Homework No.6

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Problem Statement

Show that the Fourier transforms of

(a) $f(ax)$ is $\frac{1}{a}F(\frac{u}{a})$, where a us any nonzero real number

(b) $f(x-x_0)$ is $F(u)exp(-j2\pi ux_0)$

Answer

(a)

1. (=>) ：Let $t=ax, dt=adx$ => $dx=\frac{1}{a}dt$

$$\begin{aligned} &\frac{1}{2\pi}\int_{-\infty}^{\infty} f(ax)exp(-jux)dx \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)exp(-ju\frac{t}{a})\ \frac{1}{a}dt \\ &= \frac{1}{a}\ F(\frac{u}{a}) \end{aligned}$$

2. (<=) ：Let $t=\frac{u}{a}, dt=\frac{1}{a}\ du$ => $du=adt$

$$\begin{aligned} &\int_{-\infty}^{\infty} \frac{1}{a}\ F(\frac{u}{a})\ exp(jux)\ du \\ &= \int_{-\infty}^{\infty} \frac{1}{a}\ F(t)\ exp(j\ at\ x)\ a\ dt \\ &= \int_{-\infty}^{\infty} \frac{1}{a}\ F(t)\ exp(jt\ ax)\ a\ dt \\ &= \int_{-\infty}^{\infty}\ F(t)\ exp(jt\ ax)\ dt \\ &= f(ax) \end{aligned}$$

(b)

1. (=>)：Let $t=x-x_0, dt=dx$

$$\int_{-\infty}^{\infty} f(x - x_0) exp(-j2\pi ux) dx$$
$$= \int_{-\infty}^{\infty} f(t) exp(-j2\pi u(t + x_0)) dt$$
$$= \int_{-\infty}^{\infty} f(t) exp((-j2\pi ut) + (-j2\pi ux_0)) dt$$
$$= \int_{-\infty}^{\infty} f(t) exp(-j2\pi ut) exp(-j2\pi ux_0) dt$$
$$= F(u)\ exp(-j2\pi ux_0)$$

2. (<=)

$$\int_{-\infty}^{\infty} F(u) exp(-j2\pi ux_0) exp(-j2\pi ux) du$$
$$\int_{-\infty}^{\infty} F(u) exp((-j2\pi ux_0) + (-j2\pi ux)) du$$
$$\int_{-\infty}^{\infty} F(u) exp(-j2\pi u(x + x_0)) du$$
$$= f(x + x_0)$$