin \Gamma$
		$a \notin \mathbf{labels} T$
		$A \notin \mathbf{labels} T$
		$s(x) = t$
		uniq s
		$d \in \mathbf{defs}$
$Typing$	$::=$	
		$\Gamma \vdash t : T$
		$\Gamma \vdash d : T$
		$\Gamma \vdash \mathbf{defs} : T$
		$\Gamma \vdash T_1 <: T_2$
$Inert$	$::=$	
		record T
		inert T
		inert Γ
$PreciseTyping$	$::=$	
		$\Gamma \vdash_{!} val : T$
		$\Gamma \vdash_{!} x : T_1 \gg T_2$
$TightTyping$	$::=$	
		$\Gamma \vdash_{\#} t : T$
		$\Gamma \vdash_{\#} T_1 <: T_2$

<i>InvertibleTyping</i>	$::=$ $\mid \Gamma \vdash_{\#\#} x : T$ $\mid \Gamma \vdash_{\#\#} \text{val} : T$
<i>OperationalSemantics</i>	$::=$ $\mid (s_1, t_1) \rightarrow (s_2, t_2)$
<i>judgement</i>	$::=$ $\mid \textit{Typing}$ $\mid \textit{Inert}$ $\mid \textit{PreciseTyping}$ $\mid \textit{TightTyping}$ $\mid \textit{InvertibleTyping}$ $\mid \textit{OperationalSemantics}$
<i>user_syntax</i>	$::=$ $\mid \textit{termvar}$ $\mid \textit{trmlabel}$ $\mid \textit{typlabel}$ $\mid \textit{varref}$ $\mid \textit{trm}$ $\mid \textit{val}$ $\mid \textit{defs}$ $\mid \textit{def}$ $\mid \textit{typ}$ $\mid \textit{dec}$ $\mid \textit{terminals}$ $\mid \textit{ctx}$ $\mid \textit{stack}$ $\mid \textit{formula}$

$$\boxed{\Gamma \vdash t : T}$$

$$\begin{array}{c}
\frac{\Gamma(x) = T}{\Gamma \vdash x : T} \quad \text{TY_VAR} \\
\\
\frac{(\Gamma, x : T_1) \vdash t : T_2}{\Gamma \vdash \lambda(x : T_1).t : \forall (x : T_1) T_2} \quad \text{TY_ALL_INTRO} \\
\\
\frac{\Gamma \vdash x : \forall (z : T_1) T_2 \quad \Gamma \vdash y : T_1}{\Gamma \vdash x y : [y/z] T_2} \quad \text{TY_ALL_ELIM} \\
\\
\frac{(\Gamma, x : T) \vdash \textit{defs} : T}{\Gamma \vdash \nu (x : T) \textit{defs} : \mu (x : T)} \quad \text{TY_NEW_INTRO} \\
\\
\frac{\Gamma \vdash x : \{a : T\}}{\Gamma \vdash x.a : T} \quad \text{TY_NEW_ELIM} \\
\\
\frac{\Gamma \vdash t_1 : T_1 \quad (\Gamma, x : T_1) \vdash t_2 : T_2}{\Gamma \vdash \textbf{let } x = t_1 \textbf{ in } t_2 : T_2} \quad \text{TY_LET} \\
\\
\frac{\Gamma \vdash x : T}{\Gamma \vdash x : \mu (z : T)} \quad \text{TY_REC_INTRO}
\end{array}$$

$$\frac{\Gamma \vdash x : \mu(z : T)}{\Gamma \vdash x : [x/z]T} \quad \text{TY_REC_ELIM}$$

$$\frac{\Gamma \vdash x : T_1 \quad \Gamma \vdash x : T_2}{\Gamma \vdash x : T_1 \wedge T_2} \quad \text{TY_AND_INTRO}$$

$$\frac{\Gamma \vdash t : T_1 \quad \Gamma \vdash T_1 <: T_2}{\Gamma \vdash t : T_2} \quad \text{TY_SUB}$$

$$\boxed{\Gamma \vdash d : T}$$

$$\frac{\Gamma \vdash t : T}{\Gamma \vdash \{a = t\} : \{a : T\}} \quad \text{TY_DEF_TRM}$$

$$\frac{}{\Gamma \vdash \{A = T\} : \{A : T..T\}} \quad \text{TY_DEF_TYP}$$

$$\boxed{\Gamma \vdash defs : T}$$

$$\frac{\Gamma \vdash d : T}{\Gamma \vdash d \wedge \{\} : T} \quad \text{TY_DEFS_ONE}$$

$$\frac{\Gamma \vdash d : T_1 \quad \Gamma \vdash defs : T_2}{\Gamma \vdash d \wedge defs : T_1 \wedge T_2} \quad \text{TY_DEFS_CONS}$$

$$\boxed{\Gamma \vdash T_1 <: T_2}$$

$$\frac{}{\Gamma \vdash T <: \top} \quad \text{SUBTYP_TOP}$$

$$\frac{}{\Gamma \vdash \perp <: T} \quad \text{SUBTYP_BOT}$$

$$\frac{}{\Gamma \vdash T <: T} \quad \text{SUBTYP_REFL}$$

$$\frac{\Gamma \vdash T_1 <: T_2 \quad \Gamma \vdash T_2 <: T_3}{\Gamma \vdash T_1 <: T_3} \quad \text{SUBTYP_TRANS}$$

$$\frac{}{\Gamma \vdash T_1 \wedge T_2 <: T_1} \quad \text{SUBTYP_AND11}$$

$$\frac{}{\Gamma \vdash T_1 \wedge T_2 <: T_2} \quad \text{SUBTYP_AND12}$$

$$\frac{\Gamma \vdash T_1 <: T_2 \quad \Gamma \vdash T_1 <: T_3}{\Gamma \vdash T_1 <: T_2 \wedge T_3} \quad \text{SUBTYP_AND2}$$

$$\frac{\Gamma \vdash T_1 <: T_2}{\Gamma \vdash \{a : T_1\} <: \{a : T_2\}} \quad \text{SUBTYP_FLD}$$

$$\frac{\Gamma \vdash T_1 <: T_2 \quad \Gamma \vdash T_3 <: T_4}{\Gamma \vdash \{A : T_2..T_3\} <: \{A : T_1..T_4\}} \quad \text{SUBTYP_TYP}$$

$$\frac{\Gamma \vdash x : \{A : T_1..T_2\}}{\Gamma \vdash x.A <: T_2} \quad \text{SUBTYP_SEL1}$$

$$\frac{\Gamma \vdash x : \{A : T_1..T_2\}}{\Gamma \vdash T_1 <: x.A} \text{ SUBTYP_SEL2}$$

$$\frac{\Gamma \vdash T_3 <: T_1 \quad (\Gamma, x : T_1) \vdash T_2 <: T_4}{\Gamma \vdash \forall (x : T_1) T_2 <: \forall (x : T_3) T_4} \text{ SUBTYP_ALL}$$

record T

$$\overline{\mathbf{record} \{a : T\}} \text{ RT_ONE_TRM}$$

$$\overline{\mathbf{record} \{A : T..T\}} \text{ RT_ONE_TYP}$$

$$\frac{\mathbf{record} T_1 \quad a \notin \mathbf{labels} T_1}{\mathbf{record} T_1 \wedge \{a : T_2\}} \text{ RT_AND_TRM}$$

$$\frac{\mathbf{record} T_1 \quad A \notin \mathbf{labels} T_1}{\mathbf{record} T_1 \wedge \{A : T_2..T_2\}} \text{ RT_AND_TYP}$$

inert T

$$\overline{\mathbf{inert} \forall (x : T_1) T_2} \text{ INERT_TYP_ALL}$$

$$\frac{\mathbf{record} T}{\mathbf{inert} \mu(x : T)} \text{ INERT_TYP_BND}$$

inert Γ

$$\overline{\mathbf{inert} \emptyset} \text{ INERT_EMPTY}$$

$$\frac{\mathbf{inert} \Gamma \quad \mathbf{inert} T \quad x \notin \Gamma}{\mathbf{inert} (\Gamma, x : T)} \text{ INERT_ALL}$$

$\Gamma \vdash_{!} val : T$

$$\frac{(\Gamma, x : T_1) \vdash t : T_2}{\Gamma \vdash_{!} \lambda(x : T_1).t : \forall (x : T_1) T_2} \text{ TY_ALL_INTRO_P}$$

$$\frac{(\Gamma, x : T) \vdash defs : T}{\Gamma \vdash_{!} \nu(x : T) defs : \mu(x : T)} \text{ TY_NEW_INTRO_P}$$

$\Gamma \vdash_{!} x : T_1 \gg T_2$

$$\frac{\Gamma(x) = T}{\Gamma \vdash_{!} x : T \gg T} \text{ PF_BIND}$$

$$\frac{\Gamma \vdash_{!} x : T_1 \gg \mu(z : T_2)}{\Gamma \vdash_{!} x : T_1 \gg [x/z] T_2} \text{ PF_OPEN}$$

$$\frac{\Gamma \vdash_{!} x : T_1 \gg T_2 \wedge T_3}{\Gamma \vdash_{!} x : T_1 \gg T_2} \text{ PF_AND1}$$

$$\frac{\Gamma \vdash !x : T_1 \gg T_2 \wedge T_3}{\Gamma \vdash !x : T_1 \gg T_3} \quad \text{PF_AND2}$$

$$\boxed{\Gamma \vdash_{\#} t : T}$$

$$\frac{\Gamma(x) = T}{\Gamma \vdash_{\#} x : T} \quad \text{TY_VAR_T}$$

$$\frac{(\Gamma, x : T_1) \vdash t : T_2}{\Gamma \vdash_{\#} \lambda(x : T_1).t : \forall (x : T_1) T_2} \quad \text{TY_ALL_INTRO_T}$$

$$\frac{\begin{array}{c} \Gamma \vdash_{\#} x : \forall (z : T_1) T_2 \\ \Gamma \vdash_{\#} y : T_1 \end{array}}{\Gamma \vdash_{\#} x y : [y/z] T_2} \quad \text{TY_ALL_ELIM_T}$$

$$\frac{(\Gamma, x : T) \vdash \text{defs} : T}{\Gamma \vdash_{\#} \nu(x : T) \text{defs} : \mu(x : T)} \quad \text{TY_NEW_INTRO_T}$$

$$\frac{\Gamma \vdash_{\#} x : \{a : T\}}{\Gamma \vdash_{\#} x.a : T} \quad \text{TY_NEW_ELIM_T}$$

$$\frac{\begin{array}{c} \Gamma \vdash_{\#} t_1 : T_1 \\ (\Gamma, x : T) \vdash t_2 : T_2 \end{array}}{\Gamma \vdash_{\#} \text{let } x = t_1 \text{ in } t_2 : T_2} \quad \text{TY_LET_T}$$

$$\frac{\Gamma \vdash_{\#} x : T}{\Gamma \vdash_{\#} x : \mu(z : T)} \quad \text{TY_REC_INTRO_T}$$

$$\frac{\Gamma \vdash_{\#} x : \mu(z : T)}{\Gamma \vdash_{\#} x : [x/z] T} \quad \text{TY_REC_ELIM_T}$$

$$\frac{\begin{array}{c} \Gamma \vdash_{\#} x : T_1 \\ \Gamma \vdash_{\#} x : T_2 \end{array}}{\Gamma \vdash_{\#} x : T_1 \wedge T_2} \quad \text{TY_AND_INTRO_T}$$

$$\frac{\begin{array}{c} \Gamma \vdash_{\#} t : T_1 \\ \Gamma \vdash_{\#} T_1 <: T_2 \end{array}}{\Gamma \vdash_{\#} t : T_2} \quad \text{TY_SUB_T}$$

$$\boxed{\Gamma \vdash_{\#} T_1 <: T_2}$$

$$\overline{\Gamma \vdash_{\#} T <: \top} \quad \text{SUBTYP_TOP_T}$$

$$\overline{\Gamma \vdash_{\#} \perp <: T} \quad \text{SUBTYP_BOT_T}$$

$$\overline{\Gamma \vdash_{\#} T <: T} \quad \text{SUBTYP_REFL_T}$$

$$\frac{\begin{array}{c} \Gamma \vdash_{\#} T_1 <: T_2 \\ \Gamma \vdash_{\#} T_2 <: T_3 \end{array}}{\Gamma \vdash_{\#} T_1 <: T_3} \quad \text{SUBTYP_TRANS_T}$$

$$\overline{\Gamma \vdash_{\#} T_1 \wedge T_2 <: T_1} \quad \text{SUBTYP_AND11_T}$$

$$\overline{\Gamma \vdash_{\#} T_1 \wedge T_2 <: T_2} \quad \text{SUBTYP_AND12_T}$$

$$\begin{array}{c}
\frac{\Gamma \vdash_{\#} T_1 <: T_2 \quad \Gamma \vdash_{\#} T_1 <: T_3}{\Gamma \vdash_{\#} T_1 <: T_2 \wedge T_3} \text{SUBTYP_AND2_T} \\
\\
\frac{\Gamma \vdash_{\#} T_1 <: T_2}{\Gamma \vdash_{\#} \{a : T_1\} <: \{a : T_2\}} \text{SUBTYP_FLD_T} \\
\\
\frac{\Gamma \vdash_{\#} T_1 <: T_2 \quad \Gamma \vdash_{\#} T_3 <: T_4}{\Gamma \vdash_{\#} \{A : T_2..T_3\} <: \{A : T_1..T_4\}} \text{SUBTYP_TYP_T} \\
\\
\frac{\Gamma \vdash_{!} x : T_1 \gg \{A : T_2..T_2\}}{\Gamma \vdash_{\#} x.A <: T_2} \text{SUBTYP_SEL1_T} \\
\\
\frac{\Gamma \vdash_{!} x : T_1 \gg \{A : T_2..T_2\}}{\Gamma \vdash_{\#} T_2 <: x.A} \text{SUBTYP_SEL2_T} \\
\\
\frac{\Gamma \vdash_{\#} T_3 <: T_1 \quad (\Gamma, x : T_1) \vdash T_2 <: T_4}{\Gamma \vdash_{\#} \forall (x : T_1) T_2 <: \forall (x : T_3) T_4} \text{SUBTYP_ALL_T}
\end{array}$$

$$\boxed{\Gamma \vdash_{\#\#} x : T}$$

$$\begin{array}{c}
\frac{\Gamma \vdash_{!} x : T_1 \gg T_2}{\Gamma \vdash_{\#\#} x : T_2} \text{TY_PRECISE_INV} \\
\\
\frac{\Gamma \vdash_{\#\#} x : \{a : T_1\} \quad \Gamma \vdash_{\#} T_1 <: T_2}{\Gamma \vdash_{\#\#} x : \{a : T_2\}} \text{TY_DEC_TRM_INV} \\
\\
\frac{\Gamma \vdash_{\#\#} x : \{A : T_2..T_3\} \quad \Gamma \vdash_{\#} T_1 <: T_2 \quad \Gamma \vdash_{\#} T_3 <: T_4}{\Gamma \vdash_{\#\#} x : \{A : T_1..T_4\}} \text{TY_DEC_TYP_INV} \\
\\
\frac{\Gamma \vdash_{\#\#} x : [x/z]T}{\Gamma \vdash_{\#\#} x : \mu(z : T)} \text{TY_BND_INV} \\
\\
\frac{\Gamma \vdash_{\#\#} x : \forall (z : T_2) T_3 \quad \Gamma \vdash_{\#} T_1 <: T_2 \quad (\Gamma, z : T_1) \vdash T_3 <: T_4}{\Gamma \vdash_{\#\#} x : \forall (z : T_1) T_4} \text{TY_ALL_INV} \\
\\
\frac{\Gamma \vdash_{\#\#} x : T_1 \quad \Gamma \vdash_{\#\#} x : T_2}{\Gamma \vdash_{\#\#} x : T_1 \wedge T_2} \text{TY_AND_INV} \\
\\
\frac{\Gamma \vdash_{\#\#} x : T_1 \quad \Gamma \vdash_{!} y : T_2 \gg \{A : T_1..T_1\}}{\Gamma \vdash_{\#\#} x : y.A} \text{TY_SEL_INV} \\
\\
\frac{\Gamma \vdash_{\#\#} x : T}{\Gamma \vdash_{\#\#} x : \top} \text{TY_TOP_INV}
\end{array}$$

$$\boxed{\Gamma \vdash_{\#\#} val : T}$$

$$\begin{array}{c}
\frac{\Gamma \vdash !val : T}{\Gamma \vdash \#\# val : T} \quad \text{TY_PRECISE_INV_V} \\
\\
\frac{\begin{array}{c} \Gamma \vdash \#\# val : \forall (z : T_2) T_3 \\ \Gamma \vdash \# T_1 <: T_2 \\ (\Gamma, z : T_1) \vdash T_3 <: T_4 \end{array}}{\Gamma \vdash \#\# val : \forall (z : T_1) T_4} \quad \text{TY_ALL_INV_V} \\
\\
\frac{\begin{array}{c} \Gamma \vdash \#\# val : T_1 \\ \Gamma \vdash \#\# val : T_2 \end{array}}{\Gamma \vdash \#\# val : T_1 \wedge T_2} \quad \text{TY_AND_INV_V} \\
\\
\frac{\begin{array}{c} \Gamma \vdash \#\# val : T_1 \\ \Gamma \vdash !y : T_2 \gg \{A : T_1..T_1\} \end{array}}{\Gamma \vdash \#\# val : y.A} \quad \text{TY_SEL_INV_V} \\
\\
\frac{\Gamma \vdash \#\# val : T}{\Gamma \vdash \#\# val : \top} \quad \text{TY_TOP_INV_V}
\end{array}$$

$$\boxed{(s_1, t_1) \rightarrow (s_2, t_2)}$$

$$\begin{array}{c}
\frac{\begin{array}{c} s(x) = \nu(z : T) defs \\ \{a = t\} \in [x/z] defs \end{array}}{(s, x.a) \rightarrow (s, t)} \quad \text{RED_SEL} \\
\\
\frac{s(x) = \lambda(z : T_1).t}{(s, x y) \rightarrow (s, [y/z]t)} \quad \text{RED_APP} \\
\\
\frac{}{(s, \mathbf{let} \ x = val \ \mathbf{in} \ t) \rightarrow ((s, x : val), t)} \quad \text{RED_LET_VAL} \\
\\
\frac{}{(s, \mathbf{let} \ x = y \ \mathbf{in} \ t) \rightarrow (s, [y/x]t)} \quad \text{RED_LET_VAR} \\
\\
\frac{(s_1, t_1) \rightarrow (s_2, t_2)}{(s_1, \mathbf{let} \ x = t_1 \ \mathbf{in} \ t_3) \rightarrow (s_2, \mathbf{let} \ x = t_2 \ \mathbf{in} \ t_3)} \quad \text{RED_LET_TGT}
\end{array}$$

Definition rules: 80 good 0 bad
 Definition rule clauses: 176 good 0 bad