ECM5901 Assignment #7, 2019 due January 7, 2020 Jan. 14

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1. (Equality constrained entropy maximization.) Consider the equality constrained entropy maximization problem

dom(log x) = $\mathbb{R}_+$

$$\begin{array}{ll} \text{minimize} & f(x) = \sum_{i=1}^{5} x_i \boxed{\log x_i} \\ \text{subject to} & Ax = b \end{array}$$

where $A = \begin{bmatrix} 4 & 3 & 2 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 20 \\ 15 \end{bmatrix}$ with $\operatorname{dom} f = \mathbb{R}^n_{++}$.

$\alpha = 0.01$

Compute the solution of the problem using the following methods.

$\beta = 0.5$

(a) Standard Newton method (Algorithm 10.1) with initial point $x^{(0)} = [1\ 2\ 3\ 4\ 5]^T$. (30%)

(b) Infeasible start Newton method (Algorithm 10.2) with initial point $\nu^{(0)} = \mathbf{0}$, $x^{(0)} = [1\ 2\ 3\ 4\ 5]^T$, and also $x^{(0)} = [5\ 2\ 3\ 4\ 5]^T$. (30%)

$\log x'$ x' $x + \Delta x > 0$

$x + t\Delta x$

Verify that the two methods compute the same optimal point.

Note that $\operatorname{dom} f$ is not $\mathbb{R}^5$ and thus in the update step of x, you have to check that $x + t\Delta x \in \operatorname{dom} f$.

2. You were asked to prove that $\boxed{x^\star = (1, 1/2, -1)}$ is optimal for the following optimization problem in HW#4:

$$\begin{array}{ll} \text{minimize} & f_0(x) = (1/2)x^T P x + q^T x + r \\ \text{subject to} & -1 \le x_i \le 1, \quad i = 1, 2, 3 \end{array}$$

$f_0(x) + \frac{1}{t} \sum_i \log(b_i - a x_i)$

where

$$P = \begin{bmatrix} 13 & 12 & -2 \\ 12 & 17 & 6 \\ -2 & 6 & 12 \end{bmatrix}, \quad q = \begin{bmatrix} -22 \\ -14.5 \\ 13 \end{bmatrix}, \quad r = 1.$$

Implement a barrier method for solving this QP. Assume that the initial point is $x^\star(0) = (0, 0, 0)$. Plot the $\boxed{\text{duality gap}}$ versus $\boxed{\text{Newton steps}}$ (such as Fig. 11.4). Verify that the barrier method computes the optimal point.

(40%)

$x^{(0)} = (0, 0, 0)$, $t^{(0)} = 1$, $\alpha = 0.1$, $\beta = 0.05$

strictly feasible

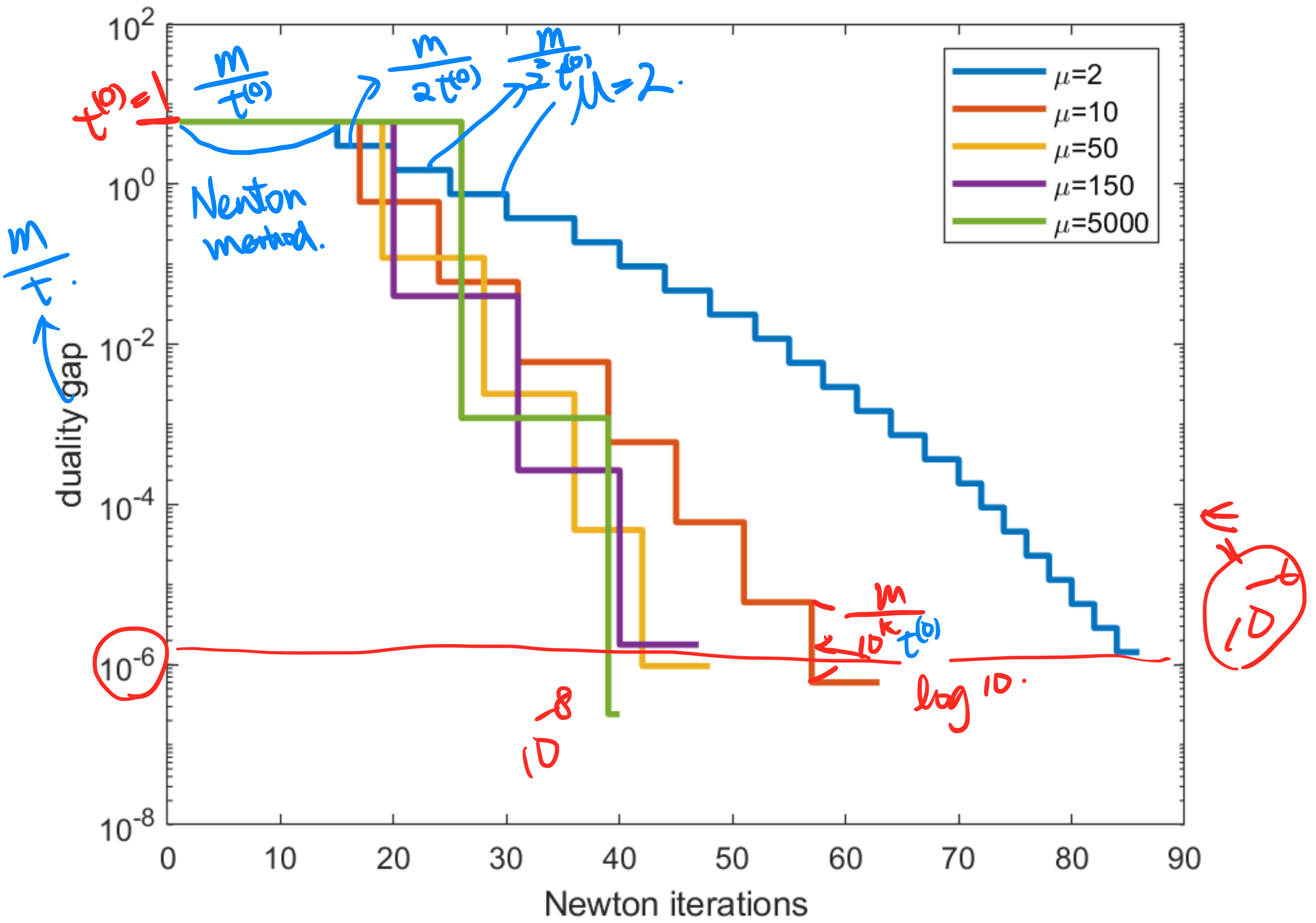
tolerance 10^{-6}

consider the feasibility of the modified objective function during the backtracking line search.

minimize $f_0(x) + \boxed{\frac{1}{t}\Phi(x)}$ (log ~)

$x^{(0)} \rightarrow$ do a Newton method

check the update is feasible



Feasibility and Phase I method.

- The barrier method requires a strictly feasible starting point $x^{(0)}$.
- When such a point is not known, the barrier method is preceded by a preliminary stage, called Phase I, in which a strictly feasible point is computed. (or the constraints are found to be infeasible.)
- The strictly feasible point found during phase I is then used as the starting point for the barrier method, which is called the Phase II stage.

Basic phase I method.

Find x such that

$$f_i(x) < 0, \quad i=1,\dots,m. \quad Ax=b. \qquad (2)$$

– phase I optimization

$$\begin{aligned} &\text{minimize} \quad s \qquad\qquad s \in \mathbb{R}. \\ &\text{s.t.} \quad f_i(x) \le s, \quad i=1,\dots,m \qquad (3) \\ &\qquad Ax = b. \end{aligned}$$

- If x, s, feasible with $s<0$, then x is strictly feasible for (2)
- If optimal value $\bar{p}^*$ of (3) is positive, the problem (2) is infeasible.
- If $\bar{p}^* = 0$. and attained, then problem (3) is feasible (but not strictly):

  If $\bar{p}^* = 0$, and not attained, then (3) is infeasible.