平面图与图着色

可平面图(Planar Graph)

- 如果图G能够被画在一个平面上且图中的任意两条边都不相交，则图G被称为可平面图。

Regions

- Exterior region
- Boundary of region

The Euler Identity

- Theorem 9.1
  - If G is a connected plane graph of order n, size m and having r regions, then n-m+r =2.

Theorem 9.2

- If G is a planar graph of order n>=3 and size m, then m <= 3n-6.

- Corollary 9.3
  - Every planar graph contains a vertex of degree 5 or less.
- Corollary 9.4
  - K_5 is nonplanar.

Theorem 9.5

- The graph $K_{3,3}$ is nonplanar.

Kuratowski's theorem

- A graph G is planar if and only if G does not contain K_5, $K_{3,3}$, or a subdivision of K_5 or $K_{3,3}$ as a subgraph.

  – A graph G' is called a subdivision of a graph G if one or more vertices of degree 2 are inserted into one or more edges of G.

Graph Coloring

- Dated back to 1852, Francis Guthrie
- → De Morgan→ Hamilton→Sylvester→Kempe→Heawood →Birkhoff→Heesch→Shimamoto→Appel & Haken & IBM 370-168 (June 1976).

Vertex Coloring

- Assignment of colors to the vertices of G, one color to each vertex, such that adjacent vertices are colored differently.
- Chromatic number, χ (G)
- k-colorable; k-coloring; k-chromatic.

The Four Color Theorem

- The chromatic number of every planar graph is at most 4.

Theorem 10.5

- For every graph G of order n,
  - $\chi(G) \geq \omega(G)$ and $\chi(G) \geq n/\beta(G)$.

Theorem 10.7

- For every graph
  - $\chi(G) <= 1 + \Delta(G)$.

Theorem 10.8 (Brooks' Theorem)

- For every connected graph G that is not an odd cycle or a complete graph,
  - $\chi(G) <= \Delta(G)$.