

Problem 5

proof:

For any integer n , we know that if n is divided by 3, the remainder is 0, 1 or 2.

It is that

$$n = 3k, \text{ or } n = 3k + 1, \text{ or } n = 3k + 2 \text{ (} k \text{ is an integer)}$$

If $n = 3k$, then:

$$3|3k|n$$

so n is divisible by 3.

If $n = 3k + 1$, then:

$$n + 2 = 3k + 3 = 3(k + 1)$$

$$3|3(k + 1)|(n + 2)$$

so $n + 2$ is divisible by 3.

If $n = 3k + 2$, then:

$$n + 4 = 3k + 6 = 3(k + 2)$$

$$3|3(k + 2)|(n + 4)$$

so $n + 4$ is divisible by 3.

So for any integer n , at least one of the integers n , $n+2$, $n+4$ is divisible by 3.

The proof is complete.