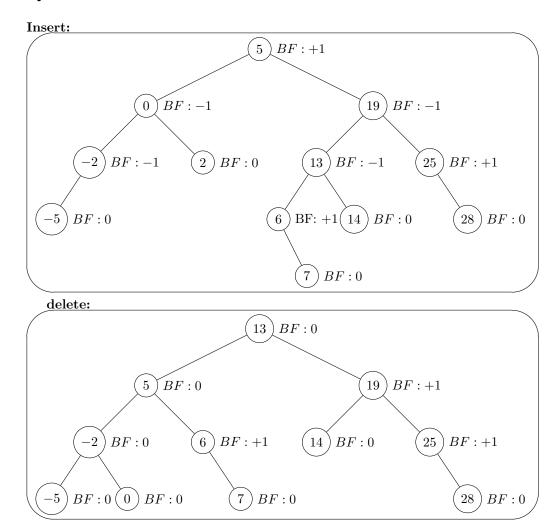
CSC263H1 Assignment 3

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Question 1



Question 2

Description:

We add one more AVL tree such that each of its node contains price (which is also the key), the pointer to its left child, the pointer points to its right child, and the max_rating that stores the maximum rating among all nodes in the subtree rotted at the current node.

```
def BestBookRating(D, p):
    if D is leaf:
        if D.price <= p:
            return D.rating
        return -1
    elif D only has right child:
        if D.price <= p:
            return max(BestBookRating(D.right, p), D.rating)
        return -1</pre>
```

```
elif D only has left child:
    if D.price <= p:
        return max(D.left.max_rating, D.rating)
    return -1
elif D has both left and right child:
    if D.price > p:
        return BestBookRating(D.left, p)
    return max(BestBookRating(D.left, p), BestBookRating(D.right, p), D.rating)
```

Question 3

a. Our data structure is based on hash table.

Idea: Put every element of set \mathbf{B} into a hash table. Then hash every element of set \mathbf{A} to a slot in the hash table, and check whether the element of \mathbf{A} is in this slot. If it is not in the slot, then it means that the element of set \mathbf{A} is not in set \mathbf{B} .

Pseudo Code:

Suppose $\alpha = 10$, that is, each slot contains a linked list with size of at most approximately 10 elements. Suppose we have a hash table T with a size of $\frac{n}{\alpha}$.

Suppose we have a hashing function h(x) that would return the index of one of the slots of **T** given x as a input. Also assume **SUHA**.

```
def h(x, num_slot):
    return x % num_slot

for element in B:
    linked_list = T[h(element, len(T))]
    linked_list.append(element)

result = []
for element in A:
    linked_list = T[h(element, len(T))]
for item in linked_list:
    if element == item:
        break
result.append(element)
```

b. Assumptions:

- The linked list in each slot of hash table has approximately a length of $\alpha = 10$
- Hash table **T** has a length of $\frac{len(A)}{\alpha}$
- SUHA for hash function $h(x, num_slot)$

Explanation: Part I: put B into T

- 1. Hashing function costs constant time
- 2. There are n elements in **B**, performing hashing function h for each element of **B** costs $\mathcal{O}(n \times 1)$ of time.

Part II: match element of **A** to **T**

- 1. Each linked list in each slot of **T** has a size of at most α (by **SUHA**), which is constant. In the worst case, we have to traverse through every linked list, which costs $\mathcal{O}(\alpha) = \mathcal{O}(1)$ of time.
- 2. There are n elements in **A**, performing step 1 for each of them costs $\mathcal{O}(\alpha n) = \mathcal{O}(n)$ of time.

Part I and Part II altogether cost $\mathcal{O}(n)$ of time.

c. In The worst case scenario, **SUHA** may not hold, and every element of B may be hashed into one single slot of **T**. Then when element of set **A** tries to find a matching element in this slot, it needs to traverse through the entire linked list and takes $\mathcal{O}(n)$ of time.

While there are n elements in set \mathbf{A} . If for each of the element in \mathbf{A} , the program traverses through the linked list of size n and do not exit early, it will take $\mathcal{O}(n \times n) = \mathcal{O}(n^2)$ of time.