Fundamentals Of Information Science	**2022 Spring**

HW 1: 信息与比特

学生: 华园（202000120027）） *时间: 2022.2.24*

Problem 1.

Proof the following atatements are true for a 0-1 Boolean algebra.

Solution.

$$
\begin{aligned}
(1) \quad a \cdot (a + (b \cdot c)) &= a \cdot a + a \cdot (b \cdot c) \\
&= a + a \cdot (b \cdot c) \\
&= a(1 + b \cdot c) \\
&= a
\end{aligned}
$$

$$
\begin{aligned}
(2) \quad (a \cdot b) + (\overline{a} + \overline{b}) &= (a \cdot b) + \overline{(a \cdot b)} \\
&= 1
\end{aligned}
$$

Problem 2.

Coin flips.A fair coin is flipped until the first head occurs. Let X denote the number of flips required. Find the entropy H(X) in bits.

Solution. *Suppose the probability distribution of random variable X is P(X).*

$$
\begin{aligned}
P(X = 1) &= \frac{1}{2} \\
P(X = 2) &= (\frac{1}{2})^2 \\
P(X = 3) &= (\frac{1}{2})^3 \\
P(X = 4) &= (\frac{1}{2})^4 \\
&\cdots \\
P(X = n) &= (\frac{1}{2})^n
\end{aligned}
$$

According to the definition of entropy, we can get the following results:

$$\begin{aligned}
H(X) &= -\sum_{n=1}^{\infty} p(x) log_2 p(x) \\
&= -\sum_{n=1}^{\infty} (\frac{1}{2})^n log_2 (\frac{1}{2})^n \\
&= \sum_{n=1}^{\infty} n(\frac{1}{2})^n \\
&= \frac{1/2}{(1-1/2)^2} \\
&= 2(bits)
\end{aligned}$$

Problem 3.

Solution.

$$m(a,b) = \overline{a} + \overline{b} = \overline{ab} \qquad P(a,b) = \overline{a}b + a\overline{b}$$

$$m(a,a) = \overline{a} \qquad m(\overline{a},b) = \overline{(\overline{a}b)} \qquad m(a,\overline{b}) = \overline{(a\overline{b})}$$

$$m(\overline{(\overline{a}b)}, \overline{(a\overline{b})}) = \overline{a}b + a\overline{b} = P(a,b)$$