HW4

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Q1.1

Since we know

$$x_2^T F x_1 = 0$$

Where x1 and x2 are the coordinates of two cameras and in this particular condition we know that

$$x_1 = x_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Thus, we get

$$x_2^T F x_1 = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 0$$

From the above equation, we can get that F_{33} = 0

Q1.2

We know that the essential matrix is calculated by translation matrix and rotation matrix as

$$E = t_c R$$

Where

$$t_c = \begin{pmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{pmatrix}$$

Since in this particular question, we know that translation is parallel to the x-axis, so y and z coordinate is zero, thus,

$$t_c = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{pmatrix}$$

Since pure rotation

$$R = \begin{pmatrix} 1\,0\,0 \\ 0\,1\,0 \\ 0\,0\,1 \end{pmatrix}$$

And

$$E = t_c R = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{pmatrix}$$

And we know the equation of epipolar lines

$$Line = x^T E$$

In this case

$$L_1 = \begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix}^T \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{pmatrix} = (0 \quad t_x \quad -y_2 t_x)$$

$$L_2 = \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix}^T \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{pmatrix} = (0 \quad t_x \quad -y_1 t_x)$$

So as we can see that the first coordinate of the above equations are both 0 so we can get that the lines in the two cameras are irrelevant to x coordinate so they are both parallel to the x-axis, for example

$$L_1 \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = t_x y - y_2 t_x = 0$$

$$L_2 \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = -t_x y + y_2 t_x = 0$$

So they are both parallel to the x-axis.

Q1.3

For example, we have some real-world point $p = (x \;\; y \;\; z)$

The points at different time in the frame of reference camera are

$$p_1 = R_1 p + t_1$$

$$p_2 = R_2 p + t_2$$

So, we can get the expression of p

$$p = R_1^{-1}(p_1 - t_1)$$

From which we can get expression of p_2

$$p_2 = R_2 R_1^{-1}(p_1 - t_1) + t_2 = R_2 R_1^{-1} p_1 - R_2 R_1^{-1} t_1 + t_2$$

So, we can see that

$$R_{rel} = R_2 R_1^{-1}$$

$$t_{rel} = -R_2 R_1^{-1} t_1 + t_2$$

The essential matrix is

$$E = t_{rel} \times R_{rel}$$

Therefore, the fundamental matrix is

$$F = K^{-T} t_{rel} \times R_{rel} K^{-1}$$

Q1.4

Assume the point is p and the corresponding points in image1 and image2 are x_1 and x_2. And its reflection in the plane mirror is p' and the corresponding points in image1 and image2 are ${x_1}'$ and ${x_2}'$ .From the question description, we know that x_1 and ${x_1}'$ are symmetric as well as x_2 and ${x_2}'$, thus we can write that

$$x_1^T F x_2 = x_2' F^T x_1' = x_1^T F^T x_2 = 0$$

Where F is the fundamental matrix and if we add them together

$$x_1^T F x_2 + x_1^T F^T x_2 = 0$$

$$x_1^T (F+F^T) x_2 = 0$$

$$(F + F^T) = 0$$

$$F = -F^T$$

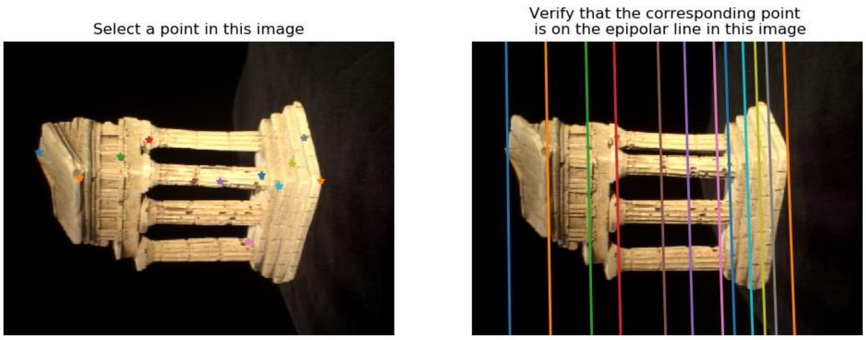
So F is a skew-symmetric matrix

Q2.1

The recovered F is shown below:

```
[[ 9.78833282e-10 -1.32135929e-07  1.12585666e-03]
 [-5.73843315e-08  2.96800276e-09 -1.17611996e-05]
 [-1.08269003e-03  3.04846703e-05 -4.47032655e-03]]
```

Below is an image of some example output of displayEpipolarF.



Q3.1

```
[[ 2.26268683e-03 -3.06552495e-01  1.66260633e+00]
 [-1.33130407e-01  6.91061098e-03 -4.33003420e-02]
 [-1.66721070e+00 -1.33210351e-02 -6.72186431e-04]]
```

Q3.2

We have

$$C_1 = \begin{bmatrix} c_{11}^1 & c_{12}^1 & c_{13}^1 & c_{14}^1 \\ c_{21}^1 & c_{22}^1 & c_{23}^1 & c_{24}^1 \\ c_{31}^1 & c_{32}^1 & c_{33}^1 & c_{34}^1 \end{bmatrix}$$

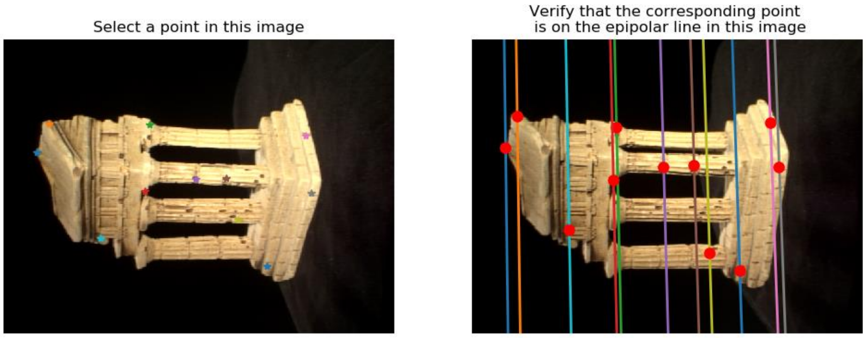
$$C_2 = \begin{bmatrix} c_{11}^2 & c_{12}^2 & c_{13}^2 & c_{14}^2 \\ c_{21}^2 & c_{22}^2 & c_{23}^2 & c_{24}^2 \\ c_{31}^2 & c_{32}^2 & c_{33}^2 & c_{34}^2 \end{bmatrix}$$

My expression for A is

$$A = \begin{bmatrix} x_1 c_{31}^1 - c_{11}^1 & x_1 c_{32}^1 - c_{12}^1 & x_1 c_{33}^1 - c_{13}^1 & x_1 c_{34}^1 - c_{14}^1 \\ y_1 c_{31}^1 - c_{21}^1 & y_1 c_{32}^1 - c_{22}^1 & y_1 c_{33}^1 - c_{23}^1 & y_1 c_{34}^1 - c_{24}^1 \\ x_2 c_{31}^2 - c_{11}^2 & x_2 c_{32}^2 - c_{12}^2 & x_2 c_{33}^2 - c_{13}^2 & x_2 c_{34}^2 - c_{14}^2 \\ y_2 c_{31}^2 - c_{21}^2 & y_2 c_{32}^2 - c_{22}^2 & y_2 c_{33}^2 - c_{23}^2 & y_2 c_{34}^2 - c_{24}^2 \end{bmatrix}$$

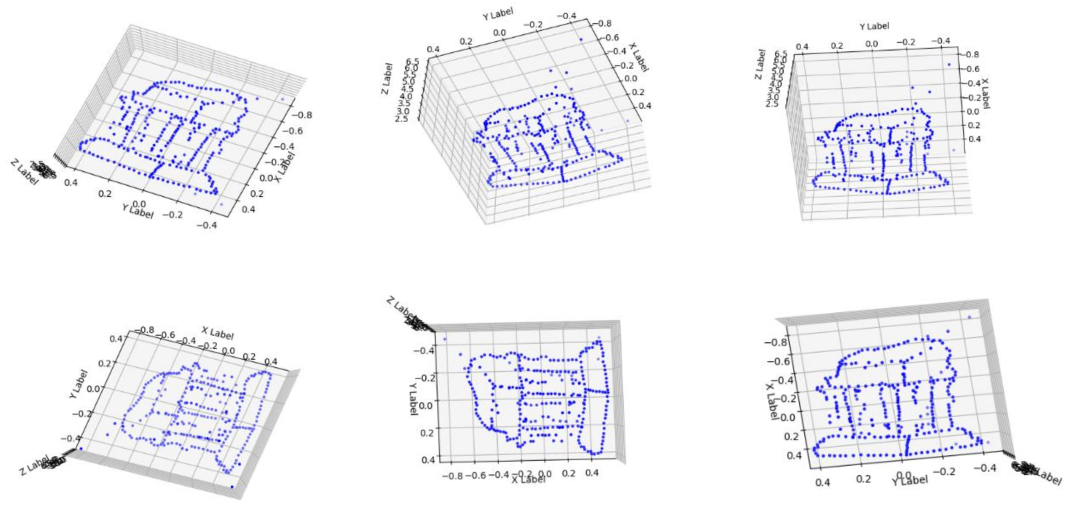
Q4.1

Below is a screenshot of epipolarMatchGUI with some detected correspondences.



Q4.2

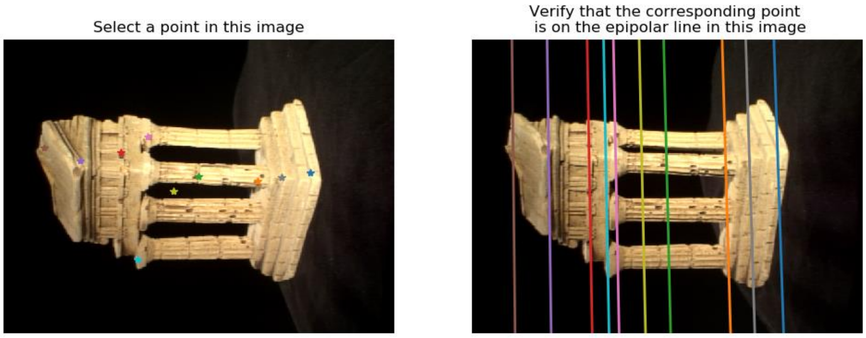
The following are some of the screenshots of temple 3Dvisualization.



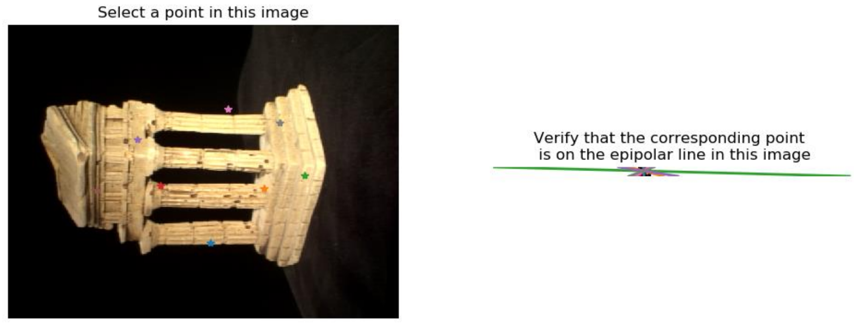
Q5.1

In ransacF method, I apply a ransac to the dataset. In each iteration, eight data sets of coordinates are randomly selected from the given points from two images, then I calculate the fundamental matrix using these eight data sets of points. Then the epipolar line on image2 coresponding to the points in image1 is calculated by the fundamental matrix. Then I calculate the distance from point in pts2 to the epipolar line using the equation of distance from point to line. If the distance is smaller than the threshold, we consider it as an inlier. So finally, out of the loop, I use inliers to pass to eightpoint function to get a refined fundamental matrix.

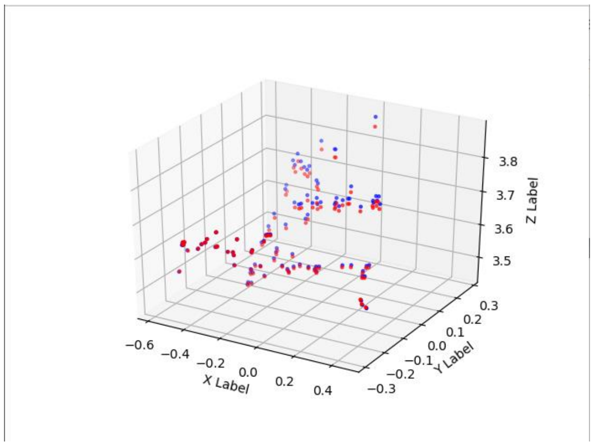
Below is the result of displayEpipolarF using Ransac with 100 iteration and the threshold is 3.8. My default iteration and threshold for Ransac function is 500 and 0.4. (The result varies a little bit in each trail so not always get such a good result, sometimes the line would tilt a little bit)



Below is the result of using eightpoint function without Ransac



5.3



Red is the original 3D points and blue is the optimized points

Reprojection error with your initial M2 and w is 117.29
Reprojection error with the optimized matrices is 9.98