The Application of R-T-S Smoothing Algorithm in the Post-processing of the Integrated Navigation

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Abstract: Integrated navigation system usually uses Kalman filter to make it possible for error compensation. In order to improve the precision of navigation and the stability of data, we introduce the R-T-S (Rauch-Tung-Striebel) optimal fixed-interval smoothing into the post-processing of data. On the basis of the forward Kalman filter, we add the backward information filter to the system and use the measured data to verify the algorithm. The results show that compared with the traditional Kalman filter, the R-T-S optimal fixed-interval smoothing can not only improve the precision of the position and posture but can also improve the precision of navigation significantly in case of lock-lose, making it possible as an effective way of data processing.

Key Words: Integrated navigation; Kalman filter; post-processing; optimal smoothing algorithm

1 INTRODUCTION

Integrated navigation is the inevitable result of the development of modern navigation theory and technology. Because each kind of navigation system has its own uniqueness and limitations, it is difficult to find the navigation system which has the qualification to do well in all aspects. But if we combine two or more navigation systems, with the application of modern information fusion theory and some specific algorithms, it is realistic to make full use of the effective information of each sensor and the advantages of each navigation system, constituting the integrated navigation system with high precision and high reliability. With the help of all the advantages of each sensor and each navigation system, the integrated navigation system is obviously better than that of the single one in the performance, which makes it easy to have wide development prospects than others [1].

Due to the swift development of micro-inertia device, MINS (Micro Inertial Navigation System)/GPS (Navigation Satellite Timing and Ranging Global Position System) has been put into application in some areas in recent years. It overcomes the defect that the error of the low-precision internal navigation system is accumulated over time while working alone and the inherent limitations of GPS. GPS can not only provide high-precision position and velocity data, but can also align and calibrate MINS continuously. MINS can provide high-accuracy data of the velocity in short term and can also help receivers to shorten the time needed to capture and recapture the satellite [2]. MINS can be directly put into use to get the acceleration and orientation data for navigation while the disruption of GPS occurs. Therefore, the precision of MINS/GPS is higher than each one of the two navigation system working alone and it improves the reliability of the system [3].

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By far, it commends to use the Kalman filter to process the data in the project. But the accuracy is not high in case of lock-lose, so does its limit precision. In this paper, the R-T-S smoothing algorithm is applied to data processing and verified by the real data of the experiment. The results show that compared with the traditional Kalman filter, R-T-S smoothing algorithm can not only improve the precision of the position and posture, but also improve the precision in case of lock-lose, which turns out to be an effective data processing algorithm [4].

2 INSTRUCTIONS POST-PROCESSING ALGORITHM

The information of GPS can be used to estimate the deviation of the inertial equipment. In case of the lock-lose of the satellite signal, the information provided by inertial navigation equipment can be used to estimate the motion of the carrier, so as to reduce the time required to recapture the satellite signal [5].

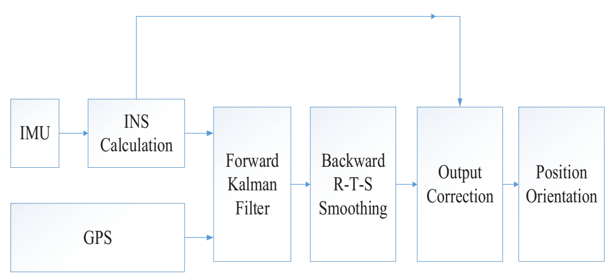

Fig 1. The flow diagram of post-processing algorithm

Post-processing algorithm contains two main procedures. First we use the acceleration and angular velocity measured by IMU for inertial navigation calculation to get the data of position and velocity. We use them together with the information of GPS to make a difference [6]. The result is used as a prior observation of Kalman filtering. Then we use the covariance matrix and state transition matrix gained from forward Kalman filtering process to do the reverse R-T-S smooth to the state estimate [7]. According to the result of the output of the smooth we correct the position, velocity and posture output by the INS, which is the final output of the system.

3 SMOOTHING TECHNOLOGY

Smoothing technology, which as a method of afterwards or real-time data processing, can significantly improve the accuracy of data processing, obtaining widespread application in the field of surveying and mapping [8]. Smoothing technology is divided into three categories, fixed-interval smoothing, fixed-point smoothing and fixed-lag smoothing. Among all the categories, the fixed-interval smoothing is the most widely applied in the data processing. The schematic diagram is shown in Fig 2. It is the inverse filtering based on the forward Kalman filter, making full use of all the time range measurements to estimate the state. This method can provide higher precision than one-way filter, and it is also a good bridging algorithm in case of lock-lose of the satellite signal [9].

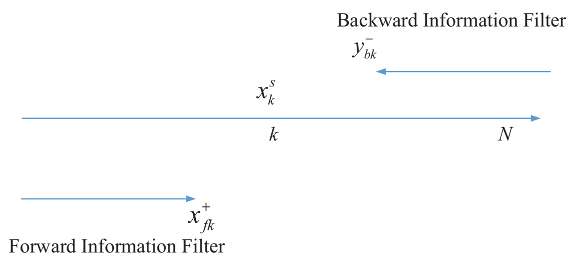

Fig 2. Schematic diagram of fixed-interval smoothing

4 RAUCH-TUNG-STRIEBEL(R-T-S) ALGORITHM

Rauch-Tung-Striebel(R-T-S) algorithm is an optimal fixed-interval smoothing algorithm. In the literature R-T-S optimal fixed interval smoothing algorithm is applied to the GPS/INS integrated navigation system and the result shows that the accuracy of the R-T-S fixed-interval smoothing algorithm is higher than the accuracy of Kalman filtering [10]. It turns out that this method is simple and easy to implement, making it possible as a kind of effective post-processing method.

R-T-S smooth is made up of forward filter and backward filter. Forward filter is a classical Kalman filter, which is used to estimate the state of each moment. Backward filter is to obtain more accurate state estimate on the basis of forward filter [11].

Considering the linear time-invariant system

$$X_k = \Phi_{k,k-1} X_{k-1} + \Gamma_{k-1} W_{k-1} \tag{1}$$

$$Z_k = H_k X_k + V_k \tag{2}$$

In the equation, the state vector $X_k \in R^n$, the observation vector is $Z_k \in R^m$, the process noise is $W_k \in R^r$, the measurement noise is $V_k \in R^m$ ($k = 0,1,\cdots,N$), W_k and V_k are white Gaussian noise with zero mean. Q and R are the variance. The original state X_0 is the Gaussian sequence with certain mean and variance and $E(X_0) = mx_0$, $\mathrm{var}(X_0) = P_0$. There is no correlation among X_0, W_k and V_k.

Assume that we know the measurements Z_0, $Z_1,\cdots,$ Z_k and want to figure out the optimal linear estimate $\hat{X}_{j,lk}$ from X_j. If $j > k$, $\hat{X}_{j,lk}$ is called the forecast estimate of the system state. If $j = k$, $\hat{X}_{j,lk}$ is called the filter estimation of the system state. If $j < k$, $\hat{X}_{j,lk}$ is called the smooth estimate of the system state. Because the method of smooth uses all the information of the measurement, it has the highest accuracy in principle [12]. Due to the changes of k and j, the optimal smoothing method is divided into three categories, fixed-interval smoothing, fixed-point smoothing and fixed-lag smoothing. The fixed-interval smoothing can make full use of all the measurements $\bar{Z}_N = \left[Z_1^T \; Z_2^T \cdots Z_N^T \right]^T$ in the fixed time interval $[0 \;\; N]$ to estimate the state X_k ($k = 0,1,\cdots,N$) of each moment.

Before performing R-T-S optimal fixed-interval smoothing algorithm, we need to filter the system shown in the equation (1) in the time interval $[0 \;\; N]$ with Kalman filter firstly [13]. The recursion formula of Kalman filter is shown as follows.

The one-step prediction of the state vector and the variance matrix are shown below

$$\hat{X}_F(k \mid k-1) = \Phi_{k,k-1} \hat{X}_F(k-1 \mid k-1) \tag{3}$$

$$P_F(k \mid k-1) = \Phi_{k,k-1} P_F(k-1 \mid k-1) \times \Phi_{k,k-1}^T + \Gamma_{k-1} Q_{k-1} \Gamma_{k-1}^T \tag{4}$$

The subscript F represents Kalman filter in the equation.

The filtering gain is shown below.

$$K_F(k) = P_F(k \mid k-1) H_k^T \times \left[H_k P_F(k \mid k-1) H_k^T + R_k \right]^{-1} \tag{5}$$

The update of state vector and the variance matrix is shown below.

$$\hat{X}_F(k \mid k) = \hat{X}_F(k \mid k-1) + K_F(k) \left[Z_k - H_k \hat{X}_F(k \mid k-1) \right] \tag{6}$$

$$P_F(k \mid k) = \left[I - K_F(k) H_k \right] P_F(k \mid k-1) \tag{7}$$

At the same time we store the estimate value and the prediction $\hat{X}_F(k \mid k)$, $\hat{X}_F(k \mid k-1)$, $P_F(k \mid k)$ and $P_F(k \mid k-1)$ of the state vector and the variance matrix of each time in the process of Kalman filter.

After the Kalman filtering, we use the data stored in the filtering process to do the R-T-S optimal fixed-interval smoothing. Before we start we need to initialize the smoother. Let $k = N$ so we can get the following equation.

$$\hat{X}_S(N|N) = \hat{X}_F(N|N) \tag{8}$$

$$P_S(N|N) = P_F(N|N) \tag{9}$$

The subscript S represents the optimal smooth in the equation. During the time interval $[N-1 \;\; 0]$, the recursion formula of R-T-S fixed-interval smoothing algorithm is shown as follows.

The gain of smooth is shown below.

$$K_S(k) = P_F(k|k)\Phi_{k+1,k}^T P_F^{-1}(k+1|k) \tag{10}$$

The smooth state vector and the updating variance matrix is shown below.

$$\begin{aligned}\hat{X}_S(k|N) = &\hat{X}_F(k|k) + \\ &K_S(k)\left(\hat{X}_S(k+1|N) - \hat{X}_F(k+1|k)\right)\end{aligned} \tag{11}$$

$$\begin{aligned}P_S(k|N) = &P_F(k|k) + K_S(k)\big(P_S(k+1|N) - \\ &P_F(k+1|k)\big)K_S^T(k)\end{aligned} \tag{12}$$

We can know from the equation (10) that the smooth recursion equation is a backward process from $k = N-1$ to $k = 0$. And in the smooth recursion progress we need to use the estimate value and the prediction $\hat{X}_F(k|k)$, $\hat{X}_F(k|k-1)$, $P_F(k|k)$ and $P_F(k|k-1)$ of the state vector and the variance matrix [14]. As a result, R-T-S algorithm is the combination of the forward filtering algorithm and the backward smoothing algorithm [15].

5 THE ERROR EQUATION AND MEASUREMENT EQUATION OF INTEGRATED NAVIGATION SYSTEM

We use the NEU coordinate system (Local Cartesian Coordinates Coordinate System) as the navigation coordinate system in this article. The state variables are shown as follows.

$$X_I = [\varphi_E \;\; \varphi_N \;\; \varphi_U \;\; \delta v_E \;\; \delta v_N \;\; \delta v_U \;\; \delta L \;\; \delta\lambda \;\; \delta h \;\; \varepsilon_x \;\; \varepsilon_y \;\; \varepsilon_z \;\; \nabla_x \;\; \nabla_y \;\; \nabla_z]^T$$

φ_E、φ_N、φ_U are mathematical platform angles. δv_E、δv_N、δv_U are velocity error of the carrier in east, north and vertical respectively. δL、$\delta\lambda$、δh are the latitude error, the longitude error and the height error respectively. ε_x、ε_y、ε_z、∇_x、∇_y、∇_z are the random constant drift of the gyroscope and the random constant zero bias of the accelerometer.

Let the system be the linear system and use the loose coupling as the working mode. The state equation is shown below.

$$\dot{X}_I(t) = F_I(t)X_I(t) + G_I(t)W_I(t)$$

The measurement equation is shown below.

$$\begin{aligned}Z(t) &= \begin{bmatrix} Z_V(t) \\ Z_P(t) \end{bmatrix} = \begin{bmatrix} H_V \\ H_P \end{bmatrix} X(t) + \begin{bmatrix} V_V(t) \\ V_P(t) \end{bmatrix} \\ &= H(t)X(t) + V(t)\end{aligned}$$

All the coefficient matrix in the equation can be figured out with navigation relation. Because of the limited passage, there is no detailed discussion on the derivations.

6 EXPERIMENT AND ANALYSIS

In order to verify the R-T-S algorithm in the application of the post-processing of data, we do the simulation with real data get from the experiment. We get three set of MIMU on board the car and conduct three groups of trials. Each set of experimental installation method is shown in Fig 3.

Fig 3. The experimental installation method

According to the installation (a), we stop at the benchmark J01 for about ten minutes to acquire the output of MIMU. Then we drive the car along the Beiqing Road until the next stop and we drive back in a straight line as far as possible. And we acquire the output of the MIMU during the journey. After that we repeat trials according to the installation (b) (c).

Fig 4. The Beiqing Road in experiment

We filter the GPS/MINS integrated navigation system with forward Kalman filter in matlab environment first and store the estimate value of each time from Kalman filter. After the experiment we use R-T-S optimal smoothing algorithm to do the post-processing to the system. Then we get the simulation diagram of the position error, the posture error and the integrated position.

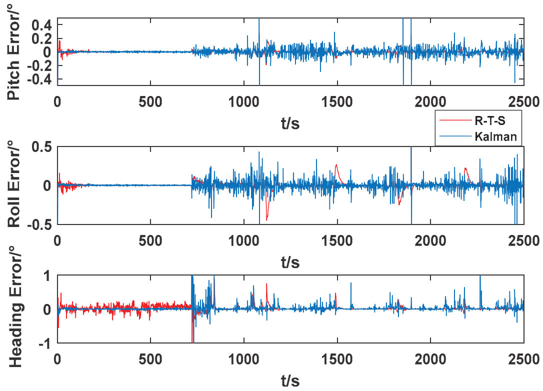

Fig 5. The posture error

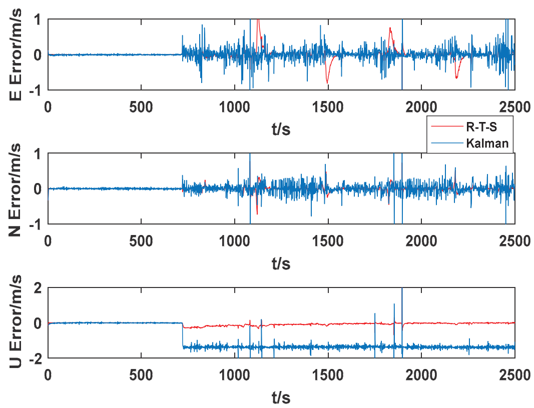

Fig 6. The velocity error

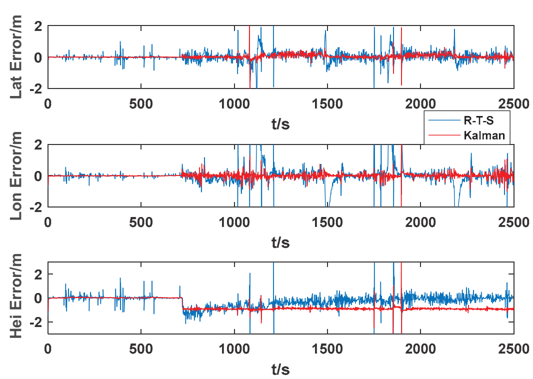

Fig 7. The position error

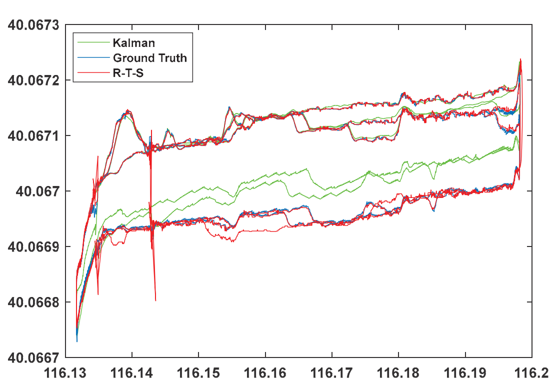

Fig 8. The position

From the figure above we can see that compared with Kalman filer, the R-T-S optimal fixer-interval smoothing algorithm can significantly reduce the posture error, the velocity error and the position error, which contribute most to the navigation. In Fig 5, there is no big difference between two methods when the car keeps stationary. The posture error is smaller with the method of R-T-S when the car begins to move from about 700 second. What's more, the method of R-T-S has an even change on the estimation about posture parameters and the astringency is better. The maximum posture error term appears at the time when lock-lose of satellite signal happens. The maximum posture error with the method of Kalman is about 1 degree of pitch error, 1 degree of roll error and 1.2 degrees of heading error. Respectively the maximum posture error with the method of R-T-S is about 0.2 degree of pitch error, 0.4 degree of roll error and 0.8 degree of heading error. The improvement is obviously to see.

In Fig 6, there is also no big difference between two methods when the car keeps stationary. The velocity error is much smaller with the method of R-T-S when the car begins to move from about 700 second. The curve in the figure with the method of Kalman is more divergent than the curve with the method of R-T-S, which proves the better astringency of R-T-S. Obviously, it also has an even change on the estimation about velocity parameters. The maximum velocity error term appears at the time when lock-lose of satellite signal happens. The maximum velocity error with the method of Kalman is about 2 meter per second in east direction, 1.8 meter per second in north direction and 2.3 meter per second in vertical direction. Respectively the maximum velocity error with the method of R-T-S is about 1 meter per second in east direction, 0.6 meter per second I north direction and 0.2 meter per second in vertical direction. The improvement is the best among all.

In Fig 7, we get the latitude error, the longitude error and the height error. Although there is a difference between two methods in the latitude error and the longitude error, it does not represent the real path of the movement only by each one of them. The combination of them makes the real path of the movement. That is what we see in Fig 8. Besides, the height error does not matter in this experiment. In Fig 8, we introduce a new standard named ground truth to evaluate the accuracy of the two algorithm. The ground true represents the real path we go through in the experiment. Then we compare the curve made by two methods with the ground truth. From the figure we can see that the curve made by the method of R-T-S is nearly coincident with the ground truth with about the maximum error 1.87 meters. While the curve made by the method of Kalman has an apparent deviation with the ground truth. In a word, R-T-S optimal fixed-interval smoothing algorithm not only has a better estimation precision than Kalman method, but also has an even change on the estimation about navigation parameter.

7 CONCLUSION

In order to improve the accuracy of data processing, getting the position and posture data of high precision, we apply

R-T-S optimal fixed-interval smoothing algorithm to data post-processing. It is proved by the experiment that R-T-S optimal fixed-interval smoothing algorithm not only has a better estimation precision than Kalman method, but also has an even change on the estimation about navigation parameter, making it possible as an effective way of data processing.

REFERENCES

[1] HOOD L L. Central magnetic anomalies of Nectarianaged lunar impact basins: Probable evidence for an early core dynamo[J]. Icarus, 2011, 211(2): 1109-1128.

[2] ROCHETTE P, GATTACCECA J, IVANOV A V, et al. Magnetic properties of lunar materials: Meteorites, Luna and Apollo returned samples[J], Earth and Planetary Science Letters, 2010, 292(3): 383-391

[3] Grewal M S, Miyasako R, Smith J. Application of fixed point smoothing to the calibration, alignment, and navigation of inertial navigation systems[A]. Second World Congress of Nonlinear Analysis [C]. Athens, Greece [s. n.]1996. 476-479.

[4] Shortelle K J, Graham W R, Rabourn Lt C. F-16 flight test of a rapid transfer alignment procedure[A]. Position Location and Navigation Symposium [C]. IEEE Plans, 1998. 379-383.

[5] Rauch H E, Tung F, Striebel C T. Maximum likelihood estimates of linear dynamic systems[J]. AIAAJ, 1965, (3): 1445-1450.

[6] SU Yan, WANG Shou-rong, ZHOU Bai-ling. Application of AR model to drift compensation of the dynamically tuned gyro (DTG) [J]. Journal of Chinese Inertial Technology, 1997, 5(3): 24-26

[7] RICHMOND N C, HOOD L L. A preliminary global map of the vector lunar crustal magnetic field based on Lunar Prospector magnetometer data[J]. Journal of Geophysical Research, 2008, 113(E2): E02010, doi: 10.1029/2007JE002933.

[8] PURUCKER M E, SABAKA T J, Halekas J, et al. The lunar magnetic field environment: Interpretation of new maps of the internal and external fields[C]//37th Annual Lunar and Planetary Science Conference. 2006, 37: 1933.

[9] PURUCKER M E. A global model of the internal magnetic field of the Moon based on Lunar Prospector magnetometer observations[J]. Icarus, 2008, 197(1): 19-22.

[10] SAMEH NASSAR. Different Algorithms for Bridging Kinematic DGPS Outages using SINS/DGPS Integration[C]//Proceeding of ION GPS2002. Portland, 2002: 1474-1480

[11] SAMEH NASSAR. Improving the Inertial Navigation System(INS) Error Model for INS and INS/DGPS Applications[D]. Canada: University of Calgary, 2003: 100-105

[12] WANG X X, PAN Q, LIANG Y, et al. Application of unscented transformation for nonlinear state smoothing[J]. Acta Automatica Sinica, 2012, 38(7): 1107-1111.

[13] Bierman G J. A new computationally efficient fixed-interval, discrete-time smoother[J]. Automatica, 1983, 19(5): 503-511

[14] Liu Jianye, Yuan Xin. The application of smoothing filter to GPS/INS integrated navigation system[J]. Aerospace Control, 1995(4):36-44.

[15] Mark L P. Square-root information filtering and fixed-interval smoothing with singularities [C]//Proceedings of 1994 America Control Conference. Pennsylvania: Philadelphia, 1998: 2744-2748.