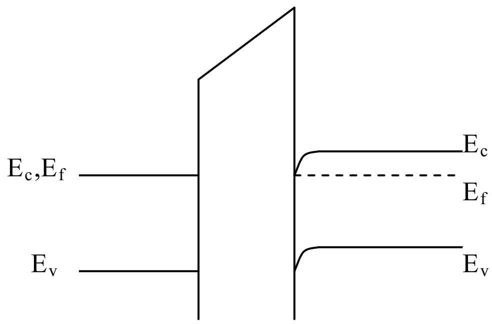
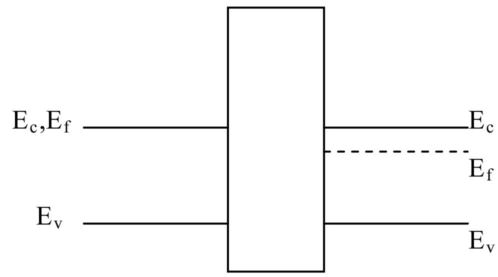
Chapter 5

Energy Band Diagram

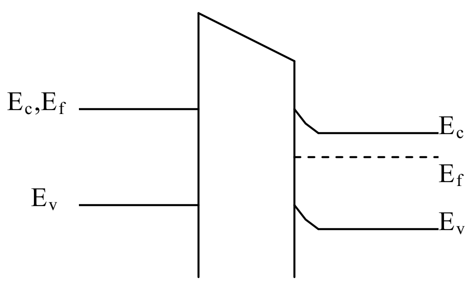
5.1



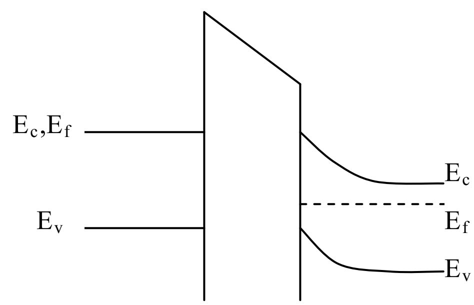
Accumulation



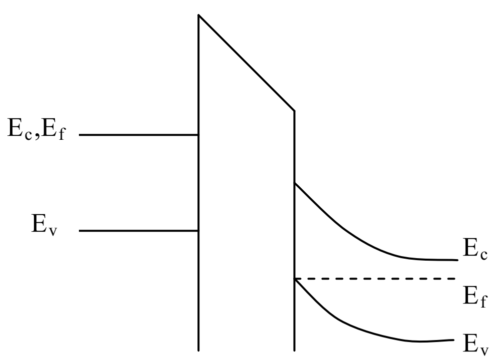
Flat-band



Depletion



Threshold



Inversion

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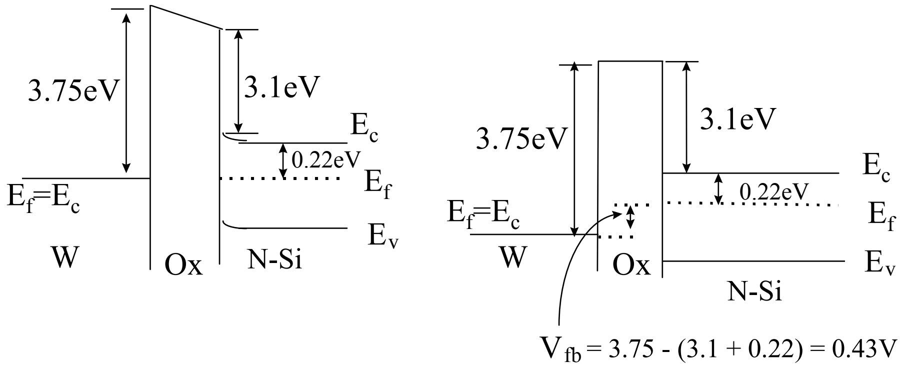
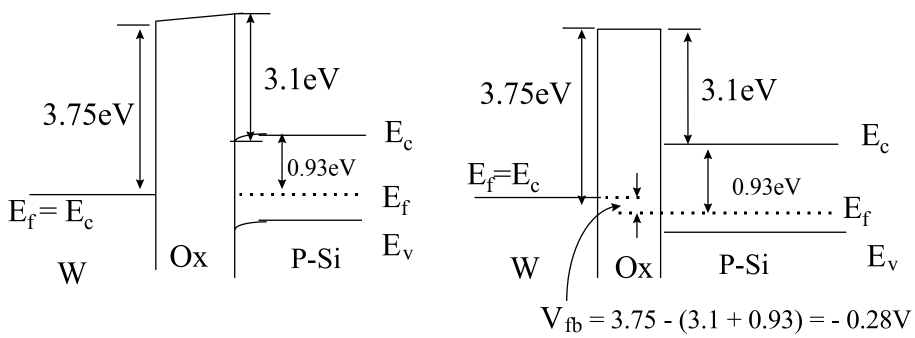
5.2 (a) 1 Ω-cm N-type silicon substrate : $N_d = 5\times10^{15}$ cm^{-3}, $E_f = E_c - 0.223eV$



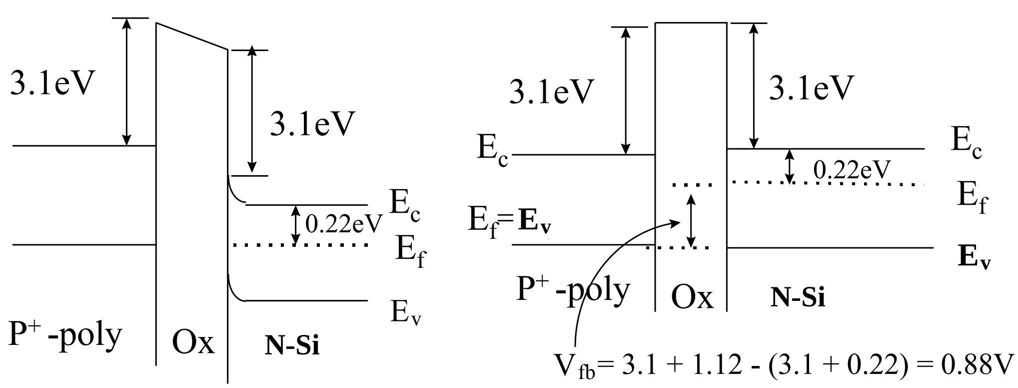
(b) 1 Ω-cm P-type silicon substrate : $N_a = 1.5\times10^{16}$ cm^{-3}, $E_f = E_v + 0.168eV$

Thermal Equil. Flat Band



(c) Heavily doped P^+-polycrystalline silicon gate with 1 *Ω-cm* N-type silicon substrate.

Thermal Equil. Flat Band



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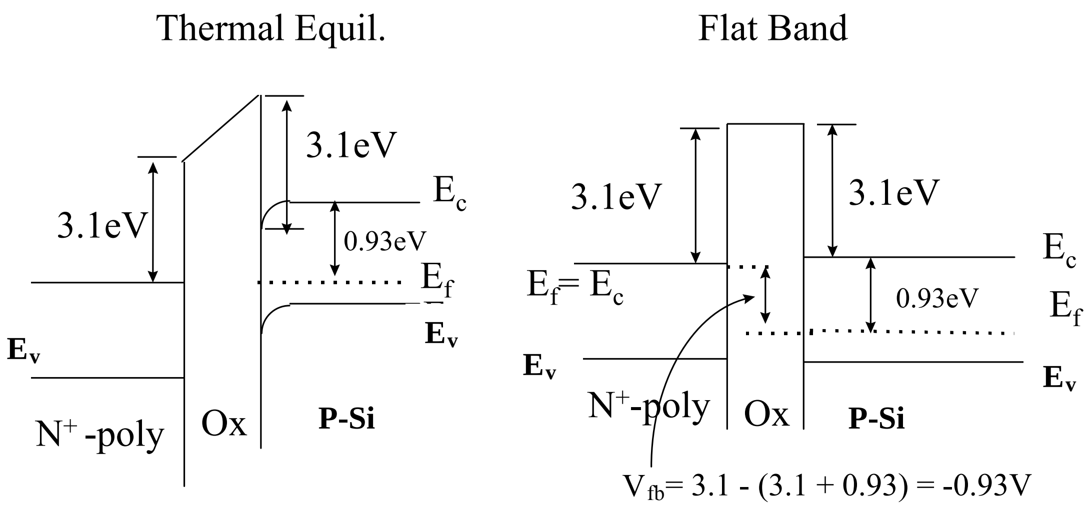
(d) Heavily doped N^+-polycrystalline silicon gate with 1 Ω-cm P-type silicon substrate.



MOS System: Inversion, threshold, depletion, and accumulation

5.3 (a) 3
(b) 2
(c) 1
(d) 4
(e) 5

5.4 (a)

$$\phi_B = \left(\frac{kT}{q}\right)\ln\left(\frac{N_a}{n_i}\right) = (0.026V)\times\ln\left(\frac{10^{18}\,cm^{-3}}{10^{10}\,cm^{-3}}\right) = 0.479V$$

$$\therefore V_{fb} = \left(\frac{\chi_{si}}{q}\right) - \left[\left(\frac{\chi_{si}}{q}\right) + \frac{1}{2}\left(\frac{E_g}{q}\right) + \phi_B\right] = -\left[\frac{1}{2}\left(\frac{E_g}{q}\right) + \phi_B\right]$$

$$= -\frac{1.12V}{2} - 0.479V = -1.039V$$

(b)

$$W_{d\max} = \sqrt{\frac{2\varepsilon_{si}\times 2\phi_B}{qN_a}} = \sqrt{\frac{2\times(11.7)\times\left(8.84\times10^{-14}\right)\times 2\times(0.479)}{\left(1.6\times10^{-19}\right)\times\left(10^{18}\right)}} = 3.521\times10^{-6}\,cm$$

(c)

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$$V_t = V_{fb} + 2\phi_B + \frac{\sqrt{2\varepsilon_{si} q N_a \times 2\phi_B}}{C_{ox}}$$
$$= (-1.039V) + 2\times(0.479V)$$
$$+ \frac{\sqrt{2\times(11.7)\times(8.85\times10^{-14}F/cm)\times(1.6\times10^{-19}C)\times(10^{18}cm^{-3})\times2\times(0.479V)}}{(3.9)\times(8.85\times10^{-14}F/cm)/(2\times10^{-7}cm)}$$
$$= 0.2455V$$

(d)
Only the flat-band voltage changes.

$$V_{fb} = \left[\left(\frac{\chi_{si}}{q}\right)+\left(\frac{E_g}{q}\right)\right]-\left[\left(\frac{\chi_{si}}{q}\right)+\frac{1}{2}\left(\frac{E_g}{q}\right)+\phi_B\right]=\frac{1}{2}\left(\frac{E_g}{q}\right)-\phi_B$$
$$= \frac{1.12V}{2} - 0.479V = 0.081V$$
$$\therefore\ V_t = (0.081V) + 2\times(0.479V)$$
$$+ \frac{\sqrt{2\times(11.7)\times(8.85\times10^{-14}F/cm)\times(1.6\times10^{-19}C)\times(10^{18}cm^{-3})\times2\times(0.479V)}}{(3.9)\times(8.85\times10^{-14}F/cm)/(2\times10^{-7}cm)}$$
$$= 1.3655V$$

5.5 (a) At V_g-V_{fb} = -1V, the MOS capacitor is in accumulation.

$$C_{ox} = \frac{Q_S}{V_{ox}} = \frac{\varepsilon_{ox}}{T_{ox}} = 4\times10^{-7}\frac{F}{cm^2}$$
$$T_{ox} = 8.62nm$$

(b) At threshold,

$$V_g - V_{fb} = 1V = \phi_s + V_{ox} = 2\phi_B + \frac{(2\varepsilon_s q N_a 2\phi_B)^{1/2}}{C_{ox}} \quad \text{where}$$

$$\phi_B = \frac{kT}{q}\ln\left(\frac{N_a}{n_i}\right).$$

Solving iteratively, we get

$$N_a = 2.9\times10^{16}cm^{-3}.$$

(c) Since $\phi_s \approx 0$, $V_{ox} = -1V$.

(d) $$V_g - V_{fb} = 1V = \phi_s + V_{ox} = \phi_s + \frac{(2\varepsilon_s q N_a \phi_s)^{1/2}}{C_{ox}}$$

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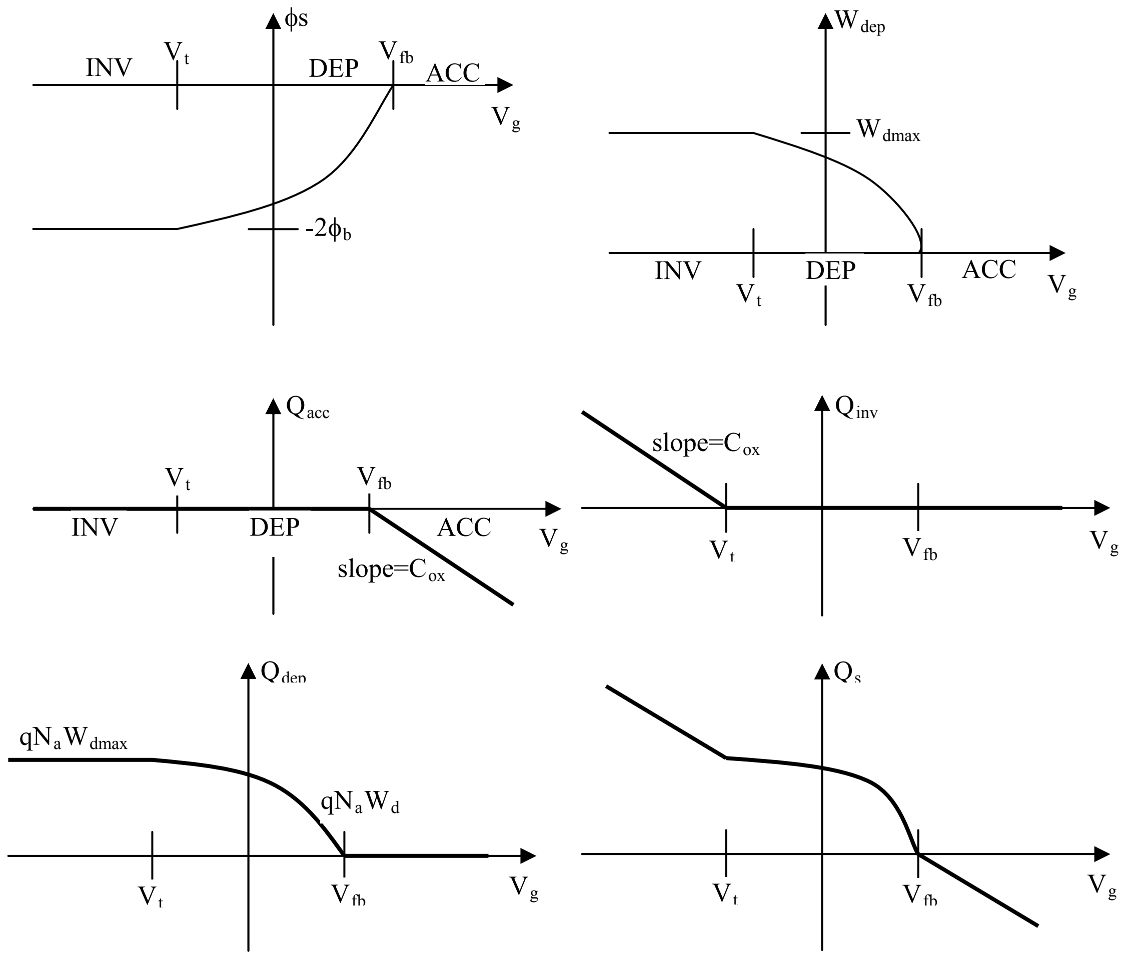
$$\phi_s + 0.245(\phi_s)^{1/2} = 0.5$$

Solving the equations above, we get

$$\phi_s = 0.354V .$$

5.6



5.7 (a) From Equation 5.3.2,

$$V_g = V_{fb} + \phi_s + \frac{1}{C_{ox}}\sqrt{2qN_a\varepsilon_s\phi_s} \text{ where } \phi_s = \left(\sqrt{\phi_s}\right)^2 .$$

Rearranging the terms, we obtain

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$$\left(\sqrt{\phi_s}\right)^2+\left(\frac{\sqrt{2qN_a\varepsilon_s}}{C_{ox}}\right)\sqrt{\phi_s}+\left(V_{fb}-V_g\right)=0\,.$$

Solving for $\sqrt{\phi_s}$ yields

$$\sqrt{\phi_s}=\frac{-\dfrac{\sqrt{2qN_a\varepsilon_s}}{C_{ox}}\pm\sqrt{\dfrac{2qN_a\varepsilon_s}{C_{ox}^2}-4\left(V_{fb}-V_g\right)}}{2}.$$

Since $\sqrt{\phi_s}$ cannot be less than 0,

$$\sqrt{\phi_s}=\frac{-\sqrt{2qN_a\varepsilon_s}}{2C_{ox}}+\sqrt{\frac{2qN_a\varepsilon_s}{4C_{ox}^2}-\left(V_{fb}-V_g\right)}=\frac{\gamma}{2}+\sqrt{\frac{\gamma^2}{4}-\left(V_{fb}-V_g\right)}$$

where

$$\gamma=\frac{\sqrt{2qN_a\varepsilon_s}}{C_{ox}}\,.$$

Hence,

$$\begin{aligned}\phi_s&=\frac{\gamma^2}{4}+\left[\frac{\gamma^2}{4}-\left(V_{fb}-V_g\right)\right]+\frac{\gamma}{2}\sqrt{\frac{\gamma^2}{4}-\left(V_{fb}-V_g\right)}\\&=\frac{\gamma^2}{2}-\left(V_{fb}-V_g\right)+\frac{\gamma}{2}\sqrt{\frac{\gamma^2}{4}-\left(V_{fb}-V_g\right)}\,.\end{aligned}$$

Or,

$$\phi_s=\left[\frac{\gamma}{2}+\sqrt{\frac{\gamma^2}{4}-\left(V_{fb}-V_g\right)}\right]^2.$$

(b) From Equation 5.3.1,

$$V_{ox}=\frac{\sqrt{2qN_a\varepsilon_s}}{C_{ox}}\sqrt{\phi_s}=\gamma\left[\frac{\gamma}{2}+\sqrt{\frac{\gamma^2}{4}-\left(V_{fb}-V_g\right)}\right].$$

(c) We can qualitatively predict that $\phi_s \propto V_g$ since $|\gamma|<<1$. Also, $V_{ox}\propto\sqrt{V_g}$.

$\phi_s\,(V)$

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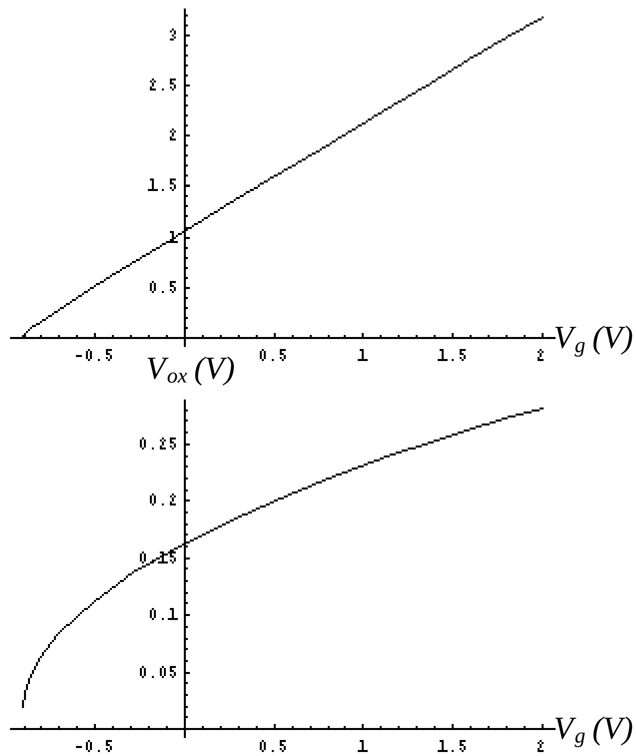

(d) From Equation 4.2.10,

$$W_{dep} = \frac{\sqrt{2q\varepsilon_s}}{C_{ox}}\sqrt{\phi_s} = \frac{\sqrt{2q\varepsilon_s}}{C_{ox}}\left[\frac{\gamma}{2} + \sqrt{\frac{\gamma^2}{4} - \left(V_{fb} - V_g\right)}\right].$$

5.8 (a) $C_{ox} = \dfrac{\varepsilon_{ox}}{T_{ox}} = 3.45\times10^{-7}\,\dfrac{F}{cm^2}$

$$\phi_B = kT\ln\frac{N_a}{n_i} = 0.4V$$

$$V_{fb} = -\frac{E_g}{2} - \phi_B = -0.96V$$

$$V_t = V_{fb} + 2\phi_B + \frac{1}{C_{ox}}\sqrt{2q\varepsilon_s N_a 2\phi_B} = \left(-0.96 + 0.8 + 0.34\right)V = 0.18V$$

(b) $Q_{acc} = -C_{ox}\left(V_g - V_{fb}\right) = 3.45\times10^{-7}\,\dfrac{C}{cm^2}$

(c) $Q_{dep} = -\sqrt{2q\varepsilon_s N_a 2\phi_B} = -1.1\times10^{-7}\,\dfrac{C}{cm^2}$

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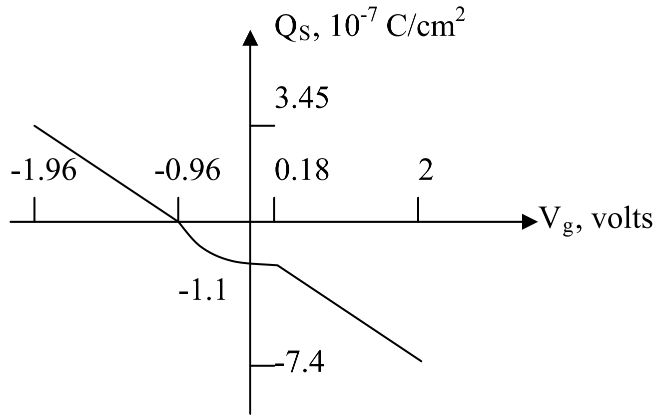
$$Q_{inv} = -C_{ox}\left(V_g - V_t\right) = -6.3\times 10^{-7}\ \frac{C}{cm^2}$$

(d)



5.9

	Parameters	Increase	Decrease	Unchanged
a	Accumulation Region Capacitance			X
b	Flat-band Voltage, Vfb	X		
c	Depletion Region Capacitance		X	
d	Threshold Voltage, Vt		X	
e	Inversion Region Capacitance		X	

(a) At accumulation: Accumulation capacitance

$$C = C_{ox} = \varepsilon_{ox} / T_{ox}$$

(b) At flat-band: Flat-band Voltage

$$V_{fb} = \varphi_g - \left(4.05 + 0.56 + 0.026\ln\left(\frac{N_a}{n_i}\right)\right)$$

$$N_a \downarrow, |\phi_B| \downarrow$$

(c) At depletion: Depletion Capacitance

$$1/C = 1/C_{ox} + W_{dep}/\varepsilon_s$$

$$N_a \downarrow, W_{dep} \uparrow, C \downarrow$$

(d) At threshold: Threshold Voltage

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$$V_t = V_{fb} + |2\phi_B| + Q_d / C_{ox}$$
$$N_a \downarrow, |\phi_B| \downarrow, Q_d \downarrow, V_t \downarrow$$

(e) At inversion: Inversion Capacitance

$$1/C = 1/C_{ox} + W_{d\,\max} / \varepsilon_s$$
$$N_a \downarrow, W_{d\,\max} \uparrow, C \downarrow$$

5.10 To find the value of the oxide capacitance,

$$C_{ox} = \frac{\varepsilon_{ox}}{T_{ox}} = 1.15\times 10^{-7} \frac{F}{cm^2}.$$

The capacitance at V_g=V_{fb} is given by

$$C_{fb} = \frac{1}{1/C_{ox} + L_D/\varepsilon_s} = 7.9\times 10^{-8} \frac{F}{cm^2}$$

where

$$L_D = \left[\frac{\varepsilon_s kT}{q^2 N_a}\right]^{1/2} = 4.09\times 10^{-6}\,cm.$$

To find the minimum value of the capacitance,

$$W_{d\,\max} = \sqrt{\frac{4\varepsilon_s \phi_B}{qN_a}} = 3\times 10^{-5}\,cm \;;\; \phi_B = \frac{kT}{q}\ln\frac{N_a}{n_i} = 0.348V$$
$$C_{\min} = \frac{1}{1/C_{ox} + W_{d\,\max}/\varepsilon_s} = 2.65\times 10^{-8} \frac{F}{cm^2}.$$

V_t is given by

$$V_t = 2\phi_B + \frac{1}{C_{ox}}\sqrt{2\varepsilon_s q N_a 2\phi_B} + V_{fb} = 1.11V$$

where $V_{fb} = \varphi_g - \varphi_s = -2.00$ V.

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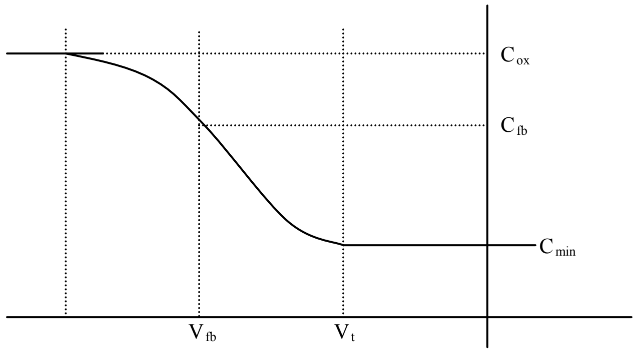

To find the effective oxide charge, we need to find

$$\varphi_{Al} = 4.1V$$
$$\varphi_s = 4.05 + 0.562 + 0.3482 = 4.96V$$
$$\varphi_{Al-s} = -0.86V$$
$$\Delta V_t = V_{fb} - \varphi_{Al-s} = -2 - (-0.86) = -1.14V = \frac{-Q_{ox}}{C_{ox}}$$

Hence,

$$Q_{ox} = 1.31\times 10^{-7}\,\frac{C}{cm^2} = 8.2\times 10^{11}\,\frac{q}{cm^2}.$$

Field Threshold Voltage

5.11 (a) $V_{fb} = \phi_{Al} - \phi_{Si} = 4.1V - (\chi_{Si} + E_g/2q + \dfrac{E_i - E_f}{q})$

$$= 4.1V - (4.05V + (1.12/2)V + (kT/q)\cdot\ln(N_a/n_i)) = -0.80V$$

(b) $\phi_B = \dfrac{E_i - E_f}{q} = kT/q\cdot\ln(N_a/n_i) = 0.290V$

$$W_{d\max} = \sqrt{\frac{2\varepsilon_s 2\phi_B}{qN_a}} = 0.866um$$
$$V_t = V_{fb} + 2\phi_B + qN_a W_{d\max}/C_{ox} \ge 5V$$
$$C_{ox} \le 2.65\times 10^{-9}\,F/cm^2$$

(c) $C_{ox} = \dfrac{\varepsilon}{T_{ox}} = \kappa\varepsilon_0/1\mu m = 2.65\times 10^{-9}\,F/cm^2$ where $K = 2.99$

(d) In accumulation,

V_g- V_{fb} = -1V,

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$C_{ox} = Q_s/V_{ox}$
$V_t \geq 5V$
$C_{ox} \leq 2.65\times10^{-9} F/cm^2$
$K \leq 2.99$

It is the maximum allowable K.

(e) $V_g - V_t = -\dfrac{Q_{inv}}{C_{ox}}$

$Q_{inv} = -C_{ox}(V_g - V_t) = -(2.65\times10^{-9} F/cm^2)(2V) = -5.3\times10^{-9} C/cm^2$

(f) $1/C = 1/C_{ox} + 1/C_{dep} = 1/C_{ox} + W_{d\max}/\varepsilon_s = 1/2.65\times10^{-9} F/cm^2 + 0.866\mu m/\varepsilon_s$

$C = 2.17\times10^{-9} F/cm^2$

(g) $V_g = V_{fb} + V_s + V_{ox}$

$(V_g \geq V_t) = V_{fb} + 2\phi_B + V_{ox}$
$7V = -0.8V + 2(0.29V) + V_{ox}$
$V_{ox} = 7.22V$

Oxide Charge

5.12 $\Delta V = -Q/C_{ox}$

$$Q = -C_{ox}\Delta V = -\frac{C_0}{A}\Delta V = -\frac{45\times10^{-12}}{6.4\times10^{-5}}\times0.05 = -3.52\times10^{-8} C/cm^2$$

$V_{fb} = \psi_g - \psi_s - Q_f/C_{ox}$

It should be negative since negative charges increase V_{fb}.

5.13 Oxide charge will change the device characteristics. Mobile charges are particularly bad as they give the device instability. That is, as the charge moves from one side of the oxide to the other, it will change the threshold voltage. This is undesirable.

Mobile charges are generally introduced into the oxide during wafer cleaning or oxidation. Strict measures of cleanliness can be achieved by using ultra-clean chemicals. And, the impurities such as sodium can be immobilized by introducing a chlorine compound during oxidation.

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C-V Characteristics

5.14 $V_g = V_{fb} + \phi_s + V_{ox}$

Using $V_{ox} = \dfrac{qN_a W_d}{C_{ox}}$ and $\phi_s = \dfrac{W_d{}^2 qN_a}{2\varepsilon_s}$, we solve for W_d

$$V_g - V_{fb} = qN_a\left(\frac{W_d{}^2}{2\varepsilon_s} + \frac{W_d}{C_{ox}}\right)$$

$$W_d = \frac{-2\varepsilon_s + \sqrt{4\varepsilon_s{}^2 - 4C_{ox}\left(2\varepsilon_s C_{ox}\right)\left(\dfrac{V_g - V_{fb}}{qN_a}\right)}}{2C_{ox}}$$

$$\frac{W_d}{\varepsilon_s} = \frac{-1 + \sqrt{1 - \dfrac{2C_{ox}{}^2\left(V_g - V_{fb}\right)}{q\varepsilon_s N_a}}}{C_{ox}}$$

Since $\dfrac{1}{\mathrm{C}} = \dfrac{1}{C_{ox}} + \dfrac{1}{C_{dep}} = \dfrac{1}{C_{ox}} + \dfrac{W_d}{\varepsilon_s}$,

$$\frac{1}{C} = \sqrt{\frac{1}{C_{ox}{}^2} - \frac{2\left(V_g - V_{fb}\right)}{q\varepsilon_s N_a}}\,.$$

5.15 (a) The substrate doping is P-type since the threshold voltage is larger than V_{fb}.

(b) $T_{ox} = A\dfrac{\varepsilon_{ox}}{C_{ox}} = \dfrac{10^{-4} \times 0.345 \times 10^{12}}{50 \times 10^{-12}} = 6.9\ nm$

(c) $V_t - V_{fb} = 2\phi_B + \dfrac{\sqrt{2\varepsilon_s qN_a 2\phi_B}}{C_{ox}} = 2\dfrac{kT}{q}\ln\left(\dfrac{N_a}{n_i}\right) + \dfrac{2\sqrt{\varepsilon_s qN_a \ln\dfrac{N_a}{n_i}}}{C_{ox}}$

Using V_{fb}=-1V and V_t = 0.5V and solving iteratively, we obtain

$N_a \cong 3 \times 10^{17}\, cm^2$.

(e) At position C, MOS has the minimum capacitance.

$$C_{\min} = \left(\frac{1}{C_{ox}} + \frac{W_{d\max}}{\varepsilon_s}\right)^{-1} = 12.7\, pF \quad \text{where} \quad W_{d\max} = \sqrt{\frac{2\varepsilon_s}{qN_a}\frac{2kT}{q}\ln\frac{N_a}{n_i}}$$

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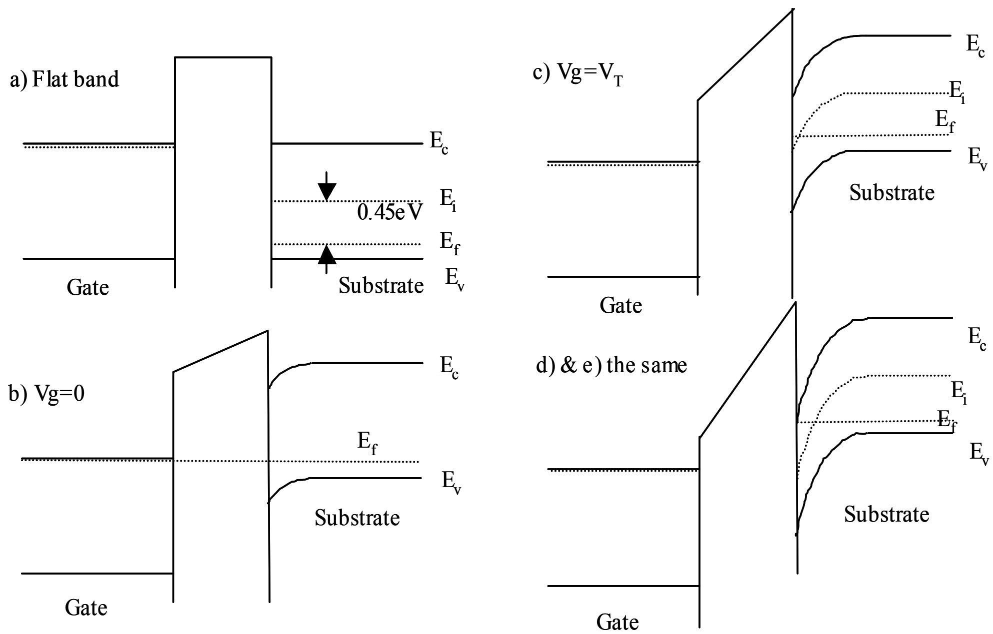
(e)



(f) $V_g = 0 = V_{fb} + \phi_s + \dfrac{1}{C_{ox}}\sqrt{2\varepsilon_s q N_a \phi_s}$

Substituting the numerical values and solving the quadratic equation, we obtain

$\phi_s + 0.644\sqrt{\phi_s} - 1 = 0$

$\phi_s = 0.53V$.

5.16 (a) P-type, since $V_t > V_{fb}$.

(b)

T_{ox} :	A	**B**	($C_{max}=\varepsilon_{ox}/T_{ox}$)
V_{fb} :	A	**B**	(directly from the graph)
W_{dmax} :		**A**	B ($1/C_{min} = 1/C_{ox}+W_{dmax}/\varepsilon_s$ and $C_{ox}^A > C_{ox}^B$)
N_{sub} :	A	**B**	($W_{dmax}\sim(N_{sub})^{-1/2}$)
V_t :	A	**B**	(directly from the graph)

5.17 $A = 100\times100\ \mu m^2 = 10^{-4}\ cm^{-2}$

For the MOS capacitor, the field in the oxide $\mathcal{E}_{max} = 8\times10^6 = \dfrac{10V}{T_{ox}}$

$\Rightarrow T_{ox} = 1.25\times10^{-6}$ cm or 12.5 nm

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$\therefore\ C_{ox} = \dfrac{\varepsilon_{ox}}{T_{ox}} A = 28.32 \text{ pF} = C_{\text{MOS}}$

For P^+N junction,

$$\phi_B = \frac{\text{kT}}{\text{q}} \ln\left(\frac{\text{N}_d}{\text{n}_i}\right) = 0.356 \text{ V for } N_\text{d} = 10^{16} \text{ cm}^{-3}$$

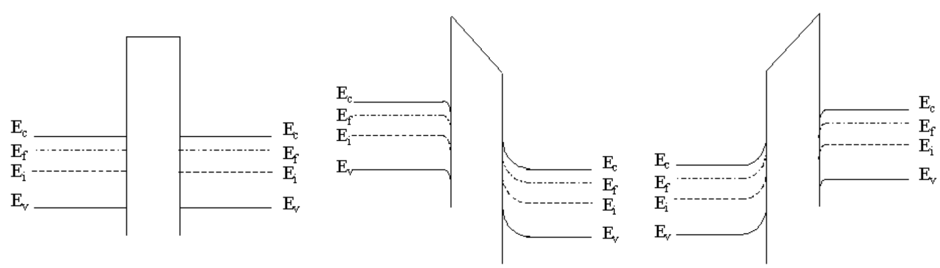
$$W_{dep} = \sqrt{\frac{2\varepsilon_s(\phi_B + V_B)}{qN_d}} = 8.35\times10^{-5} \text{ cm for } V_\text{R} = 5 \text{ V.}$$

$$C = \frac{\varepsilon_s}{W_{dep}} A = 1.25 \text{ pF} = C_{\text{P-N jn}}$$

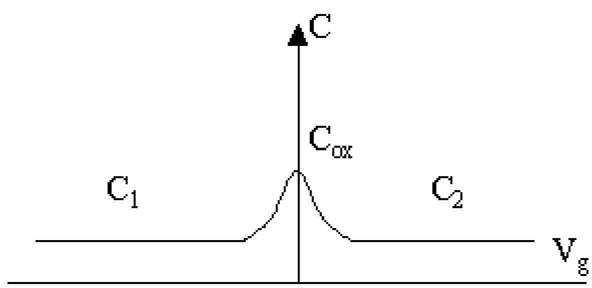
$$C_{\text{MOS}} / C_{\text{P-N jn}} = 22.66$$

5.18 (a) The flatband voltage is 0V because the 2 silicon sides are equally doped

(i)Vg =0 (ii) Vg<0 and large (iii) Vg>0 and large



(b)



As both sides are equally doped, the values of C_1 and C_2 will be equal.

$$C_1 = C_2 = \left(C_{ox}^{-1} + C_{dep}^{-1}\right)^{-1}$$

$$C_{dep} = \frac{\varepsilon_s}{W_{dep}} \quad \text{where}$$

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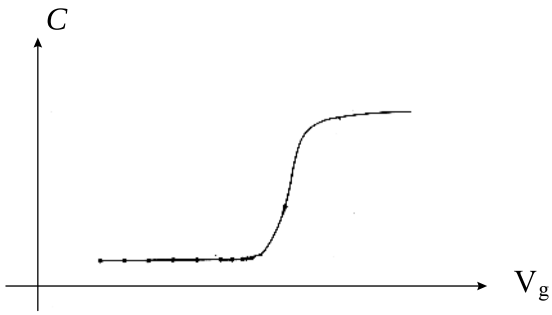
$$W_{dep} = \sqrt{\frac{2\varepsilon_s}{qN_d}\frac{2kT}{q}ln\left(\frac{N_d}{n_i}\right)} \Rightarrow C_{\text{dep}} = 34.5 \text{ nF cm}^{-2} \text{ and } C_{ox} = \frac{\varepsilon_{ox}}{T_{ox}}.$$

(c) When the left side is P-type, both the silicon layers go into accumulation and depletion for the same type of bias.

The flat band voltage will be $(\chi_{Si} + E_g/2 + \phi_{BL}) - (\chi_{Si} + E_g/2 - \phi_{BR}) = \phi_{BL} + \phi_{BR} = 0.713$ V



5.19

Bias Condition	Surface Potential	MOS cap (LF)	MOS Cap (HF)	MOSFET
accumulation	~ 0	C_{ox}	C_{ox}	C_{ox}
flatband	= 0	C_{ox}	C_{ox}	C_{ox}
threshold	$= 2\phi_B$	C_{ox}	$(C_{ox}^{-1} + W_{dep,max}/\varepsilon_s)^{-1}$	C_{ox}
inversion	$\sim 2\phi_B$	C_{ox}	$(C_{ox}^{-1} + W_{dep,max}/\varepsilon_s)^{-1}$	C_{ox}

5.20 (a) Accumulation -1 V < V_g
Depletion -3 V < V_g < -1 V
Inversion V_g < -3 V
The substrate is N-type.

(b) $C_0 = \frac{\varepsilon_{ox}}{T_{ox}} A$

Thus, $T_{ox} = 205$ nm.

(c) From the high-frequency *C-V* curve, $C_{\min} = \left(C_0^{-1} + C_{dep}^{-1}\right)^{-1}$

Plugging in $C_{\min} = 0.4C_0 \Rightarrow C_{\text{dep}} = 0.67 \times C_0 = 54.9$ pF

Now, $C_{\text{dep}} = \frac{\varepsilon_s}{W_{dep}} A$

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For uniform doping $W_{dep} = \sqrt{\dfrac{2\varepsilon_s(2\phi_B)}{qN_d}}$ at threshold, where $\phi_B = \dfrac{\mathrm{kT}}{\mathrm{q}}\ln\left(\dfrac{\mathrm{N}_d}{\mathrm{n}_i}\right)$

$$\Rightarrow N_d = \frac{4\varepsilon_s kT}{W_{dep}^2 q^2} ln\left(\frac{N_d}{n_i}\right)$$

which can be solved by iteration and we get the value of $N_\mathrm{d} \sim 10^{15}\ \mathrm{cm}^{-3}$.

(d) $V_{fb} \sim .55V + (kT/q)\ln(10^{15}/10^{10}) = .85V$

Since V_{fb} from the plot is ~ -1V, then Q_{ox} consists of positive charges. The shift in V_{fb} is 1.85V. Thus,

Q_{ox}= 1.85V × C_0 = 1.85V × 82pF / 4.75x10^{-3} cm^2 = 32 nC/cm^2.

Poly-gate Depletion

5.21 (a) Using Gauss Law,

$$Q_{poly} = qN_{poly}W_{dpoly} = \varepsilon_{poly}\mathcal{E}_{poly} = \varepsilon_{ox}\mathcal{E}_{ox},$$

where $\mathcal{E}_{ox}$ is the electric field inside the oxide and

$$W_{dpoly} = \frac{\varepsilon_{ox}\mathcal{E}_{ox}}{qN_{poly}}.$$

(b) $$\phi_{poly} = \frac{qN_{poly}W_{dpoly}^2}{2\varepsilon_{poly}} = \frac{qN_{poly}}{2\varepsilon_{poly}}\frac{\varepsilon_{ox}^2\mathcal{E}_{ox}^2}{q^2N_{poly}^2} = \frac{\varepsilon_{ox}^2\mathcal{E}_{ox}^2}{2q\varepsilon_{poly}N_{poly}}.$$

(c) $V_g = \phi_{poly} + V_{ox} + \phi_s + V_{fb}$.

$$V_{ox}C_{ox} = |Q_s| = Q_{poly} \Rightarrow V_{ox} = \frac{1}{C_{ox}}\sqrt{2q\varepsilon_{poly}N_{poly}\phi_{poly}}$$

$$V_g = \phi_{poly} + V_{fb} + \phi_s + V_{ox} = \phi_{poly} + V_{fb} + 2|\phi_B| + \frac{T_{ox}}{\varepsilon_{ox}}\sqrt{2q\varepsilon_{poly}N_{poly}\phi_{poly}}$$

$$\phi_{poly} + \frac{T_{ox}}{\varepsilon_{ox}}\sqrt{2q\varepsilon_{poly}N_{poly}}\sqrt{\phi_{poly}} - \left(V_g - V_{fb} - 2|\phi_B|\right) = 0$$

$$\sqrt{\phi_{poly}} = \frac{-\dfrac{T_{ox}}{\varepsilon_{ox}}\sqrt{2q\varepsilon_{poly}N_{poly}} \pm \sqrt{\dfrac{T_{ox}^2}{\varepsilon_{ox}^2}2q\varepsilon_{poly}N_{poly} + 4\left(V_g - V_{fb} - 2|\phi_B|\right)}}{2}$$

$$= -\frac{T_{ox}}{2\varepsilon_{ox}}\sqrt{2q\varepsilon_{poly}N_{poly}} \pm \sqrt{\frac{T_{ox}^2}{4\varepsilon_{ox}^2}2q\varepsilon_{poly}N_{poly} + \left(V_g - V_{fb} - 2|\phi_B|\right)}$$

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We know that $\sqrt{\phi_{poly}}>0$. Also, if we set

$$\alpha = \frac{T_{ox}}{\varepsilon_{ox}}\sqrt{2q\varepsilon_{poly}N_{poly}} \text{ and } \beta = \left(V_g - V_{fb} - 2|\phi_B|\right)\text{, then}$$

$$\sqrt{\phi_{poly}} = -\alpha + \sqrt{\alpha^2+\beta} \text{ and } \phi_{poly} = \beta + 2\alpha^2\left(1-\sqrt{1+\frac{\beta}{\alpha^2}}\right).$$

Hence,

$$\varphi_{poly} = \left(V_g - V_{fb} - 2|\phi_B|\right) + \frac{T_{ox}^2}{\varepsilon_{ox}^2}q\varepsilon_{poly}N_{poly}\left(1-\sqrt{1+\frac{2\varepsilon_{ox}^2\left(V_g - V_{fb} - 2|\phi_B|\right)}{T_{ox}^2 q\varepsilon_{poly}N_{poly}}}\right).$$

(d) $$W_{dpoly} = \frac{\varepsilon_{ox}\mathcal{E}_{ox}}{qN_{poly}} = \sqrt{\frac{2\varepsilon_{poly}\phi_{poly}}{qN_{poly}}} = \sqrt{\frac{2\varepsilon_{poly}}{qN_{poly}}}\left(-\alpha+\sqrt{\alpha^2+\beta}\right)$$

$$= \sqrt{\frac{2\varepsilon_{poly}}{qN_{poly}}}\left[\sqrt{\frac{T_{ox}^2}{4\varepsilon_{ox}^2}2q\varepsilon_{poly}N_{poly} + \left(V_g - V_{fb} - 2|\phi_B|\right)} - \frac{T_{ox}}{2\varepsilon_{ox}}\sqrt{2q\varepsilon_{poly}N_{poly}}\right].$$

(e) Using the equation derived in part (c), we find $\phi_{poly} = 0.28\,V$.

And, using the equation derived in part (d), we find $W_{dpoly} = 2.46\,nm$.

$$\left(|\phi_B| = \frac{kT}{q}\ln\left(\frac{N_a}{n_i}\right) = 0.41\,V, \quad V_{fb} = -0.97\,V, \quad and\ \varepsilon_{poly} = \varepsilon_s\right).$$

(f) Calculate V_t using Equation 5.4.3:

$$V_t = V_{fb} + 2|\phi_B| + \frac{1}{C_{ox}}\sqrt{4q\varepsilon_s N_a|\phi_B|} = (-0.97+0.82+0.095)V = -0.055\,V$$

Using Equation 5.8.3 with $\phi_{poly} = 0.28\,V$,

$$Q_{inv} = C_{ox}\left(V_g - V_t - \phi_{poly}\right) = \frac{\varepsilon_{ox}}{T_{ox}}(1.5+0.055-0.28)V = 2.20\times10^{-6}\,coul/cm^2.$$

(g) First, we calculate C_{oxe}.

$$C_{oxe} = \frac{\varepsilon_{ox}}{T_{ox}+W_{dpoly}/3} = 1.22\times10^{-6}\,F/cm^2.$$

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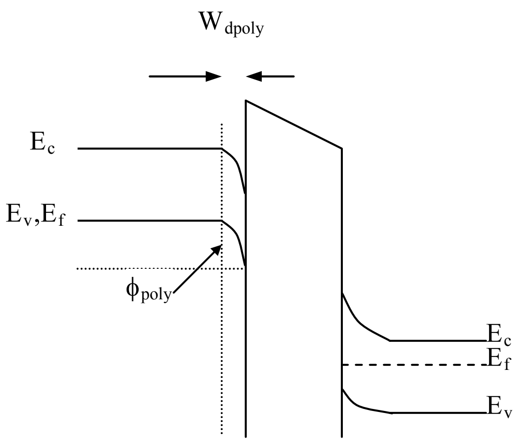
$$Q_{inv} = C_{oxe}\left(V_g - V_t\right) = 1.90\times10^{-6}\, coul/cm^2 .$$

This value is smaller than what we have found in part (f).

5.22 (a)



V_g is negative, V_{fb} is positive, ϕ_{st} is negative, V_{ox} is negative and ϕ_{poly} is negative

5.23 (a) $Q_{poly} = qN_{poly}W_{dpoly} = qN_{poly}\sqrt{\dfrac{2\varepsilon_s V_{poly}}{qN_{poly}}} = \sqrt{2\varepsilon_s qN_{poly}V_{poly}}$

(b) $C_{poly} = \dfrac{\varepsilon_s}{W_{dpoly}} = \dfrac{\varepsilon_s qN_{poly}}{Q_{poly}} = \sqrt{\dfrac{\varepsilon_s qN_{poly}}{2V_{poly}}}$

(c) $C_{total} = (C_{ox}{}^{-1} + C_{poly}{}^{-1})^{-1} = (\dfrac{1}{C_{ox}} + \sqrt{\dfrac{2V_{poly}}{\varepsilon_s qN_{poly}}})^{-1}$

Threshold Voltage Expression

5.24
$$\begin{aligned} V_t &= V_{fb} + \phi_s + V_{ox} \\ &= V_{fb} + \phi_s + \frac{qN_a W_{dep}}{C_{ox}} \\ &= V_{fb} + \phi_s + \frac{\sqrt{qN_a 2\varepsilon_s \phi_s}}{C_{ox}} \\ &= V_{fb} + 2\phi_B + \frac{\sqrt{qN_a 2\varepsilon_s 2\phi_B}}{C_{ox}} \end{aligned}$$

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