ECM5901 Assignment #7, 2019 due January 7, 2020

1. (Equality constrained entropy maximization.) Consider the equality constrained entropy maximization problem

$$\begin{array}{ll} \text{minimize} & f(x) = \sum_{i=1}^{5} x_i \log x_i \\ \text{subject to} & Ax = b \end{array}$$

where $A = \begin{bmatrix} 4 & 3 & 2 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 20 \\ 15 \end{bmatrix}$ with $\mathbf{dom}\, f = \mathbb{R}^n_{++}$.

Compute the solution of the problem using the following methods.

(a) Standard Newton method (Algorithm 10.1) with initial point $x^{(0)} = [1\ 2\ 3\ 4\ 5]^T$. (30%)

(b) Infeasible start Newton method (Algorithm 10.2) with initial point $\nu^{(0)} = \mathbf{0}$, $x^{(0)} = [1\ 2\ 3\ 4\ 5]^T$, and also $x^{(0)} = [5\ 2\ 3\ 4\ 5]^T$. (30%)

Verify that the two methods compute the same optimal point.
Note that $\mathbf{dom}\, f$ is not $\mathbb{R}^5$ and thus in the update step of x, you have to check that $x + t\Delta x \in \mathbf{dom}\, f$.

2. You were asked to prove that $x^\star = (1, 1/2, -1)$ is optimal for the following optimization problem in HW#4:

$$\begin{array}{ll} \text{minimize} & f_0(x) = (1/2)x^T P x + q^T x + r \\ \text{subject to} & -1 \le x_i \le 1, \quad i = 1, 2, 3 \end{array}$$

where

$$P = \begin{bmatrix} 13 & 12 & -2 \\ 12 & 17 & 6 \\ -2 & 6 & 12 \end{bmatrix}, \quad q = \begin{bmatrix} -22 \\ -14.5 \\ 13 \end{bmatrix}, \quad r = 1.$$

Implement a barrier method for solving this QP. Assume that the initial point is $x^\star(0) = (0, 0, 0)$. Plot the duality gap versus Newton steps (such as Fig. 11.4). Verify that the barrier method computes the optimal point.

(40%)