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Research of SINS/DVL/OD Integrated Navigation System based on Observability Analysis

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Abstract: In order to amplify the effective field of vehicle navigation, and overcome the obstacles such as poor road condition and bad weather, the SINS (Strap-down Inertial Navigation System) /DVL (Doppler Velocity Log) /OD (Odometer) integrated navigation technology is proposed. PWCS theory and SVD (singular value decomposition) are used to analyze observability of this integrated navigation system. In the process of S maneuver simulation, we contrast the calculating error between the integrated navigation and singular SINS through the Kalman filtering algorithm. The filtering result of integrated navigation is more similar to actual result than a pure SINS system. Finally, we compare the result of Kalman filter to that of observability analysis, it is concluded the method that the observable degree of the system states can be judged only by the SVD of the observability matrix is theoretically deficient.

Key Words: Kalman Filter, Observability Analysis, PWCS, SVD, Integrated Navigation

1 Introduction

SINS system is widely used in the navigation of land vehicles because of its high independence and it can provide velocity, position and attitude whole-aspect information, but its error is accumulated over time and it's high-accuracy only in short time. OD is an instrument for measuring speed and position [1]. Due to the measuring error caused by the change of tire inflation, vehicle speed, the wear of tires, the size of the load, the influence of the road condition, the wheel slip and the turning of the vehicle, high-accuracy OD can provide velocity information to compensate SINS error, calculating the position through the attitude matrix form dead reckoning system.

GPS has the characteristics of all-weather, continuity and availability, but because of its poor dynamic capabilities, no signal met the needs of modern vehicle navigation in many-storied buildings of city, also its initiative belongs to America, therefore GPS has limitation. DVL gets the vehicle's speed by measuring Doppler frequency shift of speed-radar reflected from ground [2], because of its dynamic performance and non-contact measurement independently, and could replace GPS at any time to provide navigation information. Otherwise, DVL can avoid the shortcomings of OD to adapt to the extreme circumstances of miriness and snow. In this way, DVL/SINS/OD integrated navigation system proceeds information fusion through Kalman filtering algorithm. Every sensor strengthens each other, improving the efficiency and accuracy of the actual-field application in vehicle identified navigation.

Kalman filtering is a common algorithm applied to the integrated navigation. The speed and accuracy of the navigation are determined by the speed and accuracy of the estimated-filtering system. SINS/DVL/OD integrated navigation system is a linear time-varying system. A observability analysis method, PWCS (Piece-Wise Constant System) proposed by Goshen-Meskin[4,5] and Bar-Itzhack, can make the effective application of observability analysis in the time-varying system. Based on the PWCS theory, Wan Dejun et al proposed singular value decomposition (SVD), a method for quantitative analysis of the states' observability. This method can determine the observability degree of the state components through singular value decomposition of observability matrix.

2 Model of Integrated Navigation System

2.1 State Equation of System

The navigation system of SINS uses the E-N-U geographic coordinate system [3], ignoring the influences of the vertical velocity error and height error. The error of each sub navigation system is used as the state variable of integrated navigation system, so that the state variable of the integrated-navigation-system filter is 14-dimensional, expressing specifically as follows:

$$X=\left(\phi_E\ \phi_N\ \phi_U\ \delta\lambda\ \delta L\ \delta V_E\ \delta V_N\ \ \varepsilon_x\ \varepsilon_y\ \varepsilon_z\ \nabla_x\ \nabla_y\ \delta C\ \delta K_D\right)^T$$

Where ϕ_E, ϕ_N and ϕ_U are attitude error angles, $\delta\dot{\lambda}$ the longitude error, $\delta\dot{L}$ the latitude error, δV_E and δV_N are east direction error and north direction error, ε_x、ε_y and ε_z respectively is the component of the gyro drift in the E-N-U three axes of the coordinate axis, ∇_x and ∇_y are bias errors of accelerometer in north and east direction, δC the DVL's scale factor error, $\delta\dot{K}_D$ the OD's scale factor error.

The state equation of SINS/DVL/OD integrated navigation system:

$$\dot{X}=FX+W \tag{1}$$

Where F is the system state matrix, W is the system process noise.

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$$F(t)=\begin{bmatrix} F_{SINS}(t) & 0 & 0 \\ 0 & F_{DVL}(t) & 0 \\ 0 & 0 & F_{Od}(t) \end{bmatrix},$$

$$W=\left[0\ 0\ 0\ 0\ 0\ \nabla_E\ \nabla_U\ w_x\ w_y\ w_z\ \varsigma_E\ \varsigma_N\ 0\ 0\right]^T,$$

$$F_{DVL}=\left[-\frac{1}{T_t}\right],\ F_{Od}=[0]$$

$F_{SINS}(t)$ is a matrix contained 12×12 dimensions, its non-zero elements are:

$$F_{SINS}(1,2)=\omega_{ie}\sin L+\frac{V_E}{R_N}\tan L,$$

$$F_{SINS}(1,3)=-\omega_{ie}\cos L-\frac{V_E}{R_N},$$

$$F_{SINS}(1,7)=-\frac{1}{R_M},F_{SINS}(1,8)=-C_b^n(1,1),$$

$$F_{SINS}(1,9)=-C_b^n(1,2),F_{SINS}(1,10)=-C_b^n(1,3),$$

$$F_{SINS}(2,1)=-\omega_{ie}\sin L-\frac{V_E}{R_N}\tan L,$$

$$F_{SINS}(2,3)=-\frac{V_N}{R_M},$$

$$F_{SINS}(2,5)=-\omega_{ie}\sin L,F_{SINS}(2,6)=\frac{1}{R_N},$$

$$F_{SINS}(2,8)=-C_b^n(2,1),F_{SINS}(2,9)=-C_b^n(2,2),$$

$$F_{SINS}(2,10)=-C_b^n(2,3),$$

$$F_{SINS}(3,1)=\omega_{ie}\cos L+\frac{V_E}{R_N},\ F_{SINS}(3,2)=\frac{V_N}{R_M},$$

$$F_{SINS}(3,5)=\omega_{ie}\cos L+\frac{V_E}{R_N}\sec^2 L,$$

$$F_{SINS}(3,6)=\frac{\tan L}{R_N},\ F_{SINS}(3,8)=-C_b^n(3,1),$$

$$F_{SINS}(3,9)=-C_b^n(3,2),\ F_{SINS}(3,10)=-C_b^n(3,3),$$

$$F_{SINS}(4,5)=\frac{V_E}{R_N}\sec L\tan L,$$

$$F_{SINS}(4,6)=\frac{\sec L}{R_N},F_{SINS}(5,7)=\frac{1}{R_M},$$

$$F_{SINS}(6,2)=-f_U,\ F_{SINS}(6,3)=f_U,$$

$$F_{SINS}(6,5)=2\omega_{ie}\cos LV_N+\frac{V_EV_N}{R_N}\sec^2 L,$$

$$F_{SINS}(6,6)=\frac{V_N}{R_M}\tan L,$$

$$F_{SINS1}(6,7)=2\omega_{ie}\sin L+\frac{V_E}{R_N}\tan L,$$

$$F_{SINS}(6,11)=C_b^n(1,1),\ F_{SINS}(6,12)=C_n^n(1,2),$$

$$F_{SINS}(7,1)=f_U,\ F_{SINS}(7,3)=-f_E,$$

$$F_{SINS}(7,6)=-2\omega_{ie}\sin L-\frac{V_E}{R_N}\tan L,$$

$$F_{SINS}(7,11)=C_b^n(2,1),$$

$$F_{SINS}(7,12)=C_b^n(2,2),F_{SINS}(8,8)=-\frac{1}{T_g},$$

$$F_{SINS}(9,9)=-\frac{1}{T_g},F_{SINS}(10,10)=-\frac{1}{T_g},$$

$$F_{SINS}(11,11)=-\frac{1}{T_a},F_{SINS}(12,12)=-\frac{1}{T_a}$$

2.2 Measurement Equation

Through the method of indirect estimation, we choose the difference between SINS calculative velocity and DVL metrical velocity (components in east and north two axes of the geographic coordinate system), and the difference between SINS calculative position and OD calculative position (longitude and latitude) as view measurement. Get the measurement equation:

$$Z=\begin{pmatrix} V_E-v^n_{E_{DVL}} \\ V_N-v^n_{N_{DVL}} \\ \lambda_{SINS}-\lambda_{Od} \\ L_{SINS}-L_{Od} \end{pmatrix}=HX+V \tag{2}$$

Where $H(t)=diag\{H_1(t),H_2(t)\}$,

$$H_1(t)=\begin{pmatrix} 0\ 0\ -V'_N & \vdots 0\ 0 & \vdots\ 1\ 0 & \vdots\ 0\ 0\ 0\ 0\ 0 & \vdots\ -V'_E & \vdots -V'_E \\ 0\ 0\ \ V'_E & \vdots 0\ 0 & \vdots\ 0\ 1 & \vdots\ 0\ 0\ 0\ 0\ 0 & \vdots\ -V'_N & \vdots -V'_N \end{pmatrix}_{2\times 14}$$

V'_E, V'_N respectively is actual east velocity and north velocity,

$$H_2(t)=diag\{0_{2\times 3}\ I_{2\times 2}\ 0_{2\times 9}\}_{2\times 14},$$

$V(t)=\left[m_{vE}\ m_{vN}\ m_\lambda\ m_L\right]^T$ are white noise of zero mean.

3 Observability Analysis of Integrated Navigation System

3.1 Observability Analysis Method of PWCS Theory

In a sufficiently small time interval Δt_i (i=1, 2, …, n), if the change of the coefficient matrix of Linear time-varying system is very small, real variable system can be used as constant system. This is the PWCS [6]. The steps and algorithms of this observability analysis are called PWCS observability analysis method. The observability analysis of the system is carried out using the following discrete linear system:

$$\begin{cases} X(k+1) = F_j(k)X(k) \\ Z_j(k) = H_j X(k) \end{cases} \tag{3}$$

In jth (j=1,2,..,r) time period, $X(k) \in R^n$, $Z_j(k) \in R^m$, $F_j \in R^{n\times n}$, $H_j \in R^{n\times n}$. In a certain time period, F_j and H_j also are constant value matrixes, but they exist vary in different time period. The total observability matrix (TOM) $Q(r)$ and selective observability matrix (SOM) $Q_s(r)$ in discrete PWCS are as follows:

$$Q(r) = \begin{bmatrix} Q_1 \\ Q_2 F_1^{n-1} \\ \vdots \\ Q_r F_{r-1}^{n-1} F_{r-2}^{n-1} \cdots F_1^{n-1} \end{bmatrix},\ Q_s(r) = \begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_r \end{bmatrix}$$

$$Q_j = \left[(H_j)^T \ (H_j F_j)^T \cdots (H_j F_j^{n-1})^T \right]^T .$$

According to the description of Theorem 1, Theorem 2 and Theorem 3 in [1,2], our research proves that for this SINS/DVL/OD integrated system, SOM in the discrete PWCS can be used to study the observability of PWCS instead of TOM. So we can get the conclusion [7]: the results of the SOM analysis in the discrete system of this integrated navigation system are the same as the results of the TOM analysis in the continuous system.

3.2 Analysis Method of Observability based on Singular Value Decomposition

When the vehicle runs under different maneuvering conditions, the observability of the integrated navigation system will change [8]. In order to expand the research, set up a route to meet the static (0s), uniform speed linear motion (1s-1000s) and S trajectory (1000s-5000s) three different states of the route. Set initial position: $\lambda = 118.78^\circ$, $L = 32.05^\circ$; Use $\lambda/(^\circ)$ as the horizontal coordinates, $L/(^\circ)$ as the longitudinal coordinates of the moving trajectory is shown below:

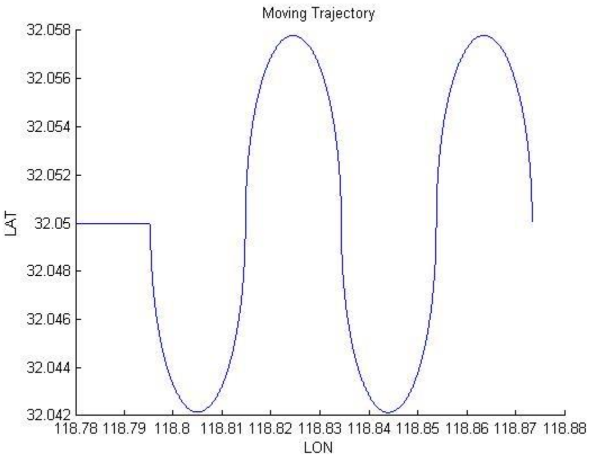

Fig. 1: Moving Trajectory

First of all, the observability of the S trajectory is set up for the later contrast between SINS and integrated navigation. There are right singular vector v_i corresponding to singular value σ_i in SVD decomposition method in the chart below. The singular value that is less than 10^{-4} is recorded as 0, which is considered as a non-observation. There are 9 observable states in the graph (under the S trajectory circumstance). $\sigma_1 \sim \sigma_9$ are assigned to ϕ_U , ε_U , ϕ_E , ε_E , $\delta\lambda$, δV_E , δV_N , δL and ε_N .

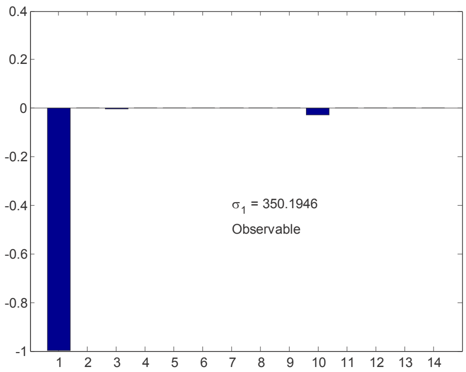

Fig. 2: Singular value: 350.1946

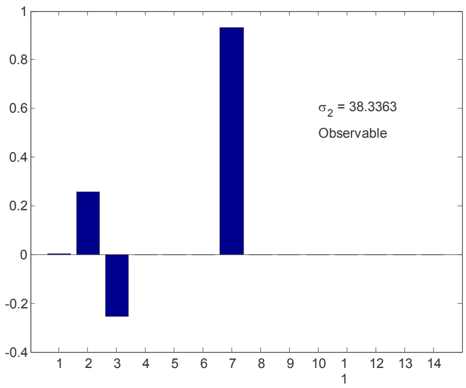

Fig. 3: Singular value: 38.3363

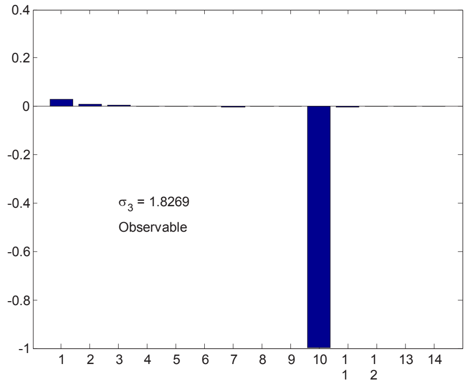

Fig. 4: Singular value: 1.8269

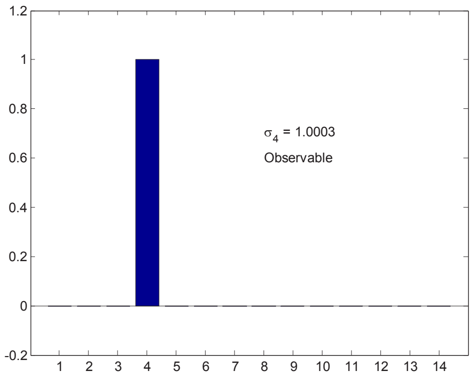

Fig. 5: Singular value: 1.0003

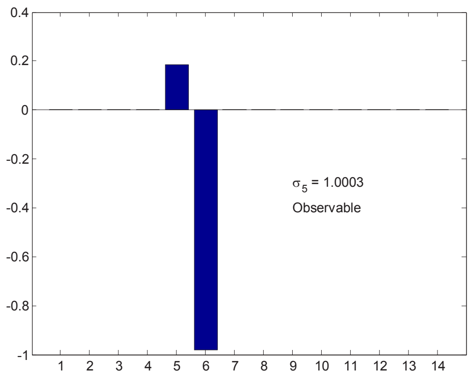

Fig. 6: Singular value: 1.000.

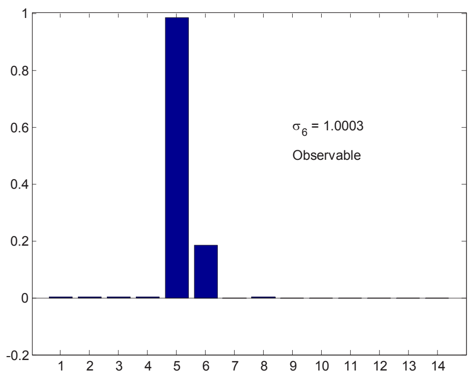

Fig. 7: Singular value: 1.0003

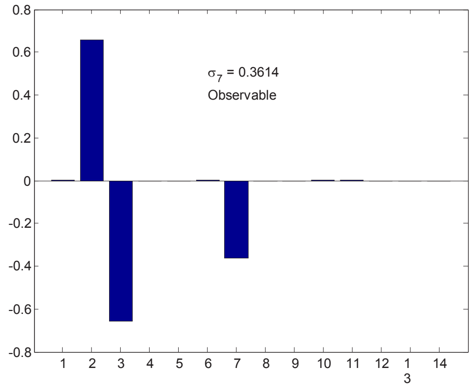

Fig. 8: Singular value: 0.3614

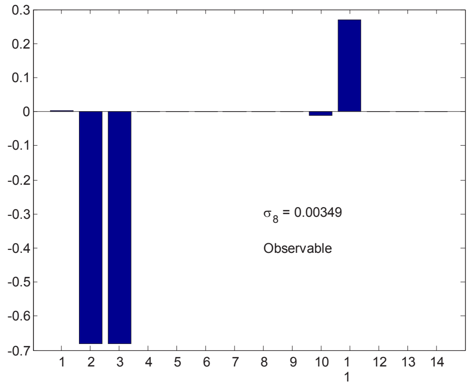

Fig. 9: Singular value: 0.00349

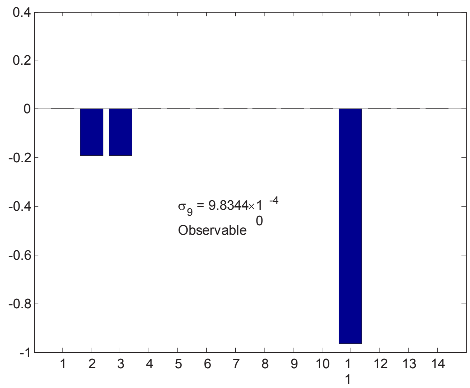

Fig. 10: Singular value: 9.8344×10^{-4}

The singular value of the observable states in the uniform speed linear motion and S maneuver are on the follow form.

Table 1: Singular Value and the Corresponding Variables

States	Static		Uniform speed linear motion	
	Value	Variable	Value	Variable
σ_1	13.88801	Φ_N	13.9161	Φ_N
σ_2	13.8520	ε_y	13.8936	ε_y
σ_3	9.8458	Φ_E	9.8655	Φ_E
σ_4	9.7948	ε_x	9.8243	ε_x
σ_5	0.9999	$\delta\lambda$	1.001	δV_N
σ_6	0.9999	δV_N	1.001	$\delta\lambda$
σ_7	0.9999	δV_E	1.001	δV_E

σ_8	0.9999	δL	1.0009	δL
σ_9	0.0067	∇E	0.0079	∇E
σ_{10}	5.5848×10^{-4}	ε_z	4.8106×10^{-4}	ε_z

Under the condition of static state, the number of observable states is 10, but the corresponding singular values are obviously smaller than that of S trajectory. The changed magnitude of the singular value reflects the observable degree of the state [9]. The greater the singular value is, the higher the corresponding state of the observable degree is, the higher the accuracy and speed of the filtering estimation is [10].

The observability analysis of the integrated navigation system in uniform acceleration motion is similar to that under the static condition. The number of observable states is 10, too. But the corresponding singular value of variables becomes larger. Compared with other two circumstances, the singular values of the corresponding states in S trajectory are increased, so the degree of observation can be greatly improved.

4 Simulation and Analysis

The simulation trajectory of the vehicle is the S trajectory same as that in 3.3, the total simulation time is 4000s. The initial values of the system are as follows: ① Initial longitude and latitude error is 10m, initial velocity error is 0.1m/s; ② Three platform error angle: 0.5’, 0.5’, 0.6’; ③ Three direction of the gyroscope: drift errors also are $0.03^\circ/h$, drift mean square values also are $0.03^\circ/h$, related time is 1800s; ④ Two directions of accelerometer: bias errors also are $10^{-4}g$, zero deviation mean square deviation also are $10^{-4}g$; ⑤OD scale factor: constant error is 0.005, mean square value is 0.0003, related time is 3600s; ⑥DVL scale factor: constant error is 0.001, mean square value is 0.0001, related time is 900s.

In the process of simulation, we put up a contrast between SINS/DVL/OD integrated navigation system calculating and pure SINS navigation system calculating in 7 kinds of errors, such as pitch error, roll error, yaw error, east position error, north position error, east velocity error and north velocity error.

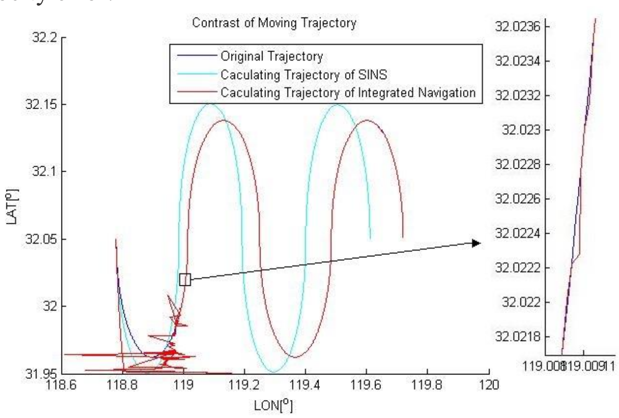

Fig. 11: Contrast of Moving Trajectory

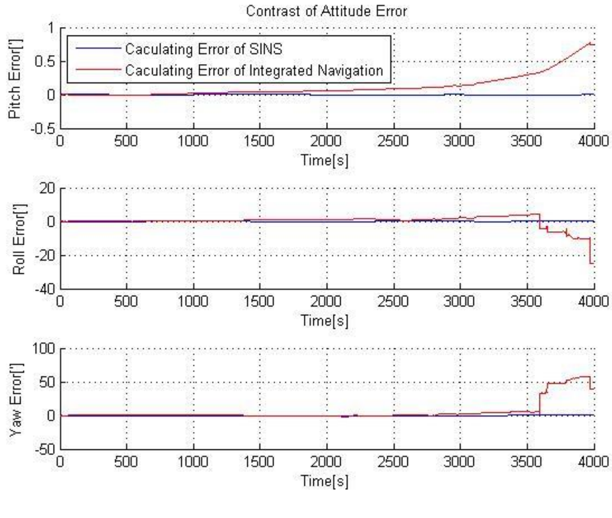

Fig. 12: Contrast of Attitude Error

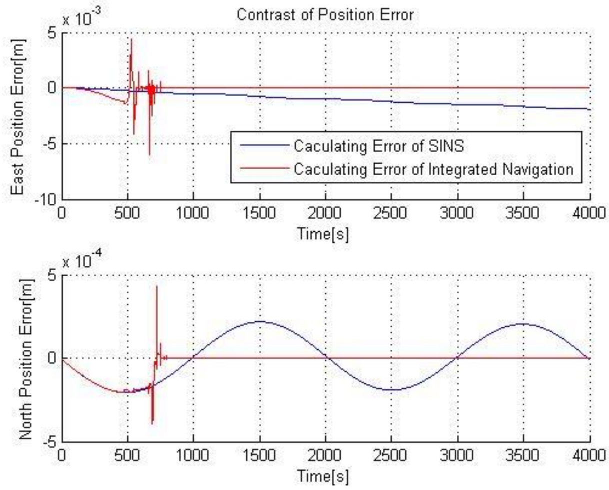

Fig. 13: Contrast of Position Error

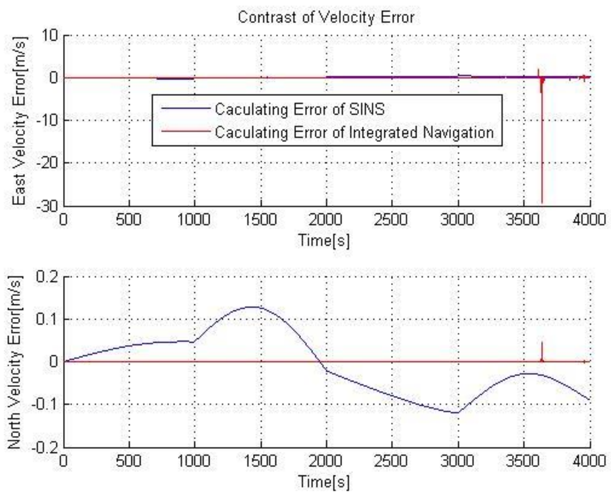

Fig. 14: Contrast of Velocity Error

On the other hand, gyro drift and accelerometer drift after Kalman filter are in the following charts.

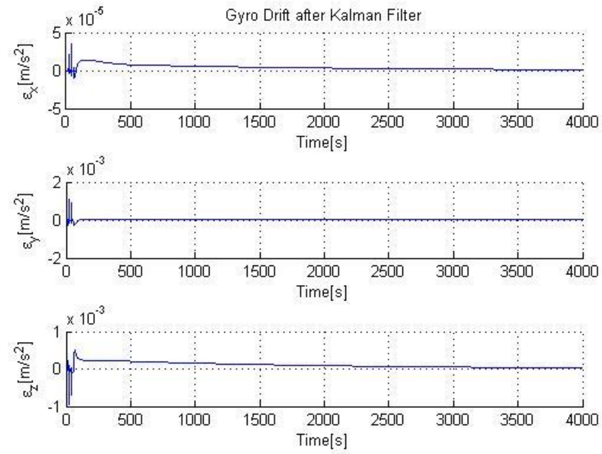

Fig. 15: Gyro Drift after Kalman Filter

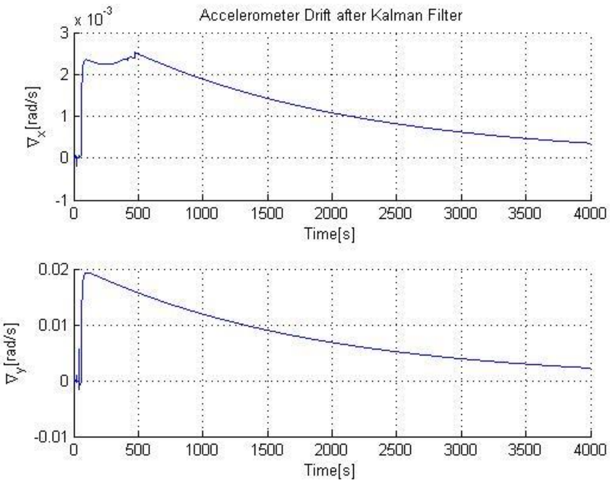

Fig. 16: Accelerometer Drift after Kalman Filter

By comparing photos can be seen, the SINS/DVL/OD integrated navigation effectively inhibits the divergent tendency over time of inertial navigation error, and sharply increase the navigation accuracy compared with the pure SINS system. It realizes the goal that DVL and OD helps to correct accelerometer error through Kalman filter.

Meanwhile, combined with the results obtained from the observability analysis, the state variables except three attitude angles become convergence after Kalman filtering algorithm. The Kalman filtering accuracy of position and velocity is also pretty high. Only the attitude errors have a divergent trend. So we can infer that PWCS theory and SVD singular value decomposition cannot reflect the observability degree of the viarable states.

5 Conclusion

Using PWCS theory and SVD singular value decomposition to analyze the observability of the SINS/DVL/OD integrated navigation system. In these two theories, when the vehicle is running in a moving of speed and attitude change largely, that can increase every state's observability degree. For the integrated navigation system, DVL and OD can work alternately to overcome their own drawback. Accordingly, the integrated navigation can keep high accuracy in many extreme weather and circumstances. By the contrast with pure SINS system, the integrated navigation through Kalman filtering algorithm increases navigation accuracy effectively. Finally, we compare the result of Kalman filter to that of observability analysis. it is obviously that the observability analysis of PWCS theory and SVD singular value decomposition is not much related to Kalman filtering accuracy and velocity. So we propose that the observable degree of the system states can be judged only by the SVD of the observability matrix is theoretically deficient.

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