Study of Aggregate and Workforce Planning

James Gates, Delin Huang, Defeng Tao
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School of Mechanical, Industrial, Manufacturing Engineering
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Introduction

Aggregate planning prescribed the process of developing a long range production plan with a typical time horizon of 1 to 3 years [1]. Aggregate planning (AP) problems involve optimizing the long term production elements such as production machines and in the case of workforce planning (WP); labor. These decisions have broad ranging effects on the production planning and control process, however due to the unique conditions of any given plant the same set of constraints and assumptions cannot be carried over from scenario to scenario.

The tool that will be described in this handout in detail for working with these AP modules is linear programming. Linear programming allows the planner to optimize the plan for minimum cost or maximum profit. It is the perfect tool for aggregate and workforce planning because it employs flexible constraints that are easily adjusted to test different scenarios that may be relevant to a relatively longer time horizon. This handout will demonstrate the process of developing a combined aggregate planning and workforce planning module using a linear programming model that meets a set of distinctive needs. The model is implemented using the Excel solver tool to maximize profit while meeting forecasted demand for a hypothetical company.

Literature Review

Aggregate and workforce planning carry such broad influence over the production process, and each individual implementation is unique due to differences in demand, production processes and other relevant factors. Therefore, there are many opportunities to apply optimization techniques such as linear programming and test their effectiveness in a case study like the example explained in detail in this handout. Rohman et al. apply this technique to the production process of coated peanut products and manage to reduce costs by an impressive 27.06% [2]. This improvement was relative to managers setting policies for aggregate planning.

Linear programming (LP) is a tool for solving optimization problems subject to linear equality and inequality constraints [3]. Aggregate planning and workforce planning problems are concerned with the optimization of maximum profit or minimum cost with a set of constraints or restrictions. As a result, linear programming is widely used to solve the AP or combined AP/WP problems [1].

Linear programming is a mathematical modeling method that attempts to optimize a linear objective function given a set of constraints. The major components of an LP are decision variables, an objective function and constraints [4]. Decision variables are the variables determined in the solution, which affect the final decision. In the aggregate planning and workforce planning problems, the decision variables could be the number of products produced or sold, the inventory level the manufacturer should hold, the number of people hired or fired and so forth. The objective function is the goal that needs to be achieved, such as maximum profit or minimum cost. It is a linear function containing the decision variables and its value varies if values of decision variables are different. Constraints are the linear equality and inequality restrictions on decision variables choices. In the AP and WP problems, these could be capacity constraints, working station availability limits, raw material restrictions, working hours rules, etc.

There are also some special constraints on variables, such as non-negativity restriction, which means the decision variables cannot be negative values [5].

Methodology

In this handout, we will develop a combined AP and WP example considering the decision variables and constraints mentioned above to optimize profit for using the Solver tools in Excel.

The scenario involves a company producing a product and selling it to their customer. The selling price is $699/unit and the raw material cost is $199/unit. In the beginning of month 3, this company plans to launch a new version of this product with the selling price $799/unit. Due to updates, the raw material cost increases to $249/unit. The price of the old version product decreases to $599/unit when the new version is launched.

The initial stock for the old version product is 100 while there is no initial stock for the new version. The holding costs of both versions are the same, which is $10/month. It takes one working station 10 hours and costs 3 units raw materials to produce the old version product. As for the new version product, it takes 12 hours and 5 units of raw materials to make one. The total work time for the working station is 2400 hours every month and the raw material limit is 1000 units per month. *Table 1* illustrates the capacity to produce the old version and new version product and the minimum and maximum demand for both versions in the following 8 months, which are predicted by selling history.

Table 1: The maximum and minimum demand for products and capacity of production

Time Period (month)	1	2	3	4	5	6	7	8
Maximum demand for old version product	150	150	120	100	100	80	80	60
Minimum demand for old version product	100	100	80	60	60	50	50	40
Maximum demand for new version product	0	0	80	150	250	200	180	150
Minimum demand for new version product	0	0	60	100	200	150	120	100
Capacity to produce old version product	150	150	150	150	120	120	100	100
Capacity to produce new version product	120	120	150	200	200	150	150	120

The cost of regular time in dollars per worker-hour is $12 while it is $15 for overtime working. The cost to increase workforce by one worker-hour is $15 and the cost to decrease workforce by one worker-hour is $9. The number of worker-hours required to produce one unit of an old version product is 12 hours and it is 15 hours for the new version product. The initial workforce in worker-hours of regular time is 1500.

The top manager has received all the information above and is going to make a decision: How to make the profit maximum for the next 8 months? In order to solve this optimization problem, some notations first need to be introduced:

i :an index of product, i = 1 (old version) and 2 (new version)
t :an index of time period, t = 1,…,8
r_{it}:revenue of product i in each time period t
I_{i0}: initial inventory of product i
h_i: holding cost of product i
$dmax_{it}$:maximum demand for product i in each time period t
$dmin_{it}$:minimum demand for product i in each time period t
C_{it}: capacity for producing product i in each time period t
w_i: working hours per unit for product i on workstation
w_{limit}: total available hours on workstation
m_i: raw material used per unit product i
m_{limit}:total available raw materials
l : cost of regular time in dollars per worker-hour
l': cost of overtime in dollars per worker-hour
e: cost to increase workforce by one worker-hour per period
e': cost to increase workforce by one worker-hour per period
b_i: number of worker-hours required to produce one unit of product i
W_0:initial workforce in worker-hours of regular time
X_{it}:quantity produced for product i in each time period t
S_{it}:quantity sold of product i in each time period t
I_{it}: inventory level of product i at the end of each time period t
W_t: workforce in worker-hours of regular time in each time period t
O_t: overtime in hours in each time period t
H_t: increase (hires)in workforce from time period $t-1$ to t in worker-hours
F_t: decrease (hires)in workforce from time period $t-1$ to t in worker-hours

The decision variables are X_{it}, S_{it}, I_{it}, W_t, O_t, H_t and F_t. Management should dictate values of these variables to maximize profit based on the sum of net profit of the products minus the holding and workforce costs. As for the constraints, the number of each product i produced in period t should be less or equal to the capacity in each time period. The capacity is determined by multiple factors, including but not limited to the available working hours on working stations, available raw materials and workforce. The sale for each product i produced in period t has an upper bound and lower bound based on the demand. The total working hour used by a working station should be less or equal to the total available limit of the working station. The total material used to produce the products should be less or equal to the total available raw material. Inventory at the end of period t equals to the inventory at the end of time period $t-1$ plus the number of each product produced minus the sales. In addition, workforce in period t equals to the workforce at the end of time period $t-1$ plus the increase in workforce minus the decrease. The total time spent on producing each product i, is the sum of regular working time and overtime in each time period t. Last but not the least, all the decision variables should be larger or equal to zero. The final LP formulation of this problem based on the objective and constraints is:

Maximize	$\sum_{t=1}^{8} \quad \sum_{i=1}^{2} \quad (r_{it}S_{it} - h_i I_{it} - lW_t - l'O_t - eH_t - e'F_t)$	
Subject to:	$X_{it} \leq C_{it}$	for all i, t
	$dmin_{it} \leq S_{it} \leq dmax_{it}$	for all i, t
	$\sum_{i=1}^{2} \quad w_i X_{it} \leq w_{limit}$	for all t
	$I_{it} = I_{i(t-1)} + X_{it} - S_{it}$	for all i, t
	$W_t = W_{t-1} + H_t - F_t$	for all t
	$\sum_{i=1}^{2} \quad b_i X_{it} = W_t + O_t$	for all t
	$X_{it}, S_{it}, I_{it}, W_t, O_t, H_t, F_t \geq 0$	for all i, t

Implementation and Results

Instance 1: This is the initial setup for this model, and variable values are shown in appendix *Figure 1*. Simulating the model with the initial data, we can see that 1020 worker-hours are needed to be hired in the first period, and the total workforce is 2520 worker-hours for all 8 periods. The total overtime varies from 420 hours to 450 hours in the first 7 periods. The net profit is $580843.33.

Instance 2: The top manager thinks the overtime is slightly higher than the expected and wants to lower the overtime. A constraint that the maximum allowable overtime is constrained to less than or equal to 15% regular working hours for all periods. Another constraint is added:

$$O_t \leq 0.15W_t \qquad \text{for all } t$$

The outcomes and variables values are illustrated in appendix *Table 2.* In the second simulation, the results show that the overtime varise from 357 hours to 369 worker-hours in the first four periods, and from 381 to 387 hours in the fifth month to 7th. In general, the overtime has reduced from the initial simulation. On the other hand, the value of other factors such as total workstation time, hiring, and fire increases from the initial model. The total work force for the first 7 periods is about 2583 worker-hours. In the first period, 1083 worker-hours should be increased by hiring people, and 67 worker-hours needed to be reduced by firing employees in the 8th period. The net profit drops slightly to $580355.51. The detailed variable values are shown in appendix *Figure 2*.

Instance 3: The top manager doesn't want to fire people because it will hurt company's interests in the long term. Then, a constraint that amount of decrease worker-hours is equal to 0 is added based on the model in instance 2:

$$F_t = 0 \qquad \text{for all } t$$

This constraint changes slightly other variables such as product production and leftover. Compared to the last simulation, the producing quantity and the leftover quantity for x1 increases in the last months. There is not much difference for other variables. The net profit continuously drops to $580115.51. The detailed variable values are shown in appendix *Figure 3*.

Instance 4: The top manager is wondering if there is an inventory storage limitation, what decision should be made. The constraint that the sum of leftover x1 and x2 production at the end of each month is less than or equal to 200 is added based on instance 3:

$$\sum_{i=1}^{2} \quad I_{it} \leq 200 \qquad \text{for all } t$$

In this simulation, the producing quantity for x1 starts from 65 quantities and increases to 135 quantities in the second month, then maintained in the range of 60 to 65 quantities from month 4 to month 8. Because the production for x1 has increased in the second and third month, production for x2 decreases, however, it mostly maintains above 60 quantities. The overtime is nearly 292 hours in the third month, and from the fourth month to 7th month, the overtime is 387 hours. The net profit is $567256.52. The detailed variable values are shown in appendix *Figure 4*.

Instance 5: In order to make full usage of the machine, another constraint is added based on the model in instance 4: the usage percentage of each working station is bigger than or equal to 90 percent of maximum available hours.

$$\sum_{i=1}^{2} w_i X_{it} \geq 0.9\, w_{limit} \qquad \text{for all } t$$

In this simulation, we can find that the regular worker-hours increases and more people are hired at the beginning of the plan. The amount of product produced is more than the instance 4 and the overtime is less. The net profit is $566524.00. The detailed variable values are shown in appendix *Figure 5*.

Discussion

The linear programming model described in this handout is a realistic example for an AP/WP scenario. In general, as more constraints are added, overall profits decrease on the margin. However, these allow the researchers to better understand the impacts of policy changes as they are put into practice and achieve better solutions. These constraints are necessary however to portray a realistic problem scenario.

With so many decision variables influencing the success of aggregate and workforce planning, it would be infeasible to expect management to be able to come up with the most profitable plan without a formalized optimization method like linear programming. Manager arbitration of these aggregate planning decisions is a potential source of waste in the production process. Because AP and WP problems have such a broad impact on PP&C this waste could be significant. Therefore, AP and WP optimization tools can be highly valuable to a producer if they are implemented effectively.

Conclusion

The detailed implementation and customization of the model presented in this handout provides a simple and conceptually easy to follow case to emphasize the value that can be added by optimizing aggregate and workforce planning. The problem includes a detailed account of not only building the model, but how to adjust it to meet changing requirements and policy decisions as would be necessary in an industry setting. The insights provided in this example can be extended to more sophisticated optimization tools such as non-linear optimization and integer programming. However, for the purposes of functional implementation in industry a linear programming approach is a highly feasible method for improving overall production planning and control.

Reference

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[3] D. Bertsimas and J. N. Tsitsiklis, Introduction to linear optimization. Belmont, MA: Athena Scientific, 1997.

[4] Bazaraa, M.S., Jarvis, J.J., and Sherali, H.D., Linear Programming and Network Flows, 4th Edition, Wiley, 2010.

[5] R. J. Vanderbei, *Linear programming: foundations and extensions*. Cham, Switzerland: Springer, 2002.

Appendix

Decision Variables:								
t	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
X_1_t	96	104	120	100	72	60	60	60
X_2_t	120	113	100	117	140	150	150	120
S_1_t	150	150	120	100	72	60	60	60
S_2_t	0	0	80	150	250	200	180	150
I_1_t	46	0	0	0	0	0	0	0
I_2_t	120	233	253	220	110	60	30	0
W_t	2520.00	2520.00	2520.00	2520.00	2520.00	2520.00	2520.00	2520.00
H_t	1020.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
F_t	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
O_t	432.00	428.00	420.00	430.00	444.00	450.00	450.00	0.00
Objective:								
Net profit	$580,543.33							

Figure 1: Initial Model Setup Solver Result

Decision Variables:								
t	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
X_1_t	96	104	120	100	72	60	60	60
X_2_t	120	113	100	117	140	150	150	120
S_1_t	150	150	120	100	72	60	60	60
S_2_t	0	0	80	150	250	200	180	150
I_1_t	46	0	0	0	0	0	0	0
I_2_t	120	233	253	220	110	60	30	0
W_t	2582.61	2582.61	2582.61	2582.61	2582.61	2582.61	2582.61	2520.00
H_t	1082.61	0.00	0.00	0.00	0.00	0.00	0.00	0.00
F_t	0.00	0.00	0.00	0.00	0.00	0.00	0.00	62.61
O_t	369.39	365.39	357.39	367.39	381.39	387.39	387.39	0.00
Objective:								
Net profit	$580,355.51							

Figure 2: Solver Result of Adjust for Feasible Overtime

Decision Variables:								
t	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
X_1_t	96	104	120	100	72	60	60	65
X_2_t	120	113	100	117	140	150	150	120
S_1_t	150	150	120	100	72	60	60	60
S_2_t	0	0	80	150	250	200	180	150
I_1_t	46	0	0	0	0	0	0	5
I_2_t	120	233	253	220	110	60	30	0
W_t	2582.61	2582.61	2582.61	2582.61	2582.61	2582.61	2582.61	2582.61
H_t	1082.61	0.00	0.00	0.00	0.00	0.00	0.00	0.00
F_t	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
O_t	369.39	365.39	357.39	367.39	381.39	387.39	387.39	0.00
Objective:								
Net profit	$580,115.51							

Figure 3: Solver Result of Workforce Firing Limit Constraint

Decision Variables:								
t	1	2	3	4	5	6	7	8
X_1_t	65	135	120	60	60	60	60	65
X_2_t	120	64	96	150	150	150	150	120
S_1_t	150	150	120	60	60	60	60	60
S_2_t	0	0	80	150	250	200	180	140
I_1_t	15	0	0	0	0	0	0	5
I_2_t	120	184	200	200	100	50	20	0
W_t	2582.61	2582.61	2582.61	2582.61	2582.61	2582.61	2582.61	2582.61
H_t	1082.61	0.00	0.00	0.00	0.00	0.00	0.00	0.00
F_t	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
O_t	0.00	0.00	292.17	387.39	387.39	387.39	387.39	0.00
Objective:								
Net profit	$567,256.52							

Figure 4: Solver Result of Inventory Storage Constraint

Decision Variables:								
t	1	2	3	4	5	6	7	8
X_1_t	72	128	120	60	60	60	60	72
X_2_t	120	75	85	150	150	150	150	120
S_1_t	150	150	120	60	60	60	60	60
S_2_t	0	0	80	150	250	200	180	140
I_1_t	22	0	0	0	0	0	0	12
I_2_t	120	195	200	200	100	50	20	0
W_t	2664.00	2664.00	2664.00	2664.00	2664.00	2664.00	2664.00	2664.00
H_t	1164.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
F_t	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
O_t	0.00	0.00	48.00	306.00	306.00	306.00	306.00	0.00
Objective:								
Net profit	$566,524.00							

Figure 5: Solver Result of Workstation usage Constraint

Constants:	Value
I_1_0	100 unit
I_2_0	0 unit
h_1	$10/unit/month
h_2	$10/unit/month
w_1	10 hours/unit
w_2	12 hours/unit
w_limit	2400 hours
m_1	3 unit/product unit
m_2	5 unit/product unit
m_limit	1000 units
l	$12/worker-hour
l'	$15/worker-hour
e	$15/worker-hour
e'	$9/worker-hour
b_1	12 worker-hour/unit
b_2	15 worker-hour/unit
W_0	1500 worker-hours

Figure 6: Initial Setup Constants

t (month)	1	2	3	4	5	6	7	8
r_1_t	$500.00	$500.00	$400.00	$400.00	$400.00	$400.00	$400.00	$400.00
r_2_t	$0.00	$0.00	$550.00	$550.00	$550.00	$550.00	$550.00	$550.00

Figure 7: Revenue of each product in different time period