Topology-Aware Segmentation Using Discrete Morse Theory

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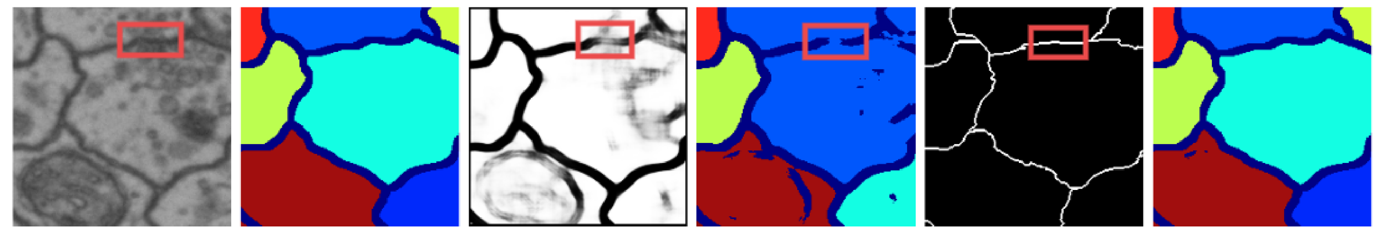
1 Introduction

Problem: segmentation of fine-scale image structures: membranes, neurons, vessels, etc.

- Existing methods are optimized for per-pixel accuracy.
- Prone to **structural errors**: broken / wrong connections.
- Affects downstream analysis.


Input Image | Ground Truth | Likelihood Map | With Topo Errors | Structures | Our Method

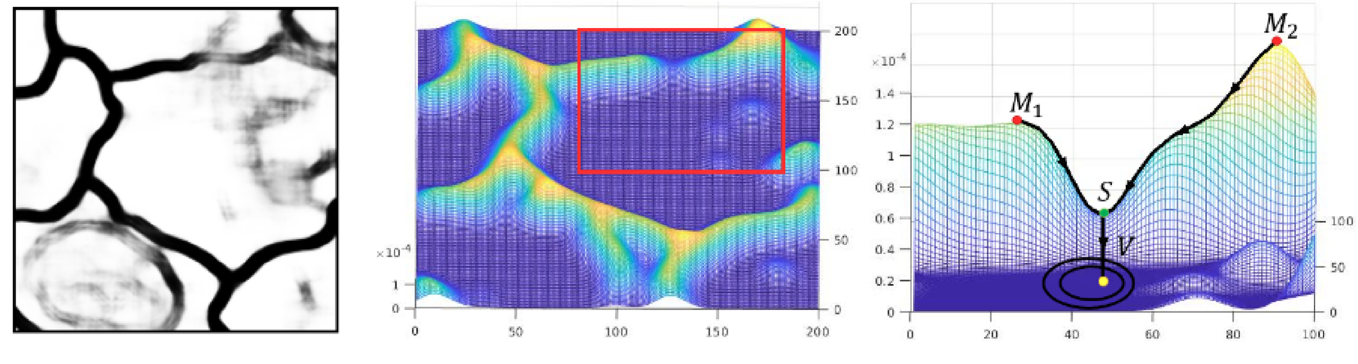
Contribution: a topology-preserving deep neural network.

- DMT loss: capturing the critical structures of the training data
- DMT-based loss function for end-to-end training of neural networks
- Efficiency: converging faster than topological loss

2 Method

Challenge: Topological loss identifies difficult locations/critical pixels during training, and it needs to fix the critical pixels one by one. **Not efficient enough**.

Key: To address this issue, instead of identifying critical points, we use discrete Morse theory to capture critical structures, consequently we will be able to fix the broken connections once at a time.


Likelihood Map | Density Map | Density Map for highlighted region

Negative Gradient:

$$-\nabla f(x) = -\left[\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, ..., \frac{\partial f}{\partial x_d}\right]^T$$

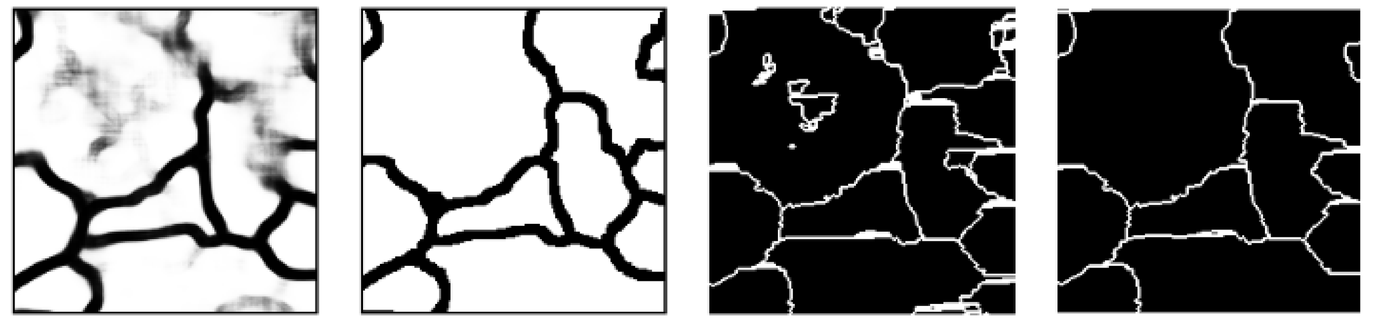
Critical Points (minimum, maximum, saddle):

$$-\nabla f(x) = 0$$

Persistence-based structure pruning

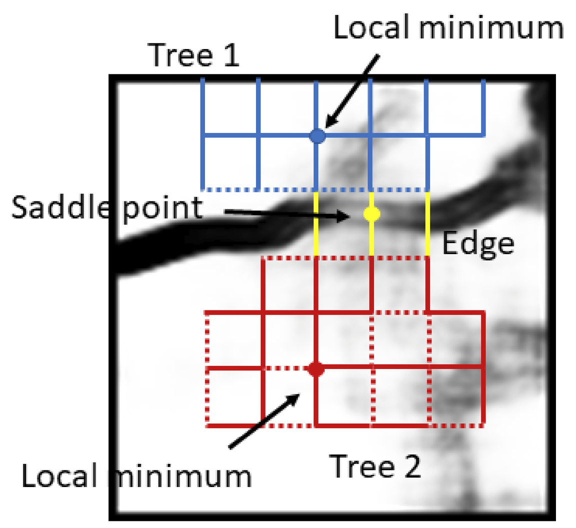
We use the tool of persistent homology to remove the noisy structures with small persistence. The persistence epsilon is a dataset dependent hyperparameter:

- If epsilon is too small, the obtained structure will remain lots of noises.
- If the epsilon is too large, there will be fewer structures/less useful information left.


Likelihood Map | Ground Truth | Improperly pruned structures | Properly pruned structures

Approximation

In terms of 3D cases, we need an efficient algorithm to compute $S_2(\epsilon)$, because this computation needs to be carried out at each epoch. To this end, we propose to approximate $S_2(\epsilon)$ by $\hat{S_2}(\epsilon)$ which intuitively comes from the "boundary" of the stable manifold for minima.



Workflow of our algorithm

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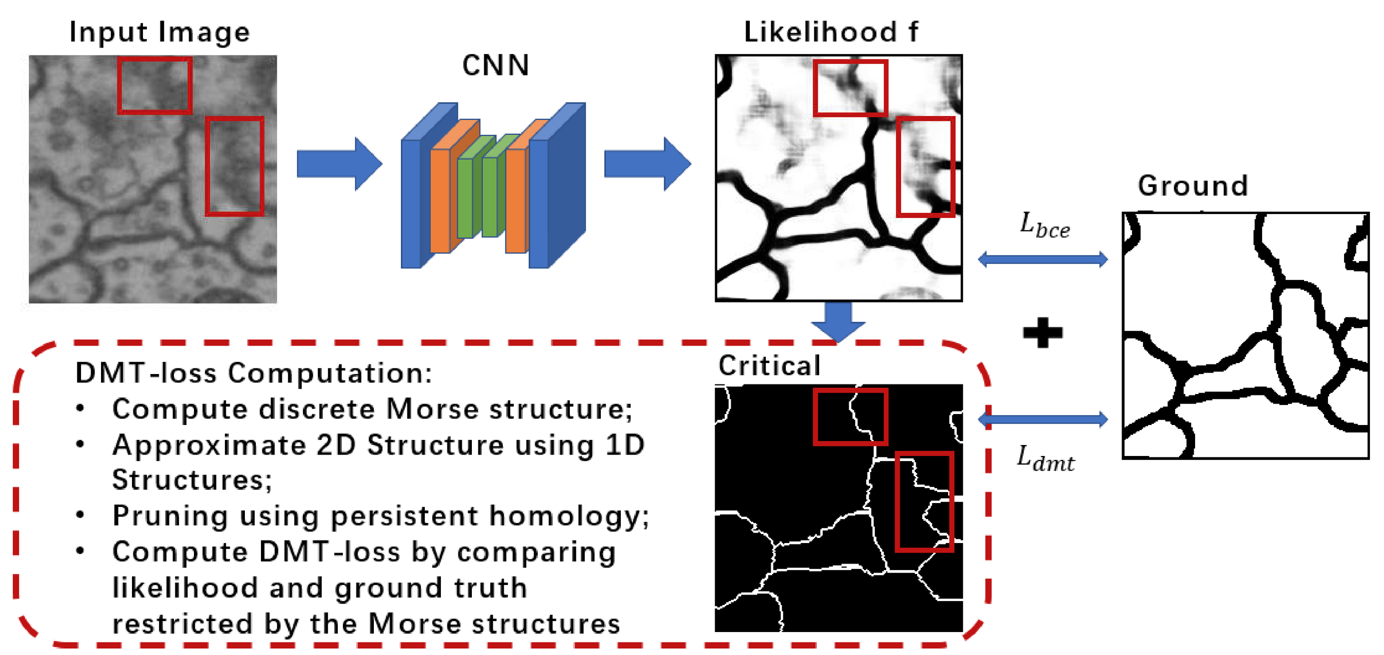
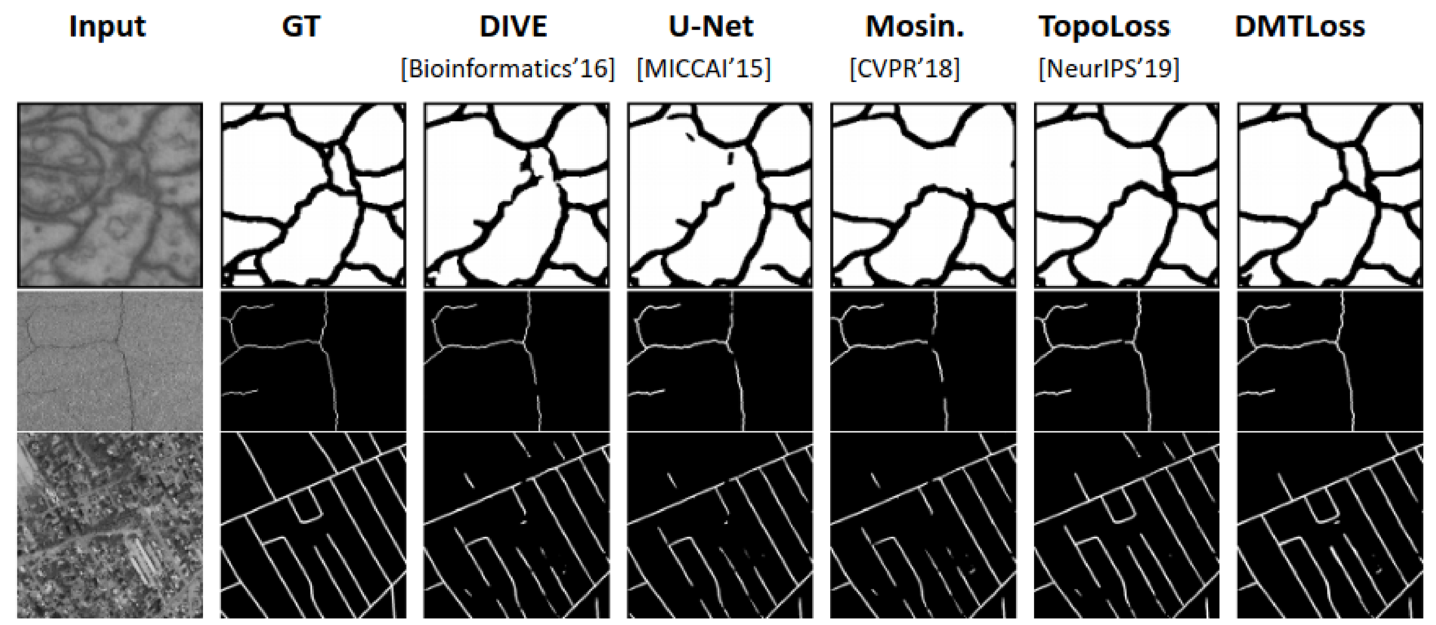


3 Results

Qualitative results



- This shows the qualitative results compared with several popular methods, such as DIVE, standard U-Net, Mosin. paper and the topological loss.
- From the results, we could find that our proposed method achieves better results in terms of structures, like recovering the broken connections.

Quantitative Results for 2D datasets

- Our method achieves superiorperformance on both the DICE score and topological metrics.

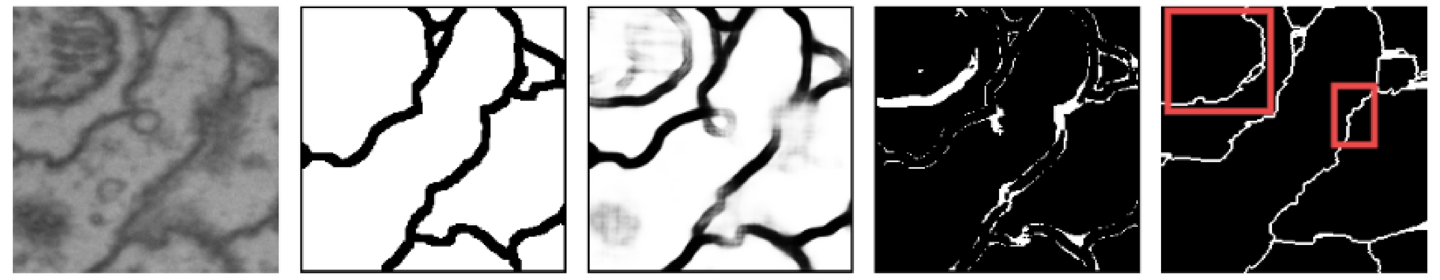
Method	Accuracy	DICE	ARI	VOI	Betti Error
			ISBI13		
DIVE	0.9642 ± 0.0018	0.9658 ± 0.0020	0.6923 ± 0.0134	2.790 ± 0.025	3.875 ± 0.326
U-Net	0.9631 ± 0.0024	0.9649 ± 0.0057	0.7031 ± 0.0256	2.583 ± 0.078	3.463 ± 0.435
Mosin.	0.9578 ± 0.0029	0.9623 ± 0.0047	0.7483 ± 0.0367	1.534 ± 0.063	2.952 ± 0.379
TopoLoss	0.9569 ± 0.0031	**0.9689 ± 0.0026**	0.8064 ± 0.0112	1.436 ± 0.008	**1.253 ± 0.172**
DMT	0.9625 ± 0.0027	**0.9712 ± 0.0047**	**0.8289 ± 0.0189**	**1.176 ± 0.052**	**1.102 ± 0.203**
			CREMI		
DIVE	0.9498 ± 0.0029	0.9542 ± 0.0037	0.6532 ± 0.0247	2.513 ± 0.047	4.378 ± 0.152
U-Net	0.9468 ± 0.0048	0.9523 ± 0.0049	0.6723 ± 0.0312	2.346 ± 0.105	3.016 ± 0.253
Mosin.	0.9467 ± 0.0058	0.9489 ± 0.0053	0.7853 ± 0.0281	1.623 ± 0.083	1.973 ± 0.310
TopoLoss	0.9456 ± 0.0053	0.9596 ± 0.0029	0.8083 ± 0.0104	1.462 ± 0.028	**1.113 ± 0.224**
DMT	0.9475 ± 0.0031	**0.9653 ± 0.0019**	**0.8203 ± 0.0147**	**1.089 ± 0.061**	**0.982 ± 0.179**
			CrackTree		
DIVE	0.9854 ± 0.0052	0.6530 ± 0.0017	0.8634 ± 0.0376	1.570 ± 0.078	1.576 ± 0.287
U-Net	0.9821 ± 0.0097	0.6491 ± 0.0029	0.8749 ± 0.0421	1.625 ± 0.104	1.785 ± 0.303
Mosin.	0.9833 ± 0.0067	0.6527 ± 0.0010	0.8897 ± 0.0201	1.113 ± 0.057	1.045 ± 0.214
TopoLoss	0.9826 ± 0.0084	0.6732 ± 0.0041	**0.9291 ± 0.0123**	0.997 ± 0.011	**0.672 ± 0.176**
DMT	0.9842 ± 0.0041	**0.6811 ± 0.0047**	**0.9307 ± 0.0172**	**0.901 ± 0.081**	**0.518 ± 0.189**
			Road		
DIVE	0.9734 ± 0.0077	0.6743 ± 0.0051	0.8201 ± 0.0128	2.368 ± 0.203	3.598 ± 0.783
U-Net	0.9786 ± 0.0052	0.6612 ± 0.0016	0.8189 ± 0.0097	2.249 ± 0.175	3.439 ± 0.621
Mosin.	0.9754 ± 0.0043	0.6673 ± 0.0044	0.8456 ± 0.0174	1.457 ± 0.096	2.781 ± 0.237
TopoLoss	0.9728 ± 0.0063	0.6903 ± 0.0038	0.8671 ± 0.0068	1.234 ± 0.037	**1.275 ± 0.192**
DMT	0.9744 ± 0.0049	**0.7056 ± 0.0022**	**0.8819 ± 0.0104**	**1.092 ± 0.129**	**0.995 ± 0.301**

Quantitative Results for 3D datasets

- We use the same evaluation metrics to demonstrate the effectiveness of the proposed method.

Method	Accuracy	DICE	ARI	VOI	Betti Error
			ISBI13		
3D DIVE	0.9723 ± 0.0021	0.9681 ± 0.0043	0.8719 ± 0.0189	1.208 ± 0.149	2.375 ± 0.419
3D U-Net	0.9746 ± 0.0025	0.9701 ± 0.0012	**0.8956 ± 0.0391**	1.123 ± 0.091	1.954 ± 0.585
MALA	0.9701 ± 0.0018	0.9699 ± 0.0013	**0.8945 ± 0.0481**	0.901 ± 0.106	1.103 ± 0.207
3D TopoLoss	0.9689 ± 0.0031	0.9752 ± 0.0045	**0.9043 ± 0.0283**	0.792 ± 0.086	0.972 ± 0.245
DMT	0.9701 ± 0.0026	**0.9803 ± 0.0019**	**0.9149 ± 0.0217**	**0.634 ± 0.086**	**0.812 ± 0.134**
			CREMI		
3D DIVE	0.9503 ± 0.0061	0.9641 ± 0.0011	0.8514 ± 0.0387	1.219 ± 0.103	2.674 ± 0.473
3D U-Net	0.9547 ± 0.0038	0.9618 ± 0.0026	0.8322 ± 0.0315	1.416 ± 0.097	2.313 ± 0.501
MALA	0.9472 ± 0.0027	0.9583 ± 0.0023	0.8713 ± 0.0286	1.109 ± 0.093	1.114 ± 0.309
3D TopoLoss	0.9523 ± 0.0043	0.9672 ± 0.0010	0.8726 ± 0.0194	1.044 ± 0.128	1.076 ± 0.206
DMT	0.9529 ± 0.0031	**0.9731 ± 0.0045**	**0.9013 ± 0.0202**	**0.891 ± 0.099**	**0.726 ± 0.187**
			3Dircadb		
3D DIVE	0.9618 ± 0.0054	0.6097 ± 0.0034	/	/	4.571 ± 0.505
3D U-Net	0.9632 ± 0.0009	0.5898 ± 0.0025	/	/	4.131 ± 0.483
MALA	0.9546 ± 0.0033	0.5719 ± 0.0043	/	/	2.982 ± 0.105
3D TopoLoss	0.9561 ± 0.0019	0.6138 ± 0.0029	/	/	2.245 ± 0.255
DMT	0.9587 ± 0.0023	**0.6257 ± 0.0021**	/	/	**1.415 ± 0.305**

Ablation study: Comparison with reweighted cross entropy loss


Input Image | Ground Truth | Likelihood map | Pixels identified by reweighted CE | Highlighted Structures by DMT

- The reweighted cross entropy loss contains lots of noise, which are useless for image structures.
- The proposed DMT loss identifies structures relevant to image topology.

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Reference

1. Hu, Xiaoling, et al. "Topology-Preserving Deep Image Segmentation." Advances in neural information processing systems 32 (2019).