Available online at www.sciencedirect.com
SCIENCE DIRECT®

Aerosol Science 36 (2005) 1444–1458

Journal of Aerosol Science

www.elsevier.com/locate/jaerosci

Wet scrubbing of polydisperse aerosols by freely falling droplets

S.H. Park[a,*], C.H. Jung[b], K.R. Jung[c], B.K. Lee[c], K.W. Lee[a]

[a] *Department of Environmental Science and Engineering, Gwangju Institute of Science and Technology, 1 Oryong-dong, Buk-gu, Gwangju 500-712, Republic of Korea*

[b] *Department of Environmental Health, Kyungin Women's College, 101 Kyesan-gil, Kyeyang-gu, Incheon 407-740, Republic of Korea*

[c] *Department of Environmental Engineering, University of Ulsan, Mugeo-dong, Nam-gu, Ulsan 680-749, Republic of Korea*

Received 15 November 2004; received in revised form 11 March 2005; accepted 29 March 2005

Abstract

In this study, analytical solutions for removal of a polydisperse aerosol by wet scrubbing were derived employing Brownian diffusion and inertial impaction as removal mechanisms. Size distribution of aerosol particles were assumed to be represented by a time-evolving log-normal function during the scrubbing process. Derived solutions were compared with the direct integration solution, which is not based on the log-normal assumption, showing good agreement. Error resulting from the log-normal assumption was shown to be greater in the impaction-dominant regime than in the diffusion-dominant regime due to higher size dependency of collision kernel which destructed log-normal shape of size distribution. The monodisperse model significantly overpredicted particle removal in the diffusion- and impaction-dominant size regimes due to its incapability of tracing average particle size change, while it underpredicted particle removal in the intermediate size range because of neglect of polydispersity effect. A new solution for the minimum collection efficiency particle diameter was also provided. The minimum efficiency diameter was shown not to be very sensitive to the scrubbing condition and to lie around 0.3 μm for wide range of size and concentration of water drops.

Keywords: Wet scrubbing; Brownian diffusion; Inertial impaction; Minimum efficiency diameter

* Corresponding author. Tel.: +82 62 970 3282; fax: +82 62 970 2434.
E-mail addresses: dawnhope@gist.ac.kr (S.H. Park), lee@gist.ac.kr (K.W. Lee).

0021-8502/$ - see front matter © 2005 Elsevier Ltd. All rights reserved.
doi:10.1016/j.jaerosci.2005.03.012

1. Introduction

The wet scrubber is a device for removing airborne particles by bringing them into contact with liquid. Removal of particles from a gas stream is achieved by collisions between water drops and aerosol particles. In humid environment of a scrubber, small particles can also grow through condensation of water vapor and thereby become easier to remove (Flagan & Seinfeld, 1988). Wet scrubbing also provides the possibility of simultaneously removing soluble gaseous components. Air cleaning ability of wet scrubbing is routinely witnessed by us during rainfall periods. One can refer to textbooks such as Flagan and Seinfeld (1988) and Licht (1988), for detailed review of different types of wet scrubbers and basic knowledge on their operations.

In wet scrubbing, the size of water drops usually lies in the range 0.1–1 mm in diameter. Airborne particles smaller than 0.1 μm can be effectively collected as a result of their Brownian diffusion. Inertial impaction and interception may be important collection mechanisms for particles larger than 1 μm. Therefore, there exists a particle size between 0.1 and 1 μm at which the collection efficiency has the minimum value, which is called "minimum collection efficiency diameter". Wet scrubbers have usually been used for removing particles larger than 1 μm (Licht, 1988). Kim, Jung, Oh, and Lee (2001) showed, however, that a wet scrubber can have sufficient ability to remove particles smaller than 1 μm in optimum conditions such as small drop size and high liquid-to-gas ratio.

The collection efficiency of a wet scrubber is calculated by considering the efficiency of a single drop and then summing over the number of drops per unit volume of gas flow. The single drop collision efficiency is defined as the fraction of particles contained within the volume swept out by a falling drop that are collected (Seinfeld & Pandis, 1998). It can be obtained by solving the Navier–Stokes equation for the air flow around a water drop. It is difficult, however, to obtain a theoretical solution of the Navier–Stokes equation for the collision efficiency because of complicated flow patterns influenced by internal circulations of drops. An alternative solution using dimensional analysis coupled with experimental data was proposed by Slinn (1983). Jung and Lee (1998) derived the collision efficiency due to Brownian diffusion and interception for a multiple fluid sphere system using the extended Kuwabara free vorticity model including the effects of induced internal circulation inside a liquid drop and of neighboring collectors. Their derivation was compared with Slinn's formula and good agreement was shown (Jung, Kim, & Lee, 2002). For the case where inertial impaction is the predominant removal mechanism, Calvert (1984) suggested another alternative formula for collision efficiency.

As the adverse health effect of fine particles is brought to light, the information on particle size distribution becomes more important. The effect of polydispersity on particle deposition rates was investigated by Rosner (Rosner, 1989; Rosner & Tassopoulos, 1989) using the moment methods in conjunction with power law deposition mechanisms. However, the changes in broadness and average particle size by deposition process, which affects the deposition rates in return, were not addressed in those studies.

Recently, the change in particle size distribution due to wet scrubbing was numerically studied (Kim et al., 2001). This problem was further investigated by Jung et al. (2002) to derive analytical solutions for the change in particle size distribution and the minimum efficiency particle diameter taking into account Brownian diffusion and interception. Their solutions, however, were valid only for small particles because inertial impaction was not accounted for.

In this study, analytical solutions for wet scrubbing removal of a polydisperse aerosol by freely falling droplets in impaction-dominant size regime as well as near the minimum efficiency diameter are derived employing Brownian diffusion, interception and inertial impaction as removal mechanisms. A new solution for the minimum efficiency particle diameter is also provided.

2. Governing equation

The removal of a polydisperse aerosol by wet scrubbing is represented by

$$\frac{\partial n(d_p, t)}{\partial t} = -n(d_p, t)\int_{D_{d,\min}}^{D_{d,\max}} \beta(d_p, D_d) n_d(D_d)\, dD_d, \tag{1}$$

where $n(d_p, t)$ is the particle size distribution function at time t, d_p is the particle diameter, $n_d(D_d)$ is the water drop size distribution function, and D_d is the drop diameter. The collision kernel β is given as

$$\beta(d_p, D_d) = \frac{\pi D_d^2}{4} U(D_d) E(d_p, D_d), \tag{2}$$

where U is the falling velocity of a drop and $E(d_p, D_d)$ is the efficiency of collision between an aerosol particle of diameter d_p and a water drop of diameter D_d.

According to Jung and Lee (1998), neglecting minor terms and simplifying the equation, the collision efficiency due to Brownian diffusion is

$$E_{\mathrm{diff}}(d_p, D_d) = 2\left(\frac{\sqrt{3}\pi}{4Pe}\right)^{2/3}\left[\frac{(1-\alpha)(3\sigma+4)}{J+\sigma K}\right]^{1/3}, \tag{3}$$

where the packing density α is defined as the volume fraction of drops, σ is the viscosity ratio of water to air, $J = 1 - \frac{6}{5}\alpha^{1/3} + \frac{1}{5}\alpha^2$, $K = 1 - \frac{9}{5}\alpha^{1/3} + \alpha + \frac{1}{5}\alpha^2$, and Pe is the Peclet number defined as

$$Pe = \frac{D_d U(D_d)}{D_{\mathrm{diff}}}. \tag{4}$$

D_{diff} is the diffusion coefficient of aerosol particles given by

$$D_{\mathrm{diff}} = \frac{k_B T C_c(d_p)}{3\pi\mu d_p}, \tag{5}$$

where k_B is the Boltzmann constant, T is the absolute temperature, μ is the air viscosity, and C_c is the Cunningham slip correction factor:

$$C_c = 1 + 2.493\frac{\lambda}{d_p} + 0.84\frac{\lambda}{d_p}\exp\left(-0.435\frac{d_p}{\lambda}\right), \tag{6}$$

where λ is the mean free path length of air molecules.

If a particle follows a streamline which approaches the drop within a distance of the particle radius, the particle is captured by the drop, which is called interception. According to Jung and Lee (1998), the collection efficiency due to interception is given by

$$E_{\mathrm{int}}(d_p, D_d) = \frac{1-\alpha}{J+\sigma K}\left\{\frac{R}{1+R} + \frac{1}{2}\left(\frac{R}{1+R}\right)^2(3\sigma+4)\right\}, \tag{7}$$

where $R = d_p/D_d$.

For large particles, inertial impaction is the dominant removal mechanism in a wet scrubber. The dimensionless number that describes the inertial impaction property of a particle is the Stokes number defined as

$$Stk = \frac{\rho_p d_p^2 U(D_d)}{18\mu D_d}, \tag{8}$$

where ρ_p is the particle density. The collision efficiency due to inertial impaction is given by (Calvert, 1984)

$$E_{imp}(d_p, D_d) = \left(\frac{Stk}{Stk + 0.35}\right)^2. \tag{9}$$

3. Approximate collision efficiency

The overall collision efficiency accounting for Brownian diffusion, interception and inertial impaction can be represented by the sum of Eqs. (3), (7) and (9). The resulting equation, however, is very complicated making it difficult to derive analytical solutions for particle size distribution change. Thus, we first introduce several approximations for the collision efficiency.

The falling velocity of water drops can be parameterized as (Kessler, 1969; Mircea & Stefan, 1998)

$$U(D_d) = c_1 D_d^{c_2}, \tag{10}$$

where c_1 and c_2 are experimentally determined constants. In this study we use the following equation suggested for atmospheric water drops by Kessler (1969):

$$U(D_d) = 130 D_d^{1/2}, \tag{11}$$

where D_d and U are in SI units.

Fig. 1 compares the collision efficiencies due to the three mechanisms as a function of particle diameter. For this figure, the drop size was set at 1 mm and the drop concentration was $50\,g/m^3$, which is equivalent to the packing density of 5×10^{-5}. It is shown that inertial impaction is dominant for large particles while Brownian diffusion dominates for small particles. We examined this for different values of the drop size ($0.1\,mm \leqslant D_d \leqslant 10\,mm$) and the packing density ($10^{-7} \leqslant \alpha \leqslant 0.1$), and confirmed that interception may always be neglected. From Fig. 1, we can notice there is a particle size for which neither Brownian diffusion nor inertial impaction actively removes particles: the minimum collision efficiency diameter.

Lee and Liu (1980) suggested an approximation of the Cunningham slip correction factor as follows:

$$C_c = \mathrm{Max}\left[1.664\left(\frac{2\lambda}{d_p}\right), 2.609\sqrt{\frac{2\lambda}{d_p}}\right]. \tag{12}$$

Using this approximation, Jung et al. (2002) derived an analytical solution for particle size distribution change by wet scrubbing in the diffusion-dominant size regime as will be presented in the next section.

For the collision efficiency due to inertial impaction, we use the following approximation (Kim et al., 2001):

$$E_{imp}(d_p, D_d) = \mathrm{Min}[3.4 Stk^{9/5}, 1]. \tag{13}$$

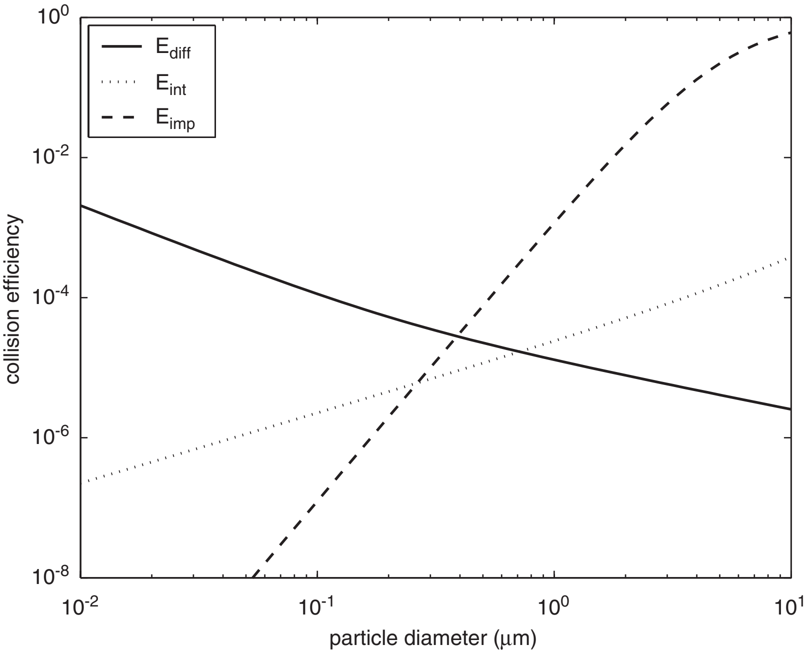

Fig. 1. Comparison of collection efficiencies due to Brownian diffusion, interception and inertial impaction. Drop mass concentration is $50\,g/m^3$ and D_d is 1 mm.

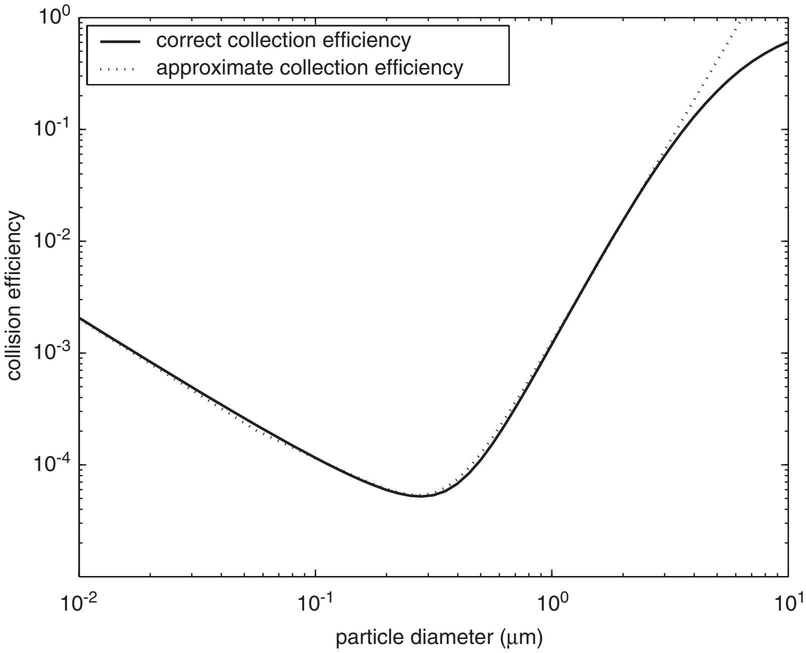

Fig. 2. Comparison between the approximate and correct collection efficiencies. Drop mass concentration is $50\,g/m^3$ and D_d is 1 mm.

Using the above approximations with the neglect of interception mechanism, the overall collision efficiency is compared with the correct one in Fig. 2. It is shown from this figure that the approximation adopted is reasonably good for $d_p \leqslant 5\,\mu m$.

It should be noted at this point that the "correct" collection efficiency appearing in Fig. 2 is not "exact" one but is itself an approximation obtained under the assumption that the removal efficiencies of different mechanisms are additive. Rigorous treatments of combined inertial and diffusive transport are available in the literature (Gutfinger & Tardos, 1979; Fernandez de la Mora & Rosner, 1982; Gupta & Peters, 1985). However, they could not provide simple analytical formulae that can be utilized in this study. The approximation adopted in this study may bring about a larger relative error than is shown in Fig. 2 near the minimum efficiency diameter, but the absolute value of the error is not expected to be considerable.

4. Derivation of analytical solutions

To derive analytical solutions for the particle size distribution change by wet scrubbing, we assume that the particle size distribution is represented by a time-evolving log-normal distribution function:

$$n(d_p, t) = \frac{1}{d_p}\frac{N(t)}{\sqrt{2\pi}ln\sigma_g(t)}\exp\left[-\frac{ln^2\{d_p/d_g(t)\}}{2ln^2\sigma_g(t)}\right], \tag{14}$$

where $N(t)$ is the total number concentration of particles, $d_g(t)$ is the geometric number mean particle diameter, and $\sigma_g(t)$ is the geometric standard deviation of particle diameter.

The kth moment of size distribution function $n(d_p, t)$ is defined as

$$M_k = \int_0^\infty d_p^k n(d_p, t)\,\mathrm{d}d_p, \tag{15}$$

where k is an arbitrary real number. Among the moments, M_0 represents the total number concentration of particles ($=N$). The properties of a log-normal distribution function are such that the following equation holds for any kth moment:

$$M_k = N d_g^k \exp\left(\frac{k^2}{2}ln^2\sigma_g\right). \tag{16}$$

If Eq. (16) is written for $k = 0$, 1 and 2, and subsequently solved for d_g and σ_g, we have

$$ln^2\sigma_g = ln\left(\frac{M_0 M_2}{M_1^2}\right), \tag{17}$$

$$d_g = \frac{M_1^2}{M_0^{3/2}M_2^{1/2}}. \tag{18}$$

4.1. Diffusion-dominant size regime

By substituting Eqs. (3)–(5), (10) and (12) into Eq. (2), we have the collision kernel for the diffusion-dominant size regime:

$$\beta_{\text{diff}}(d_{\text{p}}, D_{\text{d}}) = Ad_{\text{p}}^{-1} D_{\text{d}}^{(4+c_2)/3}, \tag{19}$$

where

$$A = \frac{\pi}{2}\left\{\frac{c_1\lambda(1-\alpha)(3\sigma+4)}{24(J+\sigma K)}\right\}^{1/3}\left(\frac{2.609k_{\text{B}}T}{\mu}\right)^{2/3}.$$

Eq. (19) has appropriate functional form for the moment method. By substituting Eq. (19) into Eq. (1), multiplying the equation by d_{p}^{k}, and integrating from 0 to ∞, one can obtain the following equation:

$$\frac{\mathrm{d}M_k}{\mathrm{d}t} = -\zeta M_{k-1}, \tag{20}$$

where $\zeta = \int_0^\infty AD_{\text{d}}^{(4+c_2)/3} n_{\text{d}}(D_{\text{d}})\,\mathrm{d}D_{\text{d}}$. Substituting Eq. (16) into Eq. (20) and rewriting the equation for $k = 0$, 1 and -1, the following equations are obtained:

$$\frac{\mathrm{d}N}{\mathrm{d}t} = -\zeta N d_{\text{g}}^{-1} b, \tag{21}$$

$$\frac{\mathrm{d}(N d_{\text{g}} b)}{\mathrm{d}t} = -\zeta N, \tag{22}$$

$$\frac{\mathrm{d}(N d_{\text{g}}^{-1} b)}{\mathrm{d}t} = -\zeta N d_{\text{g}}^{-2} b^4, \tag{23}$$

where $b = \exp(\frac{1}{2}\, ln^2\sigma_{\text{g}})$. From Eqs. (21)–(23), Jung et al. (2002) derived an analytical solution for the changes in the three particle size distribution parameters of log-normal distribution as follows:

$$\frac{N}{N_0} = \exp\left(\frac{1-\sqrt{2\zeta d_{\text{g}0}^{-1} b_0(b_0^2-1)t+1}}{b_0^2-1}\right), \tag{24}$$

$$\frac{b^2-1}{b_0^2-1} = \left\{2\zeta d_{\text{g}0}^{-1} b_0(b_0^2-1)t+1\right\}^{-1/2}, \tag{25}$$

$$\frac{d_{\text{g}}}{d_{\text{g}0}} = \frac{b(b_0^2-1)}{b_0(b^2-1)}, \tag{26}$$

where the subscript 0 represents the initial condition. From Eqs. (24)–(26) one can construct the whole particle size distribution using Eq. (14).

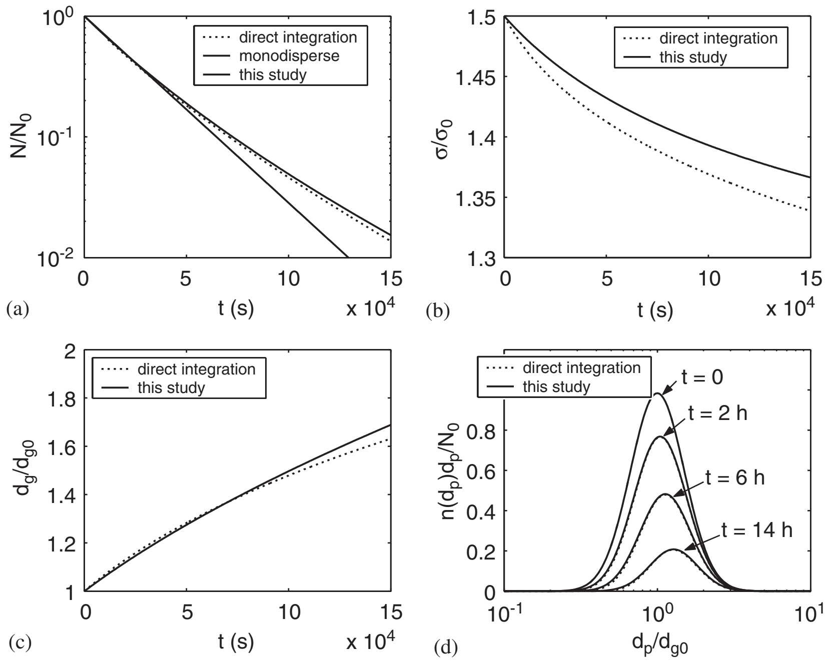

Fig. 3. Comparison of change in particle size distribution in the diffusion-dominant range; $d_{g0} = 0.1\ \mu\text{m}$, $\sigma_{g0} = 1.5$, $D_d = 1$ mm, and drop mass concentration is $50\ \text{g/m}^3$.

Eq. (24) provides with an explicit expression for the particle number concentration decay of polydisperse particles which is not given by direct time integration of Eq. (1). This is the important advantage of this alternative solution. For $b_0 = 1$ ($\sigma_{g0} = 1$), Eq. (24) reduces to the solution for the monodisperse particles.

Fig. 3 compares the results calculated for the particle number concentration, geometric mean standard deviation of particle diameter, geometric mean diameter, and size distribution with the solution calculated by direct integration of Eq. (1) with the assumption of additivity of removal efficiencies. Due to higher removal of smaller particles by diffusional process, the mean particle size increases and size distribution becomes narrower. It is shown in this figure that the particle size distribution parameters are predicted well by our solution. The solution for the removal of a monodisperse aerosol is compared together in Fig. 3a. In the monodisperse model, particle size increase cannot be taken into account, resulting in significant overprediction of particle removal.

4.2. Inertial impaction-dominant size regime

By substituting Eqs. (8), (10) and (13) into Eq. (2), we have the collision kernel for the inertial impaction-dominant size regime:

$$\beta_{\text{imp}}(d_p, D_d) = B d_p^{18/5} D_d^{(1+14c_2)/5}, \tag{27}$$

where $B = \frac{3.4\pi c_1^{14/5}}{4}(\frac{\rho_p}{18\mu})^{9/5}$. By substituting Eq. (27) into Eq. (1), multiplying the equation by d_p^k, and integrating from 0 to ∞, we have

$$\frac{\mathrm{d}M_k}{\mathrm{d}t} = -\xi M_{k+18/5}, \tag{28}$$

where $\xi = \int_0^\infty B D_d^{(1+14c_2)/5} n_d(D_d)\,\mathrm{d}D_d$. Substituting Eq. (16) into Eq. (28) and rewriting the equation for $k = 0$, $-18/5$ and 18/5, the following equations are obtained:

$$\frac{\mathrm{d}N}{\mathrm{d}t} = -\xi N d_g^{18/5} y, \tag{29}$$

$$\frac{\mathrm{d}(N d_g^{-18/5} y)}{\mathrm{d}t} = -\xi N, \tag{30}$$

$$\frac{\mathrm{d}(N d_g^{18/5} y)}{\mathrm{d}t} = -\xi N d_g^{36/5} y^4, \tag{31}$$

where $y = \exp(\frac{162}{25} ln^2 \sigma_g)$. Substituting Eq. (29) into Eqs. (30) and (31), we have

$$-\frac{18}{5} d_g^{-23/5} y \frac{\mathrm{d}d_g}{\mathrm{d}t} + d_g^{-18/5} \frac{\mathrm{d}y}{\mathrm{d}t} = \xi(y^2 - 1), \tag{32}$$

$$\frac{18}{5} d_g^{-23/5} y \frac{\mathrm{d}d_g}{\mathrm{d}t} + d_g^{-18/5} \frac{\mathrm{d}y}{\mathrm{d}t} = -\xi y^2(y^2 - 1). \tag{33}$$

Rewriting Eqs. (32) and (33) in terms of d_g and y, we have

$$\frac{\mathrm{d}d_g}{\mathrm{d}t} = -\frac{5\xi d_g^{23/5}(y^4 - 1)}{36y}, \tag{34}$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = -\frac{\xi d_g^{18/5}(y^2 - 1)^2}{2}. \tag{35}$$

Eliminating $\mathrm{d}t$ from Eqs. (34) and (35) and integrating the resulting equation, one can obtain

$$\frac{d_g}{d_{g0}} = \left\{\frac{y_0(y^2 - 1)}{y(y_0^2 - 1)}\right\}^{5/18}. \tag{36}$$

Substituting Eq. (36) into Eq. (35) and integrating the resulting equation, one can obtain the following equation for y:

$$\frac{y^2 - 1}{y_0^2 - 1} = \left\{2\xi d_{g0}^{18/5} y_0(y_0^2 - 1)t + 1\right\}^{-1/2}. \tag{37}$$

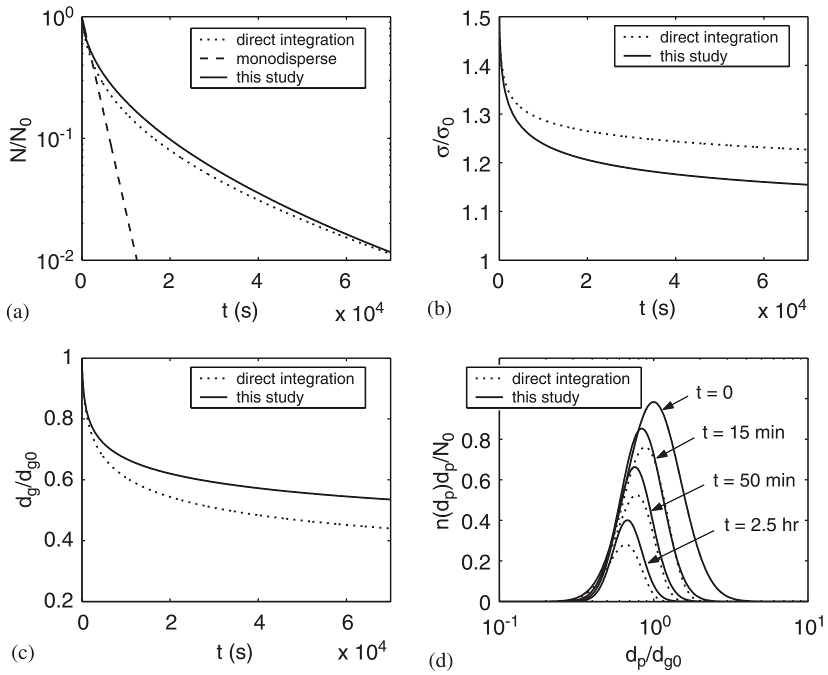

Fig. 4. Comparison of change in particle size distribution in the impaction-dominant range; $d_{g0} = 1\ \mu\text{m}$, $\sigma_{g0} = 1.5$, $D_d = 1$ mm, and drop mass concentration is $50\ \text{g/m}^3$.

Substituting Eqs. (36) and (37) into Eq. (29) and integrating the resulting equation, one can obtain the solution for N:

$$\frac{N}{N_0} = \exp\left\{\frac{1 - \sqrt{2\xi d_{g0}^{18/5} y_0 (y_0^2 - 1)t + 1}}{y_0^2 - 1}\right\}. \tag{38}$$

Eqs. (36)–(38) constitute the solution for the inertial impaction-dominant size regime.

Fig. 4 compares the results with the direct integration solution for the impaction-dominant regime. As larger particles are scavenged by water drops due to inertial impaction faster than smaller ones, the mean particle size decreases. It is noted that the particle size distribution parameters are reasonably predicted by our solution, but the discrepancy from the direct integration solution is larger than that for the diffusion-dominant regime. It is believed this is due to the higher size dependency of the collision kernel in this size range (Fig. 2). Rapid removal of large particles renders the size distribution be skewed deviating from log-normality, which leads to larger error of the alternative solution based on the log-normal assumption. Even higher size dependency of the collision kernel results in a considerable slow down of the particle removal rate as time elapses. The monodisperse model that does not reflect this change dramatically overestimates the particle removal rate (Fig. 4a).

4.3. Intermediate size range

For the intermediate size range, neither diffusion nor inertial impaction can be neglected. Thus, from Eqs. (19) and (27), the collision kernel for the intermediate size range is

$$\beta(d_\mathrm{p}, D_\mathrm{d}) = Ad_\mathrm{p}^{-1}D_\mathrm{d}^{(4+c_2)/3} + Bd_\mathrm{p}^{18/5}D_\mathrm{d}^{(1+14c_2)/5}. \tag{39}$$

Applying similar procedure to those used in diffusion- and impaction-dominant regimes, the following equation is obtained.

$$\frac{\mathrm{d}M_k}{\mathrm{d}t} = -\zeta M_{k-1} - \xi M_{k+18/5}. \tag{40}$$

It is not possible to derive an analytical solution from Eq. (40) in a similar way with that used in the inertial impaction-dominant regime. We use here the solution-combining technique used by our previous study on particle deposition problem (Park & Lee, 2002).

Eq. (1) can be rewritten for the intermediate size range as follows:

$$\frac{\partial n(d_\mathrm{p}, t)}{\partial t} = -n(d_\mathrm{p}, t)\{\theta_\mathrm{diff}(d_\mathrm{p}) + \theta_\mathrm{imp}(d_\mathrm{p})\}, \tag{41}$$

where $\theta(d_\mathrm{p}) = \int_0^\infty \beta(d_\mathrm{p}, D_\mathrm{d})n_\mathrm{d}(D_\mathrm{d})\,\mathrm{d}D_\mathrm{d}$. Let us define the kth moment distribution function as follows:

$$m_k(d_\mathrm{p}, t) = d_\mathrm{p}^k n(d_\mathrm{p}, t). \tag{42}$$

By combining Eqs. (41) and (42), we have

$$\frac{\partial m_k(d_\mathrm{p}, t)}{\partial t} = -m_k(d_\mathrm{p}, t)\{\theta_\mathrm{diff}(d_\mathrm{p}) + \theta_\mathrm{imp}(d_\mathrm{p})\}. \tag{43}$$

Integration of Eq. (43) gives

$$\frac{m_k(d_\mathrm{p}, t)}{m_k(d_\mathrm{p}, 0)} = \exp[-\{\theta_\mathrm{diff}(d_\mathrm{p}) + \theta_\mathrm{imp}(d_\mathrm{p})\}t]. \tag{44}$$

From Eq. (44), and using an additional approximation, the following equation is obtained:

$$\begin{aligned}\frac{M_k(t)}{M_k(0)} &= \frac{\int_0^\infty m_k(d_\mathrm{p}, 0)\exp[-\{\theta_\mathrm{diff}(d_\mathrm{p}) + \theta_\mathrm{imp}(d_\mathrm{p})\}t]\,\mathrm{d}d_\mathrm{p}}{\int_0^\infty m_k(d_\mathrm{p}, 0)\,\mathrm{d}d_\mathrm{p}} \\ &\approx \frac{\int_0^\infty m_k(d_\mathrm{p}, 0)\exp\{-\theta_\mathrm{diff}(d_\mathrm{p})t\}\,\mathrm{d}d_\mathrm{p}\int_0^\infty m_k(d_\mathrm{p}, 0)\exp\{-\theta_\mathrm{imp}(d_\mathrm{p})t\}\,\mathrm{d}d_\mathrm{p}}{\{\int_0^\infty m_k(d_\mathrm{p}, 0)\,\mathrm{d}d_\mathrm{p}\}^2} \\ &= \left.\frac{M_k(t)}{M_k(0)}\right|_\mathrm{diff}\left.\frac{M_k(t)}{M_k(0)}\right|_\mathrm{imp}.\end{aligned} \tag{45}$$

Eq. (45) is correct for monodisperse aerosols but it may underestimate the moment depletion for polydisperse aerosols (Park & Lee, 2002).

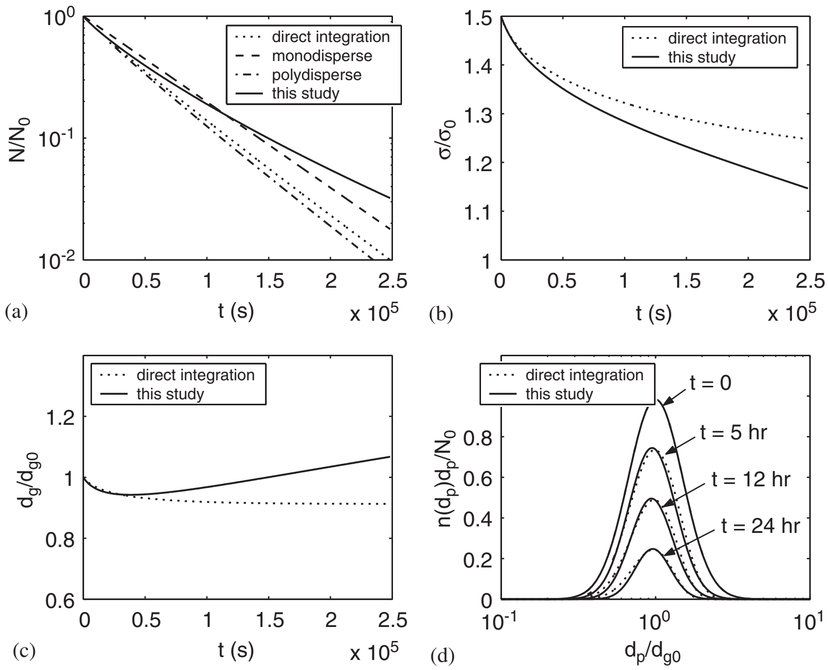

Fig. 5. Comparison of change in particle size distribution in the intermediate size range; $d_{g0} = 0.3\,\mu m$, $\sigma_{g0} = 1.5$, $D_d = 1$ mm, and drop mass concentration is $50\,g/m^3$. Size distribution presented in (d) is prepared using the polydisperse model for N.

From Eqs. (17), (18) and (45), one can easily derive the following equations:

$$\frac{N}{N_0} = \left.\frac{N}{N_0}\right|_{\text{diff}} \cdot \left.\frac{N}{N_0}\right|_{\text{imp}}, \tag{46}$$

$$\frac{d_g}{d_{g0}} = \left.\frac{d_g}{d_{g0}}\right|_{\text{diff}} \cdot \left.\frac{d_g}{d_{g0}}\right|_{\text{imp}}, \tag{47}$$

$$\frac{\exp(ln^2\sigma_g)}{\exp(ln^2\sigma_{g0})} = \left.\frac{\exp(ln^2\sigma_g)}{\exp(ln^2\sigma_{g0})}\right|_{\text{diff}} \cdot \left.\frac{\exp(ln^2\sigma_g)}{\exp(ln^2\sigma_{g0})}\right|_{\text{imp}}. \tag{48}$$

Fig. 5 compares the solution derived for the intermediate size range with the direct integration solution. The mean particle size and the broadness of size distribution are well predicted by the derived solution whereas the number concentration is poorly estimated. Large error for the number concentration stems from the approximation used in Eq. (45).

One difference noted in this figure from the diffusion- or impaction-dominant regime is that the monodisperse model underpredicts particle removal in the intermediate size range. As the mean particle size does not undergo a big change (Fig. 5c), the error of the monodisperse model in this size range is caused by neglect of polydispersity effect. With higher polydispersity, an aerosol contains more particles larger or smaller than the minimum collision efficiency diameter, which tend to be collected faster.

Substituting Eq. (16) into Eq. (40) and rewriting the equation for $k=0$, one can obtain

$$\frac{\mathrm{d}N}{\mathrm{d}t} = -\zeta N d_{\mathrm{g}}^{-1} b - \xi N d_{\mathrm{g}}^{18/5} y. \tag{49}$$

With $\sigma_{\mathrm{g}} = 1$ ($b = y = 1$), the deposition rate of a monodisperse aerosol is obtained. Therefore, the polydispersity factor PDF is defined by the following equation:

$$\theta_{\mathrm{polydisperse}} = PDF(\sigma_{\mathrm{g}}) \cdot \theta_{\mathrm{monodisperse}}, \tag{50}$$

where $PDF(\sigma_{\mathrm{g}}) = \frac{\zeta d_{\mathrm{g}}^{-1} b + \xi d_{\mathrm{g}}^{18/5} y}{\zeta d_{\mathrm{g}}^{-1} + \xi d_{\mathrm{g}}^{18/5}}$. Thus, we suggest to use the following solution in which the polydispersity factor is taken into account for the intermediate size range:

$$\frac{N}{N_0} = \exp\left\{-\theta_{\mathrm{monodisperse}} PDF(\sigma_{\mathrm{g}}) t\right\}. \tag{51}$$

Eq. (51) is referred to as "polydisperse model" in this article and compared together in Fig. 5a with good agreement with the direct integration solution.

5. Particle size with minimum collection efficiency

Jung et al. (2002) have derived an analytical solution for the minimum efficiency particle diameter under the wet scrubbing process. However, only Brownian diffusion and interception were taken into account in their derivation. As it is shown that the inertial impaction is important and interception is

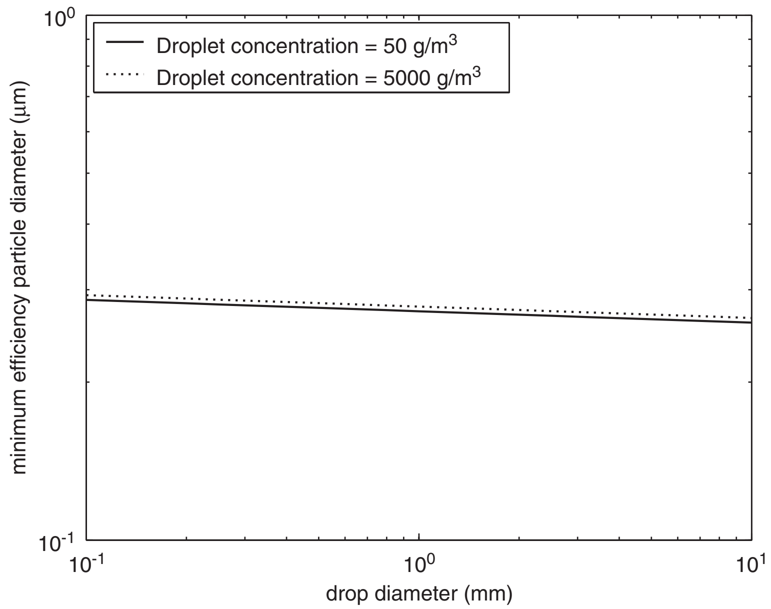

Fig. 6. Minimum collision efficiency particle diameter as a function of drop size for different values of drop concentration.

negligible, an alternative solution is presented here. By differentiating Eq. (39) and setting its value at 0, the minimum efficiency particle diameter is obtained:

$$d_{\mathrm{p,min}} = \left(\frac{5A}{18B}\right)^{5/23} D_{\mathrm{d}}^{(17-37c_2)/69}, \tag{52}$$

Fig. 6 shows the minimum efficiency diameter as a function of drop size. It is interesting to note that the minimum efficiency diameter lies around 0.3 μm for wide range of D_{d} and the packing density.

6. Conclusions

In this study, the changes in particle size distribution parameters of an aerosol scavenged by freely falling water drops were investigated with Brownian diffusion and inertial impaction taken into account. An analytical solution was derived for impaction-dominant regime under the assumption that particle size distribution retained log-normal shape during the scrubbing process. The solution obtained was compared with the direct integration solution and good agreement was shown. The error was larger than for diffusion-dominant regime because of higher dependency of collision kernel on particle size in impaction-dominant regime which leads to destruction of log-normality of size distribution. The monodisperse model severely overpredicted the particle concentration decay as it cannot account for the change in average particle size.

The solutions derived for two limiting regimes were mechanistically combined to obtain a solution for the intermediate size range in which neither Brownian diffusion nor inertial impaction can be neglected. Mean particle size and geometric standard deviation were well predicted by this method while the number concentration was significantly overestimated. Monodisperse model underestimated the particle number decay in this size range due to the neglect of polydispersity effect. The polydispersity factor was derived analytically and combined with the monodisperse model solution to yield the polydisperse model solution, which showed good agreement with the direct integration solution. Therefore, it is recommended to use the solutions for d_{g} and σ_{g} derived by log-normal model and the polydisperse model solution for N in the intermediate size range.

Acknowledgements

This work was supported by the Brain Korea 21 program from the Ministry of Education through the Graduate Program for Chemical and Environmental Engineering at Gwangju Institute of Science and Technology.

References

Calvert, S. (1984). Particle control by scrubbing. In: S. Calvert, & H.M. Englund (Eds.), *Handbook of air pollution technology* (pp. 215–248). New York: Wiley.

Fernandez de la Mora, J., & Rosner, D. E. (1982). Effects of inertia on the diffusional deposition of small particles to spheres and cylinders at low Reynolds numbers. *Journal of Fluid Mechanics*, *125*, 379–395.

Flagan, R. C., & Seinfeld, J. H. (1988). *Fundamentals of air pollution engineering*. Englewood Cliffs: Prentice-Hall.

Gupta, D., & Peters, M. H. (1985). A Brownian dynamics simulation of aerosol deposition onto spherical collectors. *Journal of Colloid and Interface Science*, *104*, 375–388.

Gutfinger, C., & Tardos, G. I. (1979). Theoretical and experimental investigation on granular bed dust filters. *Atmospheric Environment*, *13*, 853–867.

Jung, C. H., & Lee, K. W. (1998). Filtration of fine particles by multiple liquid drop and gas bubble systems. *Aerosol Science and Technology*, *29*, 389–401.

Jung, C. H., Kim, Y. P., & Lee, K. W. (2002). Analytic solution for polydispersed aerosol dynamics by a wet removal process. *Journal of Aerosol Science*, *33*, 753–767.

Kessler, E. (1969). On the distribution and continuity of water substance in atmospheric circulations. *Meteorological Monograph*, *32*, 48.

Kim, H. T., Jung, C. H., Oh, S. N., & Lee, K. W. (2001). Particle removal efficiency of gravitational wet scrubber considering diffusion, interception, and impaction. *Environmental Engineering Science*, *18*, 125–136.

Lee, K. W., & Liu, B. Y. H. (1980). On the minimum efficiency and the most penetrating particle size for fibrous filters. *Journal of the Air Pollution Control Association*, *30*, 377–381.

Licht, W. (1988). *Air pollution control engineering: Basic calculations for particulate collection*. (2nd ed.), New York: Marcel Dekker.

Mircea, M., & Stefan, S. (1998). A theoretical study of the microphysical parameterization of the scavenging coefficient as a function of precipitation type and rate. *Atmospheric Environment*, *32*, 2931–2938.

Park, S. H., & Lee, K. W. (2002). Analytical solution to change in size distribution of polydisperse particles in closed chamber due to diffusion and sedimentation. *Atmospheric Environment*, *36*, 5459–5467.

Rosner, D. E. (1989). Total mass deposition rates from polydispersed aerosols. *American Institute of Chemical Engineers Journal*, *35*, 164–167.

Rosner, D. E., & Tassopoulos, M. (1989). Deposition rates from polydispersed particle populations of arbitrary spread. *American Institute of Chemical Engineers Journal*, *35*, 1497–1508.

Seinfeld, J. H., & Pandis, S. N. (1998). *Atmospheric chemistry and physics*. New York: Wiley.

Slinn, W. G. N. (1983). Precipitation scavenging. In *Atmospheric sciences and power production–1979* (Chap. 11). Washington, DC: Division of Biomedical Environmental Research, U.S. Department of Energy.