Problem 10

We give the example that $A_n = \{x|x \in (-\frac{1}{2^n}, \frac{1}{2^n})\},\ n = 1, 2, ...$

proof:

Since

$$\frac{1}{2^{n+1}} < \frac{1}{2^n},\ -\frac{1}{2^{n+1}} > -\frac{1}{2^n}$$

We know

$$(-\frac{1}{2^{n+1}}, \frac{1}{2^{n+1}}) \subseteq (-\frac{1}{2^n}, \frac{1}{2^n})$$

So,

$$A_{n+1} \subseteq A_n,\ n = 1, 2, ...$$

Clearly,

$$\frac{1}{2^n} > 0,\ 0 < -\frac{1}{2^n},\ n = 1, 2, ...$$

So,

$$0 \in (-\frac{1}{2^n}, \frac{1}{2^n}),\ n = 1, 2, ...$$

$$0 \in A_n\ n = 1, 2, ...$$

$$0 \in \bigcap_{n=1}^{\infty} A_n$$

The proof is complete.

The exmple $A_n = \{x|x \in (-\frac{1}{2^n}, \frac{1}{2^n})\},\ n = 1, 2, ...$ has the stated property.