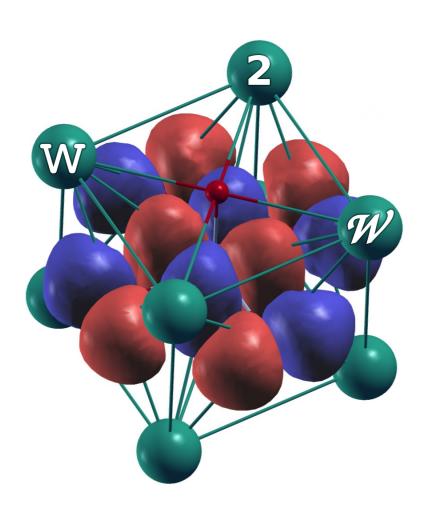
Wien2Wannier 2.0 User's Guide

From linearized augmented plane waves to maximally localized Wannier functions.

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Introduction

In the solid state theorist's tool kit, algorithms based on density functional theory (DFT) represent the backbone applications. One of the more popular codes available is the Wienzk package [1, 2]. It is based on dividing the unit cell into "muffin-tin" spheres centered on the ions on one hand and an interstitial region on the other. In the former, basis function with more or less atomic features are employed, while in the latter, plain waves are used. The combined basis functions are called (linearized) augmented plane waves (LAPW).

While in Wien2k a k-space representation of wave functions is convenient, there are many applications where a real-space picture is preferable: Determining transport properties (hopping parameters), visualization, and, especially, in codes relying on local orbital basis functions such as dynamical mean-field theory (DMFT [3]). One way to change to a real space representation is a Fourier transform of the Bloch states $\psi_{nk}(r)$ from the DFT calculation, yielding

$$w_{mR}(\mathbf{r}) = rac{V}{ au^3} \int_{BZ} dk \; \mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot\mathbf{R}} \left(\sum_n U_{nm}^{(\mathbf{k})} \psi_{n\mathbf{k}}(\mathbf{r})
ight).$$

The resulting functions $w_{mR}(r)$, parametrized by a direct lattice vector R, are called Wannier functions (wf). Due to the additional phase factors $U_{nm}^{(k)}$, which all lead to valid Wannier functions, there is considerable ambiguity in the choice of the real space basis set. This can be overcome by choosing Wannier orbitals $w_{mR}(r)$ with minimal real-space extent (spread $\langle \Delta r^2 \rangle$). These are known as maximally localized Wannier functions (MLWF). A popular program to compute the MLWF for a given set of Bloch functions is WANNIER90 [4, 5].

Unfortunately, the application of wanniergo to the LAPW basis set of Wien2k is not as straightforward as for a pure plane wave basis. In this guide, we describe the package Wien2wannier which provides an interface between Wien2k and wanniergo. In any scientific publications arising from the use of Wien2wannier, we ask that you cite Ref. [6].

J. Kuneš, R. Arita, P. Wissgott, A.Toschi, H. Ikeda, and K. Held, Comput. Phys. Commun. 181, 1888 (2010), arXiv:1004.3934

to acknowledge your use of our code. When using WANNIER90, you should cite Ref. [4]

A.A. Mostofi, J.R. Yates, Y.-S. Lee, I. Souza, D. Vanderbilt, and N. Marzari, Comput. Phys. Commun. 178, 685 (2008), arXiv:0708.0650

independently of wien2wannier (cf. wannier90 user's guide [5]).

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1 Quickstart

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This section outlines the procedure for a "standard" WIEN2WANNIER calculation. A detailed description of the individual programs can be found in Section 2. For an even more condensed reference, see the plain-text file doc/CHEATSHEET.

1.1 Preparatory steps

Before running WIEN2WANNIER, one needs a converged WIEN2k calculation. Additionally, during the setup for WIEN2WANNIER, the bands which are to be taken into account will have to be specified, and the main character (e.g., *d* bands on atom 2) of these bands should be known.

To obtain this information, a combination of partial DOS and bandstructure, or a "band character" plot is often expedient (e.g. x spaghetti's "fat bands" option, or SpaghettiPrimavera and prima.py, available in the *unsupported software* section of the Wienzk website).

Copy Required Files

As a "zeroth" step before a Wannier projection, it is recommended to use the script prepare_w2wdir subdir

to create the subdirectory subdir where the rest of the workflow will take place. Thus, one Wien2k calculation can be the starting point of several wien2wannier runs.

1.2 Interface & Wannierization

Write Input Files

The script init_w2w takes the following steps:

x kgen -fbz generates the non-symmetrized k-mesh that wanniergo requires. The density of the mesh influences the quality of localization and visualization of the Wannier functions. Normally, you should use and unshifted k-mesh.

- **x** findbands looks in *case* .output1 for bands in a given energy range $[E_{\min}, E_{\max}]$, and outputs the corresponding band indices b_{\min}, b_{\max} . To choose the energy window of interest, consult a (p)DOS and/or band structure plot.
- write_inwf writes the main input file case.inwf for the interface. The band indices b_{\min} , b_{\max} have to be specified, and initial projections A_{mn} may be given in terms of atomic sites and appropriate spherical harmonics.
- write_win writes the input file for wannier90.x, case.win, on the basis of case.inwf and other files.
- x wannier90 -pp reads the k-mesh in case.win and writes a list of nearest-neighbor k-points to case.nnkp.

Run Interface

After running init_w2w, one can proceed:

- x lapw1 before the actual interface run, the eigenvectors and eigenvalues for the new (non-symmetrized) k-mesh have to be computed.
- **x** w2w computes the overlaps M_{mn} and initial projections A_{mn} , and writes them, together with the eigenvalues E_n , to case.mmn, case.amn, and case.eig.

Run Wannierization

Finally,

 ${\tt x}$ wannier90 computes the U(k) by maximum localization. Output is stored in ${\tt case}$.wout. The Wannier orbitals should be converged to a spread which is usually smaller than the unit cell of the structure.

1.3 Verification & Postprocessing

After a successful wanniergo run, one should check if the centers and spreads of the Wannier functions (printed in <code>case.wout</code>) are sensible. Another important consistency check is to compare the Wannier-interpolated bandstructure to the one computed by Wienzk. Wienzwannier also provides programs to create a real-space plot of the Wannier functions.

Compare Bandstructures

With the option $hr_plot=T$, wanniergo writes a bandstructure derived from the Wannier-interpolated Hamiltonian H(k) to $case_band.dat$. To compare this to the bandstructure computed by spaghetti, e.g. in gnuplot, use the command (including a conversion from $Bohr^{-1}$ to $Angström^{-1}$)

```
p 'case.spaghetti_ene' u ($4*1.89):5, 'case_band.dat' w l
```

Plot Wannier Functions

- case.inwplot specifies the real-space grid on which the Wannier functions should be plotted.
- **x** wplot -wf m evaluates Wannier function number m on the real-space grid, and writes the density $|w_m(r)|^2$ to $case_m$.psink and the phase $arg\{w_m(r)\}$ to $case_m$.psiarg.
- wplot2xsf converts all case*.psink plus case*.psiarg files in the directory to XCrySDenstyle xsf files.
- xcrysden --xsf case_m.xsf opens the xsf file; pick "Tools → Data Grid" from the menu and press OK. In the isosurface controls window choose an appropriate isovalue, e.g. 0.1, and check the "Render +/- isovalue" box.

2 Detailed description

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This section lists the programs included in WIEN2WANNIER, along with brief descriptions and usage summaries. All programs have online help (-h), which may be more complete than the material here.

Programs that are described here include: the "main" wienzwannier programs w2w and wplot; several utility programs used in a typical wienzwannier run; and some programs that are needed in special cases, or grew out of wienzwannier development and are provided here in the hope that they will be useful, even if they are not needed in the context of wienzwannier.

2.1 w2w — compute overlaps

This is the main program of the interface which provides the files <code>case.mmn</code>, <code>case.amn</code> and <code>case.eig</code> for wanniergo given the output of a Wienzk run (most importantly, <code>case.vector</code>). w2w is based on lapwdm, since the main task, for the computation of the overlap matrices

$$M_{mn}^{(k,b)} = \langle \psi_{mk} | e^{-ib \cdot r} | \psi_{nk+b} \rangle$$
 (2.1)

that are stored in *case*.mmn, is to determine the basis functions $\psi_{nk}(r)$ in the entire unit cell (using appropriate boundary conditions on the muffin-tin spheres).

In addition to standard Wienzk files, a file case.nnkp is required which gives the b vectors linking each k to its nearest neighbors in Eq. (2.1). This file is written by wannier90.x -pp.

Execution

The program w2w is executed by invoking the command:

```
x w2w [-c -up|-dn -so -p] |
w2w w2w.def [#proc] | w2wc w2w.def [#proc]
```

While w2w itself is not parallelized, like x lapw2 -qtl, it accepts a -p switch (or #proc argument) to read parallel vector and energy files.

With spin-orbit coupling, w2w must be called separately for the two spins,

```
x w2w-so -up; x w2w-so -dn
```

resulting in case .mmn{up,dn}, case .amn{up,dn}, and case .eig{up,dn}. For the Wannierization, the M_{mn} and A_{mn} must be added (and the eigenvalues copied) to produce case .mmn, case .amn, and case .eig. This is done automatically by x wannier90 -so. Note: it does not make sense to run x w2w -so without either -up or -dn; conversely, x wannier90 -so does not accept -up or -dn.

Input

A sample input file for wien2wannier is given below. It can be generated using write_inwf.

```
case.inwf
BOTH
         # compute Amn | Mmn | both
         # band-lo, band-hi
 21 23
         # max LJ in exp(ibr) expansion; #(initial projections)
         # #(terms) [d-xy orbital]
2 2 -2 0 +1
               # IAT; l, m; Re(coeff), Im(coeff)
2 2 +2 0 -1
                  [d-(x2-y2) \text{ orbital}]
         #
2 2 -1 1 0
2 2 +1 1 0
                  [d-z2 \ orbital]
1
2 2 0 1 0
```

This file is read using Fortran list-oriented reads, i.e., items are separated by white space.

line 1 mode — what to calculate

```
mode Amn Only determine the initial orbital projections <code>case.amn</code>

Mmn Only determine the overlap matrices <code>case.mmn</code>

both Determine both the initial orbital projections and the overlap matrices
```

line 2 Blo, Bhi — band window

Blo, Bhi *int* the minimal and maximal Bloch band from the vector file to be taken into account, determining the lower edge of the (outer) energy window

```
line 3 LJmax, Nproj
```

LJmax int the number of terms which are used in the spherical harmonics expansion to approximate $\exp(-ikb)$, usually 3-4 is sufficient.

Nproj int the number of target Wannier functions

line 4 Nterm — number of terms (Nproj times)

Nent int the number of Y_{ℓ}^{m} terms (lines to follow for this initial projection

```
line 5 Iat, 1, m, Re, \operatorname{Im} - Y_{\ell}^{m} \operatorname{term} (\operatorname{Nterm\,times})
```

```
Iat int the atom where the entry is centered 1, m int the orbital and magnetic quantum numbers \ell and m for this entry Re, Im real the complex coefficient multiplying Y_{\ell}^{m}
```

If initial projections are given, there must be Nproj blocks 1. 4+5, one for each initial projection. Within each block, write Nterm lines 1. 5, one for each Y_{ℓ}^{m} term. Otherwise, if mode is Mmn, the initial projections can be omitted.

Note The initial projections (1.5) generated by write_inwf are normalized. For non-normalized initial projections (as in the template), the extra factor will be included in case.amn, but neutralized by the orthonormalization step in Wanniergo.

2.2 wplot — Wannier function plotter

wplot evaluates the Wannier functions on a real-space grid. It reads the transformation matrices U(k) from the file case.chk written by wannier90.x and thereby constructs the Wannier functions from the original Wien2k Bloch states.

Execution

The program wplot is executed by invoking the command:

```
x wplot [-up|-dn -c -so -p -wf m] |
wplot def [m [#proc]] | wplotc def [m [#proc]]
```

wplot is not parallelized, but accepts a -p switch (or #proc) to read parallel vector files. Moreover, as a crude form of parallelization, one can run several wplot instances in the same directory in parallel without interference (e.g. to plot several Wannier functions on fine grids).

The input file contains the index of the Wannier orbital to be plotted; m above overrides this. Output goes to $case_m.psink$ ($|w(r)|^2$) and $case_m.psiarg$ (arg w(r)).

Input

wplot is based on lapw7 and shares the general structure of the input file. A sample input file for wplot is given below. A template can be found in \$WIENROOT/SRC_templates/case.inwplot.

```
- case.inwplot -
3D ORTHO
             # grid: 3D/ANY; check axes? (ortho/non); or coord.: Cart/frac
 0 0 0 1
             # origin: x, y, z; idiv
 1 0 0 1
           # x-end: x, y, z; idiv
 0 1 0 1
            # y-end: x, y, z; idiv
 0 0 1 1 # z-end: x, y, z; idiv
20 20 20
             # #(grid points) in x, y, z dir.
             # post-processing: DEP(HASING) | NO
NO
           # units: ANG/ATU; rel. comp.: LARGE/SMALL
ANG
     LARGE
     F
             # Wannier function index; apply WF rotation?
 1
```

```
line 1 mode, flag format(A3, A1)
```

If mode=3D and flag=0, grid axes will be checked for pairwise orthogonality; set flag=N for nonorthogonal axes. If mode=ANY and flag=C, the grid points in case.grid are in cartesian coordinates (3 reals per line); if flag=F, they are in fractional coordinates (3 numbers and divisor).

```
line 3 ix, iy, iz, idv (free format) — axis end-point (mode=3D; 3 times)
ix, iy, iz, idv int Coordinates of the end points of each grid axis
```

```
line 4 nx, ny, nz (free format) — mesh size
    nx,ny,nz    int    number of mesh points in each direction (mode=3D)
    npt        int    total number of mesh points (mode=ANY)
```

If mode=3D, give l. 3 3 times, once for each direction; if mode=ANY, omit l. 2+3 and give the total number of points to be read on l. 4.

```
line 5 post format(A3) — post-processing option
```

```
post DEP "dephasing": each wave function is multiplied by a complex phase factor to align it (as far as possible) to the real axis

NO No post-processing
```

```
iwan int index of the WF to be plotted, unless overridden on command line
```

Finding the right window for plotting can be tricky. Wanniergo often yields orbitals that are not centered in the home unit cell; wplot2xsf can shift them later on, but in wplot one has to "hit" the orbitals as given by wanniergo (see section "Final State" in case. wout). Therefore, it is recommended to start with a coarse grid (for instance $10 \times 10 \times 10$), make sure the window is correct, and only then start a calculation with a finer grid.

read unitary matrix from case .wfrot and apply before plotting

The option wfrot asks wplot to read a $(N_{\rm WF} \times N_{\rm WF})$ matrix from case .wfrot, which is applied to the WFs before plotting. This is useful for some post-processing applications, where a rotation in the Wannier subspace is desired.

2.3 convham — Fourier transform Wannier Hamiltonian

This program Fourier transforms the Wanniergo real space Hamiltonian H(R) (case_hr.dat) to its k-space representation H(k) (case .ham_fine) on the k-points given by case .klist. In this way, if the real-space Hamiltonian is sufficiently localized, H(k) may be interpolated to arbitrarily fine k-grids. convham is executed by invoking the command

```
x convham [-band] | convham def
```

wfrot

logical

2.4 init_w2w — prepare wien2wannier input files

This script guides the user through the initialization of wienzwannier phase as described in Section 1.2.

```
init w2w [-up|-dn] [-b options]
```

In batch mode (-b), further options are available instead of interactive input.

2.5 findbands — get band indices inside energy interval

This program reads <code>case.output1</code> to identify the bands which lie within a certain energy window. The program findbands is executed by issuing the command:

```
x findbands window [-up|-dn -so -hf -efermi EF] | findbands def
```

window may be given as -emin e -emax E or -all e E. The energies are in eV with $E_F = 0$. The Fermi energy is read from case fermi (written by prepare_w2wdir) unless given as -efermi (in Ry).

The output is given in *case* .outputfind and consists of the bands in the interval at each k-point, as well as which bands are contained in the interval across all k-points, and which bands cross the interval at any k-point.

2.6 joinvec — join parallel vector files

energy files from a parallel calculation into one case.vector and one case.energy. It is executed by invoking the command

```
x joinvec [-up|-dn -c -so]
```

All case.vector_* files are merged into one case.vector file containing the header from case.vector_1 (and correspondingly for the .energy files).

2.7 prepare_w2wdir — copy required files

A Wien2k computation can be the starting point for various runs of wien2wannier. This script creates a new subfolder of a Wien2k directory and is invoked by

```
prepare_w2wdir subdir
```

where subdir is the name of the subdirectory to be created.

2.8 shifteig — shift energies in case.eig

to zero in *case*.eig. If needed, this program can be used to apply an additional shift by DE (in eV). shifteig is executed by invoking the command

```
x shifteig [-up|-dn] -efermi DE | shifteig def DE
```

2.9 w2waddsp — add spin directions for SOC

In calculations including spin-orbit coupling (soc), wanniergo has to be invoked on case.mmn and case.amn files which contain the sum of the overlaps / projections for the two spin channels. (This procedure is needed also for non-spin-polarized cases, cf. Section 2.1.) To this end, w2waddsp reads case.mmn{up,dn}, case.amn{up,dn} and writes case.mmn, case.amn.

There is usually no need to call this program manually, it is run by x wannier90 -so. If needed, it is executed by invoking the command

```
x w2waddsp | w2waddsp def
after running x w2w -up and -dn.
```

2.10 wannier90 — wrapper for wannier90.x

A wrapper script for wannier90.x is provided to take care of spin polarization and spin-orbit coupling. In the context of wienzwannier, wannier90.x is executed by invoking the command

```
x wannier90 [-up|-dn|-so|-pp] | wannier90 [-up|-dn|-so|-pp]
```

The wrapper script must be able to find the executable wannier90.x, i.e. you have to either have it in your \$PATH, or edit the script wannier90_lapw to provide the location.

With -pp, wannier90.x will produce case.nnkp; with -so, w2waddsp will be called to add the spin channels together before running wannier90.x.

Extensive diagnostic output goes to case. wout, error messages to case. werr; the full results (in particular the U(k) matrices) are stored in the binary file case. chk.

2.11 wplot2xsf — convert wplot output to xsf format

This program converts the files <code>case_m.psink</code> and <code>case_m.psiarg</code> produced by wplot to files <code>case_m.xsf</code> which can be opened, e.g., in XCrySDen [8] or VESTA [9].

Note that only real data can be represented in the xsf file. Therefore, $|w(r)|^2 \operatorname{sgn} \{\operatorname{Re} w(r)\}$ is saved by default. (In most cases the Wannier functions are real.)

wplot2xsf has a number of options (see wplot2xsf -h), but usually it is executed by invoking the command

```
wplot2xsf [-up|-dn]
```

If all the required files have their standard names, this will convert all the plots in the current directory.

By default, wplot2xsf reads <code>case_centres.xyz</code>, and shifts the Wannier functions so that their centers have the coordinates given in that file. If <code>translate_home_cell</code> (and <code>write_xyz</code>) is set in <code>case.win</code>, this will result in a plot where the Wannier centers lie in the "home" unit cell.

2.12 write_insp — write input file for spaghetti

Fermi energy read from case.scf2 in the input file for spaghetti, case.insp. It is invoked by

```
write_insp [-up|-dn]
```

2.13 write_inwf — prepare input file for w2w

This program prepares the main input file case.inwf for w2w. It is executed by invoking the command

```
write_inwf [-up|-dn] (interactive) |
write_inwf [-up|-dn] -bands Nmin Nmax [PROJ [PROJ ...]] (noninteractive)
```

It will ask (in interactive mode) for a range of bands, which are all the bands to be taken into account by w2w (including those for disentanglement). Then, it will ask for "projection specifications" PROJ = SITE:ORB[:XAXIS]], which consist of colon-separated parts naming an atomic site, an orbital, and, optionally, rotated z- and x-axes¹. Please see write_inwf -h and write_inwf -H axes/sites/orbitals for further information on these. Interactively, you can also use tab completion to discover input options and command line history to recall past inputs.

The program will keep asking for projections until it has accumulated as many projections as bands were specified. However, one can stop early by pressing Ctrl-D (EOF); in this case, there will be fewer Wannier functions than bands (disentanglement).

2.14 write_win — prepare input file for wannier90.x

This program reads case.inwf, case.klist, and some other files, and produces case.win, the input file for wannier90.x. It is executed by invoking the command

write_win [-up|-dn] [-fresh]

If case.win already exists, write_win updates it. Otherwise (or if -fresh is given), the template \$WIENROOT/SRC_templates/case.win is used. Note that the spin flag -up|-dn only serves to choose case.inwfup or case.inwfdn; there is normally only one case.win.

¹write_inwf uses the method of Ref. [7] to rotate the spherical harmonics.

3 Getting help

Online help for all programs can be requested via the option -h. In addition to the pointers below, the WIEN2WANNIER distribution also includes a file doc/CHEATSHEET, which concisely summarizes the usual steps of a calculation. If this does not help, send questions relating to the usage of WIEN2WANNIER to the Wien2k mailing list at http://www.wien2k.at/reg_user/mailing_list/.

For help on updating wien2wannier, see the file INSTALL. Comments on wien2wannier (bug reports, feature requests, etc.) can be submitted through the GitHub page.

For questions about WANNIER90 and the MLWF procedure, there is a mailing list at http://www.wannier.org/forum.html.

3.1 FAQ

How does one choose the initial projections?

There is no general rule to choose the initial projections which have to be prepared in the inwf file. There are, however, some hints which usually help:

- In the energy interval of interest, identify the main atoms which contribute to the partial DOS. These atoms are usually good centers for the initial projections. The character of these atomic contributions can also be seen via the partial DOS.
- A glance at the bandstructure often helps to consider the multiplicity of equivalent we and to identify certain hybridizations.

The resulting Wannier orbitals are not localized

Several possible reasons (this list is not complete)

- choosing non-optimal initial projections
- the number of k-points is too small

The bandstructure of Wien2k and wannier90 does not match

First of all, be sure to take into account the Bohr-Ångström conversion, e.g., in gnuplot:

```
p 'case.spaghetti_ene' u ($4/0.53):5, 'case_band.dat' w 1
```

3.1. FAQ

If the bands themselves differ strongly, then one might have

- non-optimal initial projections
- too few k-points
- chosen the frozen energy window in WANNIER90 in a wrong way.

When plotting a Wannier orbital in XCrySDen I can see nothing

One possible reason is that the WF centers are not necessarily in the home unit cell at $[0\,0\,0]$. The interface programs usually account for this by translating the WF to the home cell. However, the spatial mesh for wplot has to be defined with respect to the original centers which come out of WANNIER90. These centers can be found in case . wout file. Assume the WF is centered at [-0.5-0.5;-0.5] in the basis of conventional unit vectors. Then, appropriate mesh vectors might be

My Wannier orbitals are not centered at the home unit cell

It happens quite frequently that a wF is not centered within the home unit cell, which is displayed by default by XCrySDen. The interface program write_win automatically activates the option translate_home_cell in the wanniergo input file. Then, wanniergo should produce a file case_centres.xyz, where the vectors of all wF are stored which translate the orbital to the home unit cell. If this file cannot be located by wplot2xsf or the option -noshift is activated, no translation is conducted and the wF appear centered at their original position.

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