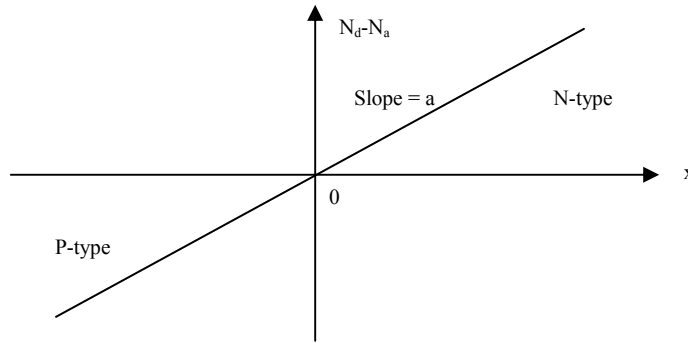


## Chapter 4. Problems

### Part I: PN Junction

#### Electrostatics of PN Junctions

4.1 The linearly graded junction is P-type for  $x < 0$  and n-type for  $x > 0$  with the junction at  $x = 0$ .



Since the doping levels are symmetric on either side of the junction, the depletion region widths into both sides must be the same ( $x_n = x_p = W_{dep}/2$ ).

(a) To find the electric field distribution, we utilize the following relation:

$$\frac{d\mathcal{E}}{dx} = \frac{\rho}{\epsilon_s} = \frac{q(N_d - N_a)}{\epsilon_s} = \frac{q \times a \times x}{\epsilon_s}.$$

Integrating the equation with the boundary conditions  $\mathcal{E}(x = x_n = x_p = W_{dep}/2) = 0$ , we find

$$\mathcal{E}(x) = \frac{q \times a}{2\epsilon_s} \left( x^2 - \frac{W_{dep}^2}{4} \right).$$

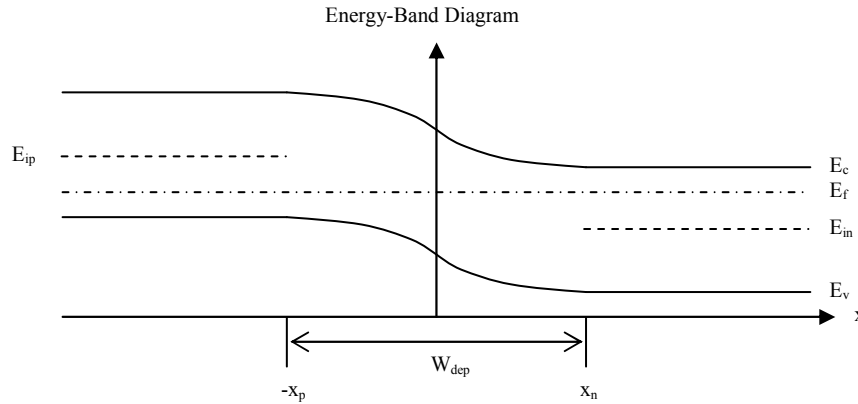
(b) The potential distribution is

$$\frac{dV}{dx} = -\mathcal{E} \Rightarrow V = -\frac{qa}{2\epsilon_s} \left( \frac{x^3}{3} - \frac{W_{dep}^2 x}{4} \right) + C.$$

For convenience, if we define  $V(x=0) = 0$  as a reference, we find  $C = 0$ . Hence,

$$\frac{dV}{dx} = -\mathcal{E} \Rightarrow V = -\frac{qa}{2\epsilon_s} \left( \frac{x^3}{3} - \frac{W_{dep}^2 x}{4} \right).$$

(c) The built-in potential is the same as  $(E_{ip}-E_{in})/q$  at thermal equilibrium.



The carrier concentrations at the edges of the depletion region are

$$n(x_n) = n_i e^{\frac{(E_f - E_{in})}{kT}} \Rightarrow E_{in} = E_i(x = x_n) = E_f - kT \ln \left[ \frac{n(x_n)}{n_i} \right] = E_f - kT \ln \left[ \frac{aW_{dep}}{2n_i} \right]$$

and

$$p(x_p) = p_i e^{\frac{(E_{ip} - E_f)}{kT}} \Rightarrow E_{ip} = E_i(x = x_p) = E_f + kT \ln \left[ \frac{p(x_p)}{n_i} \right] = E_f + kT \ln \left[ \frac{aW_{dep}}{2n_i} \right].$$

Therefore,  $\phi_{bi}$  is

$$\phi_{bi} = \frac{E_{ip} - E_{in}}{q} = 2 \frac{kT}{q} \ln \frac{aW_{dep}}{2n_i}.$$

(d) The total voltage drop takes place in the depletion region. From part (b), we know that  $|V(x_n)| = |V(x_p)| = qaW_{dep}^3 / (24\epsilon_s)$ . Remember that  $x_n = x_p = W_{dep}/2$ . Hence, the potential drop is  $\phi_{bi} - V_a = qaW_{dep}^3 / (12\epsilon_s)$ . Solving for  $W_{dep}$  gives

$$W_{dep} = \left[ \frac{12\epsilon_s}{qa} (\phi_{bi} - V_a) \right]^{1/3}.$$

Note that if we substitute this in part (c), we can iteratively solve for  $\phi_{bi}$ .

## 4.2 (a)

$$\phi_{bi} = \left( \frac{kT}{q} \right) \ln \left( \frac{N_a N_d}{n_i^2} \right) = (0.026 V) \times \ln \left( \frac{(10^{16} \text{ cm}^{-3}) \times (5 \times 10^{15} \text{ cm}^{-3})}{(10^{10} \text{ cm}^{-3})^2} \right) = 0.7 V$$

(b)

$$\begin{aligned} W_{dep} &= \sqrt{\frac{2\epsilon_s \phi_{bi}}{q} \left( \frac{1}{N_a} + \frac{1}{N_d} \right)} \\ &= \sqrt{\frac{2 \times 12 \times (8.85 \times 10^{-14}) \times 0.7}{1.6 \times 10^{-19}} \times \left( \frac{1}{10^{16}} + \frac{1}{5 \times 10^{15}} \right)} \\ &= 5.28 \times 10^{-5} \text{ cm} \\ &= 0.528 \mu\text{m} \\ &= 528 \text{ nm} \end{aligned}$$

In order to calculate  $x_n$  and  $x_p$  we need to use  $W_{dep} = x_n + x_p$  and  $x_n N_d = x_p N_a$ .

$$\begin{aligned} x_n &= W_{dep} - x_p = W_{dep} - \frac{N_d}{N_a} x_n \\ \therefore x_n &= \left( \frac{N_a}{N_a + N_d} \right) W_{dep} = \left( \frac{5 \times 10^{15} \text{ cm}^{-3}}{(5 \times 10^{15} \text{ cm}^{-3}) + (10^{16} \text{ cm}^{-3})} \right) \times (0.528 \mu\text{m}) = 0.176 \mu\text{m} \end{aligned}$$

Likewise,

$$x_p = \left( \frac{N_d}{N_a + N_d} \right) W_{dep} = \left( \frac{10^{16} \text{ cm}^{-3}}{(5 \times 10^{15} \text{ cm}^{-3}) + (10^{16} \text{ cm}^{-3})} \right) \times (0.528 \mu\text{m}) = 0.352 \mu\text{m}$$

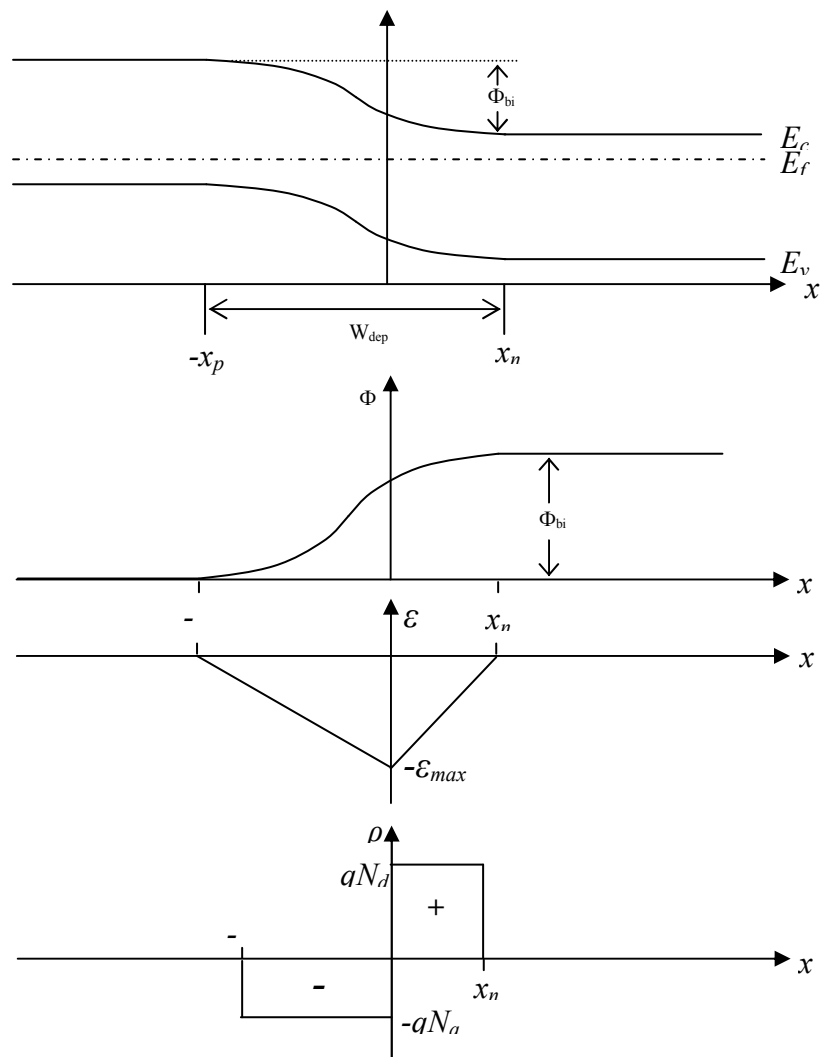
(c)

$$\mathcal{E}_{max} = \frac{q N_d x_n}{\epsilon_s} = \frac{(1.6 \times 10^{-19} \text{ C}) \times (10^{16} \text{ cm}^{-3}) \times (1.76 \times 10^{-5} \text{ cm})}{12 \times (8.85 \times 10^{-14} \text{ F/cm})} = 2.652 \times 10^4 \text{ V/cm}$$

Since the built-in potential is the integration of the electric field, the maximum electric field can also be calculated from the area of the triangle of the electric field profile.

$$\begin{aligned} \phi_{bi} &= \frac{1}{2} W_{dep} \mathcal{E}_{max} \\ \therefore \mathcal{E}_{max} &= \frac{2\phi_{bi}}{W_{dep}} = \frac{2 \times (0.7 V)}{5.28 \times 10^{-5} \text{ cm}} = 2.652 \times 10^4 \text{ V/cm} \end{aligned}$$

(d)



(e)

$$\phi_{bi} = \left( \frac{kT}{q} \right) \ln \left( \frac{N_a N_d}{n_i^2} \right) = (0.026 V) \times \ln \left( \frac{(10^{16} \text{ cm}^{-3}) \times (10^{18} \text{ cm}^{-3})}{(10^{10} \text{ cm}^{-3})^2} \right) = 0.838 V$$

$$\begin{aligned} W_{dep} &= \sqrt{\frac{2\epsilon_s \phi_{bi}}{q} \left( \frac{1}{N_a} + \frac{1}{N_d} \right)} \\ &= \sqrt{\frac{2 \times 12 \times (8.85 \times 10^{-14}) \times 0.838}{1.6 \times 10^{-19}}} \times \left( \frac{1}{10^{16}} + \frac{1}{10^{18}} \right) \\ &= 3.35 \times 10^{-5} \text{ cm} \\ &= 0.335 \mu\text{m} \end{aligned}$$

$$x_n = \left( \frac{N_a}{N_a + N_d} \right) W_{dep} = \left( \frac{10^{18} \text{ cm}^{-3}}{(10^{18} \text{ cm}^{-3}) + (10^{16} \text{ cm}^{-3})} \right) \times (0.335 \mu\text{m}) = 0.332 \mu\text{m}$$

$$x_p = \left( \frac{N_d}{N_a + N_d} \right) W_{dep} = \left( \frac{10^{16} \text{ cm}^{-3}}{(10^{18} \text{ cm}^{-3}) + (10^{16} \text{ cm}^{-3})} \right) \times (0.366 \mu\text{m}) = 0.003 \mu\text{m}$$

$$\mathcal{E}_{max} = \frac{2\phi_{bi}}{W_{dep}} = \frac{2 \times (0.838 \text{ V})}{3.35 \times 10^{-5} \text{ cm}} = 5.003 \times 10^4 \text{ V / cm}$$

The depletion width has decreased due to a higher doping and since the acceptor doping is now 100 times greater than the donor concentration, most of the depletion region is on the n-side.

- 4.3 (a)** If we apply reverse bias to the sample, the depletion width will increase. From Equation (4.2.5), we know that  $N_a x_p = N_d x_n$ . This means that the numbers of ionized dopants on the N and the P sides are equal. The side that has its dopants totally ionized is fully depleted. Hence, we need to find the total number of dopants per unit area on each side. The side with the smaller value will become depleted before the other.

$$N_a \times W_p = 5 \times 10^{16} \text{ cm}^{-3} \times 1.2 \times 10^{-4} \text{ cm} = 6 \times 10^{12} \text{ cm}^{-2} \text{ and}$$

$$N_d \times W_N = 1 \times 10^{17} \text{ cm}^{-3} \times 0.4 \times 10^{-4} \text{ cm} = 4 \times 10^{12} \text{ cm}^{-2}.$$

where  $W_p$  and  $W_N$  are the widths of the P-type region and N-type region, respectively. Since the N-type region contains less dopants per unit area, the N-type region will be fully depleted before the P-type region.

- (b) Repeating the calculation for  $N_d = 1 \times 10^{16} \text{ cm}^{-3}$ ,

$$N_a \times W_p = 1 \times 10^{16} \text{ cm}^{-3} \times 1.2 \times 10^{-4} \text{ cm} = 1.2 \times 10^{12} \text{ cm}^{-2}.$$

Hence, the P-type region will be fully depleted first.

(Note: You can also solve part (a) and (b) by finding the voltage at which each side depletes. However, this will make the problem unnecessarily complicated.)

- (c) From Equation (4.4.1), we know that

$$C_{dep / \text{unit area}} = \frac{\mathcal{E}_s}{W_{dep}}.$$

First, we need to find the total depletion region width.

For part (a), we know that the N-type region is fully depleted. And, from Equation (4.2.5),  $N_a x_p = N_d x_n$ . Hence,

$$N_d W_N = N_a x_p \Rightarrow x_p = \frac{N_d W_N}{N_a} = 0.8 \mu m \Rightarrow W_{dep} = 0.4 \mu m + 0.8 \mu m = 1.2 \mu m .$$

$$C_{dep / unit\ area} = \frac{\epsilon_s}{1.2 \mu m} = 8.63 \times 10^{-9} F / cm^2 .$$

For part (b), we know that the P-type region is fully depleted. And, from Equation (4.2.5),  $N_a x_p = N_d x_n$ . Hence,

$$N_d x_n = N_a W_P \Rightarrow x_n = \frac{N_a W_P}{N_d} = 0.12 \mu m \Rightarrow W_{dep} = 0.12 \mu m + 1.2 \mu m = 1.32 \mu m .$$

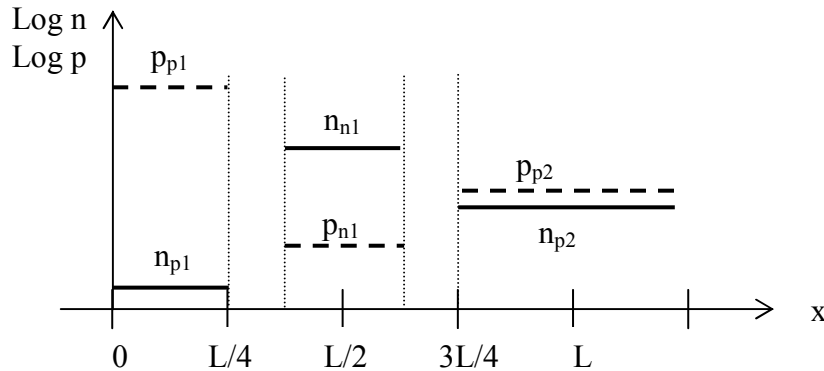
$$C_{dep / unit\ area} = \frac{\epsilon_s}{1.32 \mu m} = 7.85 \times 10^{-9} F / cm^2 .$$

**4.4 (a)** At thermal equilibrium, the Fermi level is constant throughout the system. Since this is the case for the given figure, equilibrium conditions prevail.

(b) Carrier concentrations are given by:

$$n = N_c e^{-(E_c - E_f) / kT} \text{ and } p = N_v e^{-(E_f - E_v) / kT} .$$

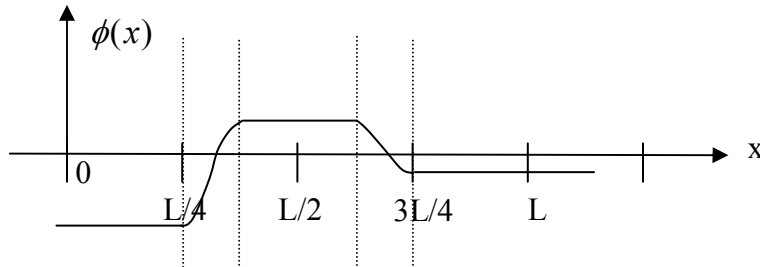
Therefore,



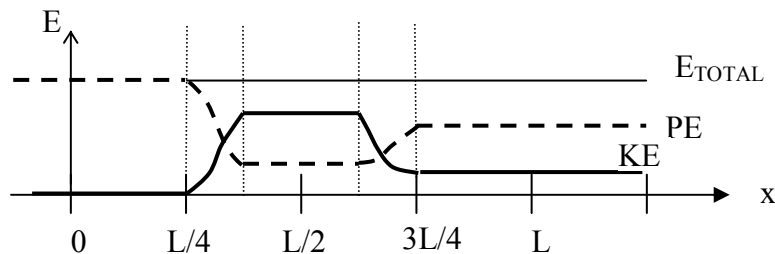
(c) The potential is the reverse of the band diagram. That is,

$$\phi = -\frac{1}{q}(E_i - E_f).$$

For this problem, sketching the qualitative shape is sufficient.



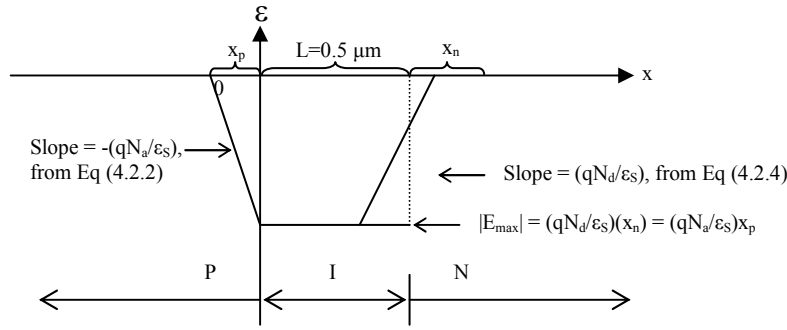
(d) We assume that the total energy is constant:  $E_{\text{TOTAL}} = E_{\text{KE}} + E_{\text{PE}} = \text{Constant}$ .



**4.5 (a)** The intrinsic region has no dopants. Consequently, there is no charge exposed in the I-region. On the N-type side, the ionized donor atoms are exposed due to the diffusion of the electrons. The same situation prevails on the P-type region due to the diffusion of holes. The field lines that start from the ionized donor atoms on the N-type side do not have any negative charge to terminate on until they reach the P-type region where ionized acceptors are present. Thus, the field is constant in the I-region. Alternatively, you may note that  $d\mathcal{E}/dx=0$  in the intrinsic region since  $\rho = 0$ . The free-carriers that are present in the I-region will be swept away by the built-in electric field. As a result, we may assume the I-region is fully depleted. To find the depletion-region width, we first need to calculate the built-in potential  $\phi_{bi}$  using Equation (4.1.2):

$$\phi_{bi} = \frac{kT}{q} \ln \left( \frac{N_d N_a}{n_i^2} \right) = 0.78V.$$

The electric field distribution in the structure is



$\phi_{bi}$  is the voltage drop across the P-I-N structure, and is the sum of the area under the electric field in each region. That is,

$$\phi_{bi} = \frac{1}{2} \mathcal{E}_{\max} x_p + \mathcal{E}_{\max} L + \frac{1}{2} \mathcal{E}_{\max} x_n \quad \text{where} \quad \mathcal{E}_{\max} = \frac{qN_d}{\epsilon_s} x_n.$$

We also know from Equation (4.2.5) that

$$x_p N_a = x_n N_d \Rightarrow x_p = \frac{N_d}{N_a} x_n.$$

Hence, the equation becomes

$$\frac{qN_d}{2\epsilon_s} \left( \frac{N_d}{N_a} + 1 \right) x_n^2 + \frac{qN_d L}{\epsilon_s} x_n - \phi_{bi} = 0.$$

Solving for  $x_n$  yields  $0.032 \mu\text{m}$ . Consequently,  $x_p$  is  $0.019 \mu\text{m}$ . Then, the depletion-region width  $W_{\text{dep}}$  is  $x_n + x_p + L = 0.551 \mu\text{m}$ .

(b) We found the maximum electric field in part (a):

$$|\mathcal{E}_{\max}| = \frac{qN_d}{\epsilon_s} x_n = \frac{qN_a}{\epsilon_s} x_p = 148 \text{ V/cm}.$$

(c) We also found the built-in potential in part (a). It was  $0.78 \text{ V}$ .

(d) The breakdown voltage is the sum of the areas under the electric field distribution with  $\mathcal{E}_{\max} = \mathcal{E}_{\text{crit}} = 2 \cdot 10^5 \text{ Vcm}^{-1}$ . The depletion-region width in each region can be written in term of  $\mathcal{E}_{\text{crit}}$  as



$$|\mathcal{E}_{crit}| = \frac{qN_d}{\mathcal{E}_S} x_n = \frac{qN_a}{\mathcal{E}_S} x_p \Rightarrow x_n = \frac{|\mathcal{E}_{crit} \mathcal{E}_S|}{qN_d} \text{ and } x_p = \frac{|\mathcal{E}_{crit} \mathcal{E}_S|}{qN_a}.$$

For P-N junction, the breakdown voltage is

$$V_B = \frac{1}{2} \mathcal{E}_{crit} x_p + \frac{1}{2} \mathcal{E}_{crit} x_n = \frac{\mathcal{E}_S}{2q} \mathcal{E}_{crit}^2 \left( \frac{1}{N_d} + \frac{1}{N_a} \right) = 6.91 \text{ V}.$$

For P-I-N junction, the breakdown voltage is

$$\begin{aligned} V_B &= \frac{1}{2} \mathcal{E}_{crit} x_p + \mathcal{E}_{crit} L + \frac{1}{2} \mathcal{E}_{crit} x_n = \\ &= \frac{\mathcal{E}_S}{2q} \mathcal{E}_{crit}^2 \left( \frac{1}{N_d} + \frac{1}{N_a} \right) + \mathcal{E}_{crit} 0.5 \mu\text{m} = 16.14 \text{ V}. \end{aligned}$$

Therefore, the breakdown voltage of the P-I-N structure is  $\sim 10\text{V}$  higher than the P-N structure with the same doping levels.

## Diffusion Equation

**4.6 (a)** At  $x = -\infty$ , all the holes generated by illumination in the region  $x > 0$  are recombined well before reaching  $x = -\infty$  due to the finite lifetime. Hence, there is no excess holes exist at  $x = -\infty$ , and the hole concentration is

$$p = p_0 = \frac{n_i^2}{N_a} = 210.25 \text{ cm}^{-3}.$$

(b) At  $x = +\infty$ , excess hole concentration won't have any position dependence. According to the continuity equations, photo generation will be balanced by thermal recombination (since the steady-state conditions prevail), therefore  $G_L = U = p'/\tau$ .

$$p = p_0 + p' = 210.25 \text{ cm}^{-3} + G_L \tau \approx 10^9 \text{ cm}^{-3}.$$

(c) “Low-level injection” implies that the minority carrier concentration is much smaller than the majority carrier concentration. For this problem, since the maximum minority carrier concentration ( $p = 10^9 \text{ cm}^{-3}$ ) is much smaller than the majority carrier concentration ( $n = N_d = 10^{18} \text{ cm}^{-3}$ ), low-level injection conditions do prevail.

(d) The continuity equation is

$$\frac{dp'}{dt} = D_p \frac{\partial^2 p'(x)}{\partial x^2} + G_L - \frac{p'}{\tau} \quad \text{and} \quad \frac{dp'}{dt} = 0 \text{ at steady state.}$$

For  $x \geq 0$ ,

$$\frac{dp'}{dt} = D_p \frac{\partial^2 p'(x)}{\partial x^2} + G_L - \frac{p'}{\tau} \quad \text{where} \quad G_L = 10^{15} \text{ cm}^{-3} \text{ sec}^{-1}$$

$$\Rightarrow p'(x) = A_1 e^{(x/\sqrt{D_p \tau})} + A_2 e^{(-x/\sqrt{D_p \tau})} + G_L \tau.$$

Since  $p' \neq \infty$  as  $x \rightarrow \infty$ ,  $A_2 = 0$ . Therefore,

$$p'_1(x) = A_1 e^{(x/\sqrt{D_p \tau})} + G_L \tau.$$

For  $x < 0$ ,

$$p'_2(x) = C_1 e^{(x/\sqrt{D_p \tau})} + C_2 e^{(-x/\sqrt{D_p \tau})} \quad (G_L = 0).$$

Similarly  $C_1 = 0$ . Therefore,

$$p'_2(x) = C_2 e^{(x/\sqrt{D_p \tau})}.$$

Now, if we apply the boundary condition or continuity to the equations above, we find

$$p'_1(x=0) = p'_2(x=0) \& \frac{\partial p'_1(0)}{\partial x} = \frac{\partial p'_2(0)}{\partial x} \Rightarrow A_1 = C_2 + G_L \tau \& A_1 = -C_2$$

$$A_1 = \frac{G_L \tau}{2} = 5 \times 10^8 \text{ cm}^{-3} \quad \text{and} \quad C_2 = -\frac{G_L \tau}{2} = -5 \times 10^8 \text{ cm}^{-3}.$$

Therefore,

$$p'_1(x) = 10^9 \text{ cm}^{-3} - 5 \times 10^8 e^{(-x/\sqrt{D_p \tau})} \quad \text{for } x \geq 0 \quad \text{and}$$

$$p'_2(x) = 5 \times 10^8 e^{(x/\sqrt{D_p \tau})} \quad \text{for } x < 0 \quad \text{where } D_p \tau = 4.07 \times 10^{-6} \text{ cm}^2.$$

- 4.7 (a)** Thermal equilibrium will be disturbed if any non-thermally generated excess carriers are present. If  $L$  (the distance from the origin to the excess carrier generation point) is much larger than the diffusion length of the holes (i.e.,  $L \gg \sqrt{D_p \tau_p}$ ), then almost all the excess carriers generated due to the light would have recombined (i.e.,  $p_N' \approx 0$ ). In this case, we could assume thermal

equilibrium. Otherwise, if  $L$  is not much larger than  $\sqrt{D_p \tau_p}$ , then significant number of excess carriers will be present at  $x=0$  and the origin will not be at thermal equilibrium.

- (b)  $p_N'(x) = n_N'(x)$  everywhere in the semiconductor because of charge neutrality inside the semiconductor and all the excess carriers due to light are generated in pairs of electrons and holes. Excess concentrations at  $x = -L$  is  $\gamma N_d$ . Total electron and hole concentrations at  $x=-L$  are  $n(x = -L) = N_d + \gamma N_d$  and  $p(x = -L) = n_i^2/N_d + \gamma N_d$ .

- (c) The excess carriers are generated at  $x = -L$  and  $x = L$ . In the rest of the semiconductor the excess carriers decay due to recombination. That is,

$$p_N'(-L) > p_N'(x) \text{ for } -L < x < L.$$

So the maximum excess carrier concentration is at  $x = -L$  and  $x = L$ . Now,  $p_N'(-L) = p_N'(L) = \gamma N_d$ . At  $\gamma = 10^{-3}$ ,  $p_N'(-L) \ll N_d$  in which  $N_d$  is the majority carrier concentration at thermal equilibrium. Therefore,  $p_N'(x) \ll N_d$  for  $-L \leq x \leq L$  which means the excess carrier concentration is always much smaller than the equilibrium majority carrier concentration. This is the condition for low-level injection.

- (d) Inside the bar, carrier generation due to light is zero. In steady state, the continuity equation simplifies to terms involving diffusion and recombination:

$$0 = D_p \frac{d^2 p_N'}{dx^2} - \frac{p_N'}{\tau_n}.$$

Rearranging terms gives us

$$\frac{d^2 p_N'}{dx^2} = \frac{p_N'}{D_p \tau}.$$

- (e) The general solution of the differential equation in part (d) is

$$\Delta p_N(x) = A \cdot e^{\frac{x}{L_p}} + B \cdot e^{-\frac{x}{L_p}}$$

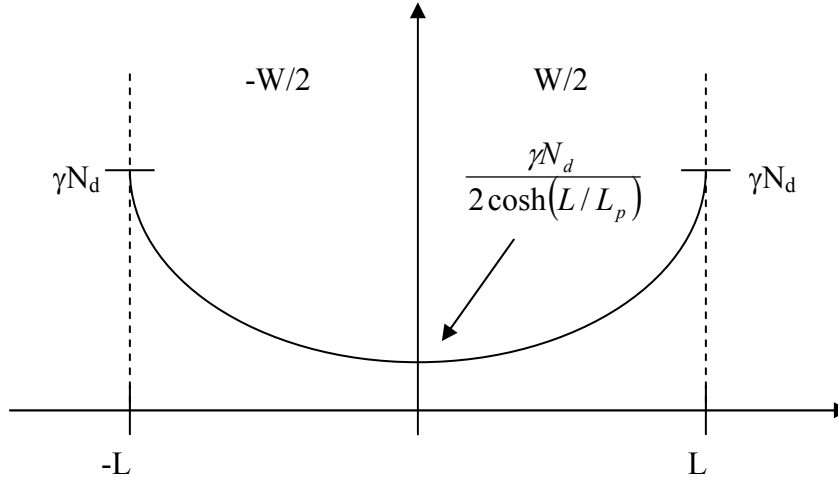
where  $L_p = \sqrt{D_p \tau}$ , and  $A$  and  $B$  are integration constants.

The boundary conditions for this problem are

$$p_N'(-L) = p_N'(L) = \gamma N_d$$

Although you do not have to solve the differential equation, the solution to this particular problem is

$$p_N'(x) = \frac{\gamma N_d}{2 \cosh\left(\frac{L}{L_p}\right)} \cdot \cosh\left(\frac{x}{L_p}\right) \quad \text{(Please note that } \cosh(z) = \frac{e^z + e^{-z}}{2} \text{.)}$$



This curve confirms what you would physically expect from carrier injection on both ends of a silicon bar.

- 4.8 (a)** From Figure 2-5,  $\mu_p$  is approximately  $450 \text{ cm}^2\text{V}^{-1}\text{sec}^{-1}$ .  $\tau_p$  is given to be  $1 \text{ }\mu\text{sec}$ . Using Equation (2.5.6b) and (4.7.6), we calculate  $D_p$  and  $L_p$ , respectively:

$$D_p = \frac{kT}{q} \mu_p = 11.7 \text{ cm}^2 / \text{sec} \quad \text{and} \quad L_p = \sqrt{D_p \tau_p} = 34.2 \text{ }\mu\text{m}.$$

- (b) Using Equation (4.6.3), we calculate the excess hole concentration:

$$p_N'(0) = p_{N0} [e^{qV/kT} - 1] = \frac{n_i^2}{N_d} [e^{qV/kT} - 1]$$

Hence,  $p_N' = n_N' = 0$  for (i), and  $p_N' = n_N' = 4.8 \times 10^{10} \text{ cm}^{-3}$  for (ii).

- 4.9 (a)** From Figure 2-8,

$$\rho = 0.2 \text{ }\Omega\text{cm}, n\text{-type} \Rightarrow N_d = 3 \times 10^{16} \text{ cm}^{-3}, \text{ and}$$

$$\rho = 1 \text{ }\Omega\text{cm}, p\text{-type} \Rightarrow N_a = 1.5 \times 10^{16} \text{ cm}^{-3}.$$

Therefore, using Equation (4.1.2), we find that the built-in voltage is

$$\phi_{bi} = kT \ln \frac{N_d N_a}{n_i^2} = 0.76 \text{ V}.$$

(b) Using Equation (4.8.2),

$$n_p(0) = \frac{n_i^2}{N_a} (e^{(qV/kT)} - 1) = 0.65 \times 10^{14} \text{ cm}^{-3}, \text{ and}$$

$$p_N(0) = \frac{n_i^2}{N_d} (e^{(qV/kT)} - 1) = 3.25 \times 10^{13} \text{ cm}^{-3}.$$

(c) P-type side:

$$N_a = 1.5 \times 10^{16} \text{ cm}^{-3} \Rightarrow \mu_n = 1150 \text{ cm}^2 \text{ V}^{-1} \text{ sec}^{-1}, \tau_n = 10^{-6} \text{ sec}, D_n = 29.9 \text{ cm}^2/\text{sec}, \\ \text{and } L_n = 5.47 \times 10^{-3} \text{ cm}.$$

Using Equation (4.9.2), we need to derive an expression for  $J_n(x)$ :

$$J_n(x) = -q \frac{D_n}{L_n} n_{p0} (e^{qV/kT} - 1) e^{x/L_n} = \\ = -q \frac{D_n}{L_n} \frac{n_i^2}{N_a} (e^{qV/kT} - 1) e^{x/L_n} = 54 e^{x/L_n} \text{ mA/cm}^2.$$

N-type side:

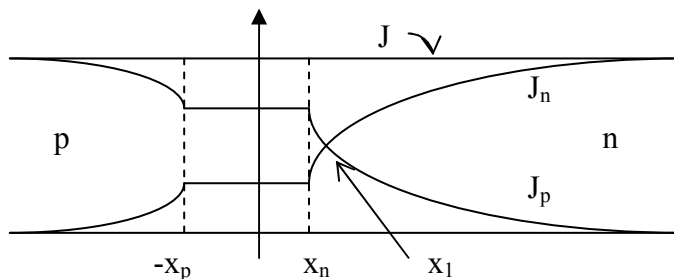
$$N_d = 3 \times 10^{16} \text{ cm}^{-3} \Rightarrow \mu_p = 400 \text{ cm}^2 \text{ V}^{-1} \text{ sec}^{-1}, \tau_p = 10^{-8} \text{ sec}, D_p = 10.4 \text{ cm}^2/\text{sec}, \\ \text{and } L_p = 3.22 \times 10^{-4} \text{ cm}.$$

Using Equation (4.9.1), we need to derive an expression for  $J_p(x)$ :

$$J_p(x) = -q \frac{D_p}{L_p} p_{N0} (e^{qV/kT} - 1) e^{-x/L_p} = \\ = -q \frac{D_p}{L_p} \frac{n_i^2}{N_d} (e^{qV/kT} - 1) e^{-x/L_p} = 168 e^{-x/L_p} \text{ mA/cm}^2.$$

And the total current density  $J$  is

$$J = J_n(0) + J_p(0) = 0.222 \text{ A/cm}^2.$$



(d) At the point ( $x_1$ ) where  $J_n$  and  $J_p$  cross each other,  $J_n = J_p = \frac{1}{2} J$ . Therefore,

$$J_p = \frac{0.222}{2} A/cm^2 = 0.111 A/cm^2 = 0.168 e^{-(x_1)/L_p} \Rightarrow x_1 = 1.38 \mu m.$$

**4.10** For this problem, please note that:

- (1)  $\Delta p_N = 0$  for  $x_b \leq x \leq x_c$  because  $\tau_p = 0$ . This yields the boundary condition  $\Delta p_N = 0$  and  $x = x_b$ .
- (2) Because we have a P<sup>+</sup>N diode, we need only deal with the N-side of the junction in establishing an expression for  $I$ . Moreover, the depletion width is all but totally on the N-side of the junction given a P<sup>+</sup>N diode (i.e.,  $x_n \cong W_{dep}$ ).
- (3) Because  $\tau_p = \infty$  for  $0 \leq x \leq x_b$ , there will be no  $I_{R-G}$  current. ( $\tau_p = \infty$  implies that there are no R-G centers.) Thus, we only need to develop an expression for the diffusion current flowing in the diode.

Since we are interested in the static state,  $\partial \Delta p_N / \partial t = 0$ . Also,  $G_L = 0$  (no light) and  $\Delta p_N / \tau_p \rightarrow 0$  because  $\tau_p = \infty$ . Thus the minority carrier diffusion equation reduces to the form

$$\partial^2 \Delta p_N / \partial x^2 = 0 \text{ for } W \leq x \leq x_b$$

which is subject to the boundary conditions

$$\Delta p_N(x_b) = 0 \text{ and } \Delta p_N(W) = \frac{n_i^2}{N_d} \left( e^{\frac{qV_A}{kT}} - 1 \right).$$

The general (narrow-base type) solution is  $\Delta p_N(x) = A_1 + A_2 x$ .  
Applying the boundary conditions

$$0 = A_1 + A_2 x_b \text{ and } \Delta p_N(W) = A_1 + A_2 W$$

giving

$$\Delta p_N(W) = -A_2(x_b - W) \text{ or } A_2 = -\Delta p_N(W)/(x_b - W) \text{ and } A_1 = -A_2 x_b = \Delta p_N(W) \frac{x_b}{x_b - W}.$$

$$\text{Thus } \Delta p_N(x) = \Delta p_N(W) \left( \frac{x_b - x}{x_b - W} \right) = \frac{n_i^2}{N_d} \left( \frac{x_b - x}{x_b - W} \right) \cdot \left( e^{\frac{qV_A}{kT}} - 1 \right) \text{ for } W \leq x \leq x_b,$$

$$J_p \cong -qD_p \frac{d\Delta p_n}{dx'} = q \frac{n_i^2}{N_d} \left( \frac{D_p}{x_b - W} \right) \cdot \left( e^{\frac{qV_A}{kT}} - 1 \right) \text{ and}$$

$$I \cong AJ_p = qA \frac{n_i^2}{N_d} \left( \frac{D_p}{x_b - W} \right) \cdot \left( e^{\frac{qV_A}{kT}} - 1 \right).$$

Please note that since  $\Delta p_N(x)$  is a linear function of  $x$ ,  $J_p$  is constant throughout the narrow-base ( $W_{\text{dep}} \leq x_b \leq x_b$ ) region.

### ***Proof of Minority Drift Current Being Negligible***

**4.11** (a)  $0.5 \Omega\text{-cm}$  n-type Si  $\Rightarrow N_d = 10^{16} \text{cm}^{-3}$ ,  $D_n = 30 \text{cm}^2\text{sec}^{-1}$ ,  $L_n = 5.5 \mu\text{m}$ ,  $D_p = 10 \text{cm}^2\text{sec}^{-1}$ ,  $L_p = 3.2 \mu\text{m}$

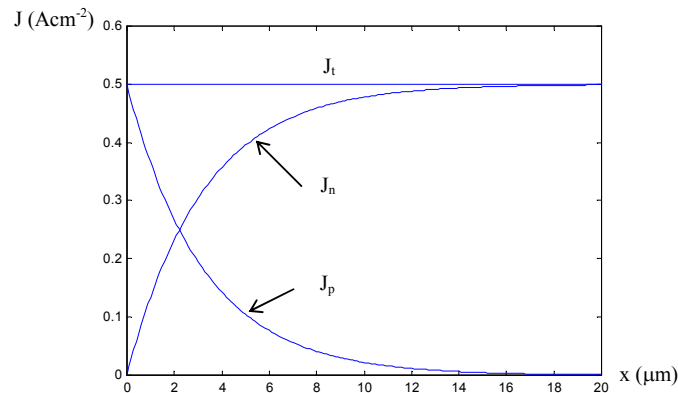
$$p(x) = p_{N0} + \frac{n_i^2}{N_d} \left( e^{qV_a/kT} - 1 \right) e^{-x/L_p} \approx 10^{14} e^{-x/L_p} \text{cm}^{-3}.$$

(b) Minority current:

$$J_p(x) = -qD_p \frac{dp(x)}{dx} = q \frac{n_i^2}{N_d L_p} D_p \left( e^{qV_a/kT} - 1 \right) e^{-x/L_p} = 0.5 e^{-x/L_p} \text{A/cm}^2.$$

Majority current:

$$J_n(x) = J_t(x) - J_p(x) = J_p(0) - J_p(x) = 0.5 \left( 1 - e^{-x/L_p} \right) \text{A/cm}^2$$



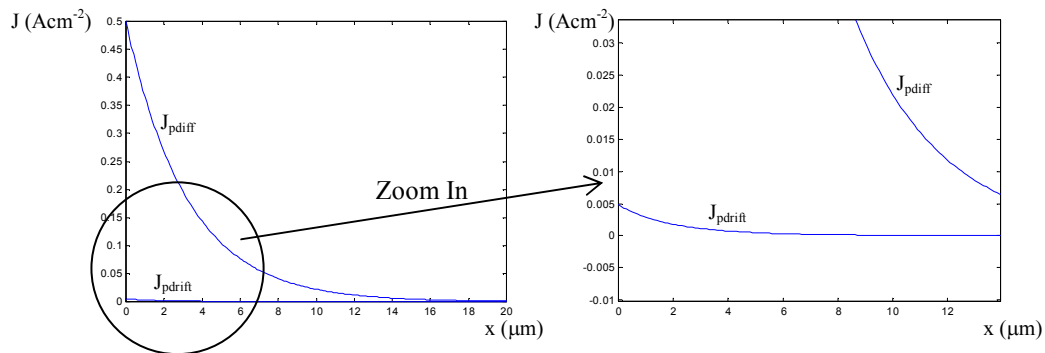
$$(c) J_{ndiff}(x) = qD_n \frac{d(n_0 + n'(x))}{dx} = qD_n \frac{dn'(x)}{dx} = -J_p(x) \frac{D_n}{D_p} = 3J_p(x) = -1.5e^{-x/L_p} A/cm^2$$

$$(d) J_{ndrift}(x) = J_n(x) - J_{ndiff}(x) = 0.5 + 1 \times e^{-x/L_p} A/cm^2.$$

$$E(x) = \frac{J_{ndrift}}{nq\mu_n} = 0.25 + 0.5 \times e^{-x/L_p} V/cm.$$

$$J_{pdrift}(x) = pq\mu_p E(x) \approx p'q\mu_p E(x) = 1.6 \times 10^{-3} e^{-x/L_p} (1 + 2e^{-x/L_p}) A/cm^2.$$

$$J_{pdrift} \ll J_{pdiff} \Rightarrow J_p \approx J_{pdiff}.$$



(e) From  $E(x)$  in (d) and the Poisson equation, Eq. (4.1.5), one the net charge density,  $\rho$  to be

$$1.7 \times 10^{-9} e^{-x/L_p} C/cm^3.$$

This  $\rho$  is negligibly small compared with the excess hole charge density,  $qp'$  in (a), which is

$$1.6 \times 10^{-5} e^{-x/L_p} C/cm^3.$$

This shows that  $p' = n'$  is a good assumption because  $\rho = qp' - qn'$ .

## Temperature Effect on IV

4.12 (a) From Equations (4.9.4) and (4.9.5),

$$I \approx I_0 \left( e^{qV/kT} \right) \quad \text{where} \quad I_0 = AqN_cN_v e^{-E_g/kT} \left( \frac{D_p}{L_pN_d} + \frac{D_n}{L_nN_a} \right) \equiv \eta e^{-E_g/kT}.$$

For simplicity, we will ignore the weak temperature dependence of  $\eta$ .



$$\begin{aligned}
V &\approx \frac{kT}{q} \ln\left(\frac{I}{I_0}\right) \Rightarrow \frac{dV}{dT} \approx \frac{k}{q} \ln\left(\frac{I}{I_0}\right) + \frac{kT}{q} \frac{d}{dT} \left( \ln I - \ln \eta + \frac{E_g}{kT} \right) \\
&\approx \frac{k}{q} \frac{qV}{kT} + \frac{kT}{q} \frac{d}{dT} \left( \frac{E_g}{kT} \right) \\
&= \frac{V}{T} - \frac{E_g}{kT^2}
\end{aligned}$$

(b) Assuming  $V=0.5\text{V}$  and  $T=300\text{K}$ ,

$$\frac{dV}{dT} = \frac{0.5 - 1.1}{300} = 0.002 \text{ V/K}.$$

(c) In Fig. 4-21, start at  $T=300\text{K}$ ,  $V=0.5\text{V}$ , or  $I = 240 \mu\text{A}$ :

At that fixed  $I$ ,  $\Delta V = -0.2 \text{ V}$  between  $-25^\circ\text{C}$  and  $75^\circ\text{C}$  ( $\Delta T = 50 \text{ K}$ ). Hence,  $\Delta V / \Delta T \sim -0.2 \text{ V} / 100^\circ\text{K} = -0.002 \text{ V/K}$ , in excellent agreement with the value found in part (b).

#### 4.13

$$I = Aq \left( \frac{n_i^2}{N_d} \frac{L_p}{\tau_p} + \frac{n_i^2}{N_a} \frac{L_n}{\tau_n} \right) = Aq \left( \frac{-p'}{\tau_p} L_p + \frac{-n'}{\tau_n} L_n \right).$$

4.14 (a) If we model the quasi-neutral regions as a resistor  $R$ , the voltage drop across the neutral regions is  $V_{\text{neutral}} = IR$  and that across the junction is

$$V - V_{\text{neutral}} = V - IR.$$

Hence, Equation 4.9.4 becomes

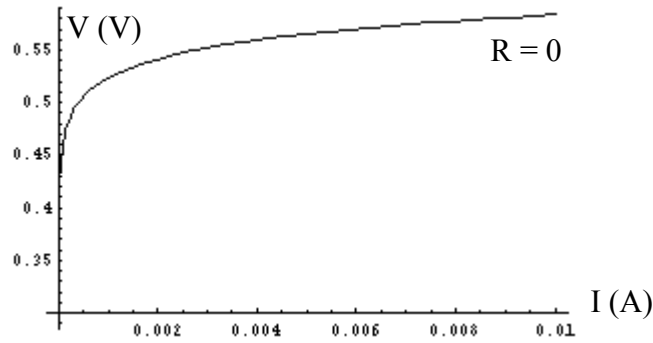
$$I = I_0 \left( e^{q(V-IR)/kT} - 1 \right).$$

(b) Solving the above equation for  $V$  yields

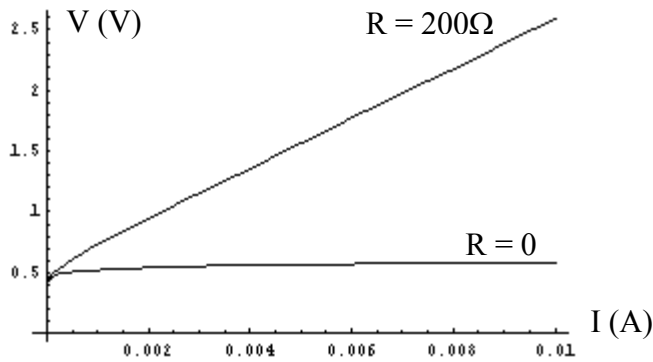
$$V = IR + \frac{kT}{q} \ln\left(\frac{I}{I_0} + 1\right).$$

(c) Let's use  $I_0 = 1.8 \times 10^{-12} \text{ A}$  from Figure 4-22.

With  $R = 0$ ,



With  $R = 200\Omega$ ,



Clearly for the case  $R = 200\Omega$ , the first term  $IR$  becomes dominant over  $\ln(I/I_0+1)$ , and the IV curve becomes very linear.

## Charge Storage

**4.15** Please refer to Section 4.9, 4.10, 8.2, 8.3, and 8.7.

(a) First, we need to find an expression for  $Q_t$ :

$$\begin{aligned}
 Q_t &= Aq \left[ \int_{x_n}^{W_B} p_N'(x) dx + \int_{-W_E}^{-x_p} n_P'(x) dx \right] = \\
 &= Aq p_{N0} \left( e^{qV_a/kT} - 1 \right) \left( -\frac{W_B'}{2} \right) \left( 1 - \frac{x - x_n}{W_B'} \right)^2 \bigg|_{x_n}^{W_B} \\
 &\quad + Aq n_{P0} \left( e^{qV_a/kT} - 1 \right) \left( \frac{W_E'}{2} \right) \left( 1 + \frac{x + x_p}{W_E'} \right)^2 \bigg|_{-W_E}^{-x_p} = \\
 &= \frac{A \cdot q}{2} \left( e^{qV_a/kT} - 1 \right) (p_{N0} W_B' + n_{P0} W_E') = \frac{1}{2} Aq n_i^2 \left( e^{qV_a/kT} - 1 \right) \left( \frac{W_B'}{N_d} + \frac{W_E'}{N_a} \right).
 \end{aligned}$$

$W_B'$  and  $W_E'$  are the actual lengths of the base and emitter regions respectively excluding the depletion regions.  $I_t$  for a short-base diode is given by

$$I_t = Aqn_i^2 \left( e^{qV_a/kT} - 1 \right) \left( \frac{D_p}{W_B'} \frac{1}{N_d} + \frac{D_n}{W_E'} \frac{1}{N_a} \right).$$

Hence,

$$\tau_s = \frac{Q_t}{I_t} = \frac{1}{2} \frac{\left( \frac{W_B'}{N_d} + \frac{W_E'}{N_a} \right)}{\left( \frac{D_p}{W_B'} \frac{1}{N_d} + \frac{D_n}{W_E'} \frac{1}{N_a} \right)} = \frac{(W_B' W_E') (N_a W_B' + N_d W_E')}{2(D_p W_E' N_a + D_n W_B' N_d)}.$$

(b)  $s = 1.8 \times 10^{-10} \text{ sec } (<< \tau_s = \tau_S = 1 \times 10^{-6} \text{ sec})$

(c) A short-based diode has a shorter storage time, thus it can operate at a higher frequency.

## ***Part II: Application to Optoelectronic Devices***

### ***IV of Photodiode/Solar Cell***

**4.16** (a) Let us examine the minority carrier diffusion equation for hole. In general,

$$\frac{\Delta \partial p_N}{\partial t} = D_p \frac{\partial^2 \Delta p_N}{\partial x^2} - \frac{\Delta p_N}{\tau_p} + G_L.$$

For the steady state problem at hand,  $\partial \Delta p_N / \partial t = 0$ . Also  $\partial^2 \Delta p_N / \partial x^2 = 0$  if one goes far from the junction on the N-side where carrier perturbation introduced by the junction has decayed to zero. Thus,

$$0 = -\frac{\Delta p_N(x \rightarrow \infty)}{\tau_p} + G_L \text{ or } \Delta p_N(x \rightarrow \infty) = G_L \tau_p. \Leftarrow \text{boundary condition}$$

(b) One simply parallels the ideal diode derivation to obtain the desired  $I-V_A$  relationship. Given a  $P^+N$  junction, however, we need consider only the lightly doped side of the junction. To simplify the mathematics, we set the origin of coordinates to the n-edge of the depletion region. Under steady state conditions and with  $x'$  as defined as above, we must solve

$$0 = D_p \frac{\partial^2 \Delta p_N}{\partial x^2} - \frac{\Delta p_N}{\tau_p} + G_L$$

subject to the boundary conditions

$$\Delta p_N(x'=0) = \frac{n_i^2}{N_d} \left( e^{\frac{qV_A}{kT}} - 1 \right) \text{ and } \Delta p_N(x \rightarrow \infty) = G_L \tau_p.$$

The general solution is

$$\Delta p_N(x') = G_L \tau_p + A_1 e^{\frac{-x'}{L_p}} + A_2 e^{\frac{x'}{L_p}}.$$

Because  $\exp(x'/L_p) \rightarrow \infty$  as  $x' \rightarrow \infty$ , the only way the second boundary condition can be satisfied is for  $A_2$  to be identically zero. With  $A_2 = 0$ , the application of the first boundary condition yields

$$\Delta p_N(x'=0) = G_L \tau_p + A_1 = \frac{n_i^2}{N_d} \left( e^{\frac{qV_A}{kT}} - 1 \right) \text{ or}$$

$$A_1 = \frac{n_i^2}{N_d} \left( e^{\frac{qV_A}{kT}} - 1 \right) - G_L \tau_p \text{ and}$$

$$\Delta p_N(x') = G_L \tau_p + \left[ \frac{n_i^2}{N_d} \left( e^{\frac{qV_A}{kT}} - 1 \right) - G_L \tau_p \right] \cdot e^{\frac{-x'}{L_p}}.$$

The associated hole current density is then

$$J_p(x') = -qD_p \frac{d\Delta p_N}{dx'} = q \frac{D_p}{L_p} \left[ \frac{n_i^2}{N_d} \left( e^{\frac{qV_A}{kT}} - 1 \right) - G_L \tau_p \right] \cdot e^{\frac{-x'}{L_p}}$$

and for a P<sup>+</sup>N diode,

$$I = AJ = A[J_n(x = -x_p) + J_p(x = x_n)] \cong AJ_p(x' = 0) \quad \text{or}$$

$$I = qA \frac{D_p}{L_p} \frac{n_i^2}{N_d} \left( e^{\frac{qV_A}{kT}} - 1 \right) - qA \frac{D_p \tau_p}{L_p} G_L.$$

Finally noting  $D_p \tau_p = L_p^2$ , we conclude

$$I = I_0 \left( e^{\frac{qV_A}{kT}} - 1 \right) - I_L$$

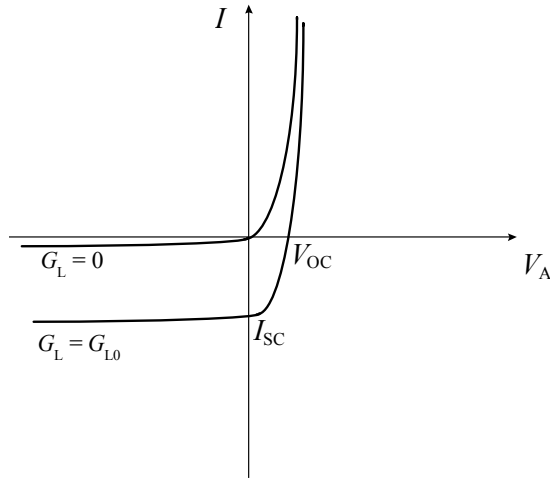
where

$$I_0 = qA \frac{D_p}{L_p} \frac{n_i^2}{N_d} \text{ and } I_L = qAL_p G_L.$$

(c) Voltage that appears when the illuminated diode is open circuit is

$$V_{OC} = \frac{kT}{q} \ln \left( 1 + \frac{I_L}{I_0} \right).$$

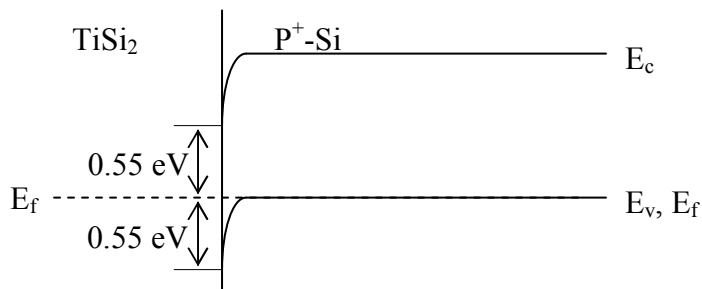
Current that flows when the illuminated diode is short-circuited is  $I_{SC} = -I_L$ .



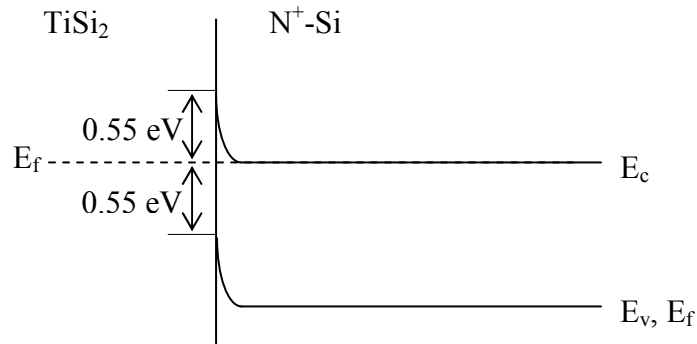
### ***Part III: Metal-Semiconductor Junction***

#### ***Ohmic Contacts and Schottky Diodes***

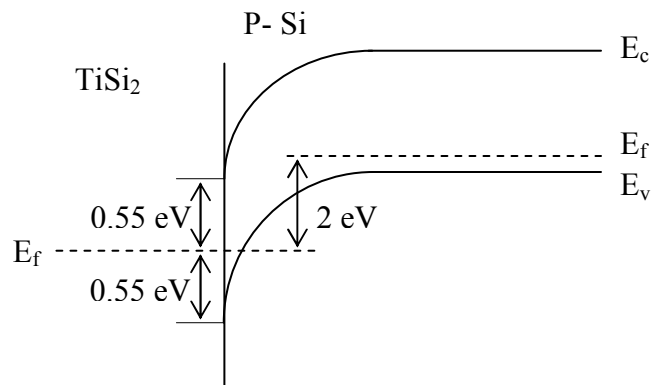
**4.17** (a) A very heavy  $P^+$  doping is important to forming an ohmic contact.



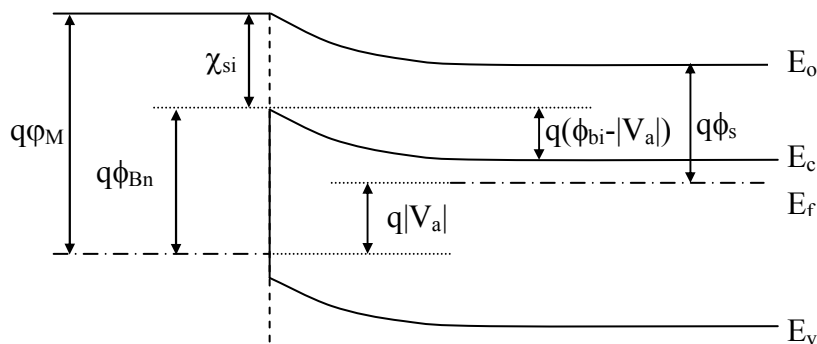
(b) A very heavy  $N^+$  doping is important to forming an ohmic contact.



(c) A very heavy doping is unacceptable to forming a rectifying contact.



4.18 (a)



$$\phi_M = 4.8V, \chi_{Si} = 4.05 eV$$

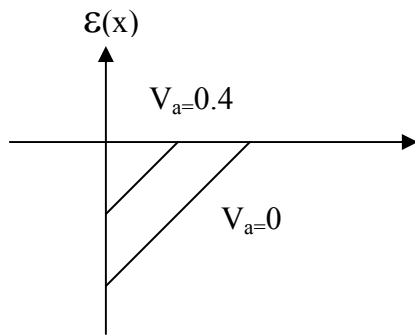
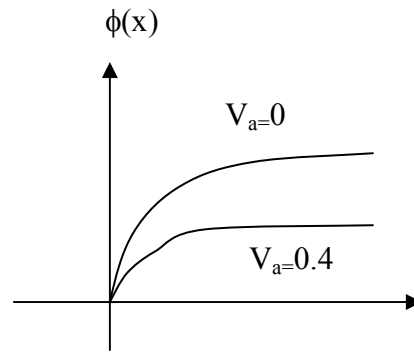
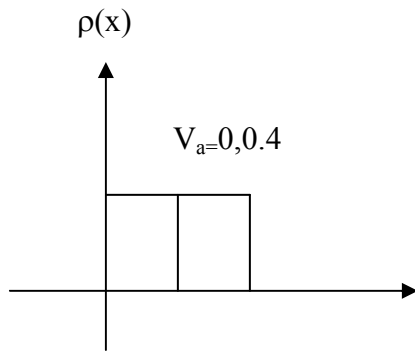
$$\phi_s = \chi_{si} + 0.026 \ln \frac{3.22 \times 10^{19}}{10^{16}} = 4.26V$$

$$\phi_{Bn} = 4.8 - 4.05 = 0.75V$$

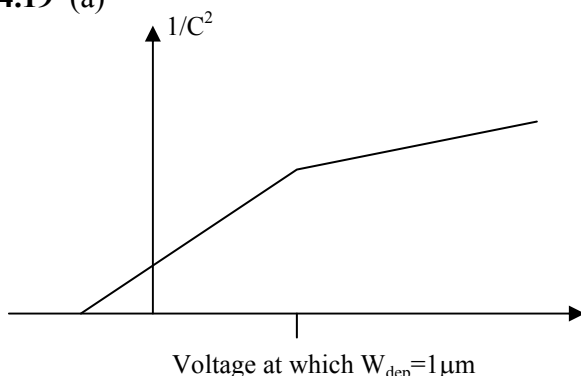
$$\phi_{bi} = \phi_M - \phi_s = 0.54V$$

$$\phi_{bi} - V_a = 0.54 - 0.4 = 0.14V$$

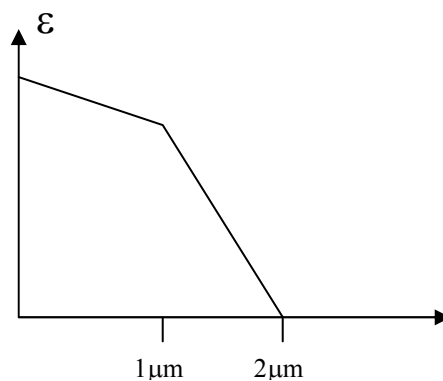
(b)



4.19 (a)



(b)



$$(c) \quad V = \int_0^{2\mu m} \mathcal{E} dx = 1 \cdot \frac{4}{5} \mathcal{E}_{\max} + \frac{1}{2} \cdot 1 \cdot \frac{1}{5} \mathcal{E}_{\max} + \frac{1}{2} \cdot 1 \cdot \frac{4}{5} \mathcal{E}_{\max} = (1.3\mu m) \mathcal{E}_{\max}$$

To find  $\mathcal{E}_{\max}$ , we use the slope when  $x_d < 1\mu m$ ,

$$\frac{-qN}{\mathcal{E}_s} = \frac{(-1.6 \times 10^{-19})(10^{16})}{11.7 \mathcal{E}_o} = 1.54 \times 10^9 \text{ V/cm}^2$$

Using this slope, we need  $x_d = 5\mu m$  for  $\mathcal{E} = 0$ ,

$$\mathcal{E}_{\max} = (1.54 \times 10^9)(5 \times 10^{-4}) = 7.723 \times 10^5 \text{ V/cm}$$

$$V = (1.3\mu m)(7.723 \times 10^5 \text{ V/cm}) = 100.4 \text{ V}.$$

(d) Express  $V$  (areas under electric field profile) in terms of  $W_{\text{dep}}$ ,

$$V = \text{area} = 30.9 W_{\text{dep}} - 23.175$$

$$W_{\text{dep}} = \sqrt{\frac{V + 23.175}{30.9}}$$

$$C = \frac{\mathcal{E}_s}{W_{\text{dep}}} = \frac{\mathcal{E}_s}{\sqrt{\frac{V + 23.175}{30.9}}}.$$

### ***Depletion-Layer Analysis for Schottky Diodes***

**4.20** (a)  $\Phi_B(\text{Platinum to Si}) = 5.3 \text{ eV}$ ,  $N_d = 10^{16} \text{ cm}^{-3}$  and  $A = 10^{-5} \text{ cm}^2$

$$\text{Now, } E_c - E_f = kT \ln(N_c/N_d) = 0.205 \text{ eV}$$

$$\therefore q\Phi_s = 4.05 + 0.205 = 4.255 \text{ eV}$$

$$\therefore \phi_{\text{bi}} = -(\Phi_s - \Phi_M) = 1.045 \text{ V (Si to Pt)}$$

The capacitance is give by

$$C = \text{charge/voltage drop} = \frac{\sqrt{qN_d 2\mathcal{E}_{\text{Si}}(\phi_{\text{bi}} - V_A)}}{(\phi_{\text{bi}} - V_A)} = \frac{\mathbf{K}}{(\phi_{\text{bi}} - V_A)^{1/2}}$$

$$\text{where } \mathbf{K} = 2.88 \times 10^{-13} \text{ F V}^{1/2}$$

$$\text{If } V_A = 0, C = 2.82 \times 10^{-13} \text{ F} = 0.282 \text{ pF}$$



(b) For a 25% reduction  $C/C(0) = 0.75 \Rightarrow \left( \frac{\phi_{bi}}{\phi_{bi} + V_A} \right)^{1/2} = 0.75$   
 $\therefore V_A = \phi_{bi}(1 - 0.75^2) = -0.813 \text{ V}$

**4.21** The development of relationships for the electrostatic variables in a linearly graded MS diode closely parallels the uniformly doped analysis.

(a) With  $N_d(x) = ax$  for  $x \geq 0$ , invoking the depletion approximation yields

$$\rho(x) = qax \quad 0 \leq x \leq W_{dep}.$$

Substituting into Poisson's equation gives

$$\frac{d\mathcal{E}}{dx} = \frac{\rho}{\epsilon_{Si}} = \frac{qa}{\epsilon_{Si}} x \quad 0 \leq x \leq W_{dep} \quad \text{and}$$

$$\int_{\mathcal{E}(x)}^0 d'\mathcal{E}' = \frac{qa}{\epsilon_{Si}} \int_x^{W_{dep}} x' dx' \quad \text{or}$$

$$\mathcal{E}(x) = -\frac{qa}{2\epsilon_{Si}} (W_{dep}^2 - x^2) \quad 0 \leq x \leq W_{dep}.$$

Turning to the electrostatic potential, we can write

$$\frac{dV}{dx} = -\mathcal{E}(x) = \frac{qa}{2\epsilon_{Si}} (W_{dep}^2 - x^2) \quad \text{and}$$

$$\int_{V(x)}^0 dV' = \frac{qa}{2\epsilon_{Si}} \int_x^{W_{dep}} (W_{dep}^2 - x'^2) dx' \quad \text{or}$$

$$V(x) = -\frac{qa}{6\epsilon_{Si}} (2W_{dep}^3 - 3W_{dep}^2 x + x^3) \quad 0 \leq x \leq W_{dep}.$$

Finally,  $V = -(\phi_{bi} - V_A)$  at  $x = 0$ , and therefore

$$-(\phi_{bi} - V_A) = -\frac{qa}{3\epsilon_{Si}} W_{dep}^3$$

$$W_{dep} = \left[ \frac{3\epsilon_{Si}}{qa} (\phi_{bi} - V_A) \right]^{1/3}.$$

(b) Paralleling the developments for the linearly graded P-N junction, the expression for  $\phi_{bi}$  will be

$$\phi_{bi} = \frac{1}{q} \left[ \Phi_B - (E_c - E_f) \right]_{x=W_0}$$

where  $W_0$  is the depletion width when  $V_A = 0$ . Since approximate charge neutrality applies for  $x > W_0$ , it follows that

$$n|_{x=W_0} = n_i e^{\left[ \frac{E_f - E_i}{kT} \right]_{x=W_0}} \cong N_d(x = W_0) = aW_0 \quad \text{or}$$

$$\left( E_c - E_f \right)_{x=W_0} \cong \frac{E_g}{2} - kT \ln \left( \frac{aW_0}{n_i} \right)$$

Thus, to determine  $\phi_{bi}$ , one must simultaneously solve the following two equations employing numerical techniques.

$$W_0 = \left[ \frac{3\epsilon_{Si}}{qa} \phi_{bi} \right]^{1/3} \quad \text{and} \quad \phi_{bi} = \frac{1}{q} \left[ \Phi_B - \frac{E_g}{2} + kT \ln \left( \frac{aW_0}{n_i} \right) \right]$$

$$(c) \quad C_J = \frac{\epsilon_{Si} A}{W_{dep}} = \frac{\epsilon_{Si} A}{\left[ \frac{3\epsilon_{Si}}{qa} (\phi_{bi} - V_A) \right]^{1/3}}$$

**4.22 (a)** From the graph, the built-in voltage is 0.8V.

(b) From Equation 4.16.7, we know that the slope of the line is inversely proportional to the doping concentration. Hence, for Region I,

$$\frac{1}{C^2} = \frac{2}{qN_d \epsilon_s} (\phi_{bi} + V) \Rightarrow \text{Slope} \equiv \frac{2}{qN_d \epsilon_s} = \frac{7 \times 10^{14} \text{ cm}^2 / \text{F}^2}{5V}.$$

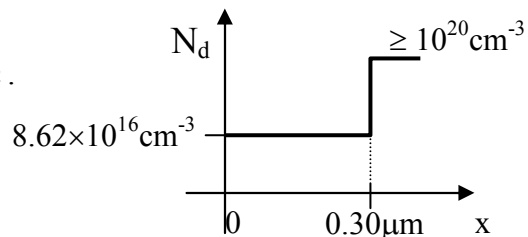
Solving for  $N_d$  yields

$$N_d = 8.62 \times 10^{16} \text{ cm}^{-3}.$$

For Region II, the slope is 0. This implies that Region II is a very heavily doped  $N^{++}$  region. Hence,  $N_d \geq 10^{20} \text{ cm}^{-3}$ .

In order to find the width of Region I, we calculate  $W_{dep}$  at  $V_a = 5V$  since Region I becomes fully depleted. Using Equation 4.3.1,

$$W_{dep} = \sqrt{\frac{2\epsilon_s (\phi_{bi} + 5V)}{qN_d}} = 0.30 \mu\text{m}.$$





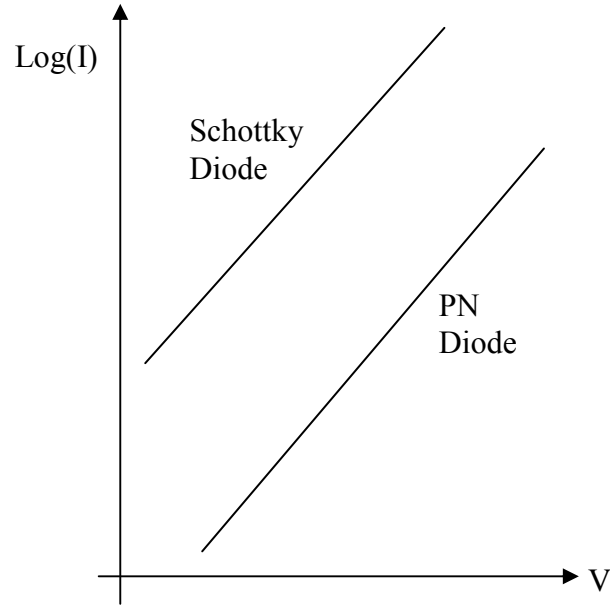
## Comparison between Schottky Diodes and PN Junction Diodes

### 4.23 (a) Similarity:

Both have the same slope of 1 decade / 60mV at room temperature.

### Difference:

The schottky diode current is several orders of magnitude larger than the PN diode current.



### (b) For Schottky diode, use Equations 4.18.2 and 4.18.6:

$$I_0 = AKT^2 e^{-q\phi_{Bn}/kT} = [(0.01)(100)(300^2) e^{-q(0.55V)/kT}] A = 58.5 \mu A$$

For PN diode, use Equation 4.9.5:

$$I_0 = Aqn_i^2 \left( \frac{D_p}{L_p N_d} \right) = (0.01 cm^2) q (10^{10} cm^{-3})^2 \left( \frac{4 cm^2 / s}{\sqrt{(4 cm^2 / s)(1 \mu s)}} \right) = 0.32 fA$$

### (c) For Schottky diode,

$$V = \frac{kT}{q} \ln \left( \frac{I}{I_0} + 1 \right) = 0.36V .$$

For PN diode,

$$V = \frac{kT}{q} \ln \left( \frac{I}{I_0} + 1 \right) = 1.03V .$$

### (d) Use a metal or silicide that yields a smaller $\phi_{Bn}$ .

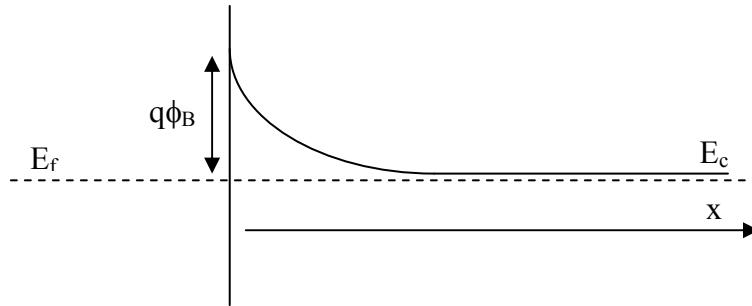
### (e) $I_0$ may be so large, especially at elevated temperatures, that the reverse leakage current

- interferes with the rectification property of the diode, and
- causes too large a power (heat) dissipation,  $I_0 \times V_r$ , under reverse bias.

## Ohmic Contacts

- 4.24 (a) The semiconductor is uniformly doped with a doping concentration of  $N_d = 10^{17} \text{ cm}^{-3}$ . Using depletion approximation, the energy band varies as  $E_c(x) = \frac{1}{2} (qN_d/\epsilon_{Si}) (x - x_0)^2$  where  $x_0$  is a constant and  $x$  is positive in the semiconductor and zero at the MS junction.  
At  $x = 0$ ,  $E_c(0) = 0.65 \text{ eV}$   
(using the equilibrium Fermi-level as the reference)  
and at  $x = 10 \text{ nm}$ ,  $E_c(0) \approx 0 \Rightarrow x_0 = 10 \text{ nm}$  and  $N_d = 8.85 \times 10^{18} \text{ cm}^{-3}$

(b)



4.25 (a)  $R_c = \frac{50 \text{ mV}}{1 \text{ mA}} \times 0.08 \mu\text{m}^2 = 4 \times 10^{-8} \Omega \cdot \text{cm}^2$

(b) Using Eq. 4.21.7,

$$R_c \equiv \frac{V}{J} = \frac{2}{qv_{thx} H \sqrt{N_d}} e^{H\phi_{Bn}/\sqrt{N_d}} = B e^{H\phi_{Bn}/\sqrt{N_d}} \Rightarrow \phi_{Bn} = \frac{\sqrt{N_d}}{H} \ln\left(\frac{R_c}{B}\right)$$

$$\begin{aligned} H &= (5.4 \times 10^{10}) \times \sqrt{(m_n/m_0)(\epsilon_s/\epsilon_0)} \\ &= (5.4 \times 10^{10}) \times \sqrt{0.26 \times 11.7} \\ &= 9.418 \times 10^{10} \text{ cm}^{-3/2} \text{ V}^{-1} \end{aligned}$$

$$v_{thx} = \sqrt{2kT/\pi m_n} = 1.057 \times 10^7 \text{ cm/s}$$

$$B = \frac{2}{qv_{thx} H \sqrt{N_d}} = 1.256 \times 10^{-9} \Omega \text{ cm}^2$$

$$\therefore \phi_{Bn} = \frac{\sqrt{N_d}}{H} \ln\left(\frac{R_c}{B}\right) = 0.3675 \text{ V}$$

This is the maximum  $\phi_{Bn}$  allowed since a larger  $\phi_{Bn}$  will result in a voltage

drop larger than 50 mV at the ohmic contact.

(c) According to the graph,  $\phi_{Bn} \approx 0.36 \text{ V}$ .

#### 4.26

$$(a) \quad \phi_{Bn} = 0.67 \text{ V} \quad \therefore R_C \approx 6 \times 10^{-7} \Omega \cdot \text{cm}^2$$

$$\phi_{Bp} = 0.43 \text{ V} \quad \therefore R_C \approx 5 \times 10^{-8} \Omega \cdot \text{cm}^2$$

$$(b) \quad \phi_{Bp} = 0.23 \text{ V} \quad \therefore R_C \approx 7 \times 10^{-9} \Omega \cdot \text{cm}^2$$

$$(c) \quad \phi_{Bn} = 0.28 \text{ V} \quad \therefore R_C \approx 1 \times 10^{-8} \Omega \cdot \text{cm}^2$$

(d) According to the graph, this contact resistance can be achieved with a barrier height of 0.3V or lower and a doping concentration of  $3 \times 10^{20} \text{ cm}^{-3}$  or higher is required. Looking at Table 4-2, we can see that this cannot be achieved using only one type of metal silicide. So different silicides should be used for contact on  $N^+$  and  $P^+$  silicon.

$$N^+ : \quad \text{ErSi}_{1.7} \quad \geq 3.5 \times 10^{20} \text{ cm}^{-3}$$

$$P^+ : \quad \text{PtSi} \quad \geq 2.5 \times 10^{20} \text{ cm}^{-3}$$