

# Using the Dynamical System Approach to Navigate in Realistic Real-World Environments

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## Abstract

*The dynamical system approach to behaviour based robotics [8] provides a sound mathematical framework to behaviour design and integration. So far this approach has been used in various simulation work and in simple real world settings. This paper is a first step towards application in a more realistic scenario such as a typical office environment. The robot perceives its environment via a set of sonar sensors. Simple geometric representations of the corridor and obstacles are extracted from the perceptual information. Obstacle avoidance and corridor following are implemented as the basic behaviours, which allow the robot to navigate safely through its environment. The robot can reliably distinguish between passages that are either wide enough to pass or too narrow to be traversed. Coordination among the primitive behaviours allows the robot to cope with more complex situations, such as a corridor that is blocked by obstacles, in a flexible manner.*

## 1 Introduction

A broad variety of behaviour based systems have been proposed and implemented on different robots for a diverse set of tasks [2]. However, most of these approaches lack a theoretical underpinning that would allow a thorough analytical design and verification of properties such as stability and convergence. The dynamical system approach introduced by Schöner and Dose [8] provides a sound mathematical framework to analytically design behaviours and their coordination depending on the particular constraints imposed by the robotic task at hand.

This methodology has been applied to design a simple navigation behaviour, composed of an obstacle avoidance and target behaviour, on simulated [8, 9, 6] as well as real world robots [4, 3]. Previous simulation

approaches assume that the obstacles are of circular shape and that their exact location and radius are observable. However, in the real world this assumption is difficult to fulfill in particular in case of sonar sensors that only provide partial and imprecise information about the environment. The earlier mentioned work on real robots utilizes no coordination among different behaviours, which therefore restricts navigation to less complex environments.

The work presented in this paper is part of a larger project that has the objective to develop an intelligent service robot that is able to perform complex tasks such as fetch and deliver objects in an office environment [1]. These tasks require a navigation scheme that is able to cope with uncertainty and complexity inherent to realistic offices and corridors. As a first step towards that objective we designed a navigation behaviour that integrates obstacle avoidance and corridor following. The novel corridor behaviour employs a Hough transform to estimate the relative pose between the robot and the parallel walls that form the corridor. The coordination scheme activates and prioritizes the behaviours according to the currently perceived context and robot goal. The scheme is able to resolve critical situations in which the basic behaviours conflict with each other, e.g. obstacles that prevent the robot from passing the corridor.

The next section provides a brief overview on the dynamical systems approach. The individual components of our system and their integration are introduced in section 3, followed by the presentation of experimental results in section 4, and a discussion in section 5.

## 2 Dynamical System Approach

The conceptual framework of this approach is based on the theory of nonlinear dynamical systems [7]. In

the following we only provide a brief outline of this framework and refer the interested reader to [9] for a more detailed description.

A behaviour  $b$  emerges from the time evolution of the *behavioural variables* described by the vector  $\vec{x}$ . In a navigation task for example the robot heading and velocity constitute the set of behavioural variables. In the dynamical system described by

$$\dot{\vec{x}} = \vec{f}_b(\vec{x}) \quad (1)$$

the function  $f_b$  can be interpreted as a *force* acting on the behavioural variables. This force is designed such that the desired values of  $\vec{x}$  (e.g. direction of a target) form an attractor and undesired values (e.g. direction of an obstacle) form a repeller in the dynamics of the behavioural variables. The function  $f_b$  depends on the relative pose between the robot and its environment. However, the dynamics of  $\vec{x}$  take place at a much faster time scale than the gradual changes that emerge in  $f_b$  as a result of the robots motion. This property assures that the dynamic variables remain close to the attractor state at all times. Multiple behaviours are aggregated by weighted addition of the individual contributions  $f_b$ .

$$\dot{\vec{x}} = \sum_b |w_b| \vec{f}_b(\vec{x}) + \text{noise} \quad (2)$$

The weights  $w_b \in [-1, 1]$  define the strength of each behaviour and are computed based on the perceived context of operation. The noise has a small amplitude and merely assures that the dynamics escapes unstable fix-points (repellers). Coordination among behaviours is modeled by means of an additional competitive dynamics that controls the weights  $w_b$ , which evolve in the following fashion:

$$\tau_b \dot{w}_b = \alpha_b (w_b - w_b^3) - \sum_{b' \neq b} \gamma_{b',b} w_{b'}^2 w_b + \text{noise} \quad (3)$$

The first term constitutes a pitchfork bifurcation, i.e. the dynamics possesses stable fix-points at

$$w_b = \begin{cases} \pm 1 & \text{if } \alpha_b > 0 \\ 0 & \text{if } \alpha_b < 0 \end{cases} \quad (4)$$

The factors  $\alpha_b \in [-1, 1]$  are called *competitive advantages*. They determine to which degree a behaviour is appropriate and desirable in the present context. The second term in equation 3 captures the competitive dynamics in that an active behaviour  $b'$  of higher priority suppresses the activation of another conflicting behaviour  $b$ . Hence, the factors  $\gamma_{b',b} \in [0, 1]$  are called

*competitive interactions*. For  $|w_{b'}| \sim 1$  and  $\gamma_{b',b} > \alpha_b$ , the point  $w_b = 0$  becomes the new stable fix-point of behaviour  $b$ , despite a positive competitive advantage  $\alpha_b > 0$ . A detailed analysis of how the stability of fix-points varies across different values of competitive advantages and interactions is given in [6]. The time constant  $\tau_b$  determines the rate at which the behaviours are switched on and off. Similar to the behavioural dynamics, the noise term helps the system to escape unstable fix-points.

### 3 System Design

In our experiments we used the Scout robot from Nomadic Technologies (Figure 1). The platform has a cylindrical shape with a diameter of 38 cm and moves at a speed of up to 1 m/s. The robot is equipped with a ring of 16 evenly spaced ultrasonic sensors. The robot possesses a two wheel differential drive located at the geometric center which allows omni-directional steering at zero turn radius.



Figure 1: The scout robot standing in the corridor of our institute, which we used as the environment for our experiments (see Figure 7).

### 3.1 Extracting Geometric Representations from Raw Sensor Data

The perception and geometric reconstruction of walls and obstacles is solely based on the information provided by the 16 ultrasonic sensors. Each sonar has a beam width of  $25^\circ$  and a detection range of 6 to 255 inches.

In order to estimate the orientation and distance of the two parallel walls that form the corridor, the 200 most recent sonar readings are kept in a FIFO buffer. A Hough transform [5] is invoked on the sonar data every few seconds in order to extract the pair of parallel lines (one on either side of the robot) that coincide with the largest number of sonar echos. The corridor behaviour is based on the orientation and distance to these lines relative to the robot, to navigate along the corridor.<sup>1</sup>

Due to the limited angular resolution of sonar sensors, the geometric representation of obstacles is rather simple and closely linked to the actual perception of the robot. Out of the 50 most recent sonar readings that lie within the boundaries described by the corridor estimate, the ones in the frontal half plane of the current robot heading are considered for obstacle reconstruction. Obstacles are reconstructed from the detected echos in ascending order of their distance to the robot. The echo closest to the robot defines the first obstacle which orientation in the robot frame is given by the axis of the sensor that received the echo. A new obstacle is recorded for every subsequent echo which orientation differs by an angle of at least  $22.5^\circ$  from any previously identified obstacle. New obstacles are added in an incremental fashion until the sonar buffer contains no further echos. Obstacle reconstruction is invoked at every control cycle of the robot. Notice, that our representation only considers the distance to an obstacle but ignores its shape or size. Despite its simplicity the chosen representation is powerful enough to successfully navigate the robot around obstacles.

While collecting the sonar readings in the two FIFO buffers, the robot is driving a short distance. Odometry is used, to calculate the relative location of sonar readings taken at different robot positions, which introduces further uncertainty in the sonar data. These errors, however, are comparatively small and hardly influence the performance of the behaviours.

<sup>1</sup>The behaviour is only active if the robot is actually located within a corridor which is either inferred by means of localization with respect to an external map or by observation of the distribution of sonar echos.

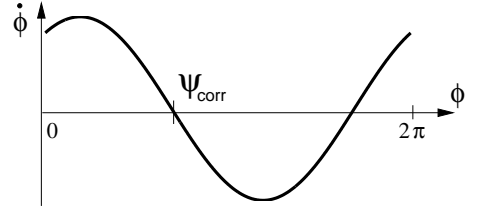


Figure 2: The dynamics of CORRIDOR FOLLOWING. At the direction  $\psi_{corr}$  in which the corridor has to be followed an attractor is generated.

### 3.2 Design of Individual Behaviours

We choose the robot heading  $\phi$  as the behavioural variable of the dynamical system as it offers the advantage that the corridor and obstacle behaviours can be naturally expressed in this variable. Furthermore, the commanded turn rate  $\dot{\phi}$  can be directly applied as a control action to the robot. The translational velocity is regulated by an external control loop, which reduces the robot speed based on the proximity of nearby obstacles. The behaviours CORRIDOR FOLLOWING and WALL AVOIDANCE navigate the robot along an empty corridor. The behaviour OBSTACLE AVOIDANCE prevents the robot from colliding with static or dynamic obstacles such as objects or people.

The behaviour CORRIDOR FOLLOWING is expected to align the robots heading with the corridor direction  $\psi_{corr}$  the robot is supposed to follow. Hence the behavioural dynamics has to possess an attractor at  $\psi_{corr}$ . To guarantee the continuity of the dynamics over the entire range of heading direction, the function  $f_{corr}$  is designed with a periodicity of  $2\pi$ . The simplest form that meets these criteria is given by (Figure 2):

$$\dot{\phi} = f_{corr}(\phi) = -\lambda_{corr} \sin(\phi - \psi_{corr}) \quad (5)$$

The strength of the attractor is defined by  $\lambda_{corr}$ .

The behaviour WALL AVOIDANCE is supposed to guide the robot towards the center of the corridor. Hence, for each wall the dynamics contains a repeller located at the direction of the wall  $\psi_{wall-1}$ ,  $\psi_{wall-2}$  resp. and an attractor along the opposite direction. The magnitude of the repeller should decrease with increasing distance to the wall at a rate determined by a gain  $C_{wall}$ . The stronger contribution of the closer wall dominates the repulsive force of the remote wall in a way that results in a repeller generated along the direction of the former. Again we require the function  $f_{wall}$  to be  $2\pi$ -periodic. These criteria are met by a

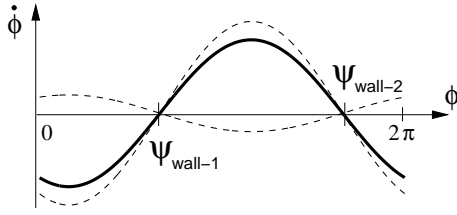


Figure 3: The dynamics of WALL AVOIDANCE. Each wall provides a contribution (dashed curves) with an attractor repeller pair. The sum of the two contributions forms a repeller in the direction of the closer wall ( $\psi_{wall-1}$  in this case). An attractor occurs at the opposite direction  $\psi_{wall-2}$ .

dynamics of the following form (Figure 3):

$$\dot{\phi} = f_{wall}(\phi) = \lambda_{wall} \sum_{l=1}^2 \left[ \sin(\phi - \psi_{wall-l}) \cdot \exp \left( -C_{wall} \frac{d_{wall-l} - R_r}{R_r} \right) \right] \quad (6)$$

$d_{wall-1}$  and  $d_{wall-2}$  denote the distances between the robot and the two walls and the term  $R_r$  represents the robot radius.

The behaviour OBSTACLE AVOIDANCE is expected to turn the robot away from the direction of nearby obstacles. In case of a single obstacle  $i$  the dynamics should create a repeller along the obstacle direction  $\psi_i$ . Since remote obstacles are less important than nearby obstacles, the magnitude of the repeller should decrease with increasing distance to the obstacle. An angular decay term captures the observation that obstacles along the current direction of motion pose a bigger threat than the ones on the side. All these criteria are met by the following dynamics (Figure 4):

$$f_i(\phi) = \lambda_{obst}(\phi - \psi_i) \cdot \exp \left( -C_{obst} \frac{d_i - R_r}{R_r} \right) \cdot \exp \left( -\frac{(\phi - \psi_i)^2}{2\sigma_i^2} \right) \quad (7)$$

The distance from the center of the robot to the obstacle is denoted by  $d_i$ . The angular range of the repeller is defined by  $\sigma_i$ .  $C_{obst}$  defines the decay of the strength of the repeller with increasing distance to the obstacle.

In case of multiple obstacles the resulting force  $f_{obst}(\phi)$  is computed by adding the contributions of individual obstacles.

$$\dot{\phi} = f_{obst}(\phi) = \sum_i f_i(\phi) \quad (8)$$

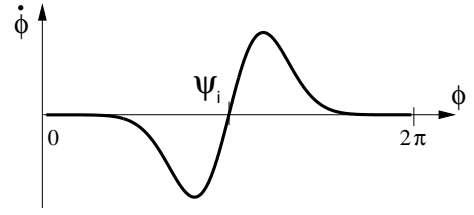


Figure 4: The dynamics of OBSTACLE AVOIDANCE for a single obstacle. At the direction  $\psi_i$  of the obstacle a repeller is generated.

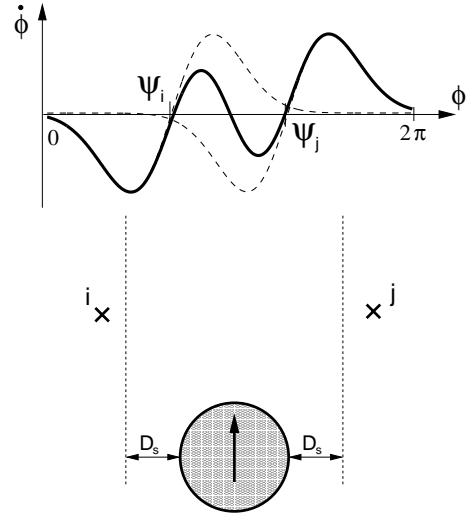


Figure 5: The dynamics in the case of two obstacles  $i$  and  $j$  (dashed curves). If the gap between the two allows to stay a safety distance  $D_s$  away from them, an attractor in the middle of the two is created.

The robot is supposed to pass between two obstacles, only if it is able to maintain a safety distance  $D_s$  to the obstacles located on either side of the robot. In other words, if the obstacles are too close to each other the dynamics should create a repeller along the direction of the gap (Figure 5). On the other hand, if the gap is sufficiently wide the dynamics should instead generate an attractor (Figure 6). By analyzing the fix-points of equation 8 it can be seen that this property can be achieved by an appropriate choice of the angular range  $\sigma_i$  of each individual obstacle contribution in equation 7.

$$\sigma_i = \arcsin \left( \frac{R_r + D_s}{d_i} \right) \quad (9)$$

The parameters of the behaviour WALL AVOIDANCE are determined in a similar fashion, according to the width of a gap between an obstacle and the corridor

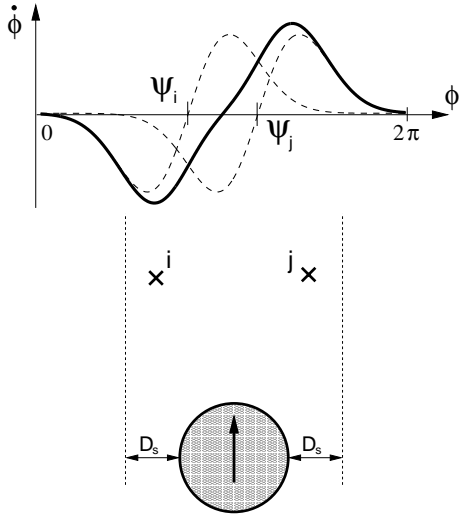


Figure 6: The dynamics in the case of two obstacles  $i$  and  $j$  (dashed curves). If the gap between the two does not allow to stay a safety distance  $D_s$  away from them, a repeller is created.

wall.

Finally the overall dynamics of the system is obtained from the weighted summation of individual behaviours based on equation 2:

$$\dot{\phi} = \sum_b |w_b| f_b(\phi) + \text{noise} \quad , b \in \{corr, wall, obst\} \quad (10)$$

### 3.3 Design of the Behavioural Coordination

For the coordination of the behaviours the competitive advantages and competitive interactions in equation 3 have to be chosen appropriately. The competitive advantages reflect the relevance and applicability of a behaviour in a particular context. Obviously, CORRIDOR FOLLOWING and WALL AVOIDANCE should be activated whenever the agent finds itself in a corridor and be turned off otherwise.

$$\alpha_{corr} = \alpha_{wall} = \begin{cases} 0.5 & \text{if in corridor} \\ -0.5 & \text{if not in corridor} \end{cases} \quad (11)$$

The relevance of OBSTACLE AVOIDANCE depends on the number and proximity of the obstacles currently surrounding the robot. The competitive advantage of OBSTACLE AVOIDANCE is related to the obstacle density

$$\rho = \sum_i \exp \left( -C_{obst} \frac{d_i - R_r}{R_r} \right) \quad (12)$$

and is computed according to

$$\alpha_{obst} = \tanh(\rho - \rho_0) \quad (13)$$

The constant  $\rho_0$  determines the density at which obstacle avoidance becomes relevant (i.e.  $\alpha_{obst} > 0$ ). The tangent hyperbolic assures that the magnitude of  $\alpha_{obst}$  is limited to the interval  $[-1, 1]$ . In some situations OBSTACLE AVOIDANCE and CORRIDOR FOLLOWING might interfere with each other in an undesirable, counterproductive manner. For example the attractor of corridor following might conceal the obstacle avoidance repeller generated by the narrow gap in Figure 6. Hence, if the obstacle density (equation 12) exceeds a critical threshold  $\rho_c$ , CORRIDOR FOLLOWING should be suppressed. This type of prioritization is achieved by appropriately choosing the competitive interaction  $\gamma_{obst,corr}$ .

$$\gamma_{obst,corr} = \frac{1}{2}(1 + \tanh(\rho - \rho_c)) \quad (14)$$

The functional form of the above term is chosen such that  $\gamma_{obst,corr} \in [0, 1]$ . Since there exist no potential conflicts among any other pair of behaviours all other competitive interactions  $\gamma_{b',b}$  are set to zero.

Finally, the time constants  $\tau_{obst}$  and  $\tau_{wall}$  are chosen, such that the robot reacts almost immediately if a new obstacle is perceived. The dynamics of  $w_{corr}$  evolves at a slower rate  $\tau_{corr} \gg \tau_{obst}$ . Once OBSTACLE AVOIDANCE becomes less relevant, e.g. after the robot circumnavigates an obstacle, CORRIDOR FOLLOWING switches on gradually rather than causing jitter among both behaviours. This concludes the design of our system.

## 4 Results

Figure 7 shows the trajectory of the robot driving along the corridor of our institute. The corridor is partially blocked by three open office doors and a number of cluttered obstacles such as chairs and boxes. During its passage the robot encounters different situations denoted by the symbols A–I. Figure 8 depicts the evolution of the activations  $w_{corr}$  and  $w_{obst}$  of the CORRIDOR FOLLOWING and OBSTACLE AVOIDANCE behaviours. The labeled ticks on the time axis refer to the corresponding location of the robot in the corridor. Notice, that the time difference between two successive ticks is not proportional to the path length between the corresponding events, as the robot does not move at a constant speed.

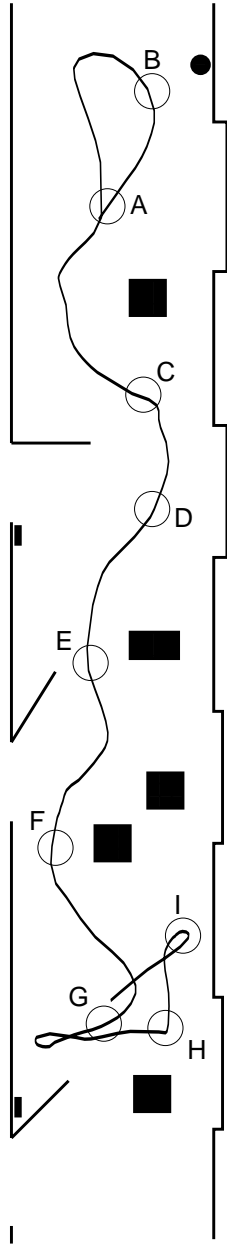


Figure 7: The trajectory of the robot traveling along the corridor of our institute. There are three open doors to offices. The black rectangles denote obstacles (chairs and boxes). The situations labeled by the symbols A–I are explained in the text. The circles at these locations depict the size of the robot.

A: The starting position: The robot is supposed to travel the corridor along the downward direction, but is initially headed in the opposite direction. The CORRIDOR FOLLOWING behaviour

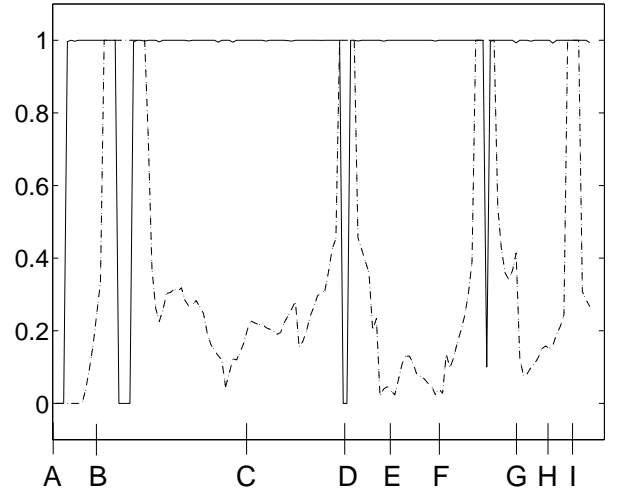


Figure 8: Time plot of the absolute values of the weights:  $|w_{corr}|$  (dashed curve) and  $|w_{obst}|$  (solid curve) (see equation 10). The time instances labeled by the symbols A–I correspond to the situations in Figure 7. See text for detailed explanations.

remains switched off until a sufficient number of sonar readings is collected in order to compute the Hough transform and extract the walls (section 3.1).

- B: Enough sonar readings are collected: Once enough sonar data is collected to reliably estimate the orientation of the walls, CORRIDOR FOLLOWING is switched on and the robot starts to align its heading with the the desired corridor direction. After the 180°-turn is completed, OBSTACLE AVOIDANCE is turned off for a brief period, as in that region the corridor contains no obstacles.
- C: The robot circumnavigates around the obstacle in the middle of the corridor and the open door located to its right. Due to the presence of obstacles  $|w_{corr}|$  decreases, but nevertheless the CORRIDOR FOLLOWING behaviour remains active to some extent.
- D: The robot travels through an uncluttered area. CORRIDOR FOLLOWING is switched on, whereas OBSTACLE AVOIDANCE becomes inactive.
- E: The robot reaches a narrow passage, which however is wider than the critical distance  $2(R_r + D_s)$  and therefore considered passable. The robot is able to traverse the gap using OBSTACLE AVOIDANCE alone (compare to Figure 5).

- F: The robot faces another narrow passage between a wall and an obstacle, which again is wide enough to allow passage.
- G: The robot encounters two obstacles, which are located closer to each other than the critical safety distance  $2(R_r + D_s)$ . CORRIDOR FOLLOWING is temporarily switched off. OBSTACLE AVOIDANCE takes over control and steers the robot away from the gap (compare to Figure 6).
- H: The corridor is blocked: Since CORRIDOR FOLLOWING is switched off, the robot momentarily heads off in the direction opposite to the commanded bearing.
- I: The robot resumes its original heading: Once the robot drives away from the obstacles, CORRIDOR FOLLOWING is switched back on and the robot resumes its original direction towards the obstacles. This sequence of the three steps  $G - I$  will now be repeated until the blockage is removed such that the robot can continue to pursue its intended path.

## 5 Discussion

We presented a control scheme which successfully navigates a mobile robot through a cluttered real-world corridor environment. The dynamic system approach provides a suitable means to the analytical design of robotic behaviours to solve navigation tasks. The behaviours rely on an approximate, simple geometric representation of the environment that directly anchors on the information provided from low level sensors.

The behaviours are designed in a way that enables the robot to distinguish between those passages that are too narrow to pass and gaps that are wide enough to traverse safely. The coordination among multiple behaviours allows the robot to cope with more complex situations, such as partially or entirely blocked corridors, in a flexible manner.

Future research in this project will be directed towards the design and integration of additional behaviours such as door traversal and docking at a charging station. Once the environment becomes more complex, for example a room cluttered with a large number of obstacles, memory and planning become a necessity to overcome the limitations of purely reactive behaviours. We consider to integrate planning and behaviour coordination into a coherent framework that

allows the robot to explore alternative strategies in order to achieve a particular task. A particular plan is formulated as an ordered sequence of subsets of desirable and simultaneously applicable behaviours.

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