

Problem 4

proof:

We know every odd natural number a can be represented as the form $2k + 1$, where k is a natural number.

And it is obvious that k is either odd or even.

If k is odd, it means that k has the form

$$k = 2n + 1 \text{ (} n \text{ is a natural number)}$$

Then:

$$a = 2k + 1 = 4n + 3$$

If k is even, it means that k has the form

$$k = 2n \text{ (} n \text{ is a natural number)}$$

Then:

$$a = 2k + 1 = 4n + 1$$

So every odd natural number has the form either $4n + 3$ or $4n + 1$.

The proof is complete.