

Problem 3

It is true.

proof:

For any integer n , we know

$$n^2 + n + 1 = n(n + 1) + 1$$

And it is obvious that n is either odd or even.

If n is odd, then $n + 1$ is even, and $2|(n + 1)|n(n + 1)$, $n(n + 1)$ is even.

If n is even, then $2|n|n(n + 1)$, $n(n + 1)$ is even.

Above all, we know $n(n + 1)$ is always even. So $n(n + 1) + 1$ is odd.

The proof is complete.