

科目 統計學 二年 甲 班 座號 姓名 甘明天 得分

6.4

$$E(\bar{X}) = \mu$$

$$V(\bar{X}) = \frac{\sigma^2}{n} = E(\bar{X}^2) - \mu^2$$

$$E(\hat{\theta}_1) = E\left(\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}\right) = \frac{1}{n} E\left(\sum_{i=1}^n x_i^2 - n\bar{x}^2\right)$$

$$= \frac{1}{n} (n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2) = \frac{n-1}{n} \sigma^2$$

$$E(\hat{\theta}_2) = E\left(\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}\right) = \frac{1}{n-1} E\left(\sum_{i=1}^n x_i^2 - n\bar{x}^2\right) = \frac{1}{n-1} (n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2) = \sigma^2$$