**Fact 3.2.** If a blockchain protocol  $(\Pi, \text{extract})$  satisfies  $(T_0(\cdot), \delta)$ -fairness in  $\Gamma$  environments, then

it satisfies  $(T_0, \mu)$ -chain quality, where  $\mu(\kappa, n, \rho, \Delta) = (1 - \delta)(1 - \rho) \ge 1 - (1 + \delta)\rho$  when  $\rho \le \frac{1}{2}$ .