Project #1

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1 Language and Platform Description

1. I use C++ to write my program, and use following dependencies.
   If possible, please use linux machine to run my code.
   (a) C++ feature is required.
   (b) Eigen: C++ template library for linear algebra
   (c) Qt5: Cross-platform software development for embedded and desktop
2. Intall Dependencies
   Eigen3: `sudo apt install libeigen3-dev`
   Qt5: Please refer to `https://www.qt.io/download`
3. Build and Run

```
mkdir build
cd build
cmake ..
make
./ project1_main_cli 1 <params>
or
./ project1_main_cli 2 <params>
or
./ project1_main_gui
```

2 Program Architecture

1. project1.hpp: computation functions
   - IsValidRange: check if the angle is not NaN and is in valid range
   - MakeVector6: make a 6 x 1 vector in Eigen
   - MakeA: given $(d_n, a_n, \alpha_n, \theta_n)$, make matrix $\mathbf{A}_n$

- DoTask1: given (n, o, a, p), compute joint variables
- DoTask2: given joint variables, compute (n, o, a, p) and $(x, y, z, \phi, \theta, \phi)$
- PrintAnswer: print task1 answer for command line interface

2. mainwindow.ui, mainwindow.h, mainwindow.cc: GUI codes

3. project1_main_cli: CLI version of main

4. project1_main_gui: GUI version of main

5. The implementation is based on the equations derived below, along with some functions in Eigen library. See some comments in project1.cc

3 Equations Derivation

In lecture, $\mathbf{A}_n = \begin{bmatrix} c_n & -s_n c_{\alpha_n} & s_n s_{\alpha_n} & a_n c_n \\ s_n & -c_n c_{\alpha_n} & -c_n s_{\alpha_n} & a_n s_n \\ 0 & s\,\alpha_n & c\,\alpha_n & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix}$ where $s_n = \sin(n)$ and $c_n = \cos(n)$

Based on the kinematic table, we have

$$\mathbf{A}_1 = \begin{bmatrix} c_1 & 0 & -s_1 & a_1 c_1 \\ s_1 & 0 & c_1 & a_1 s_1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{A}_2 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2 c_2 \\ s_2 & c_2 & 0 & a_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{A}_3 = \begin{bmatrix} c_3 & -s_3 & 0 & a_3 c_3 \\ s_3 & c_3 & 0 & a_3 s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{A}_4 = \begin{bmatrix} c_4 & 0 & -s_4 & 0 \\ s_4 & 0 & c_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{A}_5 = \begin{bmatrix} c_5 & 0 & s_5 & 0 \\ s_5 & 0 & -c_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{A}_6 = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Also calculate inverse of each matrix,

$$\mathbf{A}_1^{-1} = \begin{bmatrix} c_1 & s_1 & 0 & -a_1 \\ 0 & 0 & -1 & 0 \\ -s_1 & c_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{A}_2^{-1} = \begin{bmatrix} c_2 & s_2 & 0 & -a_2 \\ -s_2 & c_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{A}_3^{-1} = \begin{bmatrix} c_3 & s_3 & 0 & -a_3 \\ -s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{A}_4^{-1} = \begin{bmatrix} c_4 & s_4 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -s_4 & c_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{A}_5^{-1} = \begin{bmatrix} c_5 & s_5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ s_5 & -c_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{A}_6^{-1} = \begin{bmatrix} c_6 & s_6 & 0 & 0 \\ -s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{T} = \mathbf{A}_1 * \mathbf{A}_2 * \mathbf{A}_3 * \mathbf{A}_4 * \mathbf{A}_5 * \mathbf{A}_6 = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1. Calculate θ_1

$$\mathbf{A}_1^{-1}\,\mathbf{T} = \mathbf{A}_2\,\mathbf{A}_3\,\mathbf{A}_4\,\mathbf{A}_5\,\mathbf{A}_6$$

$$\begin{bmatrix} c_1c_2 & s_1c_2 & -s_2 & -a_1c_2 - a_2 \\ -c_1s_2 & -s_1s_2 & -c_2 & a_1s_2 \\ -s_1 & c_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{T} = \begin{bmatrix} \dots & \dots & \dots & a_3c_{23} + a_2c_2 \\ \dots & \dots & \dots & a_3s_{23} + a_2s_2 \\ \dots & \dots & \dots & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

That is,

$$\begin{cases} (c_1p_x + s_1p_y - a_1)c_2 - p_zs_2 = a_2 + a_3c_3 \\ p_zc_2 + (c_1p_x + s_1p_y - a_1)s_2 = -a_3s_3 \\ -s_1p_x + c_1p_y = 0 \end{cases} \tag{1}$$

From third equation of 1, we know that

$$\theta_1 = \arctan(\frac{p_y}{p_x})$$

2. Calculate θ_3

$$\mathbf{A}_3^{-1}\,\mathbf{A}_2^{-1}\,\mathbf{A}_1^{-1}\,\mathbf{T} = \mathbf{A}_4\,\mathbf{A}_5\,\mathbf{A}_6$$

$$\begin{bmatrix} c_1c_{23} & s_1c_{23} & -s_{23} & -a_1c_{23} - a_2c_3 - a_3 \\ -c_1s_{23} & -s_1s_{23} & -c_{23} & a_1s_{23} + a_2s_3 \\ -s_1 & c_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{T} = \begin{bmatrix} \dots & \dots & c_4s_5 & 0 \\ \dots & \dots & s_4s_5 & 0 \\ -s_5c_6 & s_5s_6 & \dots & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

From the forth column of the right side of 2, we get

$$\begin{cases} (c_1p_x + s_1p_y - a_1)c_{23} - p_zs_{23} = a_2c_3 + a_3 \\ p_zc_{23} + (c_1p_x + s_1p_y - a_1)s_{23} = a_2s_3 \end{cases} \tag{3}$$

Square 3 and sum,

$$(c_1p_x + s_1p_y - a_1)^2 + p_z^2 = a_2^2 + a_3^2 + 2a_2a_3c_3$$

Therefore,

$$\theta_3 = \arccos\left(\frac{(c_1p_x + s_1p_y - a_1)^2 + p_z^2 - a_2^2 - a_3^2}{2a_2a_3c_3}\right)$$

3. Calculate θ_2
   Let

$$\begin{cases} r\sin(\phi) = p_z \\ r\cos(\phi) = c_1 p_x + s_1 p_y - a_1 \end{cases}$$

   where

$$\phi = \arctan(\frac{p_z}{c_1 p_x + s_1 p_y - a_1})$$

   From first two equations of 1, we have

$$\begin{cases} r\cos(\phi)c_2 - r\sin(\phi)s_2 = a_2 + a_3 c_3 \\ r\sin(\phi)c_2 + r\cos(\phi)s_2 = -a_3 s_3 \end{cases} \Rightarrow \begin{cases} r\cos(\phi + \theta_2) = a_2 + a_3 c_3 \\ r\sin(\phi + \theta_2) = -a_3 s_3 \end{cases}$$

   Therefore,

$$\begin{aligned} \theta_2 &= \phi - \theta_3 \\ &= \arctan(\frac{-a_3 s_3}{a_2 + a_3 c_3}) - \arctan(\frac{p_z}{c_1 p_x + s_1 p_y - a_1}) \end{aligned}$$

4. Calculate θ_4
   From third column of the right side of 2,

$$\begin{cases} (c_1 a_x + s_1 a_y)c_{23} - a_z s_{23} = c_4 s_5 \\ a_z c_{23} + (c_1 a_x + s_1 a_y)s_{23} = -s_4 s_5 \end{cases}$$

   After deviding, we get

$$\theta_4 = \arctan\left(\frac{a_z c_{23} + (c_1 a_x + s_1 a_y)s_{23}}{a_z s_{23} - (c_1 a_x + s_1 a_y)c_{23}}\right)$$

5. Calculate θ_5

$$\mathbf{A}_4^{-1}\,\mathbf{A}_3^{-1}\,\mathbf{A}_2^{-1}\,\mathbf{A}_1^{-1}\,\mathbf{T} = \mathbf{A}_5\,\mathbf{A}_6$$

$$\begin{bmatrix} c_1 c_{234} & s_1 c_{234} & -s_{234} & \dots \\ s_1 & -c_1 & 0 & \dots \\ -c_1 s_{234} & -s_1 s_{234} & -c_{234} & \dots \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{T} = \begin{bmatrix} \dots & \dots & s_5 & 0 \\ \dots & \dots & -c_5 & 0 \\ \dots & \dots & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

   From third column of the right side of 4,

$$\begin{cases} (c_1 a_x + s_1 a_y)c_{234} - a_z s_{234} = s_5 \\ s_1 a_x - c_1 a_y = -c_5 \end{cases}$$

   Therefore,

$$\theta_5 = \arctan\left(\frac{(c_1 a_x + s_1 a_y)c_{234} - a_z s_{234}}{-s_1 a_x + c_1 a_y}\right)$$

6. Calculate θ_6
   From third row of the right side of 2,

$$\begin{cases} -s_1 n_x + c_1 n_y = -s_5 c_6 \\ -s_1 o_x + c_1 o_y = s_5 s_6 \end{cases}$$

   Therefore,

$$\theta_6 = \arctan(\frac{s_1 o_x - c_1 o_y}{-s_1 n_x + c_1 n_y})$$

4 Difference Between Algebra and Geometry Approach

1. Algreba Approach
   pros:

   - equation support, rarely wrong
   - all solutions provided

   cons:

   - complex computation
   - not intuitive
   - need to check validity of each angle

2. Geometry Approach
   pros:

   - intuitive
   - simple computation

   cons:

   - need to think more for complicate robots
   - if not consider well, the result may be wrong