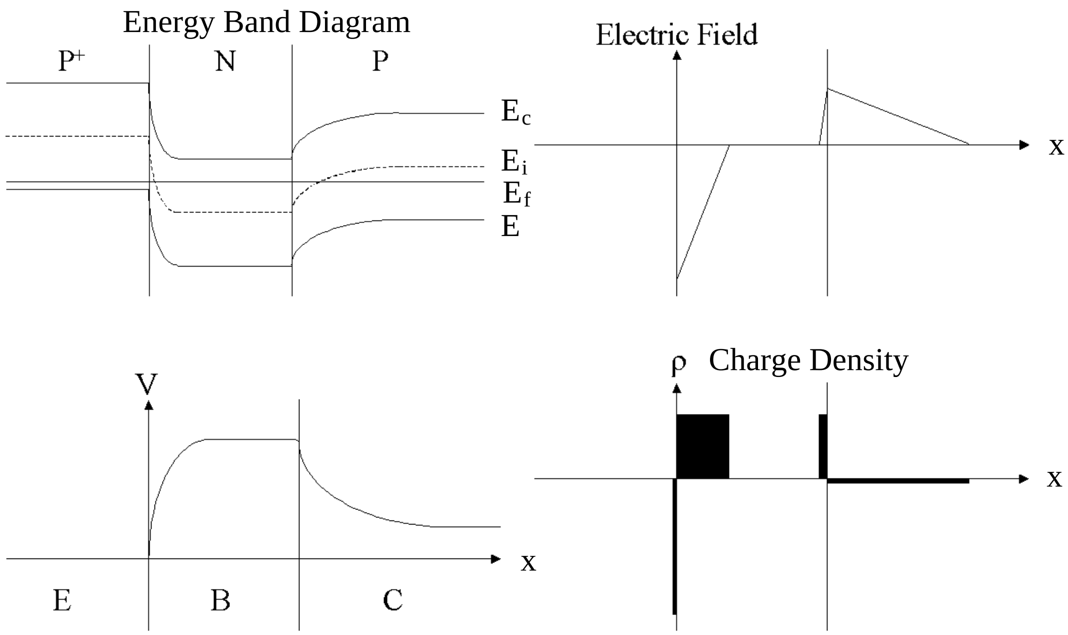
Chapter 8

Energy Band Diagram of BJT

8.1 (a) & (b)

For the given doping concentrations, one computes E_f-E_i = -0.521 eV, 0.419 eV, and –0.299 eV in the emitter, base and collector, respectively. Also with N_{aE}>>N_{dB}, the E-B depletion width will lie almost exclusively in the base. Likewise, the majority of the C-B depletion width will lie in the collector.



(c) The built-in potential between the collector and emitter is

$$\Delta V_{CE} = \frac{kT}{q}\ln\left(\frac{N_{aE}}{N_{aC}}\right) = 0.026\times\ln\left(\frac{5\times10^{18}}{1\times10^{15}}\right) = 0.221V\ .$$

(d) $W = W_n - x_{nEB} - x_{nCB}$

$$x_{nEB} \cong \left[\frac{2\varepsilon_{Si}}{qN_{dB}}V_{biEB}\right]^{1/2} = 0.112\mu m$$

$$x_{nCB} = \left[\frac{2\varepsilon_{Si}}{qN_{dB}}\frac{N_{aC}}{N_{aC}+N_{dB}}V_{biCB}\right]^{1/2} = 0.01\mu m$$

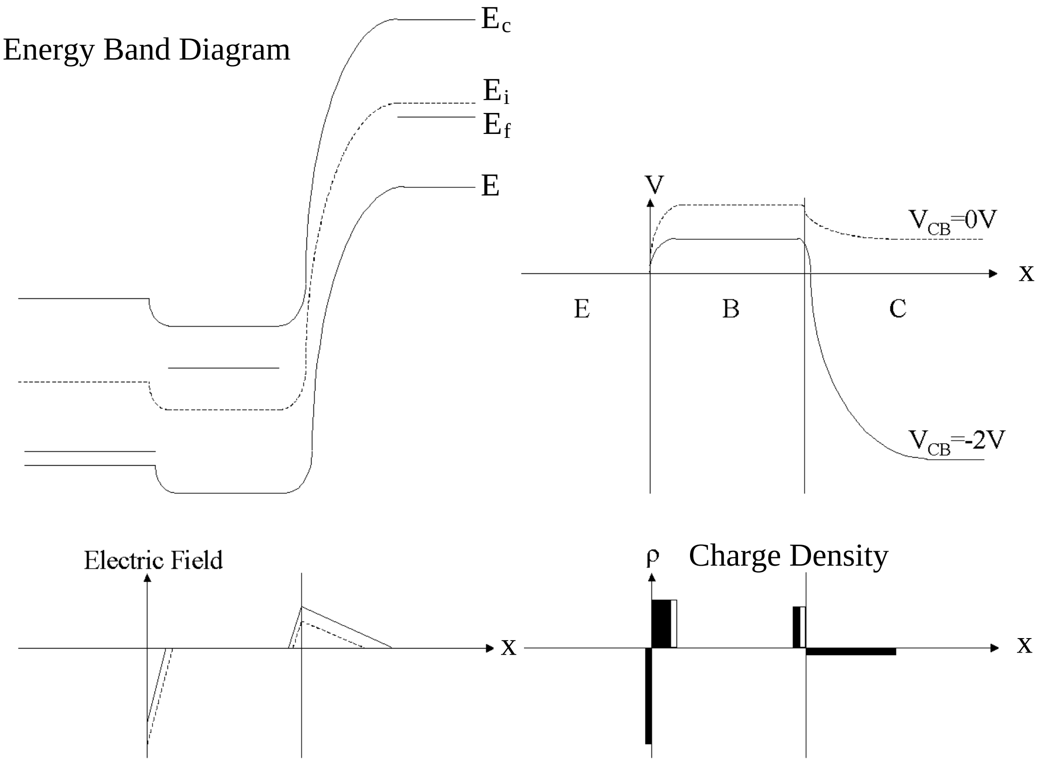
Therefore,

$$W = 2.878\mu m\ .$$

(e) $\left|Electric\ Field\right|_{\max}(E-B)=\left(\dfrac{qN_{dB}}{\varepsilon_{Si}}\right)x_{nEB}=1.72\times10^{5}\ Vcm^{-1}$.

$\left|Electric\ Field\right|_{\max}(C-B)=\left(\dfrac{qN_{dB}}{\varepsilon_{Si}}\right)x_{nCB}=1.54\times10^{4}\ Vcm^{-1}$.

(f)



IV Characteristics and Current Gain

8.2 From Eq. 8.4.1 and Eq. 8.4.2,

$$\beta_F \equiv \frac{I_C}{I_B} \Rightarrow I_B = \frac{I_C}{\beta_F}$$

$$I_C = \alpha_F I_E = \alpha_F\left(I_C + I_B\right) = \alpha_F I_C + \alpha_F I_B.$$

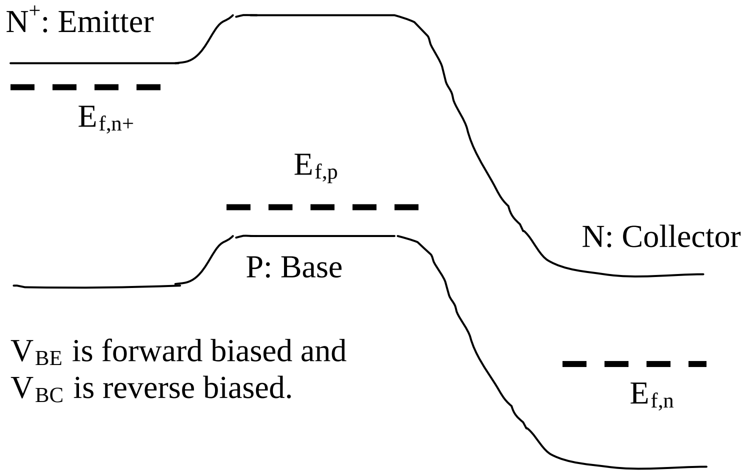
Substituting I_C/β_F into I_B, we obtain

$$I_C = \alpha_F I_C + \alpha_F \frac{I_C}{\beta_F} \Rightarrow 1 = \alpha_F + \frac{\alpha_F}{\beta_F}.$$

Solving for β_F yields

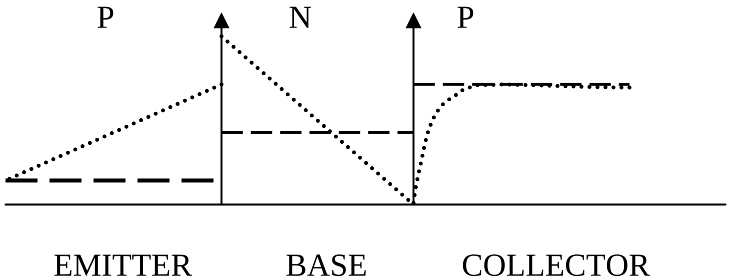
$$I_C = \frac{\alpha_F}{1-\alpha_F}.$$

8.3 (a)



V_{BE} is forward biased and
V_{BC} is reverse biased.

(b)



The P,N,P on the diagram refer to the minority carrier type in each region. The horizontal dotted lines refer to the equilibrium minority concentration (i.e. p_{N0}, n_{P0}). The remaining dotted curves correspond to the excess minority carrier concentrations.

Assumptions made here: W_E & W_B are shorter than the diffusion lengths of the holes & electrons respectively, resulting in a linear decay of excess minority carriers in the emitter and base. You should also notice that the scale for the y-axis differs for each region.

(c) Base current consists of injection of holes into the emitter and recombination with a very small part of the collector current (remember that $I_E \sim I_C$). The collector current consists almost entirely of electrons emitted from the forward-biased BE junction which travel across the CB junction. Incidentally, an easy way to remember how a BJT works is to associate the names emitter and collector with the physical emission and collection of the minority electrons in the base.

8.4 $n_B(x) = n_B(0)\left(1 - \frac{x}{x_B}\right)$ *and* $p_E(x') = p_E(0')\left(1 - \frac{x'}{x_E}\right)$.

In the base region,

$$J_n(x) = qD_B\left(\frac{dn}{dx}\right) = \frac{qD_B n_B(0)}{x_B}.$$

In Emitter region,

$$J_p(x) = -qD_E\left(\frac{dp}{dx}\right) = \frac{qD_E p_E(0)}{x_E}.$$

$n_B(0) = 10^{13}$ cm^{-3} , $p_E(0') = 10^{11}$ cm^{-3}, and $p_C(x_B^+) = p_{CO} = 10^5$ cm^{-3}
$D_B = 20.8$ cm^{2}s^{-1}, D_E=1.8 cm^{2}s^{-1}, and D_C=11.9 cm^{2}s^{-1}.

(a) $J_{n,B}(x=0) = qD_B n_B(0)/x_B = 0.666\ A/cm^2$,
$J_{p,E}(x'=0') = qD_E p_E(0)/x_E = 3.6\times10^{-4}\ A/cm^2$,

and

$$J_{p,C}(x = 0.5\mu m^+) = qD_C p_{CO}/x_C = 8.7\times10^{-10}\ A/cm^2.$$

Therefore,

$$J_{TOTAL,\,BE} = 0.666 A/cm^2 + 3.6\times10^{-4} A/cm^2 = 0.666 A/cm^2$$

and

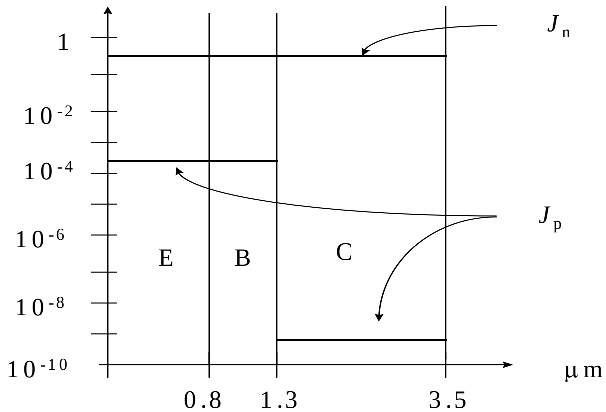
$$J_{TOTAL,\,BC} = J_n(x_B^-) + J_p(x_B^+).$$

For short diode approximation, we assume $J_n(x_B^-) = J_n(x=0)$. More accurate relationship is $J_n(X_B^-) = \alpha_T J_n(x=0)$. However, since $\alpha_T \approx 1$, we can still say $J_n(X_B^-) = J_n(x=0)$.

Hence,

$$J_{TOTAL,\,BC} = 0.666\ A/cm^2$$
$$J_{n,E} = J_{TOTAL,\,BE} - J_p(x'=0') = 0.666\ A/cm^2$$

$$J_{n,C} = J_{TOTAL,\,BC} - J_p(x'=x_B^+) = 0.666\ A/cm^2$$
$$J_{p,E} = J_{p,B} = 3.6\times10^{-4}\ A/cm^2$$



(b) $\alpha_T = \dfrac{1}{1+\dfrac{1+W_B^2}{2L_B^2}} = 0.99875 \quad (L_B = 10\mu m)$

$$\gamma_E = \frac{J_n(0)}{J_n(0)+J_p(0)} = \frac{0.666}{0.666+3.6\times 10^{-4}} = 0.999325$$

$$\beta_F = \frac{\alpha_F}{1-\alpha_F} = 519 \quad (\alpha = \alpha_T \gamma_E)$$

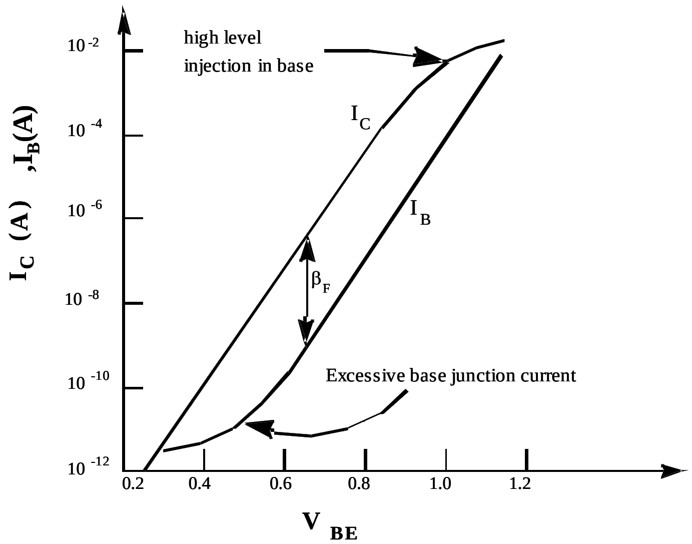
8.5 (a) To improve the emitter injection efficiency, and to reduce the back injected carriers from B-E.

(b) "Small" means $W_B << L_B$, typically $\leq 1\mu m$

(c) ΔW_B would become a larger percentage of W_B. Therefore, it would increase the slope of output characteristics.

(d) At very small values of I_C, the recombination current (excessive base junction current) in the E-B depletion region becomes a less significant part of I_B as I_C increases.

(e) At larger values of I_C, due to the high level injection in base, I_C does not increase exponentially as it does in the moderate level injection region.



(f)

Region	V_{EB}	V_{CB}
Active	+	-
Saturation	+	+
Cutoff	-	-

Schottky Emitter and Collector

8.6 (a) As you will find out in the latter part of this problem the injection of majority carrier of the semiconductor into the metal is much higher than the injection of minority carrier into the semiconductor region from the metal. This would make the emitter efficiency in the BJT very small! Hence, it would not be desirable to use a metal as an emitter in a BJT.

(b) We know that the hole diffusion current is given by

$$I_{\text{diff}} = I_{diff\,0}\left(e^{qV_A/kT}-1\right) = qA\frac{D_P}{L_P}\frac{n_i^2}{N_d}\left(e^{qV_A/kT}-1\right).$$

From Section 4.17 and 4.18,

$$I_{\text{te}} = I_{\text{te0}}\left(e^{qV_A/kT}-1\right) = AKe^{q\Phi_B/kT}\left(e^{qV_A/kT}-1\right).$$

Noting that $\dfrac{D_P}{L_P} = \sqrt{\dfrac{D_P}{\tau_p}} = \sqrt{\dfrac{(kT/q)\mu_p}{\tau_p}}$,

we have

$$\frac{I_{\mathrm{diff}}}{I_{\mathrm{te}}}=\frac{I_{diff\,0}}{I_{\mathrm{te0}}}=\frac{q\sqrt{\frac{(kT/q)\mu_{\mathrm{p}}}{\tau_{\mathrm{p}}}}\frac{n_{\mathrm{i}}^{2}}{N_{\mathrm{d}}}}{Ke^{-\Phi_{\mathrm{B}}/kT}}=\frac{\left(1.6\times10^{-19}\right)\left[\frac{(0.0259)(437)}{10^{-6}}\right]^{1/2}\left(\frac{10^{20}}{10^{16}}\right)}{(140)e^{-(0.72)/(0.0259)}}$$

$$=5.05\times10^{-7}.$$

(c) A Schottky base-collector junction (the collector being the metal) would be functional in a BJT. The energy diagram of the base-collector junction would be similar to Fig. 4-34(b). It would be effective collecting the electrons arriving at this junction from the P type base to the metal. The field at the Schottky junction sweeps the electrons into the metal collector just as in the PN base-collector junction shown in Fig. 8-1(b).

Gummel Number and Gummel Plot

8.7 (a) $\beta=J_C/J_B=100.$

(b) Intercept of J_C is 10^{-10} A/cm^2= $qn_i^2 D_n / N_B W_B$

$$N_B=\frac{qn_i^2 D_n}{10^{-10}\cdot W_B}=8\times10^{16}\,cm^{-3}.$$

(c) Peak concentration

$$\approx\frac{n_i^2}{N_B}e^{\frac{qV_{BE}}{kT}}=10^{17}$$

$$e^{\frac{qV_{BE}}{kT}}=8\times10^{13}$$

$$V_{BE}=\ln(8\times10^{13})\times26mV=832mV.$$

(d) $\tau_n=\dfrac{W_B^2}{2D_n}=\dfrac{(0.2\times10^{-4})^2}{2\times10}=2\times10^{-11}\,\mathrm{sec}.$

Ebers-Moll Model

8.8 (a) Consider that n' = $n_{P0}(e^{qVa/kT}-1)$ and p' = $p_{N0}(e^{qVa/kT}-1)$. Next, take a look at the BC junction ,

⇨ n'/p' = N_B/N_C (1)

Similarly, at the BE junction,

⇨ p'/n' = N_E/N_B (2)

Multiply (1) and (2) to get N_E/N_C = (8/4)×(10/2) = 10.

Thus, $N_C = 0.1N_E$ = *$10^{17}cm^{-3}$*.

(b) The BJT is operating in saturation because both BE and BC junctions are forward-biased and the resulting minority carrier concentrations are larger than the equilibrium values.

(c) The stored minority charge is equal to the area under the curve in the base.

$$Q = \frac{Aq}{2}\left[p'(x = 1\mu m) + p'(x = 2\mu m)\right]W_B =$$
$$= 10^{-5} \times 1.6 \times 10^{-19} \times 0.5 \times 14 \times 10^{14} \times 10^{-4} = 1.12 \times 10^{-13}\ coul\,.$$

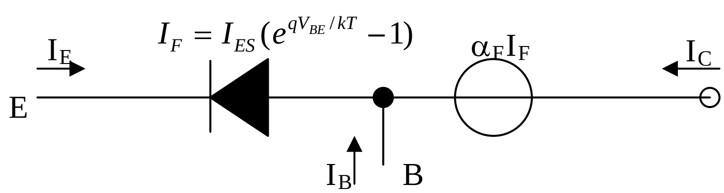
(d) $I_E = I_{nE} + I_{pE} =$

$$= Aq[D_n \frac{n'(x = 1\mu m) - n'(x = 0\mu m)}{W_E} + D_p \frac{p'(x = 1\mu m) - p'(x = 2\mu m)}{W_B}] =$$
$$= 10^{-5} \times 1.6 \times 10^{-19} \times \left[30\left(\frac{2 \times 10^{14}}{1 \times 10^{-4}}\right) + 10\left(\frac{6 \times 10^{14}}{1 \times 10^{-4}}\right)\right] mA = 0.192\ mA\,.$$

(e) $\beta = \dfrac{D_B W_E N_E}{D_E W_B N_B} = \dfrac{10 \times 10^{-4} \times N_E}{30 \times 10^{-4} \times N_B} = \dfrac{1}{3} \times 1 \times 5 = \dfrac{5}{3} = 1.7$ (Not much gain.)

8.9 If the NPN BJT is biased at the boundary between active mode and saturation mode, then forward-biased emitter-base junction ($V_{BE}>0$) and unbiased collector-base junction ($V_{BC}=0$). So $I_R=0$.

(a) At the given operating point, we can simplify the Ebers-Moll model as follows:



(b) Since $I_C + I_B + I_F = 0 \Rightarrow I_B = I_F(1 - \alpha_F)$

$$\Rightarrow I_B = I_{ES}(e^{qV_{BE}} - 1)(1 - \alpha_F)$$
$$\Rightarrow V_{BE} = \frac{kT}{q}\ln\left[\frac{I_B}{I_{ES}(1 - \alpha_F)}\right]$$
$$\therefore V_{EC} = V_{BC} - V_{BE} = 0 - \frac{kT}{q}\ln\left[\frac{I_B}{I_{ES}(1 - \alpha_F)}\right] = \frac{kT}{q}\ln\left[\frac{I_{ES}(1 - \alpha_F)}{I_B}\right].$$

Drift-Base Transistors

8.10 (a) I_B is independent of changes in base parameters. I_C is dependent on base parameters. You should convince yourself that this is the case by referring to the current equations in the reader. If a SiGe base is used, I_c increases as a result of the base bandgap narrowing. Coupled with a graded base that shortens base transport time, SiGe-base BJTs are a simple and attractive alternative to conventional Si-base BJTs.

* This solution ignores the case of increased hole barrier between base and emitter. If you wish to include this effect, then I_B(SiGe) is increased by $e^{\Delta Eg/kT}$ over I_B (Si)

(b) In this case, n_{iB}(SiGe) varies along the base region. This requires that we do an integration to find J_{c0}(SiGe).

$$J_{c0}(\text{SiGe}) = \frac{qD_B n_i^2}{N_B} \frac{1}{\int_0^{W_B} \exp(\frac{-\Delta E_{g,SiGe}}{kT}\frac{x}{W_B})dx}$$

$$= \frac{qD_B n_i^2}{N_B W_B} \frac{\Delta E_{g,SiGe}/kT}{1-\exp(-\Delta E_{g,SiGe}/kT)}.$$

Divide J_{c0}(SiGe) by J_{c0}(Si) and you get the following:

$$\beta(\text{SiGe})/\beta(\text{Si}) = \frac{\Delta E_{g,SiGe}/kT}{1-\exp(-\Delta E_{g,SiGe}/kT)} = 4.$$

8.11 (a) Find where $N_{dE}(x) = N_{aB}(x)$ to obtain the first junction.

$$10^{20} e^{-\frac{x}{0.106}} = 4\times10^{18} e^{-\frac{x}{0.19}} \quad \Rightarrow \quad x_1 = 0.77\,\mu m.$$

To obtain the second junction, equate $N_{aB}(x)$ to the background concentration.

$$4\times10^{18} e^{-\frac{x}{0.19}} = 5\times10^{19} \quad \Rightarrow \quad x_2 = 1.27\ \mu m.$$

Therefore, the base width is x_2-x_1 = 0.5 μm.

(Please note that the depletion widths have been ignored in this case. In general, you must subtract the depletion region widths in the base in both the junctions from the metallurgical base width.)

(b) Base Gummel Number:

$$\int_{x_1}^{x_2} N_{aB}(x)dx = -\left(4\times10^{18}\right)\left(0.19\times10^{-4}\right)\left[\exp\left(-x/_{0.19}\right)\right]_{x_1}^{x_2} cm^{-2} = 1.23\times10^{12}\,cm^{-2}.$$

From Eq. 8.2.12, the base Gummel number is the number above divided by the electron diffusivity in the base, which we shall assume to be around 30 cm^2/s. The result is 4×10^{10} s-cm^{-4}

Emitter Gummel Number:

$$\int_0^{x_1} N_{dE}(x)dx = -(10^{20})(0.106\times10^{-4})\left[\exp\left(-\frac{x}{0.106}\right)\right]_0^{x_1} cm^{-2} = 1.06\times10^{15}\,cm^{-2}.$$

From Eq. 8.3.2, the emitter Gummel number is the number above divided by the hole diffusivity in the base, which we shall assume to be around 2 cm^2/s. The result is 2×10^{14} s-cm^{-4}

(c) Since the doping level is not constant, we use the average doping densities to estimate the diffusivities.

Average base doping density:

$$\frac{GN_B}{W_B} = \frac{1.23\times10^{12}}{0.5\times10^{4}}cm^{-3} = 2.46\times10^{16}\,cm^{-3}.$$

Average emitter doping density:

$$\frac{GN_E}{x_E} = \frac{1.06\times10^{19}}{0.77\times10^{4}} = 1.38\times10^{19}\,cm^{-3}.$$

Average electron diffusivity in the base:

$$D'_{nB} = \mu_n(2.46\times10^{16})kT/q = (1150\times0.026)cm^2s^{-1} = 29.9\,cm^2s^{-1}.$$

Average hole diffusivity in the emitter:

$$D'_{pE} = \mu_p(1.38\times10^{19})kT/q = (70\times0.026)cm^2s^{-1} = 1.82\,cm^2s^{-1}.$$

$$\gamma = \frac{1}{1+\dfrac{GN_B D'_{pE}}{GN_E D'_{nB}}} = \frac{1}{1+\dfrac{1.23\times10^{12}\times1.82}{1.06\times10^{15}\times29.9}} = 0.99993.$$

(d) Diffusion current:

$$J_{diff} = -q\mu\upsilon = -qD\frac{dp}{dx} \quad\Rightarrow\quad \upsilon = -\frac{D}{n}\frac{dn}{dx} = -D\frac{d\ln n}{dx}.$$

$$p \approx N_B = 4\times10^{18} e^{-x/\lambda}, \quad \lambda = 0.19\mu m \quad \Rightarrow \quad \upsilon = +\frac{D}{\lambda} \qquad (+x \quad direction).$$

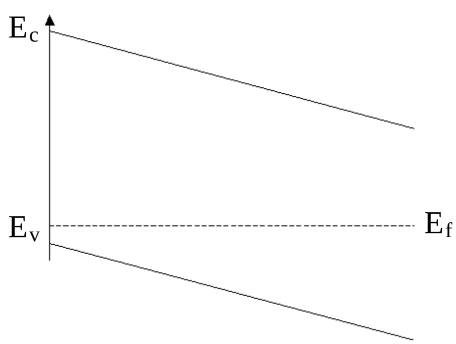
Drift current:

$$J_{drift} = q\mu_p pE = qp\upsilon = J_{diff}$$
$$\mu_p = \left(\frac{q}{kT}\right)D \quad and \quad \upsilon = +D/\lambda = \mu_p E = E\left(\frac{q}{kT}\right)D$$

Therefore,

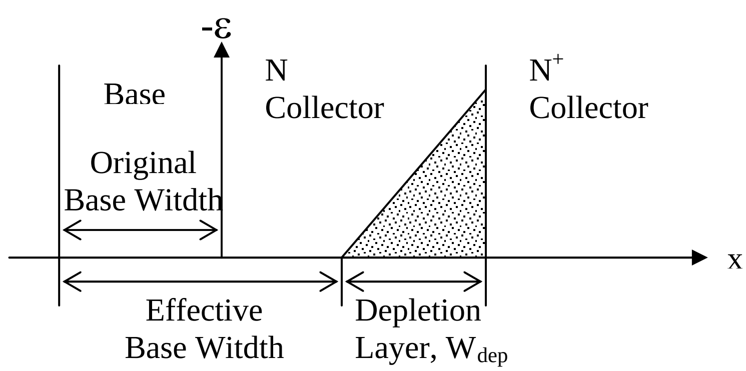
$$E_{bi} = \frac{kt}{q\lambda} = \frac{0.06\,V}{0.19\,\mu m} = 3.16\times10^5\ V/m.$$

Note: E_{bi} should be "-x" direction so that diffusion current and drift current will balance.



Kirk Effect

8.12



Clearly, $W_{B\_Effective} = W_{B\_Original} + (W_C - W_{dep})$. $W_{B\_Original}$ and W_C are assumed to be known. So, in order to find $W_{B\_Effective}$, we need to calculate W_{dep}.

$\rho = qN_C - \frac{I_C}{A_E v_{sat}}$ where v_{sat} is the saturation velocity. The length of the depletion region becomes

$$W_{dep} = \sqrt{\frac{2\varepsilon_s (V_{BC} + \phi_i)}{\frac{I_C}{A_E v_{sat}} - qN_C}}.$$

Therefore,

$$W_{B\_Effective} = W_{B\_Original} + \left(W_C - \sqrt{\frac{2\varepsilon_s (V_{BC} + \phi_i)}{\frac{I_C}{A_E v_{sat}} - qN_C}} \right).$$

Charge Control Model

8.13 The equation describing the system is

$$\frac{dQ_F(t)}{dt} = I_B(t) - \frac{Q_F(t)}{\tau_F \beta_F}.$$

Since $I_B(t) = 0$ for $t<0$ and $I_B(t) = I_{B0}$ for $t \geq 0$, the equation becomes

$$\frac{dQ_F(t)}{dt} = \left(-\frac{1}{\tau_F \beta_F} \right) Q_F(t) + I_{B0}$$

with the initial condition $Q_F(0) = 0$. Solving this equation yields

$$Q_F(t) = \tau_F \beta_F I_{B0} \left(1 - e^{-t/\tau_F \beta_F} \right).$$

Hence,

$$I_C(t) = Q_F(t)/\tau_F = \beta_F I_{B0} \left(1 - e^{-t/\tau_F \beta_F} \right).$$

8.14 $\frac{dQ_B}{dt} = i_B - \frac{Q_B}{\beta \tau_F} \Rightarrow Q_B = A + Be^{-t/\beta \tau_F}$

Boundary Conditions: F

$$Q_B(\infty) = A = \beta \tau_F i_{B2}, \quad Q_B(0^-) = \beta \tau_F i_{B1} = \beta \tau_F i_{B2} + B$$

Hence, $B = \beta \tau_F (i_{B1} - i_{B2})$

Therefore,

$$Q_B = \beta \tau_F i_{B2} + \beta \tau_F \left(i_{B1} - i_{B2}\right) e^{-t/\beta \tau_F}$$

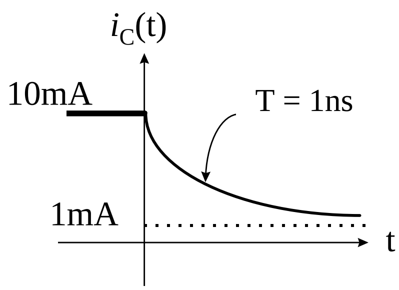
$$i_C = \frac{Q_B}{\tau_F} = \beta i_{B2} + \beta \left(i_{B1} - i_{B2}\right) e^{-t/\beta \tau_F},$$

$$\beta = \frac{\alpha_F}{1-\alpha_F} = 100, \quad \beta \tau_F = 1ns (Chracteristic\ time\ T)$$

And,

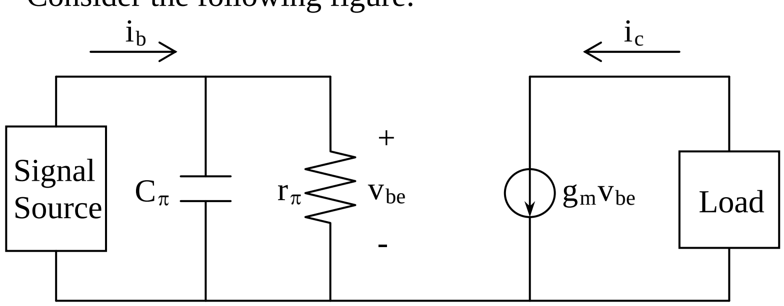
$$i_C = 1\, mA + (9\, mA) e^{-t/1ns} \quad \text{for } t \ge 0, \text{ and}$$

$$i_C = 10\, mA \quad \text{for } t < 0.$$



Cutoff Frequency

8.15 Consider the following figure:



i_b and i_c are given by

$$i_b = v_{be} / input\ impedance = v_{be} \times input\ admittance = v_{be}\left(1/r_\pi + j\omega C_\pi\right)$$

$$i_c = g_m v_{be}.$$

The gain is

$$\beta(\omega) \equiv \left|\frac{i_c}{i_b}\right| = \frac{g_m}{\left|1/r_\pi + j\omega C_\pi\right|} = \frac{1}{\left|1/g_m r_\pi + j\omega\tau_\pi + j\omega C_{dBE}/g_m\right|}$$

$$= \frac{1}{\left|1/\beta_F + j\omega\tau_\pi + j\omega C_{dBE} kT/qI_C\right|}.$$

If $\beta_F >> 1$ so that $1/\beta_F$ becomes negligible, the equation above shows that $\beta_F(\omega) \propto 1/\omega$, and β becomes 1 at

$$f_T = \frac{1}{2\pi\left(\tau_F + C_{dBE} kT / qI_C\right)}.$$