ECM5901 Assignment #5, 2019 due December 3

Using CVX (Matlab software) `http://cvxr.com/cvx/`

(1) Download. Example: download cvx-w64.zip if you are using Windows. Then unzip the file.

(2) Execute MATLAB. Run cvx_setup.m and cvx_startup.m in MATLAB.

(3) Start to program.

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1. Let $A_0 = \begin{bmatrix} 10 & 8 & 12 & 15 & 15 \\ 8 & 14 & 8 & 7 & 9 \\ 12 & 8 & 10 & 13 & 9 \\ 15 & 7 & 13 & 4 & 10 \\ 15 & 9 & 9 & 10 & 4 \end{bmatrix}$, $A_1 = \begin{bmatrix} 12 & 11 & 14 & 10 & 3 \\ 11 & 14 & 10 & 14 & 6 \\ 14 & 10 & 16 & 18 & 4 \\ 10 & 14 & 18 & 18 & 8 \\ 3 & 6 & 4 & 8 & 8 \end{bmatrix}$, $A_2 = \begin{bmatrix} 4 & 13 & 12 & 16 & 6 \\ 13 & 4 & 14 & 9 & 15 \\ 12 & 14 & 6 & 5 & 5 \\ 16 & 9 & 5 & 2 & 6 \\ 6 & 15 & 5 & 6 & 8 \end{bmatrix}$.

   Suppose $A : \mathbb{R}^2 \to S^5$ is defined by

   $$A(x) = A_0 + x_1 A_1 + x_2 A_2.$$

   Let $\lambda_1(x) \geq \lambda_2(x) \geq \lambda_3(x) \geq \lambda_4(x) \geq \lambda_5(x)$ denote the eigenvalues of $A(x)$.

   (a) Formulate the problem of minimizing the spread of the eigenvaules $\lambda_1(x) - \lambda_5(x)$ as an SDP. (15%)

   (b) Solve (a) by using MATLAB with the CVX tool. What are the optimal point and optimal value? (25%)

2. Consider the optimization problem

   $$\begin{array}{ll} \text{minimize} & x^2 + 1 \\ \text{subject to} & (x+1)(x+4) \leq 0 \end{array}$$

   with variable $x \in \mathbb{R}$.

   (a) (Analysis of primal problem.) Give the feasible set, the optimal value, and the optimal solution. (5%)

   (b) Derive the Lagrange dual function g. (5%)

   (c) State the dual problem, and verify that it is a concave maximization problem. (5%)

   (d) Find the dual optimal value and dual optimal solution? Does the strong duality hold? (5%)

3. (Dual of general LP). Find the dual function of the LP

   $$\begin{array}{ll} \text{minimize} & c^T x \\ \text{subject to} & Gx \preceq h \\ & Ax = b. \end{array}$$

   Give the dual problem, and make the implicit equality constraints explicit. (20%)

4. Derive a dual problem for

   $$\text{minimize} \quad -\sum_{i=1}^{m} \log(b_i - a_i^T x)$$

   with domain $\{x : a_i^T x < b_i, i = 1, \ldots, m\}$. First introduce new variables y_i and equality constraints $y_i = b_i - a_i^T x$. (20%)