

## Homework2

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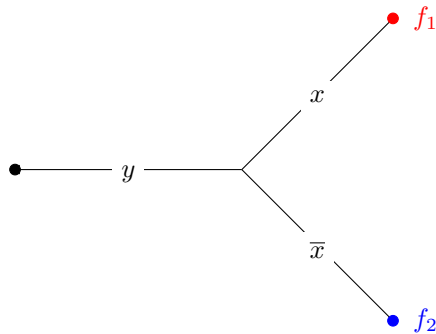
## Problem 1.

(a) Construct a circuit with 3 relays that implements the functions

$$f_1 = x \cdot y$$

$$f_2 = \bar{x} \cdot y$$

Solution.

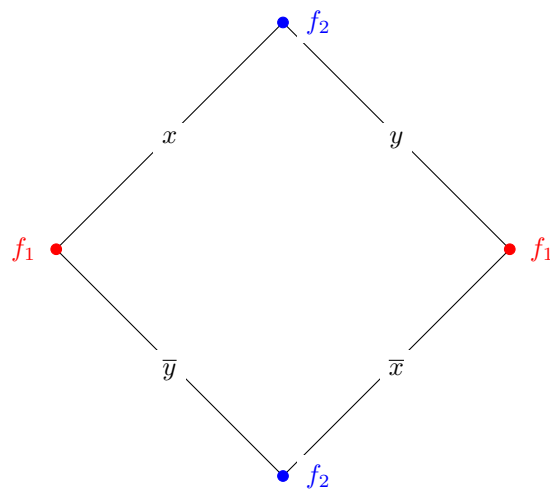


(b) Construct a circuit with 4 relays that implements the functions

$$f_1 = x \cdot y + \bar{x} \cdot \bar{y}$$

$$f_2 = \bar{x} \cdot y + x \cdot \bar{y}$$

Solution.



(c) Construct a circuit with 6 relays that implements the functions

$$f_1 = x \cdot (y + z)$$

$$f_2 = y \cdot (x + z)$$

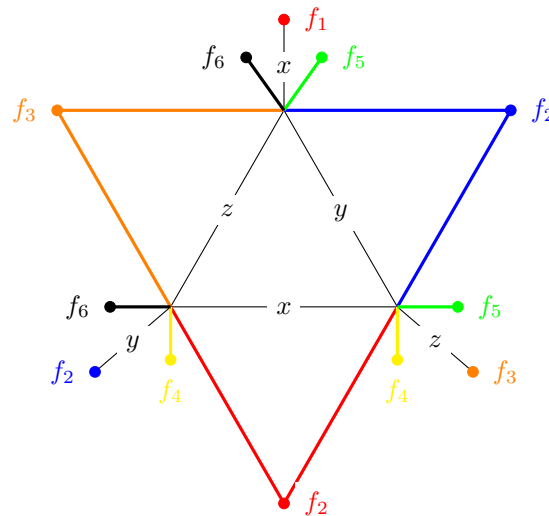
$$f_3 = z \cdot (x + y)$$

$$f_4 = x + y \cdot z$$

$$f_5 = y + x \cdot z$$

$$f_6 = z + x \cdot y$$

**Solution.**



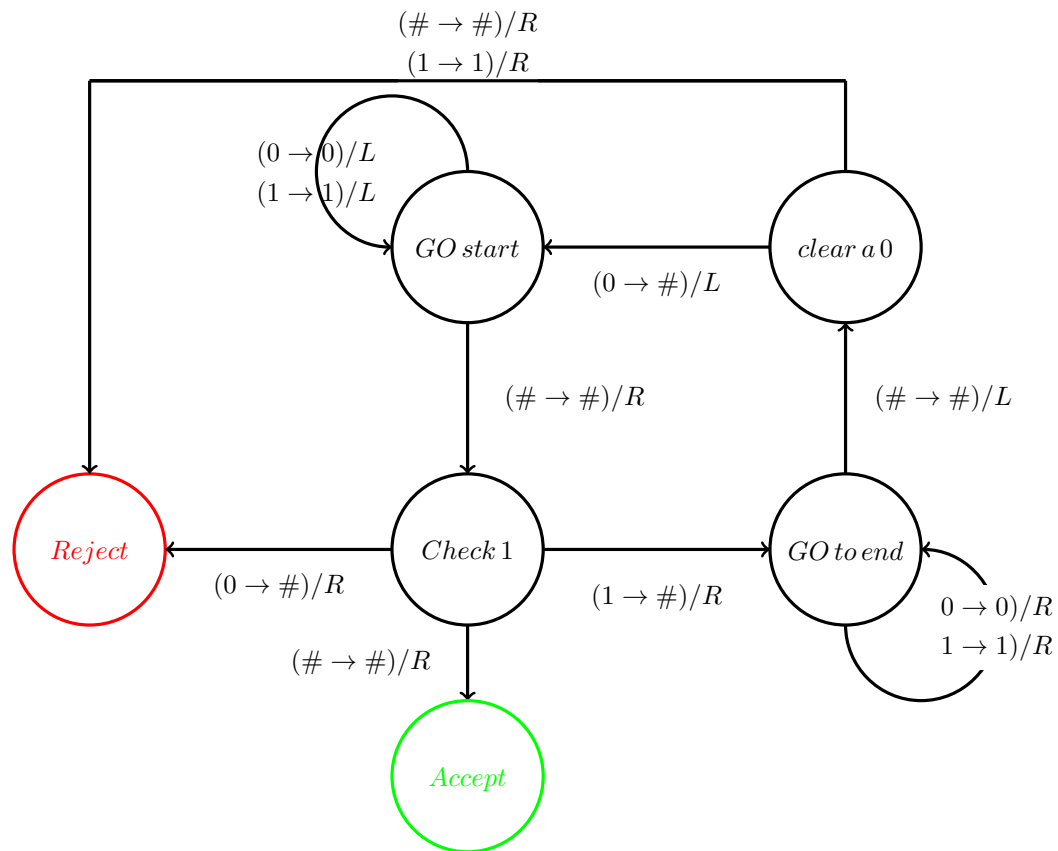
## Problem 2.

You are asked to design a Turing machine that accepts the following languages: some number of 1's followed by the same number of 0's.

[#10#, #1100#, #111000#, #11110000#, ...]

Explain your design.

**Solution.**



### Problem 3.

(1) write  $CQ(X)$  with 6 inputs function  $|X|$ . Note that there 7 entries in the table.

**Solution.**

$ X $	$CQ(X)$
0	0
1	0
2	1
3	1
4	0
5	0
6	1

(2) For an arbitrary  $n$ , express  $CQ(X)$  as a function of  $|X|$ . Namely, you need to specify, as a mathematical expression, the value of  $|X|$  for which  $CQ(X)=1$ . Justify your solution.

**Solution.**

XOR is a symmetric function, when the number of 1 in the input is odd, the output is 1. thus:

$$\text{when : } \quad C_{|x|}^2 = \frac{|x| \cdot (|x| - 1)}{2} = 2k + 1 \quad (|x| \geq 2, k \in N)$$
$$CQ(X) = 1$$