Topology-Preserving Deep Image Segmentation

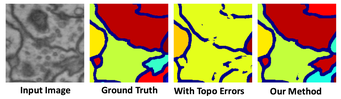
Xiaoling Hu[1], Fuxin Li[2], Dimitris Samaras[1] and Chao Chen[1]

1, Stony Brook University, New York, USA 2, Oregon State University, Oregon, USA

1 Introduction

Problem: segmentation of fine-scale image structures: membranes, neurons, vessels, etc.

- Existing methods are optimized for per-pixel accuracy.
- Prone to **structural errors**: broken / wrong connections.
- Affects downstream analysis.



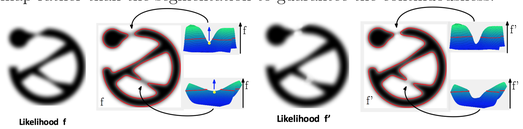
Contribution: a topology-preserving deep neural network.

- Learn topological structure from large amount of fine-structure data
- Make full use of topological information: connectivity, number of loops, etc. – available from ground truth.
- **Topological loss** for training: differentiability due to the theory of persistent homology.

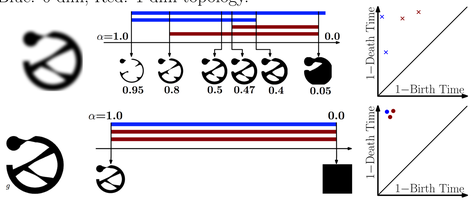
2 Method

Challenge: Betti number can be used to compute number of topology structures, however comparison of Betti number (discrete) is not differentiable.

Key: The minimum effort of moving predicted likelihood map to ground-truth can be defined as topological loss function. And we use the likelihood map rather than the segmentation to guarantee the continuousness.



Persistent homology:
track topology of superlevel sets $f^\alpha = \{x | f(x) \geq \alpha\}$ at all α's.
Output: diagrams. $Dgm(f) = \{(1 - birth(p), 1 - death(p))\}$
Blue: 0-dim, Red: 1-dim topology.



As α decreases, the topological information can be encoded in the barcode and persistence diagram. And the loss can be defined as the effort which is needed to move the predicted persistence diagram to groundtruth.

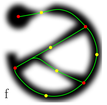
Computation and Matching: We use classic algorithms (like gudhi) to compute the persistence diagram $\mathrm{Dgm}(f)$, and the optimal matching of persistence diagrams can be computed by a modified version of the Hangarian algorithm.

Topological loss: matching distance between diagrams.

$$\min_{\gamma \in \Gamma} \sum_{p \in \mathrm{Dgm}(f)} \|p - \gamma(p)\|^2 = \sum_{p \in \mathrm{Dgm}(f)} [\mathrm{birth}(p) - \mathrm{birth}(\gamma^*(p))]^2 + [\mathrm{death}(p) - \mathrm{death}(\gamma^*(p))]^2$$

γ^* -- the optimal matching

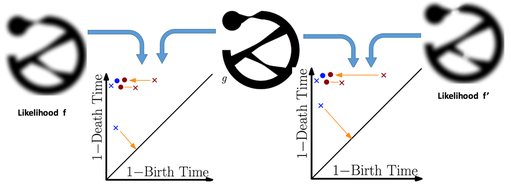
- The loss function depends on crucial thresholds at which topological changes happen, e.g., birth and death times of different dots in the diagram



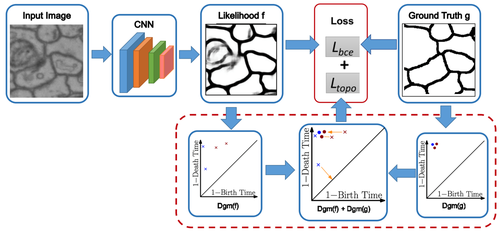
- These crucial thresholds are uniquely determined by the locations at which the topological changes happen
- For f, a critical point (right map) is always a pixel with zero gradient, since topological changes happen at pixels.

Topological gradient:.

$$\sum_{p \in \mathrm{Dgm}(f)} 2[f(c_b(p)) - \mathrm{birth}(\gamma^*(p))] \frac{\partial f(c_b(p))}{\partial \omega} + 2[f(c_d(p)) - \mathrm{death}(\gamma^*(p))] \frac{\partial f(c_d(p))}{\partial \omega}$$

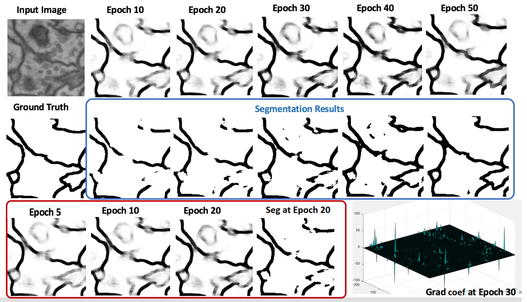


2.1 Workflow of our algorithm



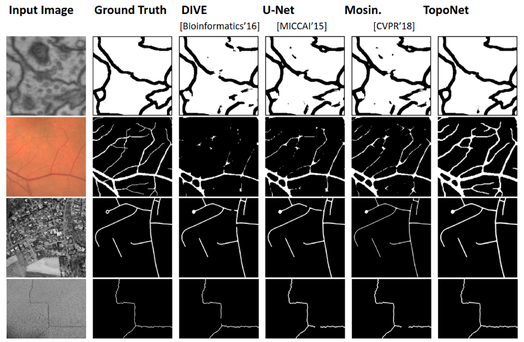
3 Results

3.1 lh map and corresponding segmentation results during training



Topological loss essentially identifies difficult locations during training, Topology structure of predicted likelihood map becomes more accurate as training goes on.

3.2 Qualitative results



The qualitative results show that the proposed method can achieve better performance compared to other methods in terms of topology-aware evaluation metrics.

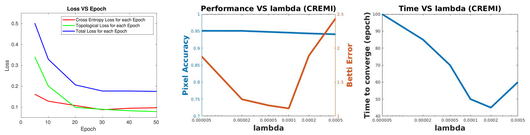
3.3 Quantitative Results

- 6 different datasets: EM (neuron membrane), retinal (vessel), aerial images (road), pavement (cracks)
- Outperform existing methods especially on topology-relevant metrics: Betti Error, Adapted Rand Index (ARI), Variation of Information (VoI).

Dataset	Method	Accuracy	ARI	VOI	Betti Error
ISBI12	**DIVE**	0.9640	0.9434	1.235	3.187
	U-Net	**0.9678**	0.9338	1.367	2.785
	Mosin.	0.9532	0.9312	0.983	1.238
	TopoNet	0.9626	**0.9444**	**0.782**	**0.429**
ISBI13	**DIVE**	**0.9642**	0.6923	2.790	3.875
	U-Net	0.9631	0.7031	2.583	3.463
	Mosin.	0.9578	0.7483	1.534	2.952
	TopoNet	0.9569	**0.8064**	**1.436**	**1.253**
CREMI	**DIVE**	**0.9498**	0.6532	2.513	4.378
	U-Net	0.9468	0.6723	2.346	3.016
	Mosin.	0.9467	0.7853	1.623	1.973
	TopoNet	0.9456	**0.8083**	**1.462**	**1.113**

Dataset	Method	Accuracy	ARI	VOI	Betti Error
DRIVE	**DIVE**	**0.9549**	0.8407	1.936	3.276
	U-Net	0.9452	0.8343	1.975	3.643
	Mosin.	0.9543	0.8870	1.167	2.784
	TopoNet	0.9521	**0.9024**	**1.083**	**1.076**
CrackTree	**DIVE**	**0.9854**	0.8634	1.570	1.576
	U-Net	0.9821	0.8749	1.625	1.785
	Mosin.	0.9833	0.8897	1.113	1.045
	TopoNet	0.9826	**0.9291**	**0.997**	**0.672**
Road	**DIVE**	0.9734	0.8201	2.368	3.598
	U-Net	**0.9786**	0.8189	2.249	3.439
	Mosin.	0.9754	0.8456	1.457	2.781
	TopoNet	0.9728	**0.8671**	**1.234**	**1.275**

3.4 Ablation study: loss weights



- Cross Entropy loss, Topological loss and total loss in terms of training epochs.
- Ablation studies of lambda on CREMI w.r.t. accuracy, Betti error.
- Ablation study of lambda on CREMI w.r.t. convergence rate.

Rationale of topology analysis:

- Topological loss essentially identifies difficult locations during training (as critical pixels)
- Topological loss cannot succeed alone. Without cross-entropy loss, inferring topology from a completely random likelihood map is meaningless.

Future work:

- Analyze (quantify) complex spatiotemporal biomedical systems.
- Feature / loss / prior.