

A Unified Closed-Loop Motion Planning Approach for an I-AUV in Cluttered Environment with Localization Uncertainty

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I. Introduction

Background:

- **I-AUV:** a 6 Degree of Freedom (DOF) free-floating base with a n-DOF manipulator.
- **Applications:** conduct cleaning, data collection or inspection on underwater infrastructures or algae growing on the underside of sea ice in polar area.

Challenges:

- Non-uniform motion and observation noise (flow, landmark).
- Unified solution (planning, control, manipulator + base).
- Efficient optimal planning in high-dimensional belief space.

II. Problem Formulation

1. System model

$$\begin{aligned} \mathbf{x}_{t+1} &= f(\mathbf{x}_t, \mathbf{u}_t, \boldsymbol{\omega}_t) \\ \mathbf{z}_t &= h(\mathbf{x}_t, \boldsymbol{\nu}_t), \end{aligned} \xrightarrow{\text{Linearization}} \begin{aligned} \tilde{\mathbf{x}}_{t+1} &= \mathbf{A}_t^p \tilde{\mathbf{x}}_t + \mathbf{B}_t^p \tilde{\mathbf{u}}_t + \mathbf{G}_t^p \boldsymbol{\omega}_t \\ \tilde{\mathbf{z}}_t &= \mathbf{H}_t^p \tilde{\mathbf{x}}_t + \mathbf{M}_t^p \boldsymbol{\nu}_t \end{aligned}$$

\mathbf{x}_t is manipulator and base state, \mathbf{G}_t^p and \mathbf{M}_t^p are weight matrices on motion and observation noise.

2. Problem definition

- ❖ **Problem 1.** Find optimal control policy for stochastic optimal control.

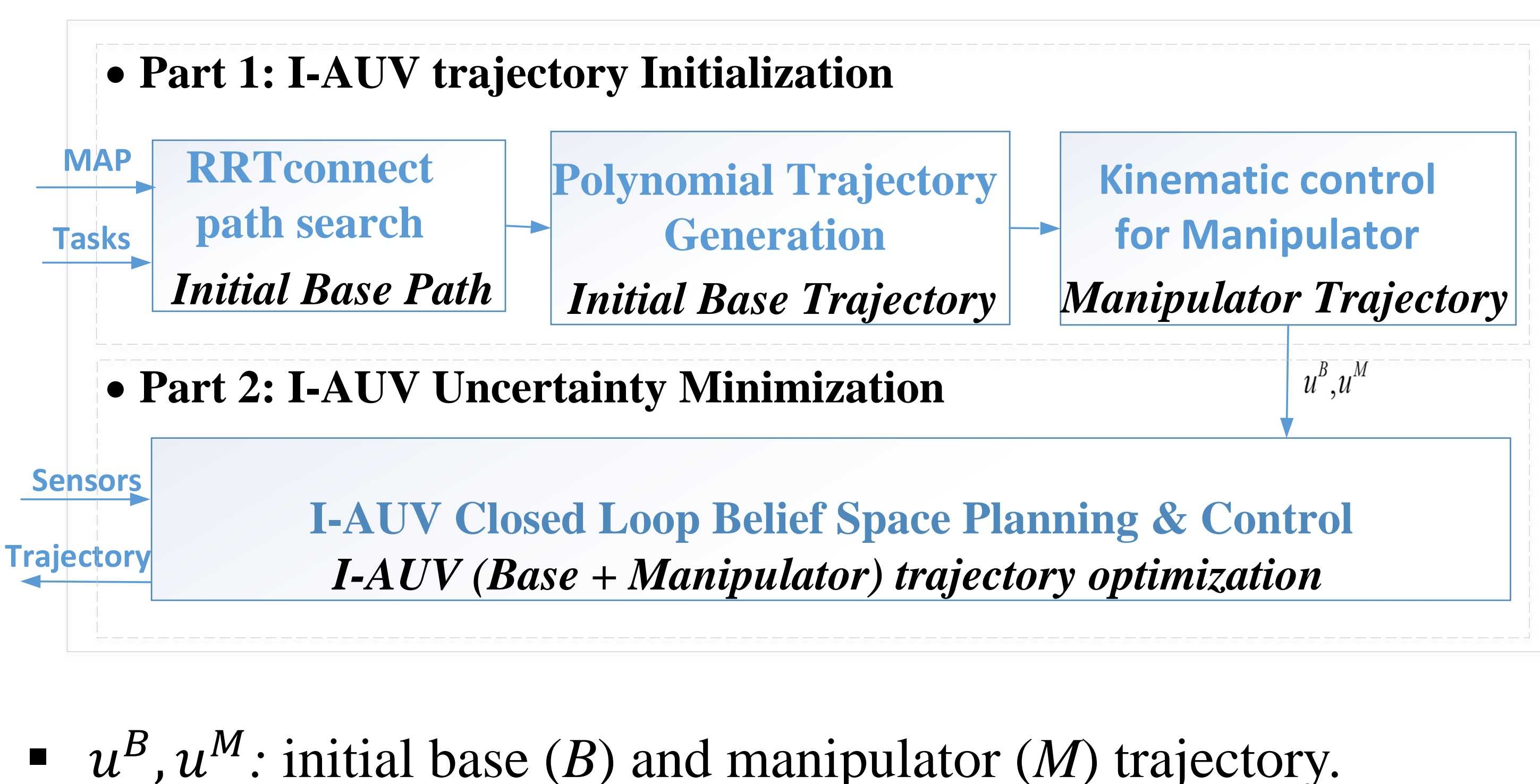
$$\begin{aligned} \min_{\pi} J^{\pi} &:= \mathbb{E} \left[\sum_{t=0}^{K-1} c_t^{\pi}(\mathbf{b}_t, \mathbf{u}_t) + c_K^{\pi}(\mathbf{b}_K) \right] \\ \text{s.t. } \mathbf{b}_{t+1} &= \tau(\mathbf{b}_t, \mathbf{u}_t, \mathbf{z}_{t+1}), \\ g_i(\mathbf{b}_f, \mathbf{u}_t) &\leq 0, \quad i = 1, \dots, m \end{aligned}$$

Based on the separation principle of Linear Quadratic Gaussian, estimation cost is dominant. **Problem 1** is reduced to optimal trajectory planning with best estimation performance [1].

- ❖ **Problem 2.** Unified I-AUV motion planning under uncertainty.

$$\begin{aligned} \min_{\mathbf{u}_{0:K-1}^p} & \sum_{t=1}^K [-\log \text{Det}(\mathbf{W}_t \boldsymbol{\Omega}_t \mathbf{W}_t^T) + (\mathbf{u}_{t-1}^p)^T \mathbf{W}_t^u \mathbf{u}_{t-1}^p] \\ \text{s.t. } \bar{\boldsymbol{\Omega}}_t &= (\mathbf{A}_t \boldsymbol{\Omega}_{t-1}^{-1} \mathbf{A}_t^T + \mathbf{G}_t^p \boldsymbol{\Sigma}_{\omega_t} (\mathbf{G}_t^p)^T), \\ \boldsymbol{\Omega}_t &= \mathbf{H}_t^T (\mathbf{M}_t^p \boldsymbol{\Sigma}_{\nu_t} (\mathbf{M}_t^p)^T)^{-1} \mathbf{H}_t + \bar{\boldsymbol{\Omega}}_t, \text{ /*belief propagation*/} \\ g_i(\mathbf{u}_t) &\leq 0, \quad i = 1, \dots, m, \text{ /*start, goal, control limit, etc.*/} \end{aligned}$$

III. Solution framework



IV. Solution

Part 1: I-AUV collision-free trajectory initialization

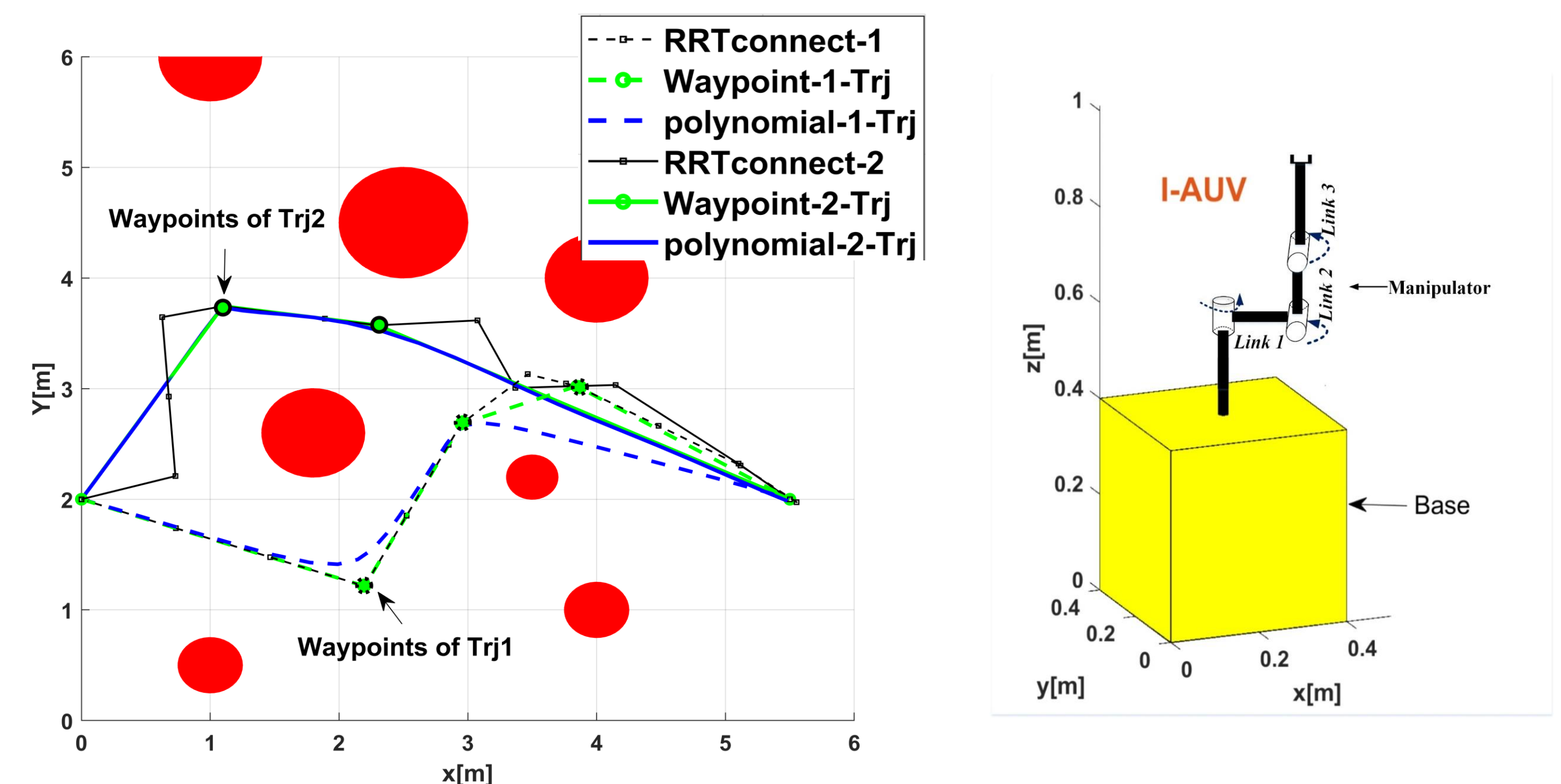
- 1). **First step (base only):** Waypoints search via RRT-connect.
- 2). **Second step (base only):** Polynomial trajectory optimization, linearly constrained by waypoints.

$$f^k(t) = a_0^k + a_1^k t + a_2^k t^2 + a_3^k t^3 + \dots + a_N^k t^N$$

$$J_p = \min_{\mathbf{d}_p} \omega_s J_s + \omega_c J_c + \omega_w J_w$$

- 3). **Third step:** Manipulator trajectory adaptation via Null-space control given the base motion, **primary task:** base trajectory tracking, **secondary task:** collision avoidance of manipulator.

$$\zeta_r = \mathbf{J}_a^+(\eta, q) (\dot{\sigma}_{a,d} + \mathbf{k}_a \tilde{\sigma}_a) + \mathbf{N}_a \mathbf{J}_b^+(\eta, q) (\dot{\sigma}_{b,d} + \mathbf{k}_b \tilde{\sigma}_b)$$

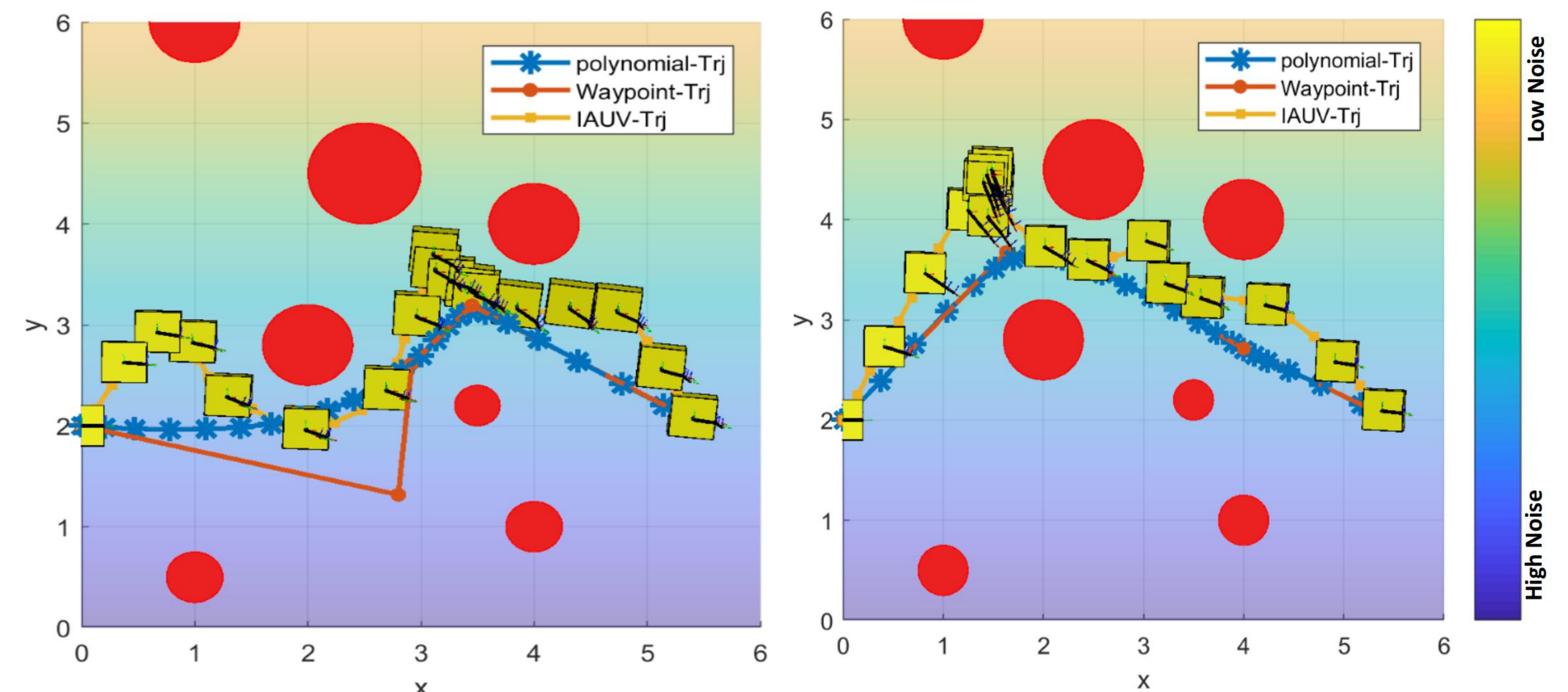


Part 2: I-AUV uncertainty minimization

- 1). Solve **Problem 2** via initials from part 1 and a SQP solver.
- 2). Compare the optimized trajectories and select an optimal one for LQR control.

V. Results

(a) *Case 1: Locally optimal solutions with a observation noise distribution (top-down view).*



(b) *Case 2: Optimal solution in cluttered scenario with same noise distribution.*

