

- 作业讲解
  - TJ第8章练习6、7、8、9、11、13、18、19、21、22、23

# TJ第8章练习18

- 如果不全为0:
  - 其中为0的codeword构成非空集合 $S_0$
  - 其中为1的codeword构成非空集合 $S_1$
- 欲证 $S_0$ 和 $S_1$ 之间存在双射:  
任取 $S_1$ 中元素 $z$ ,  $f(x)=x+z: S_0 \rightarrow S_1$ 
  - 单射:  $f(x_1)=f(x_2) \rightarrow x_1+z=x_2+z \rightarrow x_1=x_2$
  - 满射: 任取 $S_1$ 中元素 $y$ , 构造 $x=y+z$   
 $\rightarrow x$ 在 $S_0$ 中 (因为:  $x$ 是codeword, 且第 $i$ 位为0)  
且 $f(x)=f(y+z)=y+z+z=y$

- 教材讨论
  - TC第32章

# 问题1: naive

- 你能够基于naïve算法高效地解决这个问题吗？

Suppose we allow the pattern  $P$  to contain occurrences of a *gap character*  $\diamond$  that can match an *arbitrary* string of characters (even one of zero length). For example, the pattern  $ab\diamond ba\diamond c$  occurs in the text  $cabccbacbacab$  as

$$\begin{array}{ccccccc} c & ab & cc & ba & cba & c & ab \\ & \underbrace{\hspace{1.5cm}} & & \underbrace{\hspace{1.5cm}} & & & \\ & ab & \diamond & ba & \diamond & c & \end{array}$$

and as

$$\begin{array}{ccccccc} c & ab & ccbac & ba & & c & ab . \\ & \underbrace{\hspace{1.5cm}} & & \underbrace{\hspace{1.5cm}} & & & \\ & ab & \diamond & ba & \diamond & c & \end{array}$$

Note that the gap character may occur an arbitrary number of times in the pattern but not at all in the text. Give a polynomial-time algorithm to determine whether such a pattern  $P$  occurs in a given text  $T$ , and analyze the running time of your algorithm.

- 分段匹配

# 问题1: naïve (续)

- 我们稍稍改一改问题，你还能够高效地解决吗？
  - P occurs in a given text T  $\rightarrow$  P matches T（即必须与整个T匹配）
  - $\diamond \rightarrow ?$ 和\*
- 动态规划
  - if:  $P[i] == T[j] \mid \mid P[i] == '?' \ \&\& T[j] != \text{EMPTY}$ 
    - $\text{ans}[i-1, j-1]$
  - if:  $P[i] == '*'$ 
    - $\text{ans}[i, j-1] \mid \text{ans}[i-1, j] \mid \text{ans}[i-1, j-1]$

# 问题2: Rabin-Karp

- 这是对naïve和Rabin-Karp的另一种叙述方式，你理解了吗？

```
1 function NaiveSearch(string s[1..n], string pattern[1..m])
2   for i from 1 to n-m+1
3     for j from 1 to m
4       if s[i+j-1] ≠ pattern[j]
5         jump to next iteration of outer loop
6   return i
7 return not found
```

```
1 function RabinKarp(string s[1..n], string pattern[1..m])
2   hpattern := hash(pattern[1..m]); hs := hash(s[1..m])
3   for i from 1 to n-m+1
4     if hs = hpattern
5       if s[i..i+m-1] = pattern[1..m]
6         return i
7     hs := hash(s[i+1..i+m])
8 return not found
```

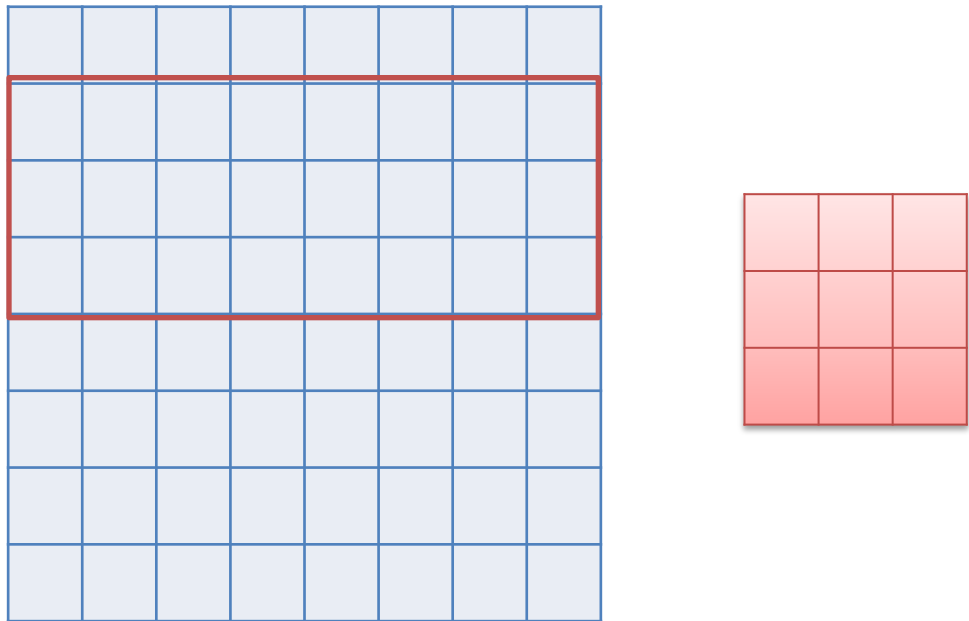
- 按照这种叙述，Rabin-Karp的效率一定比naïve高吗？
- 使Rabin-Karp的效率高于naïve的原因是什么？
  - rollinghash

## 问题2: Rabin-Karp (续)

- How would you extend the Rabin-Karp method to the problem of searching a text string for an occurrence of any one of a given set of  $k$  patterns? Start by assuming that all  $k$  patterns have the same length. Then generalize your solution to allow the patterns to have different lengths.
- hash table

## 问题2: Rabin-Karp (续)

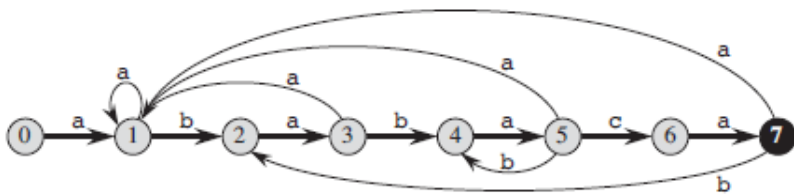
- Show how to extend the Rabin-Karp method to handle the problem of looking for a given  $m \times m$  pattern in an  $n \times n$  array of characters. (The pattern may be shifted vertically and horizontally, but it may not be rotated.)
- $m \times n$  和  $m \times m \rightarrow 1 \times n$  和  $1 \times m \rightarrow$  string matching (2维rolling hash, 先列后行)





# 问题3: automaton

- 自动机的5个组成部分是什么？

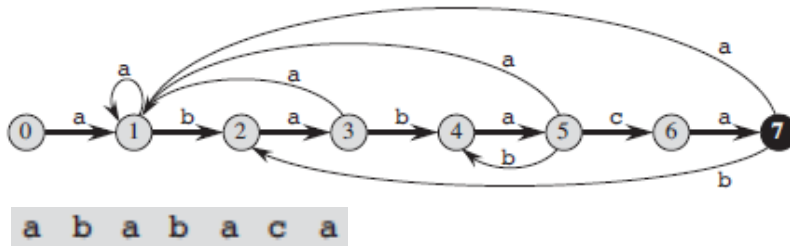


- $Q$  is a finite set of *states*,
- $q_0 \in Q$  is the *start state*,
- $A \subseteq Q$  is a distinguished set of *accepting states*,
- $\Sigma$  is a finite *input alphabet*,
- $\delta$  is a function from  $Q \times \Sigma$  into  $Q$ , called the *transition function* of  $M$ .

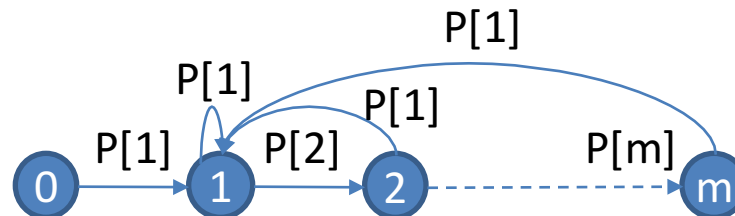
- 粗线和细线分别表示什么意思？

# 问题3: automaton (续)

- 你理解这个transition function了吗?  $\delta(q, a) = \sigma(P_q a)$
- 这个自动机始终维护的invariant是什么含义?  $\phi(T_i) = \sigma(T_i)$



- We call a pattern  $P$  *nonoverlappable* if  $P_k \sqsubset P_q$  implies  $k = 0$  or  $k = q$ . Describe the state-transition diagram of the string-matching automaton for a nonoverlappable pattern.
  - 任选一些nonoverlappable pattern, 给出其string-matching automaton
  - 你能总结出规律吗?



其它情况都回到0