MAGRATHEA
Tutorial and Practice Problems

A **1D planet structure code** written in **C++** which considers the case of fully differentiated interiors. The code integrates the hydrostatic equation using **a shooting method**. The code returns the pressure, temperature, density, phase, and radius at **steps of enclosed mass**. The code supports **4 layers: core, mantle, hydrosphere, and atmosphere**. Each layer has a phase diagram with equations of state (EOS) chosen for each phase.

The code was developed by Chenliang Huang, David R. Rice, and Jason H. Steffen at the Univerisity of Nevada, Las Vegas starting in 2017. Paper for complete info: https://ui.adsabs.harvard.edu/abs/2022MNRAS.513.5256H.

Installation Instructions (all systems):

1. Open a Terminal/Shell.
2. Check C++ compiler is installed: `g++ --version`
3. Install the GNU Scientific Library*
    a. Check if installed already: `which gsl`
    b. Linux: `sudo apt-get install libgsl-dev`
    c. Mac/Windows/Other: Download "current stable version" tar file from https://www.gnu.org/software/gsl/
        1.) Navigate to tar file location in terminal: `cd ~/Downloads`
        2.) Unzip tar: `tar -xvxf gsl-latest.tar.gz`
        3.) Navigate into gsl: `cd gsl-2.7`
        4.) `sudo ./configure && make && make install`
4. Navigate to where you want to work. `mkdir Workshop`
5. Clone the repository: `git clone https://github.com/Huang-CL/Magrathea.git`
6. Navigate to code: `cd Magrathea`
7. Compile the code for first time: `make -B`
8. If "error while loading shared libraries" or "gsl/gsl_odeiv2.h: No such"
    a. Record path for GSL cflags : `gsl-config --cflags`
    And GSL LDflags: `gsl-config --libs`
    b. Open *Makefile* in text editor. Remove "#"s before -I and -L (Line 4 &5) and update the path (i.e. ~/gsl/include or ~/Downloads/gsl/include; repeat same location for ~/gsl/lib).

If something goes wrong:

- Email drice986@gmail.com for GSL tar file and Magrathea zip file.
- *Backup instructions for GSL Installation: https://gist.github.com/TysonRayJones/af7bedcdb8dc59868c7966232b4da903

To run your first planet:

1. Open *run/mode0.cfg* in a text editor.
2. Line 12-14: set mass of the core, mantle, hydrosphere, and atmosphere in Earth-masses.
3. Line 16-20: set surface temperature and any temperature discontinuities.
4. Line 21: set output file name and path.
5. In terminal, compile changed file with `make`.
6. Run MAGRATHEA with `./planet run/mode0.cfg`.
7. Check planet's radius: `tail -2 result/Structurefilename.txt`

   Outputs a file with step number, radius (Earth radius), pressure (GPa), enclosed mass (Earth mass), density (g cm^{-3}), temperature (K), and phase of composition

Star at https://github.com/Huang-CL/Magrathea if you find our code useful!

Further instructions for reference to complete projects.

Bulk Input Mode – `mode3.cfg`:

If **run with ./planet run/mode3.cfg**, Magrathea will require a space separated file, where each row of the file lists the **total mass in Earth-masses and fraction of mass in each layer** for a planet. Any remaining mass is put in the atmosphere layer.

Example:

```
Mass  fCore  fMantle  fWater
2     0.2    0.4      0.4
1.5   0.5    0.39     0.1
```

The config file requires the input file name, output file name, and Tgap similar to mode 0. Example input files in /input directory. MAGRATHEA will generate an output file with mass of core, mantle, water, and atmosphere and the radius of the core, mantle, water, and planet for each line in the input file.

Changing Phase Diagrams for Each Layer:

The user-defined mass of each layer determines when the integrator changes layers. But within a layer, the pressure and temperature conditions determine the material and phase of material used.

The file, **src/*phase.cpp***, contains the definition for three functions which determines which equations of state are used in each layer at P,T conditions.

Change the corresponding return values of `find_phase_water _default`, `find_phase_Fe_default`, or `find_phase_Si_default` in *phase.cpp* using conditionals to set the desired pressure and temperature where the material/phase will exist. Equations of State names defined in *src/EOSlist.cpp*.

Example:

```
// Fe Default: hcp and Liquid iron
EOS* find_phase_Fe_default(double P, double T)
{
 if (P <= 0 || T <= 0)
 {
  return NULL;
 }
 P /= 1E10;              // convert microbar to GPa
 // Default Core
 if( T > 12.8*P + 2424 && T > 13.7*P + 2328)   // melting curve
from Dorogokupets et al. 2017, Scientific Reports. fcc and hcp
Fe melting curve.
        return Fe_liquid;
 else
        return Fe_hcp;     // use hcp Iron for all regions.
}
```

Changing Equations of State:

The file *src/EOSlist.cpp* contains many equations of state for planet materials and different measurements for the same material. They can be returned in *phase.cpp* to build your planet model (see previous). Individual EOS can also be changed and added to.

Example EOS:

```
double Fe_hcp_array[][2] = {{0,2}, {1,mFe/8.43}, {2,177.7},
{3,5.64}, {5,mFe}, {7,322}, {8,2.09}, {9,1.01}, {10,0.05},
{14,1}, {15,26}};

EOS *Fe_hcp = new EOS("Fe hcp (Smith)", Fe_hcp_array,
sizeof(Fe_hcp_array)/2/sizeof(Fe_hcp_array[0][0]));
```

The dictionary contains keys to indexes of variables for the EOS and their corresponding value. Example: above {0,2} means Vinet formulation, {2,177.7} is the bulk modulus of 177.7 GPa.

Index	Variable	Unit	Comment
0	EOS formula type		(0) 3rd order Birch-Murnaghan, (1) 4th order Birch-Murnaghan, (2) Vinet, (3) Holzapfel, (4) Keane, (6) Ideal Gas, (7) Density-Pressure input table from file for interpolation, (8-12) same as 0-4 in combination with RTPress.
1	V0	cm^3 mol^-1	Molar volume at reference point
2	K0	GPa	Bulk modulus
3	K0'		Pressure derivative of the bulk modulus. Default 4
4	K0"	GPa^-1	Second pressure derivative
5	m_mol	g mol^-1	Molar mass
6	P0	GPa	The minimum pressure, corresponding to V0. Default 0
7	Theta0	K	Fitting parameter of Einstein or Debye temperature. Default 1
8	gamma_0		Fitting parameter of Gruneisen parameter
9	beta		Fitting parameter of Gruneisen parameter
10	gamma_inf		Fitting parameter of Gruneisen parameter. Default 2/3
11	gamma_0'		Volume derivative of the Gruneisen parameter
12	e0	10^-6$ K^-1	Electronic contribution to Helmholtz free energy. Default 0
13	g		Electronic analogue of the Gruneisen parameter
14	n		Number of atoms in the chemical formula. Default 1
15	Z		Atomic number (number of electron)
16	T0	K	Reference temperature for the thermal pressure. Default 300
17	alpha0	10^-6 K^-1	The zeroth order coefficient of thermal expansion at a reference pressure P0
18	alpha1	10^-6 K^-2	The first order coefficient of thermal expansion at a reference pressure P0
19	xi		Power law index in the coefficient of thermal expansion. Default 0
20	c_p0	10^7 erg g^-1 K^-1	Specific heat capacity at constant pressure
21	c_p1	10^7 erg g^-1 K^-2	Coefficient for specific heat capacity
22	c_p2	10^7 erg g^-1 K	Coefficient for specific heat capacity
23	Debye approx		Positive number for Debye, otherwise Einstein
24	thermal type		See Paper

Adding a new EOS:

Define a new EOS in *src/EOSlist.cpp* and list it in *src/EOSlist.h*.

Practice Problems

Choose one project below for your team to work on.

Project 1: Find Possible Compositions of LHS 1140b and HD 137496b

Goal: Find the possible interior structures of two Super Earth's.

LHS 1140 b parameters from Lillo-Box et al. 2020 https://arxiv.org/abs/2010.06928

HD 137496 b parameters from Silva et al. 2022 https://arxiv.org/abs/2111.08764

LHS 1140 b		
Orbital period, P_b [days]	$\mathcal{G}$(24.736959,0.0004)	$24.73694^{+0.00041}_{-0.00040}$
Time of mid-transit, $T_{0,b} - 2400000$ [days]	$\mathcal{U}$(58399.0,58401.0)	$58399.9303^{+0.0012}_{-0.0013}$
Planet mass, M_b [$M_\oplus$]	$\mathcal{U}$(0.0,50.0)	$6.38^{+0.46}_{-0.44}$
Planet radius, R_b [$R_\oplus$]	$\mathcal{T}$(1.727,0.1,0,10)	$1.635^{+0.046}_{-0.046}$
Orbital inclination, i_b [deg.]	$\mathcal{T}$(89.89,0.05,70,90)	$89.877^{+0.049}_{-0.045}$
Planet density, ρ_b [$g \cdot cm^{-3}$]	(derived)	$8.04^{+0.84}_{-0.80}$
Transit depth, Δ_b [ppt]	(derived)	$4.93^{+0.27}_{-0.27}$
Orbit semi-major axis, a_b [AU]	(derived)	$0.0957^{+0.0019}_{-0.0019}$
Relative orbital separation, $a_b/R_\star$	(derived)	$96.4^{+2.2}_{-2.1}$
Transit duration, $T_{14,b}$ [hours]	(derived)	$2.055^{+0.048}_{-0.049}$
Planet surface gravity, g_b [$m \cdot s^{-2}$]	(derived)	$23.4^{+1.9}_{-2.0}$
Incident Flux, $F_{\mathrm{inc},b}$ [$F_{\mathrm{inc},\oplus}$]	(derived)	$4.98^{+0.23}_{-0.22}$
Stellar effective incident flux, S_b [$S_\oplus$]	(derived)	$0.477^{+0.022}_{-0.021}$
Stellar luminosity, $L_\star$ [$L_\odot$]	(derived)	$0.477^{+0.022}_{-0.021}$
Equilibrium temperature, $T_{\mathrm{eq},b}$ [K]	(derived)	$378.9^{+4.3}_{-4.2}$

	HD 137496 b
Period P_b [days]	$1.62116^{+7.91\,10^{-5}}_{-8.06\,10^{-5}}$
RV semi-amplitude K_b [m s^{-1}]	2.14 ± 0.29
Radius ratio $R_{p,b}/R_*$	$0.0076^{+0.00030}_{-0.00029}$
Impact parameter b_b	$0.27^{+0.14}_{-0.17}$
Eccentricity e_b	0
Time of inferior conjunction $t_{ic,b}$ [BJD-2450000]	$8039.1317^{+0.0028}_{-0.0029}$
Transit depth [ppm]	$57.5^{+4.61}_{-4.28}$
Transit duration T_b [hr]	$3.28^{+0.10}_{-0.14}$
Ingress to egress duration τ_b [hr]	$3.23^{+0.11}_{-0.15}$
Scaled semi-major axis a_b/R_*	3.70 ± 0.07
Semi-major axis a_b [AU]	0.02732 ± 0.00019
Inclination i_b [°]	$85.8^{+2.6}_{-2.2}$
Radius R_b [$R_\oplus$]	$1.31^{+0.06}_{-0.05}$
Mass M_b [$M_\oplus$]	4.04 ± 0.55
Density ρ_b [g cm^{-3}]	$10.49^{+2.08}_{-1.82}$
Equilibrium temperature $T_{eq,b}$ [K]	2130^{+30}_{-29}

1. For each planet, use `mode0.cfg` to keep the total mass the same but vary core and mantle mass percentages
    a. Start 6.38 Earth-mass and 32% of mass in core: `mass_of_core=2.0416 mass_of_mantle=4.3384`
    b. Save output file with name to remember percentages. (Output file will be appended to every time if output file name is not changed.)
2. Check output for planet radius in last line of file.
3. Change core and mantle percentages until radius matches observed values.
4. Report planet's Core Mass Fraction.

Suggested Roles: Divide in teams for each planet, 1+ group member runs codes (1 take median mass, 1 take +1σ, 1 take -1σ), 1 group member records radii found,

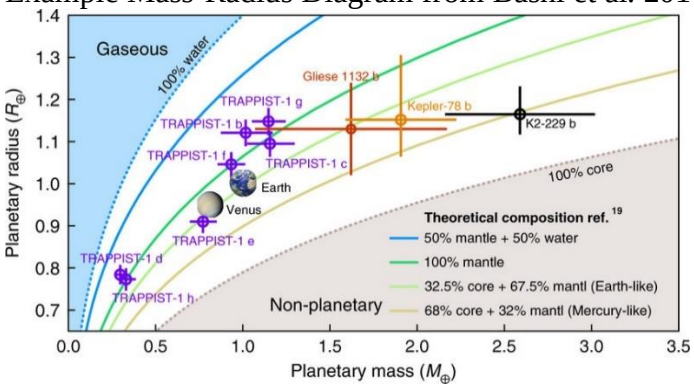
Extension: Add hydrosphere/atmosphere and find more possible interior structures

Side note: We wanted to give you a chance to feel the effect of internal structure on planet observables. However, a root solver using a secant method is available in mode7.cfg.

Project 2: Mass-Radius Diagram

Goal: Create mass-radius lines of constant interior structure for mass-radius diagram.

Example Mass-Radius Diagram from Bashi et al. 2017



Use mode3.cfg to run planets from 0.1 to 4 Earth-masses with the 6 compositions in figure: 100% water, 100% core, 100% mantle, 50/50 core/mantle, 32.5/67.5 core/mantle, 68/32 core/mantle.

1.) 3 input files for 100% planets are already in the input directory. Create remaining 3 files with text editor or run with python the script, *input/massradiusinput.py*.

2.) Set up mode3.cfg. Then compile by running: `make`

3.) Run: ./planet run/mode6.cfg

b) Save output with differing filenames
b) Use full solver: solver=`1`
c) Set temperature profile. Surface_temp=300 all temp jumps equal to 0

4.) Check output file for planet radius in final column.

5.) Plot the radius with mass of each resulting file. Suggestion to use Python.

6.) Add Trappist-1 system's mass and radius from Agol et al. 2020 https://arxiv.org/abs/2010.01074:

Planet:	b	c	d	e	f	g	h
R $[R_\oplus]$	$1.116^{+0.014}_{-0.012}$	$1.097^{+0.014}_{-0.012}$	$0.788^{+0.011}_{-0.010}$	$0.920^{+0.013}_{-0.012}$	$1.045^{+0.013}_{-0.012}$	$1.129^{+0.015}_{-0.013}$	$0.755^{+0.014}_{-0.014}$
M $[M_\oplus]$	1.374 ± 0.069	1.308 ± 0.056	0.388 ± 0.012	0.692 ± 0.022	1.039 ± 0.031	1.321 ± 0.038	0.326 ± 0.020
ρ $[\rho_\oplus]$	$0.987^{+0.048}_{-0.050}$	$0.991^{+0.040}_{-0.043}$	$0.792^{+0.028}_{-0.030}$	$0.889^{+0.030}_{-0.033}$	$0.911^{+0.025}_{-0.029}$	$0.917^{+0.025}_{-0.029}$	$0.755^{+0.059}_{-0.055}$

Python: plt.errorbar(x,y, yerr=num, xerr=num)

Suggested roles: Each take a line of constant composition to create, one starts a python program to plot data.

Extension: Add curves for planets with thin atmospheres. Default atmosphere is 3 g mol^{-1} for H/He.

Project 3: Vary Core's Composition

Goal: Find the change in a one Earth-mass planet's radius depending on the percentage of lighter elements in the core. The storage structure for EOS is described on Page 4.

1. Using mode0.cfg run an Earth-like planet with 33% of mass in core and 67% in mantle. Check radius of planet in last line of output file.
2. Open *EOSlist.cpp* and find the default hcp iron equation of state.
    a. Parameters are from: Smith et al. 2018
3. Let's assume adding lighter elements has little effect on the high-pressure parameters but adjusts the reference volume and molar mass/atomic weight.
    a. {1,mFe/8.43} is the reference volume. mFe is atomic weight (55.847 g/mol) of Fe divided by the reference density of 8.43 g cm^-3 from Smith et al. 2018.
    b. Add 5% sulfur (15.53 cm^3/mol, 32.06 g/mol) to the core by changing the parameter to {1,0.95*mFe/8.43+0.05*15.53}.
    c. Change m_mol, index 5, to {5,0.95*mFe+0.05*32.06}
4. Change output file's name in mode0.cfg to reflect your changed core.
5. Recompile with `make` and run with `./planet run/mode0.cfg`.
6. Record planet's radius. Last line in output structure file. Find the radius of the core-mantle boundary (CMB) in output file.
7. Vary core composition and report changes in radius and CMB radius.

Extension: In EOSlist.cpp is an iron-silicate alloy EOS. Fe_7Si and Fe_15Si which are 7 and 15 per cent silicate by weight. Change src/phase.cpp to use these EOS (`return Fe_hcp3` -> `return Fe_7Si`). Compile and run a one Earth-mass planet. Compare to sulfur core.

Project 4: Adding Graphite Mantle, *Difficult*

Goal: Add a graphite EOS to the code and make planets out of graphite.

1. Pick a planet mass and run a 100% mantle planet with run/mode0.cfg
2. Seager et al. 2007 (https://arxiv.org/abs/0707.2895) use a graphite equation of state from Hanfland et al. 1989 to model exoplanets. They used a 3rd order Birch-Murnagham EOS. The bulk modulus, derivative of bulk modulus, and reference density are listed:

Atom or Compound	K_0 (GPa)	K_0'	ρ_0 (Mg m^{-3})	Fit
C (graphite)	33.8±3	8.9±1.0	2.25	BME

3. Reference Molar Volume (index 1) is 12.011/2.25 cm^3/mol
4. Create a new EOS in EOSlist.cpp following instructions on Page 4. Don't forget to add to EOSlist.h also.
5. Change phase.cpp to return the graphite EOS in the mantle. Replace "`Si_PPv_Sakai`" and "`Si_Pv`" with the name of your graphite EOS.
6. Change output file name in mode0.cfg.
7. Compile (`make`) and run (`./planet run/mode0.cfg`) a 100% graphite mantle planet. Report the change in planet radius (last line of output file).

Extension: Test the large error bars on bulk modulus and derivative of bulk modulus reported in table. Change your new EOS by one sigma and test the change in radius.