Lemma. The expected time r_i for a random walk starting at node i to return to i is the reciprocal of the stationary probability of i. That is

$$r_i = \frac{1}{\pi_i}.$$

- Intuitively
 - A long walk always ends up in stationary distribution π
 - Suppose the walk length is T, then the expected number of times it visits i is $\pi_i \cdot T$
 - The average length between two visits is $\frac{T}{\pi_i \cdot T} = \frac{1}{\pi_i}$
 - A rigorous proof requires the Strong Law of Large Numbers

Theorem. The probability that a random walk starting at node i visits j before returning to i, which equals $ep(i \rightarrow j)$, satisfies

$$ep(i \to j)c(i,j) = \frac{1}{\pi_i},$$

where c(i, j) is the commute time between i and j.

- Proof
 - Consider a random walk w starting at i, and random variables
 - * X = the first time w returns to i
 - * Y =the first time w returns to i after visiting j
 - By definition $E(X) = \frac{1}{\pi_i}$ and E(Y) = c(i, j)
 - Clearly X < Y, and $\Pr[X = Y] = p \triangleq \exp(i \rightarrow j)$

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$$E(Y - X) = p \cdot 0 + (1 - p) \cdot E(Y) = (1 - p)c(i, j)$$

- Also
$$E(Y - X) = E(Y) - E(X) = c(i, j) - \frac{1}{\pi_i}$$

- $\operatorname{ep}(i \to j) + \operatorname{ep}(j \to i) = \frac{1}{c(i,j)} \left(\frac{1}{\pi_i} + \frac{1}{\pi_j}\right)$
- Recall that h(i, j) is small whenever π_j is large, which is bad for personalization
 - Sarkar et al. (2008) alleviate this by restricting the length of random walk
 - Tong et al. (2007) alleviate this by reducing the dependence on stationary distribution