Homework #2

Shao Hua, Huang
ECM9032 - Machine Learning

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Sorry, I haven't finished the whole homework, and I will hand in new version this week.

1 Bayesian Inference for Gaussian (30%)

1. Derive posterior distribution by the following equations.

$$\begin{aligned}
p(\mathbf{\Lambda} \mid \mathbf{X}) &\propto \left[p(\mathbf{\Lambda}) \prod_{n=1}^{N-1} p(\mathbf{x}_n \mid \mathbf{\Lambda}) \right] p(\mathbf{x}_N \mid \mathbf{\Lambda}) \\
&= p(\mathbf{\Lambda}) \prod_{n=1}^{N} p(\mathbf{x}_n \mid \mathbf{\Lambda}) \\
&= \mathcal{W}(\mathbf{\Lambda} \,| W_0, \nu_0) \prod_{n=1}^{N} \mathcal{N}(\mathbf{x}_n \mid \mu, \mathbf{\Lambda}^{-1}) \\
&= B_0 |\, \mathbf{\Lambda} \,|^{\frac{\nu_0 - D - 1}{2}} \exp^{-\frac{1}{2} Tr(W_0^{-1} \mathbf{\Lambda})} \times \frac{1}{(2\pi)^{\frac{ND}{2}}} |\, \mathbf{\Lambda} \,|^{\frac{N}{2}} \exp^{-\frac{1}{2} \sum_{n=1}^{N} (\mathbf{x}_n - \mu)^T \mathbf{\Lambda} (\mathbf{x}_n - \mu)} \\
&\propto |\, \mathbf{\Lambda} \,|^{\frac{\nu_0 - D + N - 1}{2}} \exp^{-\frac{1}{2} Tr(W_0^{-1} \mathbf{\Lambda})} \exp^{-\frac{1}{2} Tr(\mathbf{\Lambda} \mathbf{S})} \\
&= |\, \mathbf{\Lambda} \,|^{\frac{\nu_0 - D + N - 1}{2}} \exp^{-\frac{1}{2} Tr(W_0^{-1} \mathbf{\Lambda} + \mathbf{\Lambda} \mathbf{S}))}
\end{aligned}$$

where

$$\mathbf{S} = \sum_{n=1}^{N} (\mathbf{x}_n - \mu)(\mathbf{x}_n - \mu)^T$$

Therefore, we know that posterior is also a Wishart distribution, that is,

$$p(\mathbf{\Lambda} \mid \mathbf{X}) = \mathcal{W}(\mathbf{\Lambda} \,| W_{\mathbf{\Lambda}}, \nu_{\mathbf{\Lambda}})$$

2 Bayesian Linear Regression (30%)

See program hw2_2.py

3 Logistic Regression (40%)