Chapter 6

MOSFET V_t

6.1 15 Ω-cm $=10^{15}$ cm^{-3}, $\therefore E_f = E_v + 0.26$eV oxide trap density = $8\text{x}10^{10}$ cm^{-2}, $Z = 50$ µm, $L = 2$ µm, $T_{ox} = 5$nm

(a) $V'_{fb} = V_{fb} + \Delta V$. $V_{fb} = 3.1 - (3.1 + 0.86) = -0.86$eV, $\Delta V = Q_f/C_{ox} = 18.5$ mV.
Therefore $V'_{fb} \cong V_{fb} = -0.86$ eV
When oxide thickness is thin, the trap charge effect can be ignored.

(b) $V_t = V'_{fb} + V_{ox} + V_s \cong V_{fb} + 2\phi_B + (2\varepsilon_s qN_a 2\phi_B)^{1/2}/C_{ox}$
$= -0.86 + 0.6 + 0.02\text{V} = -0.24\text{V}$

(c) To make $V_t = 0.5$V, one should implant boron into silicon substrate such that $\Delta V_t = Q_{imp}/C_{ox}$. Therefore ion implant dose should be
$(0.5\text{V}+0.24\text{V}) \times C_{ox} \div q = 3.2\times10^{12}$ cm^{-2}.

6.2 (a) Using Equation 4.16.4 and referring to Table 1-4, we find

$$\phi_{bi} = \phi_{Bn} - kT\ln\left(\frac{N_c(GaAs)}{N_d}\right) = 1V - kT\ln\left(\frac{4.7\times10^{17}}{1\times10^{17}}\right) = 0.96V\ .$$

Then,

$$W_{dep} = \sqrt{\frac{2\varepsilon_s\phi_{bi}}{q}\frac{1}{N_d}} = \sqrt{\frac{2(13\varepsilon_0)\phi_{bi}}{q}\frac{1}{N_d}} = 0.12\,\mu m\ .$$

(b) $$W_{dep} = 0.2\,\mu m = \sqrt{\frac{2(13\varepsilon_0)(\phi_{bi}+V)}{q}\frac{1}{N_d}} \Rightarrow V = \frac{qN_dW_{dep}^2}{2(13\varepsilon_0)} - \phi_{bi} = 1.82V\ .$$

A negative V_g is need to increase W_{dep} and turn-off the channel. (A metal/N-type semiconductor Schottky diode exhibits the same forward/reverse bias properties as an P^+/N diode.)

(c) Yes. If the positive V_g is kept small (say 0.5V), the forward current of the Schottky gate maybe comparable to the subthreshold drain leakage current. A positive V_g would reduce W_{dep} and therefore raise I_{ds}.

(d) The channel thickness or doping concentration must be reduced so that $W_{dep} \geq$ channel thickness at $V_g = 0$.

6.3 $C_{ox} = 6.9\times10^{-7}\dfrac{F}{cm^2}$, $\phi_B = \dfrac{\mathrm{kT}}{\mathrm{q}}\ln\dfrac{\mathrm{N_{sub}}}{\mathrm{n_i}} = 0.47\mathrm{eV}$

(a)` $V_t = V_{fb} + 2\phi_B + \dfrac{1}{C_{ox}}\sqrt{2q\varepsilon_s N_{sub} 2\phi_B}$

$$V_t = -\frac{E_g}{2} - \phi_B + 2\phi_B + \frac{1}{C_{ox}}\sqrt{2q\varepsilon_s N_{sub} 2\phi_B} = -0.09 + 0.61 = 0.52V$$

(b) $V_t = V_{fb} - 2\phi_B - \dfrac{1}{C_{ox}}\sqrt{2q\varepsilon_s N_{sub} 2\phi_B}$

$$V_t = -\frac{E_g}{2} + \phi_B - 2\phi_B - \frac{1}{C_{ox}}\sqrt{2q\varepsilon_s N_{sub} 2\phi_B} = -0.56 - 0.47 - 0.61 = -1.64V$$

(c) $V_t = V_{fb} - 2\phi_B - \dfrac{1}{C_{ox}}\sqrt{2q\varepsilon_s N_{sub} 2\phi_B}$

$$V_t = \frac{E_g}{2} + \phi_B - 2\phi_B - \frac{1}{C_{ox}}\sqrt{2q\varepsilon_s N_{sub} 2\phi_B} = -0.52V$$

(d) $V_b = 0V$
$V_s = 0V$
$V_d = 2.5V$
$V_g = 2.5V$

(e) $V_b = 2.5V$
$V_s = 2.5V$
$V_d = 0V$
$V_g = 0V$

(f) $I_{dsat} = \dfrac{\mu_n W C_{ox}}{2L}(V_{gs} - V_t)^2$

$$\frac{I_{dsatc}}{I_{dsatb}} = \frac{(-2.5-(-0.52))^2}{(-2.5-(-1.64))^2} \approx 5.3$$

The transistor with the lower absolute value of threshold voltage has a higher saturation current. That is why P^+ poly-gate PMOSFETs are typically used in IC.

(g) The ratio of the current is the ratio of the mobilities.
To find μ_n,
$\left(V_{gs} + V_t + 0.2\right)/6T_{oxe} = \left(2.5 + 0.52 + 0.2\right)V/\left(6\times5\times10^{-7}\right)cm = 1.07MV/cm$
and $\mu_n = 250\mathrm{cm^2V^{-1}s^{-1}}$.

To find μ_p,

$$-\left(V_{gs}+1.5V_t-0.25\right)/6T_{oxe} = -\left(-2.5+1.5\times(-0.52)-0.25\right)V/\left(6\times5\times10^{-7}\right)cm = 1.01MV/cm$$

and $\mu_p = 63\text{cm}^2\text{V}^{-1}\text{s}^{-1}$.

$$\frac{I_{dsat(c)}}{I_{dsat(a)}} = \frac{\mu_p}{\mu_n} \approx \frac{1}{4}$$

Basic MOSFET IV Characteristics

6.4 (a) Due to the highly doped regions nearby, transistor C-V always approaches C_{ox} in inversion. Hence, it is impossible to determine the frequency. Either high or low frequency could have been used.

(b) Since $V_t > V_{fb}$, this is a NMOS.

(c) From the I_d-V_g curve, V_t is 0.55V. More precisely,

$$I_d = \mu C_{ox}\frac{W}{L}V_{ds}\left(V_g - V_t - \frac{V_{ds}}{2}\right) \Rightarrow 0.55 - V_t - \frac{V_{ds}}{2} = 0$$

$$V_t = 0.5\text{V}$$

(d) $\textit{Slope of curve } I_d\text{ -}V_g \textit{ line} = \mu C_{ox}\frac{W}{L}V_{ds} = 5\times10^{-3}\Omega^{-1}$.

$$V_{ds} = 0.1\text{V}$$

From the CV curve,

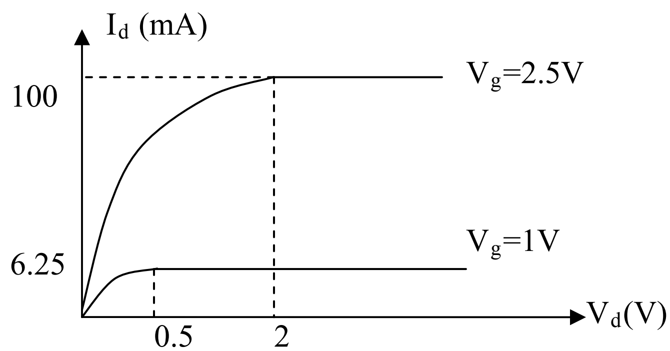
$$C_{ox}\text{WL} = 1\text{pF} \Rightarrow \text{C}_{\text{ox}}\frac{\text{W}}{\text{L}} = 10^{-4}\frac{\text{F}}{\text{cm}^2}$$

$$\text{Thus, } \mu = \frac{5\times10^{-3}}{0.1\times10^{-4}} = 500\frac{cm^2}{Vs}$$

(e) $V_{dsat} = V_g - V_t$

$$I_{dsat} = \frac{\mu C_{ox}W}{2L}(V_{dsat})^2 = 0.025\frac{A}{V^2}.V_{dsat}{}^2$$

V_g	1V	2.5V
V_{dsat}	0.5V	2V
I_{dsat}	6.25mA	100mA



6.5 (a) For V_{gs}=4V, $V_{dsat} \sim 3V = (V_{gs} - V_t)$ and V_t=1V

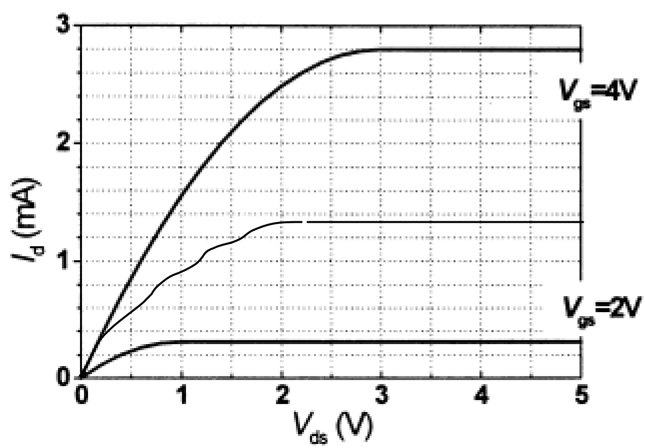
(b) $I_{dsat} = \mu_n C_{ox} W / 2L \cdot (V_g - V_t)^2$

$C_{ox} = 3.45 \times 10^{-7} F / cm^2$

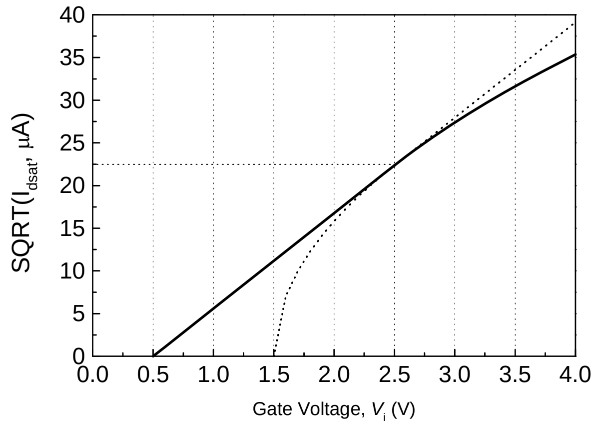
$$\mu_n = \frac{2I_{dsat}}{C_{ox}\left(\frac{W}{L}\right)(V_g - V_t)^2} = 361 cm^2 / Vs$$

(c) At V_{gs}=3V, V_{dsat}=(3-1)V=2V

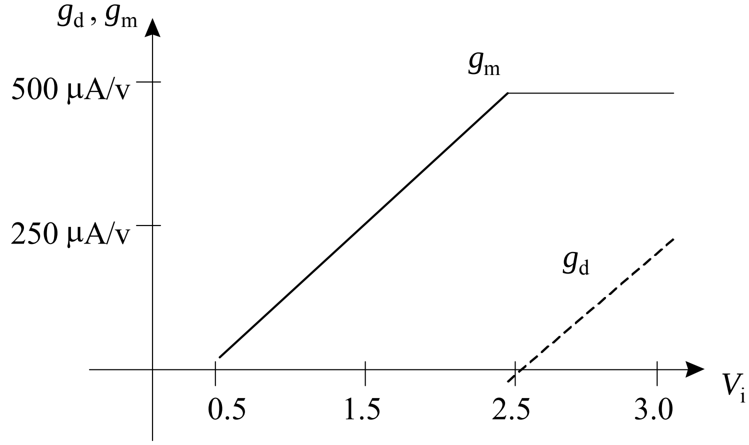
$I_{dsat} = 361 \times 3.45 \times 10^{-7} \cdot (10/2) \cdot (3-1)^2 / 2 = 1.25 \times 10^{-3} A$



6.6 (a) At saturation, $V_d = V_{dsat} = V_g - V_t$. V_{dd}=2V. Therefore the transistor is in saturation mode when V_g<2.5V. $I_{dsat} = 125(V_g - 0.5)^2$ μA. When V_g>2.5V, the transistor is in linear region with $I_d = 500(V_g - 1.5)$ μA.



(b) & (c) Transconductance: solid line, Output Conductance: dotted line



6.7 (a) $V_{gs} - V_t = 2V$

$V_{gs} = 2.5V$

(b) $Q_n = -C_{ox}(V_{gs} - V_t - V_c) = 0$ (Pinch-Off)

(c) $I_{ds} @ V_{ds} = 4V$

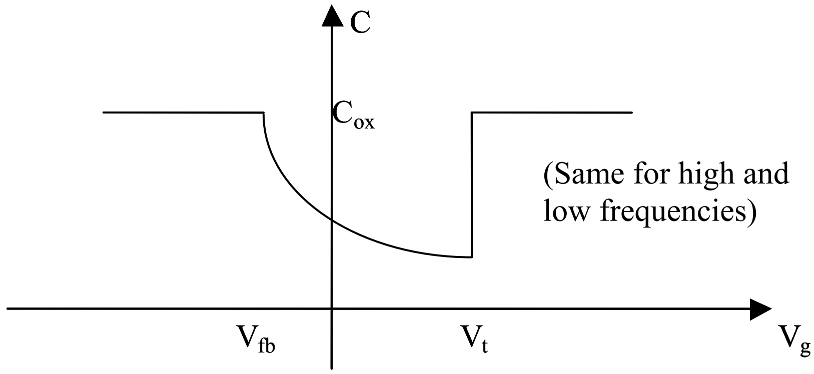
$V_{dsat} = V_{gs} - V_t = 3V$

(in saturation)

$I_{ds} \propto (V_{gs} - V_t)^2$

$I_{ds} = 10^{-3} \times \frac{3^2}{2^2} = 2.25 \times 10^{-3} A$

(d)



6.8 (a) $\phi_B = \dfrac{kT}{q}\ln\dfrac{N_a}{n_i} = 0.297V$

$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = 7.08\times10^{-8}\ F/cm^2$$

$$V_{fb} = \chi_{Si} - (\chi_{Si} + \frac{E_g}{2} + \phi_B) = -0.857\text{V}$$

$$V_g = V_{fb} + V_s + V_{ox} \Rightarrow V_t = V_{fb} + 2\phi_B + \frac{\sqrt{2\varepsilon_s qN_a 2\phi_B}}{C_{ox}} = -0.064V$$

(b) $I_{dsat} = \dfrac{\mu_n C_{ox} W}{2L}(V_g - V_t)^2 = 1.21\text{mA}$

(c) Since V_d is less than (V_g-V_t), it is in the linear region.

$$I_d = \frac{\mu C_{ox} W}{L}\left[(V_g - V_t)V_d - \frac{V_d^2}{2}\right]$$

$$g_d = \frac{\partial I_d}{\partial V_D} = \frac{\mu C_{ox} W}{L}\left[(V_g - V_t) - V_d\right] = 1.17mS$$

(d) Since V_d is less than (V_g-V_t), it is in the linear region.

$$I_d = \frac{\mu C_{ox} W}{L}\left[(V_g - V_t)V_d - \frac{V_d^2}{2}\right]$$

$$g_d = \frac{\partial I_d}{\partial V_g} = \frac{\mu C_{ox} W}{L}V_d = 1.13mS$$

Potential and Carrier Velocity in MOSFET Channel

6.9 $$I_d = -Q_n \mu_n \frac{dV_c}{dx} W = (V_g - V_t - V_c) C_{ox} \mu_n \frac{dV_c}{dx} W$$

$$\therefore \int_0^x \frac{I_d}{\mu_n C_{ox} W} dx = \int_0^{Vc} (V_g - V_t - V_c) dV_c$$

$$\rightarrow I_d \cdot x / (\mu_n C_{ox} W) = (V_g - V_t) V_c - 1/2 V_c^2$$

Solving this quadratic equation of V_c, we get

$$\therefore V_c(x) = (V_g - V_t) \pm \sqrt{(V_g - V_t)^2 - \frac{2 I_d x}{\mu_n C_{ox} W}}$$

Choosing "-" so that $V_c(0)=0$,

$$\therefore V_c(x) = (V_g - V_t)\left[1 - \sqrt{1 - \frac{2 I_d x}{\mu_n C_{ox} W (V_g - V_t)^2}}\right]$$

$$= (V_g - V_t)\left[1 - \sqrt{1 - \frac{2 x \mu_n C_{ox} \frac{W}{2L} (V_g - V_t)^2}{\mu_n C_{ox} W (V_g - V_t)^2}}\right]$$

$$= (V_g - V_t)\left[1 - \sqrt{1 - x/L}\right].$$

6.10 (a) $$I_{ds} = W C_{oxe} (V_{gs} - m V_{cs} - V_t) \mu_{es} d V_{cs} / dx$$

$$\int_0^x I_{ds} dx = W C_{oxe} \mu_s \int_0^{V_{cs}} (V_{gs} - m V_{cs} - V_t) dV_{cs}$$

$$I_{ds} x = W C_{oxe} \mu_s (V_{gs} - V_t - \frac{m V_{cs}}{2}) V_{cs}$$

Equating the expression above with

$$\mathrm{I}_{ds} = \frac{W}{L} C_{oxe} \mu_s (V_{gs} - V_t - \frac{m}{2} V_{ds}) V_{ds},$$

we get

$$\frac{\mathrm{x}}{\mathrm{L}}\left(V_{gs}-V_t-\frac{mV_{ds}}{2}\right)V_{ds}=(V_{gs}-V_t-\frac{mV_{cs}}{2})V_{cs}$$

$$m{V_{cs}}^2-2(V_g-V_t)V_{cs}+\frac{x}{L}(2(V_g-V_t)-mV_{ds})V_{ds}=0$$

Solving the quadratic equation, we get

$$\mathrm{V_{cs}}=\frac{V_{gs}-V_t}{m}\pm\frac{\sqrt{(V_g-V_t)^2-m\frac{x}{L}(2(V_g-V_t)-mV_{ds})V_{ds}}}{m}$$

$$V_{cs}=\frac{V_{gs}-V_t}{m}(1-\sqrt{1-\frac{x}{L}})$$

(b) $$Q_{inv}(x)=C_{oxe}(V_{gs}-mV_{cs}-V_t)=C_{oxe}\left[V_{gs}-V_t-(V_{gs}-V_t)(1-\sqrt{1-\frac{x}{L}}\right]$$

$$=C_{oxe}(V_{gs}-V_t)\left[1-\left(1-\sqrt{1-\frac{x}{L}}\right)\right]=C_{oxe}(V_{gs}-V_t)\left(\sqrt{1-\frac{x}{L}}\right)$$

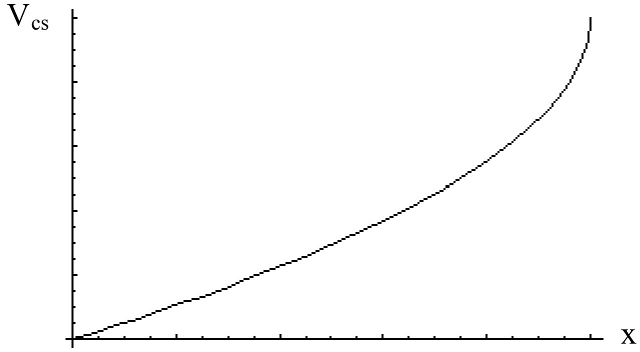
(c) $$\frac{dV_{cs}}{dx}=\mathcal{E}(x)=\frac{(V_g-V_t)}{m}\left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{1-x/L}}\right)\left(\frac{1}{L}\right)=\frac{(V_g-V_t)}{2mL}\left(\frac{1}{\sqrt{1-x/L}}\right)$$

$$\upsilon(x)=\mu_n\frac{dV_{cs}}{dx}=\mu_n\mathcal{E}(x)=\frac{\mu_n(V_g-V_t)}{2mL}\left(\frac{1}{\sqrt{1-x/L}}\right)$$

(d) $$WQ_{inv}\mu_n\mathcal{E}=W.C_{oxe}(V_{gs}-V_t)\left(\sqrt{1-\frac{x}{L}}\right).\frac{\mu_n(V_g-V_t)}{2mL}\left(\frac{1}{\sqrt{1-x/L}}\right)$$

$$=\frac{WC_{oxe}\mu_n}{2mL}(V_{gs}-V_t)^2=I_{dsat}$$

(e)



IV Characteristics of Novel MOSFET

6.11 (a) $I_d = -Q_n \mu_n \frac{dV_c}{dx} W = (V_g - V_t - V_c) C_{ox}(x) \mu_n \frac{dV_c}{dx} W$

$$= (V_g - V_t - V_c) \frac{\varepsilon_{ox}}{Ax^2 + B} \mu_n \frac{dV_c}{dx} W$$

$$\therefore \int_{-L/2}^{L/2} I_d \cdot (Ax^2 + B) dx = \int_0^{V_{ds}} (V_g - V_t - V_c) \varepsilon_{ox} \mu_n W dV_c$$

$$\to I_d \cdot [\frac{A}{3} x^3 + Bx]_{-L/2}^{L/2} = \varepsilon_{ox} \mu_n W [(V_g - V_t) V_c - 1/2 V_c^2]_0^{V_{ds}}$$

$$\therefore I_d = \frac{W}{L} \cdot \frac{\varepsilon_{ox} \mu_n}{\frac{AL^2}{12} + B} \cdot [(V_g - V_t) V_{ds} - 1/2 V_{ds}^2]$$

(b) $V_{dsat} = V_{ds} @ \frac{\partial I_d}{\partial V_{ds}} |_{V_{gs}} = 0 \qquad \therefore V_{dsat} = V_g - V_t$

(c) It suggests a large W_{dmax}. $V_{ox} = Q_n / C_{ox}$

6.12 (a) $I_d = -Q_n \mu_n \frac{dV_c}{dx} W(x) = (V_g - V_t - V_c) C_{ox} \mu_n \frac{dV_c}{dx} W(x)$

$$\therefore \int_0^L I_d / W(x) dx = \int_0^{V_{ds}} (V_g - V_t - V_c) \mu_n C_{ox} dV_c$$

$$\to I_d \cdot \ln[(W_0 + L)/W_0] = \mu_n C_{ox} [(V_g - V_t) V_{ds} - 1/2 V_{ds}^2]$$

$$\therefore I_d = \frac{\mu_n C_{ox}}{\ln(1 + L/W_0)} [(V_g - V_t) V_{ds} - 1/2 V_{ds}^2]$$

(b) $V_{dsat} = V_{ds} @ \frac{\partial I_d}{\partial V_{ds}}|_{V_{gs}} = 0 \qquad \therefore V_{dsat} = V_g - V_t$

$$\therefore I_{dast} = \frac{\mu_n C_{ox}}{\ln(1+L/W_0)} \cdot \frac{(V_g - V_t)^2}{2}$$

CMOS

6.13 (a) $V_{fb,NMOS}$ = -(E_g/2) – (kT/q * ln(5e15/1e10)) = - 0.55 -0 .4 = -0.95V
$V_{fb,PMOS}$ = -0.55 + 0.4 = -0.15V
Not symmetrical

(b) $V_{fb,NMOS}$ = 0.55 - 0.4 = 0.15V
$V_{fb,PMOS}$ = 0.55 + 0.4 = 0.95V
Not symmetrical

(c) Since V_{ox} and V_s will be symmetrical, I would use a mid-gap gate material such as tungsten.
So the workfuction will be 4.05 eV + $E_{g,Si}$/2 = 4.6eV. However, processing issues makes tungsten (or any metal gates for that matter) a challenge to implement.

(d) In the same process, the NMOS and PMOS will have same oxide thickness. If the substrate doping levels for n and p flavors are the same, then I would use P^+ gates for PMOS devices and N^+ gates for NMOS devices. In this way, the flatband voltages will be symmetrical and the resulting $|V_t|$ small.

6.14 (a) PMOS, N-type substrate:

$$\phi_n = kT \ln \frac{N_d}{n_i} = 0.38V$$

$$\mathrm{V_{fb}} = \phi_m - \chi_{si} - \frac{E_g}{2} + \phi_n = 4.1 - 4.05 - 0.55 + 0.38 = -0.12\mathrm{V}\,.$$

NMOS, N-type substrate:

$$\phi_p = kT \ln \frac{N_a}{n_i} = 0.42V$$

$$\mathrm{V_{fb}} = \phi_m - \chi_{si} - \frac{E_g}{2} - \phi_p = 4.1 - 4.05 - 0.55 - 0.42 = -0.92\mathrm{V}$$

(b) $C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = 6.9 \times 10^{-7} \frac{F}{cm^2}$

PMOS:

$$V_t = V_{fb} - 2\phi_n - \frac{1}{C_{ox}}\sqrt{2\varepsilon_s qN_d 2\phi_n} = -0.12 - 0.76 - 0.10 = -0.98V$$

NMOS:

$$V_{fb} + 2\phi_p + \frac{1}{C_{ox}}\sqrt{2\varepsilon_s qN_d 2\phi_p} = -0.92 + 0.84 + 0.24 = 0.16V$$

(c) The threshold voltage must be changed by

$$\Delta V_t = -\frac{Q_{impl}}{C_{ox}} = 0.82\text{V}.$$

Hence,

$$Q_{impl} = -5.7\times 10^{-7}\frac{C}{cm^2}.$$

6.15 $$I_{ds} = WC_{oxe}(V_{gs} - mV_{cs} - V_t)\frac{\mu_s\, dV_{cs}/dx}{1 + \frac{dV_{cs}}{dx}\Big/\mathcal{E}_{sat}}$$

$$\int_0^L I_{ds}dx = \int_0^{V_{ds}}\left[WC_{oxe}\mu_s\left(V_{gs} - mV_{cs} - V_t\right) - I_{ds}/\mathcal{E}_{sat}\right]dV_{cs}$$

$$I_{ds}L + I_{ds}\frac{V_{ds}}{\varepsilon_{sat}} = WC_{oxe}\mu_s(V_{gs} - V_t - \frac{m}{2}V_{ds})V_{ds}$$

$$I_{ds} = WC_{oxe}\mu_s(V_{gs} - V_t - \frac{m}{2}V_{ds})V_{ds}\Bigg/\left(L + \frac{V_{ds}}{\mathcal{E}_{sat}}\right)$$

$$I_{ds} = \frac{W}{L}C_{oxe}\mu_s(V_{gs} - V_t - \frac{m}{2}V_{ds})V_{ds}\Bigg/\left(1 + \frac{V_{ds}}{L\mathcal{E}_{sat}}\right) = \frac{I_{ds}(Long\ channel)}{1 + V_{ds}/\mathcal{E}_{sat}L}.$$

6.16

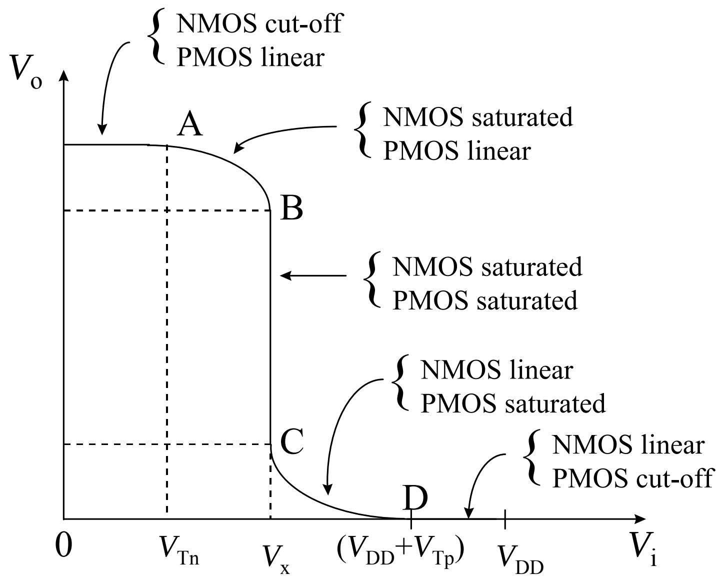
	NFET Operation Mode	PFET Operation Mode
A	**Cut-off**	**Linear**
B	**Saturation**	**Linear**
C	**Linear**	**Saturation**
D	**Linear**	**Cut-off**

A: Vgs<Vth for NFET, therefore it is cut off. For PFET |Vgs| > |Vth| and |Vds|<|Vdsat| (|Vds|~0V, |Vdsat| ~ 1.05V), so it operates in linear mode.

B: For NFET Vgs > Vth and Vds>Vdsat (Vds~1.75V, Vdsat ~ 0.3V), so it operates in saturation mode. For PFET |Vgs| > |Vth| and |Vds|<|Vdsat| (|Vds|~0.25V, |Vdsat| ~ 0.6V), so it operates in linear mode.

The answers to C and D can be worked out through the same procedure.

6.17 (a)



(b) At the point B where $V_i=V_x$, the NMOS is just becoming saturated from the linear region. Since NMOS is in the linear region

$$I_{dn} = K_N\left[(V_x - V_{tn})(V_x - V_{tn}) - \frac{(V_x - V_{tn})^2}{2}\right]$$

Since PMOS is saturated

$$I_{dp} = \frac{K_P}{2}(V_{dd} - V_x + V_{tp})^2$$

But $I_{DN} = I_{DP}$

$$\therefore \quad 40\left[(V_x - 1)^2 - \frac{(V_x - 1)^2}{2}\right] = \frac{35}{2}(5 - V_x - 1)^2$$

$$40(V_x - 1)^2 = 40(4 - V_x)^2 \Rightarrow V_x = 2.45\text{ V}$$

Thus,

Point	V_i(V)	V_o(V)
A	1	5
B	2.45	3.45
C	2.45	1.45
D	4	0

Body Effect

6.18 For a P-channel MOSFET, we have

$$V_t = V_{fb} + 2\phi_B + \frac{\sqrt{2\varepsilon_s q N_d (2\phi_B + V_{bs})}}{C_{ox}}$$

$$\Delta V_t = \frac{\sqrt{2\varepsilon_s q N_d}}{C_{ox}}\left(\sqrt{2\phi_B + V_{bs}} - \sqrt{2\phi_B}\right)$$

(a) For 100 nm oxide, $C_{ox} = 3.45\times10^{-8}$ F/cm^2.
If V_{bs} = 5V, ΔV_t = -0.8V.
By iteration, using initial guess of ϕ_B = 0.3V, we obtain
$N_d = 8.9\times10^{14}$ /cm^3 and ϕ_B = 0.284V.

(b) If V_{sb} is -2.5 V, ΔV_t = -0.497V.
V_t = -1.5 - 0.497 = 2.0 V

Velocity-Saturation Effect

6.19 In all 3 cases, use the general equation $I=WQ_{inv}v_{drift}$.

Case A:
The NMOS is in the triode region.
On source side, $Q_{inv}=C_{ox}(V_g-V_t)$ = 138e-9(5-.7) = 593 nC/cm^2.
So $\upsilon_{drift} = I/(WQ_{inv})$ = 1.5e-3/(15e-4 × 593e-9) = 1.7 x 10^6 cm/sec.
On drain side, $Q_{inv} = C_{ox}(V_g-V_t-V_d)$ = 138e-9(5-.7-.5) = 524 nC/cm^2.
Thus, υ_{dr} = 1.5e-3/(15e-4 × 524e-9) = 1.9 x 10^6 cm/sec.

Case B:
The NMOS enters saturation region.
On source side, υ_{dr} = 3.75e-3/(15e-4×593e-9) = 4.2 x 10^6 cm/sec.
On drain side, the electron velocity is saturated.
Thus, $\upsilon_{dr} = \upsilon_{sat}$ = 8 x 10^6 cm/sec.

Case C:
Similar to case B.
On source side, υ_{dr} = 4e-3/(15e-4×593e-9) = *4.5 x 10^6 cm/sec.*
On drain side, $\upsilon_{dr} = \upsilon_{sat}$ = *8 x 10^6 cm/sec.*

6.20

	T_{ox}	W	L	V_t	V_g
V_{dsat}	↑	No change	↓	↑	↓
I_{dsat}	↑	↓	↑	↑	↓

Reducing T_{ox} means smaller V_t => larger V_{dsat} $(1/(V_g-V_t) + 1/(E_{sat}L))^{-1}$ & larger I_{dsat} ($Q_{inv} \propto C_{ox}$).
Reducing W has no effect on V_{dsat} and decreases I_{dsat} since $I_{dsat} = WQ_{inv}v_{sat}$.
Reducing L reduces V_{dsat} (as discussed in lecture) and increases I_{dsat}. If you want to consider very short-channel length devices (L => 0), then essentially I_{dsat} is independent of L.
Reducing V_t => larger V_{dsat} & larger I_{dsat}.
Reducing V_g => smaller V_{dsat} & smaller I_{dsat}.

6.21
$$I_d = \mu_s C_{ox} W\left(V_{gs} - V_t - m\frac{V_{ds}}{2}\right)V_{ds} \Big/ \left(L + \frac{V_{ds}}{\varepsilon_{sat}}\right)$$
$$= WC_{ox}\left(V_{gs} - V_t - mV_{ds}\right)V_{sat}$$
$$= WC_{ox}\left(V_{gs} - V_t - mV_{ds}\right)\mu_s\frac{\varepsilon_{sat}}{2}$$

$$\left(V_{gs} - V_t - m\frac{V_{ds}}{2}\right)V_{ds} = \left(V_{gs} - V_t - mV_{ds}\right)\varepsilon_{sat}/2\left(L + \frac{V_{ds}}{\varepsilon_{sat}}\right)$$
$$= \left(V_{gs} - V_t - mV_{ds}\right)\frac{\varepsilon_{sat}L}{2} + \left(V_{gs} - V_t - mV_{ds}\right)\frac{V_{ds}}{2}$$

$$V_{gs}V_{ds} - V_tV_{ds} = \left(V_{gs} - V_t - mV_{ds}\right)\varepsilon_{sat}L$$
$$V_{ds}\left(V_{gs} - V_t - m\varepsilon_{sat}L\right) = (V_{gs} - V_t)\varepsilon_{sat}L$$

$$V_{ds} = \frac{(V_g - V_t)\varepsilon_{sat}L}{\left(V_{gs} - V_t - m\varepsilon_{sat}L\right)} = \left[\frac{m}{(V_{gs} - V_t)} + \frac{1}{\varepsilon_{sat}L}\right]^{-1}$$

6.22
$$I_{ds} = \frac{\frac{W}{L}C_{oxe}\mu_{ns}\left(V_{gs} - V_t - \frac{m}{2}V_{ds}\right)V_{ds}}{1 + \frac{V_{ds}}{\varepsilon_{sat}L}} = \frac{\frac{W}{L}C_{oxe}\mu_{ns}\left(\left(V_{gs} - V_t\right)\left(\frac{1}{V_{ds}}\right) - \frac{m}{2}\right)}{\frac{1}{V_{ds}^2} + \left(\frac{1}{\varepsilon_{sat}L}\right)\frac{1}{V_{ds}}}$$

$$\frac{1}{V_{dsat}} = \frac{m}{V_{gs} - V_t} + \frac{1}{\varepsilon_{sat}L}$$

6.23 (a) We know that

$$V_{dsat} = |\varepsilon_c L| \left[\left(1 + 2 \cdot \frac{(V_g - V_t)}{|E_c L|} \right)^{\frac{1}{2}} - 1 \right]$$

$$|\varepsilon_c L| = 0.1 \text{ V} \quad \Rightarrow \quad V_{\text{dsat}} = 0.1 \cdot \left[\left(1 + 2 \cdot \frac{4}{0.1} \right)^{\frac{1}{2}} - 1 \right] = 0.54 \text{ V}$$

(b) $$|\varepsilon_c L| = 10 \text{ V} \quad \Rightarrow \quad V_{\text{dsat}} = 10 \cdot \left[\left(1 + 2 \cdot \frac{4}{10} \right)^{\frac{1}{2}} - 1 \right] = 1.83 \text{ V}$$

(c) We know that

$$I_{\text{dsat}} = \frac{\mu_{\text{n0}} C_{\text{ox}} Z}{2L} V_{\text{dsat}}{}^2 \quad \text{and} \quad C_{\text{ox}} Z = \frac{10\text{fF}}{L}.$$

$$7\text{mA} = \frac{\mu_{\text{n0}} 10\text{fF}}{2L^2} 0.54^2$$

$$\mu_{\text{n0}} = 480 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$$

6.24 $V_{dsat} = \varepsilon_{sat} L(V_g - V_t) / (V_g - V_t + \varepsilon_{sat} L)$

What is ε_{sat}? $2v_{sat}/\mu_s = \varepsilon_{sat}$. μ_s is given by the universal mobility curve.

At T_{ox}=60A, $(V_g+V_t+0.2)/6T_{ox} = \varepsilon_{eff}$ = .9MV/cm.
From the curve, $\mu_s \sim 250$ cm^{2}V^{-1}s^{-1}.

This yields $\varepsilon_{sat} \sim 2(8\text{x}10^6)/250$ V/cm = $6.4\text{x}10^4$V/cm. Plug this back into the expression for V_{dsat} to get L ~ 0.19um.

$I_{dsat}/W = Q_{inv} v_{sat} = C_{ox}(V_g - V_t - V_{dsat}) v_{sat}$
$= (3.9\varepsilon_o/60\text{e-}8) \times (2.5\text{-}.5\text{-}.75) \times 8\times10^6 =$ *575uA/um width.*

Note: You will often find in literature that the saturation current is stated in units of uA/um instead of amperes. Also, notice that the Q_{inv} at V_c=V_{dsat} is not zero. That is, I_{dsat} is limited by velocity saturation instead of pinch-off.

6.25 (a)

$$1+\frac{V_{gs}-V_t}{mE_{sat}L}=2$$

$$V_{gs}-V_t=mE_{sat}L=\frac{2mv_{sat}L}{\mu_{ns}}=\frac{2\cdot 1.2\cdot 8\times 10^6\,cm/s\cdot 1\times 10^{-5}\,cm}{300cm^2/V-s}=0.64V$$

(b)

$$V_{gs}-V_t=mE_{sat}L$$

$$L=\frac{V_{gs}-V_t}{mE_{sat}}=\frac{\mu_{ns}\left(V_{gs}-V_t\right)}{2mv_{sat}}=\frac{300cm^2/Vs\cdot 0.2V}{2\cdot 1.2\cdot 8\times 10^6\,cm/s}=3.13\times 10^{-6}\,cm=31.3nm$$

Effective Channel Length

6.26 (a) For very small V_{ds},

$$R_{channel}=\frac{V_{ds}}{I_{ds}}=\frac{L}{\mu_s C_{ox}W(V_g-V_t)}$$

In a short-channel device, S/D resistance can seriously degrade saturation current. Note that series resistance is worse for higher currents because $R_{channel}$ is the lowest under these bias conditions.

(b) $$R_{total}=R_s+R_d+R_{channel}=R_{sd}+R_{channel}$$

$$=R_{sd}+\left[\frac{L_{eff}}{\mu_s C_{ox}W(V_g-V_t)}\right]=R_{sd}+\left(\frac{L_{gate}-\Delta L}{\mu_s C_{ox}W(V_g-V_t)}\right)$$

Think of R_{total} as the y-value, L_{gate} as the x-value, and $(\mu_s C_{ox}W(V_g-V_t))^{-1}$ as the slope. This fits nicely into the standard equation of the line: y = mx + b. You can choose devices with several gate lengths and measure the current from these devices at discrete gate voltages. Remember, that you are assuming V_{ds} is small (<100mV) in these measurements.

From the current, you can plot R_{total} versus L_{gate}. One sample data line is taken with the same V_g at different gate lengths. For example, if you measure your current at 5 different V_g's, you will get 5 separate curves. Ideally, all the lines will intersect at the same point on your plot. This intersection point occurs at $L_{gate}=\Delta L$ and $R_{total}=R_{sd}$.

In practice, it is not always straightforward to make such a plot. For instance, V_t can be difficult to determine accurately. Also, there is a strong dependence of mobility on gate voltage for thin-oxide MOSFETs. It's a good idea to check your data by taking measurements at several different V_g instead of at 2 or 3 gate voltages.

(c) $I_{dsat} = k(V_{gs}\text{-}I_{dsat}R_s\text{-}V_t)$, where k is a constant of proportionality

$I_{dsat}(1+kR_s) = k(V_{gs}\text{-}V_t) = I_{dsat0}$ *, notice here that* $k = I_{dsat0}/(V_{gs} - V_t)$

$I_{dsat} = I_{dsat0}/(1+kR_s) = I_{dsat0}/(1+I_{dsat0}R_s/(V_{gs}\text{-}V_t))$

(d) $\mathcal{E}_{sat} = (V_{gs}+V_t+0.2)/6T_{ox} = 1.1$ MV/cm.

μ_s~225cm^{2}V^{-1}s^{-1} is picked out from the universal mobility plot.

$\mathcal{E}_{sat} = 2v_{sat}/(\mu_s) = 7\text{x}10^4$V/cm.

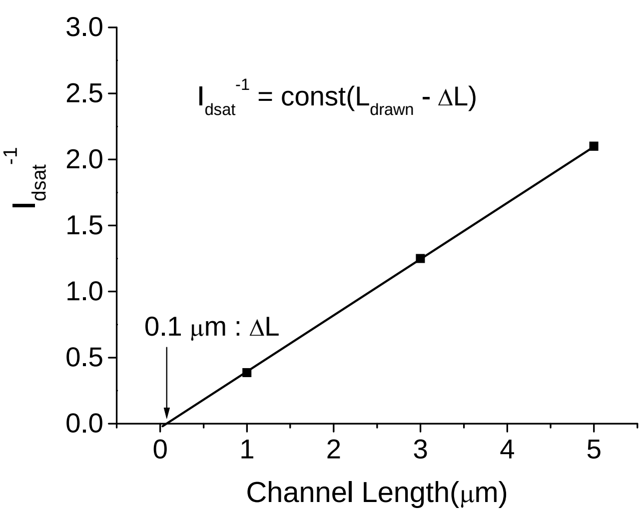
I_{dsat0} = (long channel I_{dsat}) / (1+ (V_{gs}-V_t)/($\mathcal{E}_{sat}$L))
= 1.6mA / (1 + (1.1/.7)) = 0.622mA.

Plug in 0.622mA into the expression derived in part c and get the following:
@ R_s = 0ohms, I_{dsat} = .62mA
@ R_s = 100ohms, I_{dsat} = .59mA
@ R_s = 1000ohms, I_{dsat} = .40mA

6.27 (a) Choose three transistors with same channel width, *Z*, and different channel length, L_1, L_2,and L_3. Measure I_{dsat} at saturation condition for the 3 transistors to

get I_{d1}, I_{d2}, and I_{d3}. Solve the 3 equations to get μ, C_{ox}, *and* L_{eff}.

(b) $\Delta L = L\text{-}L_{eff}$ when gate oxide thickness is 4.5nm. Z =10 µm, μ = 300 cm^2/Vs. Using approach of (a), $\Delta L \simeq$ 0.1 µm.



(c) 2.59mA(L_1- L_{eff}) = $Z\mu C_{ox}(V_{gs}\text{-}V_t)$, V_t = 0.5V.