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Filtration of Fine Particles by Multiple Liquid Droplet and Gas Bubble Systems

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ABSTRACT. The collection of small particles by a system consisting of multiple fluid spheres such as water droplets or gas bubbles was studied analytically. Kuwabara's free vorticity model based on the solid particle system was extended to include the effects of induced internal circulation inside a liquid droplet or gas bubble system on the flow and the mass transfer rate. Using the resolved flow field, analytic solutions were obtained for the particle collection efficiencies due to diffusion and interception. The results were applied to the problem of particle capture by gas bubbles in liquid or by droplets suspended in gas. The results indicated that the particle collection efficiency by the multiple fluid sphere system is higher than that by a solid sphere system. This is due to the internal circulation which develops inside the fluid spheres. AEROSOL SCIENCE AND TECHNOLOGY 29:389–401 (1998) **© 1998 American Association for Aerosol Research**

NOMENCLATURE

a = radius of sphere
b = radius of boundary
c, c' = dimensional and dimensionless transferred coordinate particle concentration
c_o = transferred coordinate particle concentrations at the outside boundary layer
d_p = diameter of the particle
d_g = diameter of the sphere
D = particle diffusion coefficient
E = overall particle collection efficiency
J = hydrodynamic factor ($J = 1 - 6\alpha^{1/3}/5 + \alpha^2/5$)
K = hydrodynamic factor ($K = 1 - 9\alpha^{1/3}/5 + \alpha - \alpha^2/5$)
L = length of collector system
m = positive number
M = dimensionless diffusion rate of particle to sphere
M_∞ = maximum value of M
n, n' = dimensional and dimensionless particle concentration
n_δ, n_i = particle concentrations at outside boundary layer and at $r = a$
Pe = Peclet number ($= 2ua/D$)
R = interception parameter ($= d_p/d_g$)
r, r' = dimensional and dimensionless radial position coordinates
u = flow velocity
u_r, u_θ = radial and circumferential components of flow velocity
α = volume fraction, packing density, or solidity of packed bed
δ, δ' = dimensional and dimensionless radii of boundary layer

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η = single sphere efficiency
η_D, η_R = single particle collection efficiencies due to diffusion, interception
μ^o, μ^i = viscosity of fluids outer and inner boundary
σ = viscosity ratio of inner to outer spheres ($= \mu^i/\mu^o$)
ω = vorticity
ξ = distance between center line and particle position in region far from collection sphere
θ = circumferential position coordinate
ψ, ψ' = dimensional and dimensionless stream functions
ψ_ξ = stream function at a distance ξ
$\tau_{r\theta}$ = shearing stress

INTRODUCTION

The problem of removing small particles from gases or liquids has become increasingly important, and many theoretical and experimental studies have been conducted (Yeh and Liu, 1974; Gebhart et al., 1973; Paretsky et al., 1971; Liu and Wang, 1997; Tien et al., 1979; Lee and Gieseke, 1979). The motion and capture of particles by an ensemble of fluid spheres can be applied directly to the problem of wet scrubbers or to the capture of particles by bubbles in a water pool. Wet collection devices collect particles by direct contact with liquid droplets. The motion and the effects of bubbles can be applied to the flow of immiscible fluids in vertical passages such as tubes, fluidized beds, absorption of gases in liquid columns, sewage purification processes, direct contact heat exchangers, etc. In the case of granular filters consisting of solid spheres or packed beds, Lee and Gieseke (1979) obtained the particle collection efficiencies due to Brownian diffusion and interception using the flow field of Kuwabara's cell model. Later Lee (1981) provided an analytical expression for the maximum penetration of aerosol particles by adding the gravitational efficiency. In the case of wet scrubbers or a system of bubbles, little has been studied quantitatively on collection of particles by diffusion, interception, and gravitation. In the present study, we have extended the flow field of Kuwabara's cell model to treat the flow field around multiple liquid droplets or gas bubbles by taking into account the effects of the internal circulation inside the droplets or bubbles on the outside flowfield and on the particle capture efficiency. The particle removing efficiencies due to Brownian diffusion and interception are considered.

In filtration theory, it is customary to introduce the concept of single sphere efficiency. The single sphere efficiency is defined as the ratio of the cross-sectional circular area surrounded by limiting streamlines of the flow approaching the collecting sphere to the projected area of the sphere. In the present study, the collecting sphere can be either a gas bubble in liquid or a liquid droplet suspended in gas. The limiting streamlines are such that the center of mass of all the particles passing within them will be collected by the bubble or liquid droplet while all the particles outside the streamlines will escape the collector. With this representation of the single bubble or droplet efficiency, the concentration ratio of the particles collected by a system of bubbles or droplets to those approaching it, or the overall efficiency of particle collection by the system is related to the single bubble or droplet efficiency as following (Lee and Gieseke, 1979).

$$E = 1 - \exp\left\{\frac{-3\alpha\eta L}{4(1-\alpha)a}\right\} \tag{1}$$

where E is the overall collection efficiency of a system consisting of many bubbles or liquid drops, η is the single bubble or liquid sphere efficiency, L is the length of the collector system consisting of bubbles or liquid drops, α is the volume fraction of the bubbles or drops, and a is the radius of the bubble or liquid drop. Thus, if η is known, the overall collection efficiency of a system can be calculated. In the present study, we have extended the flow field of Kuwabara's cell model to treat the flow field around multiple liquid droplets by taking into account the effects of the internal circulation inside the droplets on the flow field and on the particle capture efficiency. The particle removing efficiencies due to Brownian diffusion and interception are considered.

FLOW FIELD

For the flow around a system consisting of multiple solid spheres, Kuwabara (1959) used a zero velocity condition at the inner sphere surface and employed the condition that the vorticity becomes zero at the outer boundary. Applying the following boundary conditions at the surface of the multiple liquid spheres, we write

$$\begin{aligned} u_r^i &= u_r^o = 0 \\ u_\theta^i &= u_\theta^o \qquad \text{at } r = a \\ \tau_{r\theta}^i &= \tau_{r\theta}^o \end{aligned} \tag{2}$$

and

$$\omega = 0 \qquad \text{at } r = b$$

where u_r and u_θ are the velocities in the radial and the tangential directions respectively. $\tau_{r\theta}$ is the shearing stress, ω is the vorticity that is defined as $\omega = \nabla \times u$, i and o indicate the inner and the outer phases of the collection sphere respectively, r is the distance from the center of the sphere, a is the radius of the collection sphere, and b is the radius of the outer boundary. It is noted that in Eq. (2) the circumferential velocities, u_θ^i and u_θ^o at the boundary are not zero, since the sphere is liquid or bubble in this case. The solution of the flow field around the multiple fluid system using the boundary conditions shown in Eq. (2) was given as below by Gal-or (1970).

$$\psi^o = \left[\frac{1}{4}ua^3\frac{\left\{\sigma\left(1-\frac{2\alpha}{5}\right)+\frac{2\alpha}{5}\right\}}{J+\sigma K}\frac{1}{r} - \frac{1}{4}ua\frac{(2+3\sigma)}{J+\sigma K}r + \frac{1}{2}u\frac{\left\{1+\sigma\left(1+\frac{\alpha}{2}\right)\right\}}{J+\sigma K}r^2 - \frac{u}{20}\frac{\alpha}{a^2}\frac{2+3\sigma}{J+\sigma K}r^4\right]\sin^2\theta \tag{3}$$

and the induced flow field inside the fluid sphere is written

$$\psi^i = \left(-\frac{1}{4}u\frac{1-\alpha}{J+\sigma K}r^2 + \frac{1}{4}\frac{u}{a^2}\frac{1-\alpha}{J+\sigma K}r^4\right)\sin^2\theta \tag{4}$$

where $\alpha = (a/b)^3$ is the sphere concentration, $\sigma = \mu^i/\mu^o$ is the viscosity ratio of the inner to the outer fluid spheres, ψ^o and ψ^i are the stream functions outside and inside collecting spheres, respectively,

$$J = 1 - 6\alpha^{1/3}/5 + \alpha^2/5 \tag{5}$$

and

$$K = 1 - 9\alpha^{1/3}/5 + \alpha - \alpha^2/5 \tag{6}$$

The spheres are assumed solid as the viscosity ratio ($\sigma = \mu^i/\mu^o$) approaches infinity ($\sigma \to \infty$) and the solution reduces to the original Kuwabara (1959) result that is the solution of the flow field around multiple solid spheres. A system of spherical gas bubbles rising in a liquid reservoir is represented as the viscosity ratio approaches zero ($\sigma \to 0$) in Eqs. (3) and (4).

Due to the influence of the internal flow, the outer flow velocity around the fluid spheres becomes larger than that around solid spheres. For this reason, the streamlines of the flow outside a fluid sphere pass around the fluid sphere more closely than those around a solid sphere. This is illustrated in Fig. 1 where the streamlines starting from the left-hand side in Fig. 1a and 1b were initially positioned identically. As the

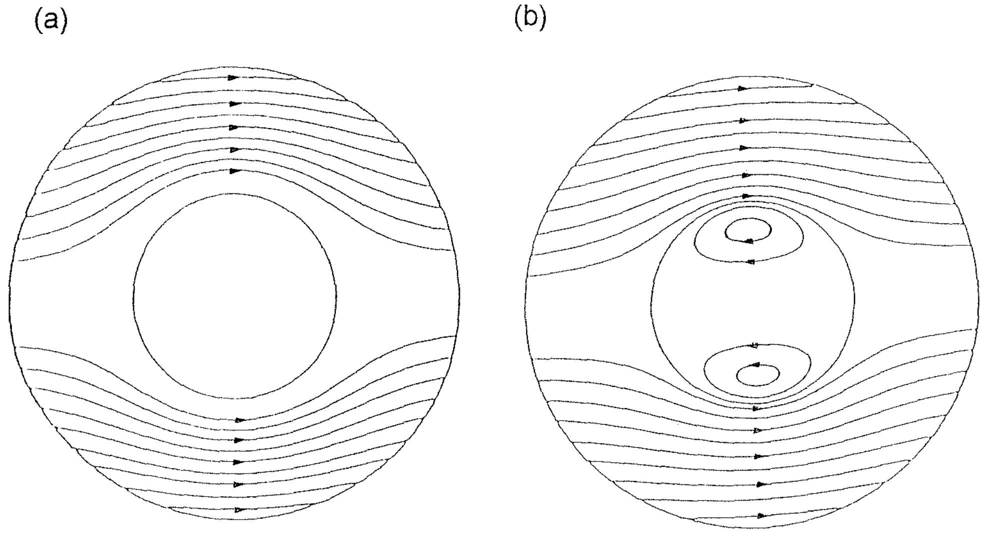

FIGURE 1. Comparison of the flow field of the spheres (α = 0.1). (a) σ = infinity; solid (b) σ = 0; bubble

packing density of collecting droplets and bubbles increases, the radius of the outer boundary, b, is reduced in this model. The flow field for the outer cell as shown in Eq. (3) is written in the dimensionless form as follows:

$$\psi'^{o} = \frac{1}{J+\sigma K}\left[\frac{1}{4r'}\left\{\sigma\left(1-\frac{2}{5}\alpha\right)+\frac{2}{5}\alpha\right\} - \frac{1}{4}(2+3\sigma)r' + \frac{1}{2}\left\{1+\sigma\left(1 + \frac{\alpha}{2}\right)\right\}r'^2 - \frac{\alpha r'^4}{20}(2+3\sigma)\right]\sin^2\theta \tag{7}$$

where $\psi'^{o} = \psi^{o}/a^2u$ and $r' = r/a$.

Noting that the flow field given by Eq. (7) is rather complicated to be used for subsequent mass transfer analysis, we attempt to simplify Eq. (7) without sacrificing the accuracy significantly. For the purpose of reducing the flow field to a form convenient for further theoretical analysis, we write each $r'^{\pm m}$ term in the following series form.

$$r'^{\pm m} = 1 \pm m\left(\frac{r'-1}{r'}\right) + \frac{m(m\pm 1)}{2!}\left(\frac{r'-1}{r'}\right)^2 \pm \frac{m(m\pm 1)(m\pm 2)}{3!}\left(\frac{r'-1}{r'}\right)^3 + \cdots \tag{8}$$

where m is the positive number. Substituting Eq. (8) for the $r'^{\pm m}$ terms in Eq. (7), we obtain

$$\psi'^{o} = \frac{1-\alpha}{2(J+\sigma K)}\left\{\left(\frac{r'-1}{r'}\right) + \frac{1}{2}\left(\frac{r'-1}{r'}\right)^2(3\sigma+4)\right\}\sin^2\theta + \Delta\left(\frac{r'-1}{r'}\right)^3 \tag{9}$$

It is noted that Eq. (9) is shown to be significantly simpler than Eq. (7) while the deviation from Eq. (7) is only on the order of

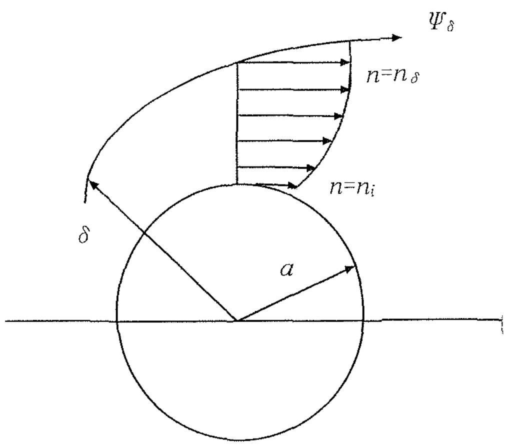

FIGURE 2. Schematic diagram of diffusion boundary layer around a liquid drop or a gas bubble.

$\Delta\{(r' - 1)/r'\}^3$. Eq. (9) will be used for performing subsequent theoretical analysis for aerosol filtration below.

DIFFUSIONAL DEPOSITION

Figure 2 shows a schematic diagram of the diffusion boundary layer around a liquid drop or a gas bubble. Unlike the solid sphere case, the particle concentration at the surface of liquid drops or bubbles are not zero. The single sphere efficiency due to diffusional deposition is defined as the ratio of the rate at which particles diffuse to the sphere surface to that at which particles approach the sphere surface within the cross-sectional area of the sphere. In dimensionless terms, it is written as (Lee and Gieseke, 1979)

$$\eta_D = \frac{4}{Pe(1 - n_i')} \int_0^\pi \left(\frac{\partial n'}{\partial r'}\right)_{r'=1} \sin\theta d\theta \tag{10}$$

with the boundary conditions

$$n' = n_i' \quad \text{at} \quad r' = 1$$
$$n' = 1 \quad \text{at} \quad r' = \delta' \tag{11}$$

where η_D is the single sphere efficiency due to diffusion, Pe is the Peclet number $(= 2au/D)$, D is the diffusion coefficient, δ and $\delta'(= \delta/a)$ are respectively the dimensional and the dimensionless distances from the center of the sphere to the boundary layer, n_i and n_δ are the particle concentrations at $r = a$ and $r = \delta$, $n'(= n/n_\delta)$ and $n_i'(= n_i/n_\delta)$ are their dimensionless form. Following Lee and Gieseke (1979) and Friedlander (1957) together with the boundary condition given by Eq. (11), we write the particle concentration balance equation in the following dimensionless form.

$$d\int_{n_i'}^1 \psi'\, dn' = -\frac{2}{Pe}\left(\frac{\partial n'}{\partial r'}\right)_{r'=1} \sin\theta\, d\theta$$
$$= -\frac{2(1 - n_i')}{Pe}\left(\frac{\delta'}{\delta' - 1}\right) \sin\theta\, d\theta \tag{12}$$

Let us write,

$$M = \int_{n_i'}^1 \psi'\, dn' \tag{13}$$

Here, M is the dimensionless rate of particle diffusion to sphere. From Eq. (3) and the expression used by Friedlander (1957), we obtain the following:

$$M = (1 - n_i')\left[\frac{1 - \alpha}{4(J + \sigma K)}\left(\frac{\delta' - 1}{\delta'}\right) \cdot \left\{1 + \frac{(3\sigma + 4)}{3}\left(\frac{\delta' - 1}{\delta'}\right)\right\} \sin^2\theta\right] \tag{14}$$

From Eq. (10) and Eq. (12), we obtain the following,

$$\eta_D = \frac{2M_\infty}{1 - n_i'} \tag{15}$$

where M_∞ is the maximum value of M.

The detailed procedures for the derivation of Eq. (12) through Eq. (15) can be referred to in the Appendix.

(a) solid sphere case ($\sigma \to \infty$)

As mentioned earlier, the case of a solid sphere system in a gas can be solved by setting the viscosity ratio ($\sigma = \mu^i/\mu^o$) as infinity. From Eq. (14), we have

$$M = \frac{(1-\alpha)(1-n_i')}{4K}\left(\frac{\delta'-1}{\delta'}\right)^2 \sin^2\theta \tag{16}$$

In the case of the solid sphere case, n_i, the concentration at $r = a$ should be zero.

From the Eqs. (12), (15) and (16), we obtain the diffusion efficiency for the solid sphere system.

$$\eta_D = \frac{2M_\infty}{1-n_i'} = 2\left(\frac{3\pi}{4}\right)^{2/3} \cdot \left(\frac{1-\alpha}{K}\right)^{1/3} Pe^{-2/3} \quad \text{for} \quad \sigma \to \infty \tag{17}$$

Eq. (17) is the same as the solution previously obtained by Lee and Gieseke (1979).

(b) bubble case ($\sigma \to 0$)

The procedure for obtaining the efficiency of the bubble is similar to the procedure for obtaining Eq. (17), and this will be described below. To simulate the case of bubbles in a liquid, the viscosity ratio is set to zero. From Eq. (9), we write a flow field around the bubbles in a gas or liquid. The simplified flow field around bubbles is written as below after setting $\sigma = 0$ in Eq. (9).

$$\psi' = \frac{1-\alpha}{2J}\left\{\left(\frac{r'-1}{r'}\right) + 2\left(\frac{r'-1}{r'}\right)^2\right\} \sin^2\theta \tag{18}$$

We write from Eq. (14)

$$M = \frac{(1-n_i')(1-\alpha)}{4J}\left(\frac{\delta'-1}{\delta'}\right) \cdot \left\{1 + \frac{4}{3}\left(\frac{\delta'-1}{\delta'}\right)\right\} \sin^2\theta \tag{19}$$

For a thin boundary layer, the first term of the right-hand side is much larger than the second term of the right-hand side. Thus, we neglect the second term to simplify Eq. (19) as,

$$M = \frac{(1-n_i')(1-\alpha)}{4J}\left(\frac{\delta'-1}{\delta'}\right) \sin^2\theta$$

or

$$\left(\frac{\delta'}{\delta'-1}\right) = \frac{(1-n_i')(1-\alpha)}{4J} M^{-1} \sin^2\theta \tag{20}$$

Inserting Eq. (20) into Eq. (12), we write

$$MdM = -\frac{(1-n_i')^2(1-\alpha)\sin^3\theta}{2J} Pe^{-1}\, d\theta \tag{21}$$

Integrating Eq. (21) between $M = 0$ at $\theta = \pi$ and $M = M_\infty$ at $\theta = 0$, we obtain

$$M_\infty = \frac{2(1-n_i')}{\sqrt{3}}\left(\frac{1-\alpha}{J}\right)^{1/2} Pe^{-1/2} \tag{22}$$

From Eqs. (15) and (22) we have the diffusional particle collection efficiency by bubble as

$$\eta_D = \frac{2M_\infty}{1-n_i'} = \frac{4}{\sqrt{3}}\left(\frac{1-\alpha}{J}\right)^{1/2} Pe^{-1/2} \quad \text{for} \quad \sigma \to 0 \tag{23}$$

Figure 3 shows the collection efficiency due to diffusion as a function of the packing density (α) and the Peclet number (Pe) for the cases of solid spheres and bubbles. From the definition of Peclet number ($Pe = 2au/D$), we find that the collection efficiency due to diffusion increases as a or u decreases. Figure 3 also shows that as the packing density (α) increases, the collection efficiency due to diffusion increases. Bubbles have a higher efficiency than that of solid spheres due to the effects of the flow circulation inside the bubbles. In obtaining the analytic solution, Eq. (23), we neglected the relatively small term in Eq. (19). To validate the analytic solution just obtained, we solved $\{\delta'/(\delta'-1)\}$ in Eq. (19) without neglecting the second term on the right-hand side and then obtained η_D numerically using Eqs.

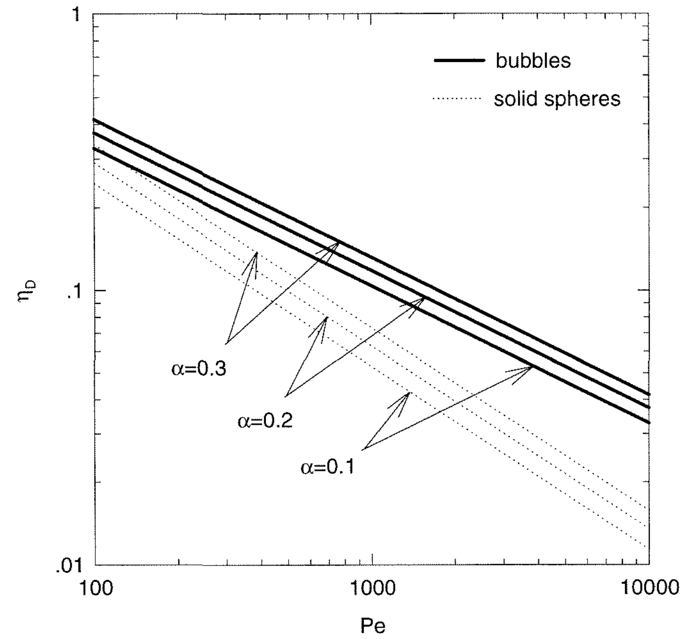

FIGURE 3. Single sphere efficiency due to diffusion (η_D) as a function of solidity (α) and Peclet number (Pe).

(12), (13), and (15). Here, we will call the results obtained numerically without neglecting any terms as "numerical solution." Figure 4 compares the collection efficiency calculated numerically with that calculated using the analytic method for the bubble case. It is seen from this comparison that the results are very close to each other. In the low *Pe* region in Fig. 4, the analytic solution is seen to give a little lower value than the numerical solution. This deviation has occurred due to neglection of the term that is relatively small during the development of the analytic solutions.

(c) liquid sphere case (1 < σ < 100)

So far, we have obtained the diffusional collection efficiency for the cases of solid spheres and bubbles. For the liquid sphere case whose viscosity ratio is intermediate, it does not appear to be possible to obtain an analytic expression. Therefore, we will take a path for obtaining a semiempirical correlation equation. In Eq. (14) the ratio of the first term to the second term of the right-hand side varies by the σ value. Though we assume the boundary layer is very thin, it is not straightforward to derive an analytic solution since both terms on the right-hand side of Eq. (14) are operating. If the first term of the right-hand side is dominant, the ratio of the second term to the first term is

$$\frac{(3\sigma + 4)}{3}\left(\frac{\delta' - 1}{\delta'}\right) \ll 1 \tag{24}$$

In this case, we deduce that the viscosity ratio (σ) should be small. If the second term of the right-hand side is dominant,

$$\frac{(3\sigma + 4)}{3}\left(\frac{\delta' - 1}{\delta'}\right) \gg 1 \tag{25}$$

in which case the viscosity ratio (σ) should be large.

In the case of Eq. (24), we have

$$M = \frac{(1 - n_i')(1 - \alpha)}{4(J + \sigma K)}\left(\frac{\delta' - 1}{\delta'}\right)\sin^2\theta \tag{26}$$

Eq. (26) has the same pattern as Eq. (20). Thus, we obtain the collection efficiency due

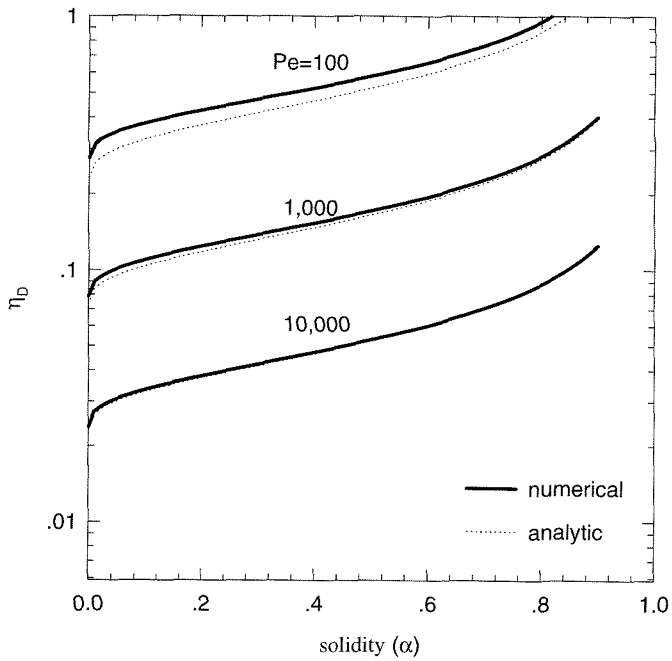

FIGURE 4. Comparison of the single sphere efficiency due to diffusion (η_D) in bubbles (analytic and numerical method).

to diffusion following the procedure taken for obtaining the collection efficiency for the bubble case. The resulting collection efficiency due to diffusion is

$$\eta_D = \frac{2M_\infty}{1 - n_i'} = \frac{4}{\sqrt{3}}\left(\frac{1-\alpha}{J+\sigma K}\right)^{1/2} Pe^{-1/2}$$

for $\sigma \leq 1$ **(27)**

It is noted that Eq. (27) becomes identical to Eq. (23) for the bubble case if we let the viscosity ratio (σ) approach 0.

In the case of Eq. (25), we write

$$M = \frac{(1 - n_i')(1-\alpha)(3\sigma+4)}{12(J+\sigma K)} \cdot \left(\frac{\delta' - 1}{\delta'}\right)^2 \sin^2\theta \quad \textbf{(28)}$$

Eq. (28) has the same pattern as the solid sphere case in Eq. (16). The collection efficiency becomes

$$\eta_D = \frac{2M_\infty}{1 - n_i'} = 2\left(\frac{\sqrt{3}\pi}{4Pe}\right)^{2/3} \cdot \left\{\frac{(1-\alpha)(3\sigma+4)}{J+\sigma K}\right\}^{1/3}$$

for $\sigma \geq 100$ **(29)**

Eq. (29) has the same formula of the solid spheres shown by Eq. (17) if we let the viscosity ratio (σ) approach infinity. In an attempt to fill the $1 < \sigma < 100$ gap, we empirically combine Eqs. (27) and (29) as below.

$$\eta_D = 0.7\left[\frac{4}{\sqrt{3}}\left(\frac{1-\alpha}{J+\sigma K}\right)^{1/2} Pe^{-1/2} + 2\left(\frac{\sqrt{3}\pi}{4Pe}\right)^{2/3}\left\{\frac{(1-\alpha)(3\sigma+4)}{J+\sigma K}\right\}^{1/3}\right]$$

for $1 < \sigma < 100$ **(30)**

It should be noted that an empirical factor of 0.7 was introduced in Eq. (30). This factor is needed to fit the results to the numerically obtained values. The procedure for obtaining the numerical solution in the intermediate viscosity ratio region is the same as that for obtaining the numerical solution of bubble case. Thus, we solved for $\{\delta'/(\delta' - 1)\}$ in Eq. (14) without neglecting any terms and then obtained η_D numerically. Figure 5 shows the collection efficiency as a function of the viscosity ratio as calculated numerically and as plotted using the analytic equations given by Eqs. (27), (29), and (30). Generally speaking, the bubble efficiency is higher than that of solid spheres with the other conditions set equal. We find that the solutions coincide well in the region between $\sigma = 1$ and $\sigma = 100$, and that they closely identify with the numerical solution. It is also seen that the collection efficiency due to diffusion increases as the viscosity ratio decreases. The case of fluid spheres, which have a smaller viscosity ratio than the solid sphere, yields a higher collection efficiency due to increased diffusion.

INTERCEPTION

The collection efficiency due to direct interception is defined by

$$\eta_R = \frac{\pi\xi^2}{\pi a^2} = \frac{2\pi\psi_\xi}{\pi a^2 u} \tag{31}$$

where ξ is the radius of the circle within which all the streamlines are contained that later pass the region between a projected circle having a diameter of $d_g + d_p$ and the sphere where d_g is the diameter of the sphere, d_p is the diameter of the particle, and ψ_ξ is the stream function at a distance ξ.

Let $r = d_g/2 + d_p/2$ and $R\ (= d_p/d_g)$ be the interception parameter. We define the dimensionless number

$$r' = \frac{r}{a} = \frac{r}{d_g/2} = 1 + \frac{d_p}{d_g} = 1 + R \tag{32}$$

Therefore, we may write the single sphere efficiency, η_R due to interception as

$$\eta_R = \frac{2\psi_{d_p/2+d_g/2,\pi/2}}{(d_g/2)^2 u} = 2\psi'_{1+R,\pi/2} \tag{33}$$

From the flow field given by Eq. (9) and from Eq. (33), the collection efficiency due to interception is

$$\eta_R = 2\psi'_{1+R,\pi/2} = \frac{1-\alpha}{(J+\sigma K)}\left\{\left(\frac{R}{1+R}\right) + \frac{1}{2}\left(\frac{R}{1+R}\right)^2(3\sigma+4)\right\} \tag{34}$$

From Eq. (34) above, the efficiency of solid spheres can be expressed when the viscosity ratio (σ) is set to infinity. The efficiency of bubbles can be expressed when the viscosity ratio (σ) becomes zero. The efficiency due to direct interception is found to increase as the particle diameter and the packing density increase and as the sphere diameter decreases (Fig. 6). These also have the same result as in the solid sphere case (Lee and Gieseke, 1979). From Fig. 6 we find the collection efficiency of bubbles due to interception is larger than the solid sphere case. Due to the internal circulation of the inner spheres, the streamlines outside the sphere pass closer to the inner sphere than in the case of solid spheres. Thus, the collection efficiency of interception is found to become larger. Figure 7 shows the particle collection efficiency due to interception as a function of the viscosity ratio. The collection efficiency has the same pattern as illustrated in Fig. 5. As a final note, it may be added that we were not able to find any experimental work that can be used for validating the present analytic results. The analysis given in this study is based on the use of the extended Kuwabara flow field that is basically a creeping flow equation. Thus, it is possible that the analysis results obtained in this study may not be applicable for high Reynolds number flows. However, a similar flow field around solid spheres was found to successfully apply to granular bed filtrations operating at a moderately high flow (Lee and Gieseke, 1979; Lee, 1981). Therefore, the limitations of the results given in this study need to be evaluated by comparing them with future experiments and with numerical calculations.

CONCLUSION

In this study the flow field and aerosol transfer around multiple liquid drops and gas bubbles were studied analytically. A boundary layer approach using a simplified form of the viscous flow field for a system consisting of multiple fluid spheres was taken successfully to derive an analytical solution valid for solid sphere, liquid drops, and gas bubbles.

According to the results, the collection efficiency due to diffusion increases with increasing packing density and with decreasing Peclet number. The collection efficiency due to interception increases with increasing packing density and particle size, which is the same results as the solid sphere case. The effect of the induced flow within the fluid spheres was found to be very important. When liquid drops and gas bubbles are considered, both diffusion and interception effects become higher than those for the solid sphere case due to the induced internal flow of the inner spheres. It is believed that this work represents the first effort for presenting a series of analytic solutions for particle capture by solid sphere, liquid droplets, and gas bubbles.

APPENDIX

In this Appendix, Eq. (12) through Eq. (15) are derived.

From Lee and Gieseke (1979), the single sphere efficiency due to diffusional deposition is written as

$$\eta_D = \frac{4}{Pe}\int_0^{\pi}\left(\frac{\partial c'}{\partial r'}\right)_{r'=1}\sin\theta\,d\theta \qquad \textbf{(A1)}$$

with the boundary conditions

$$c' = 0 \quad \text{at} \quad r' = 1$$

$$c' = 1 \quad \text{at} \quad r' = \delta' \qquad \textbf{(A2)}$$

where c' is the dimensionless transferred concentration. Following the boundary layer approach utilized previously by Lee and Gieseke (1979) and Friedlander (1957), we write the particle concentration balance equation in the following dimensionless form,

$$d\int_0^1 \psi'\,dc' = -\frac{2}{Pe}\left(\frac{\partial c'}{\partial r'}\right)_{r'=1}\sin\theta\,d\theta \qquad \textbf{(A3)}$$

In the present analysis, we write the concentration profile having the following form (Friedlander, 1957), which satisfies the boundary conditions shown as Eq. (A2).

$$c' = \frac{\ln r'}{\ln \delta'} = \frac{(r'-1)/r'}{(\delta'-1)/\delta'} \qquad \textbf{(A4)}$$

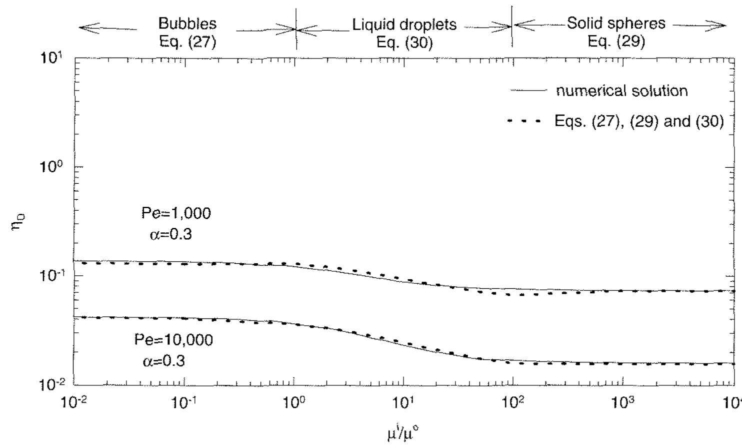

FIGURE 5. Single sphere efficiency due to diffusion (η_D) as a function of the viscosity ratio ($\sigma = \mu^i/\mu^o$) value.

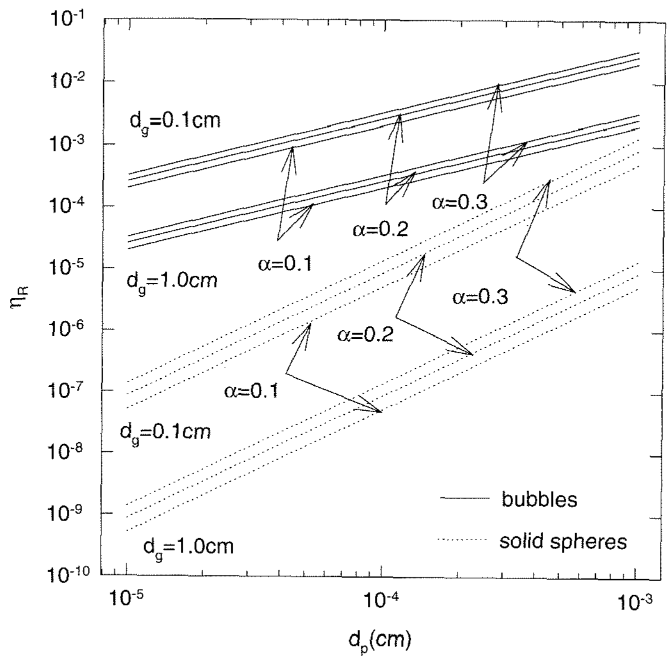

FIGURE 6. Single sphere efficiency due to interception (η_R) as a function of particle diameter (d_p).

and

$$dc' = \frac{d(\ln r')}{\ln \delta'} \tag{A5}$$

Combining Eq. (9) and Eq. (A5), we write

$$\psi' dc' = \frac{1-\alpha}{2(J+\sigma K)}\left\{\ln r' + \frac{1}{2}\ln^2 r'(3\sigma + 4)\right\}\frac{\sin^2\theta}{\ln \delta'}\, d(\ln r') \tag{A6}$$

Integrating both sides of Eq. (A6) over the diffusion boundary layer thickness with the boundary conditions of $c' = 0$ at $\ln r' = 0$ and $c' = 1$ at $\ln r' = \ln \delta'$, we have

$$\int_0^1 \psi'\, dc' = \frac{1-\alpha}{4(J+\sigma K)}\left(\frac{\delta'-1}{\delta'}\right)\left\{1 + \frac{(3\sigma+4)}{3}\left(\frac{\delta'-1}{\delta'}\right)\right\}\sin^2\theta \tag{A7}$$

In the presented analysis, the concentration at $r' = 1$ is not zero but constant n_i. Thus, we related particle concentration n to transferred particle concentration c.

$$c' = \frac{c}{c_0} = \frac{n - n_i}{n_\delta - n_i} = \frac{n' - n_i'}{1 - n_i'} \tag{A8}$$

where c_0 $(= n_\delta - n_i)$ is transferred coordinate particle concentrations at the outer boundary layer. From Eq. (A8) that gives the relationship between c' and n', Eq. (A1) now can be written as a function of the dimensionless particle concentration n'.

$$\eta_D = \frac{4}{Pe(1-n_i')}\int_0^\pi \left(\frac{\partial n'}{\partial r'}\right)_{r'=1} \sin\theta\, d\theta \tag{A9}$$

then boundary conditions become

$$n' = n_i' \quad \text{at} \quad r' = 1$$
$$n' = 1 \quad \text{at} \quad r' = \delta' \tag{A10}$$

Thus, from Eqs. (A5) and (A8), Eqs. (A3) and (A7) become

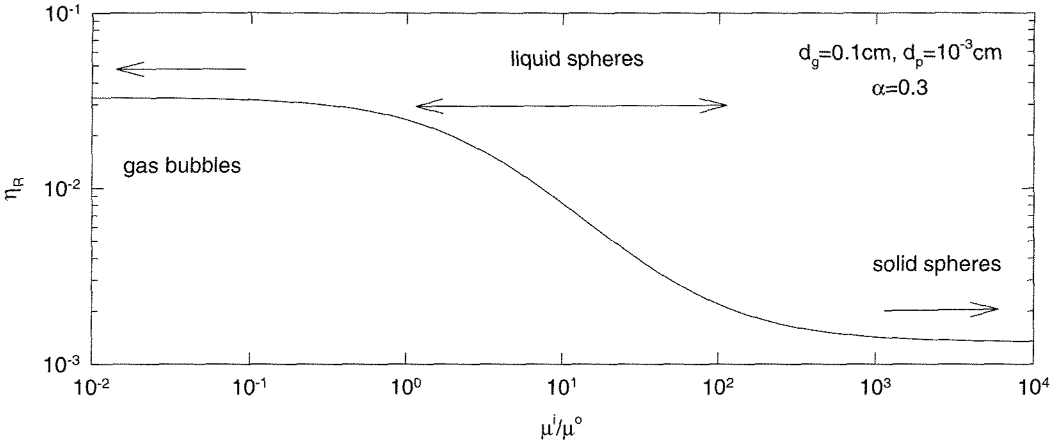

FIGURE 7. Single sphere efficiency due to interception (η_R) as a function of the sigma ($\sigma = \mu^i/\mu^o$).

$$d\int_{n_i'}^{1} \psi'\, dn' = dM = -\frac{2}{Pe}\left(\frac{\partial n'}{\partial r'}\right)_{r'=1} \sin\theta\, d\theta = -\frac{2(1-n_i')}{Pe}\left(\frac{\delta'}{\delta'-1}\right) \quad \textbf{(A11)}$$

$$M = \int_{n_i'}^{1} \psi'\, dn' = (1-n_i')\int_0^1 \psi'\, dc' = (1-n_i')\left[\frac{1-\alpha}{4(J+\sigma K)}\left(\frac{\delta'-1}{\delta'}\right) \cdot \left\{1+\frac{(3\sigma+4)}{3}\left(\frac{\delta'-1}{\delta'}\right)\right\}\sin^2\theta\right] \quad \textbf{(A12)}$$

From Eq. (A11) we can obtain

$$dM = -\frac{2}{Pe}\left(\frac{\partial n'}{\partial r'}\right)_{r'=1} \sin\theta\, d\theta$$

or

$$\left(\frac{\partial n'}{\partial r'}\right)_{r'=1} \sin\theta\, d\theta = -\frac{Pe}{2}\, dM \quad \textbf{(A13)}$$

Integration of Eq. (A13) between $M = 0$ at $\theta = \pi$ and $M = M_\infty$ at $\theta = 0$ yields,

$$\int_{M_\infty}^{0} dM = -\frac{2}{Pe}\int_0^{\pi}\left(\frac{\partial n'}{\partial r'}\right)_{r'=1} \sin\theta\, d\theta \quad \textbf{(A14)}$$

From Eqs. (A9) and (14) we obtain the following equation, which is the same as Eq. (15) in the main text.

$$\eta_D = \frac{2M_\infty}{1-n_i'} \quad \textbf{(A15)}$$

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