Problem 7

proof:

For $n = 1$,we know

$$2^1 = 4 - 2 = 2^{1+1} - 2$$

So the equality is valid for $n = 1$.

We assume that the equality is valid for $n = k$, where $k \geq 1$.

It is that

$$2 + 2^2 + ...2^k = 2^{k+1} - 2$$

For $n = k + 1$,

$$\begin{aligned} 2 + 2^2 + ...2^k + 2^{k+1} &= 2^{k+1} - 2 + 2^{k+1} \ (Assumption) \\ &= 2^{k+2} - 2 \\ &= 2^{(k+1)+1} - 2 \end{aligned} \tag{1}$$

So the equality is valid for $n = k + 1$.

So for any natural number n, the equality is valid .And the **Theorem** is right.

The proof is complete.