- 书面作业讲解
  - TC第4.1节练习5
  - TC第4.3节练习3、7
  - TC第4.4节练习2、8
  - TC第4.5节练习4
  - TC第4章问题1、3、4

TC第4.1节练习5

```
int sequence(std::vector<int>& numbers)
{
	// Initialize variables here
	int max_so_far  = numbers[0], max_ending_here = numbers[0];
	size_t begin = 0;
	size_t begin_temp = 0;
	size_t end = 0;
	// Find sequence by looping through
	for(size_t i = 1; i < numbers.size(); i++)
	{
		// calculate max_ending_here
		if(max_ending_here < 0)
		{
			max_ending_here = numbers[i];
			begin_temp = i;
		}
		else
		{
			max_ending_here += numbers[i];
		}
		// calculate max_so_far
		if(max_ending_here > max_so_far )
		{
			max_so_far  = max_ending_here;
			begin = begin_temp;
			end = i;
		}
	}
	return max_so_far ;
}
```

动态规划

- max_so_far: A[1..j]上的最大值
- max_ending_here: 最大的A[i…j+1]
- 两者中较大者为A[1..j+1]上的最大值

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
13	–3	–25	20	–3	–16	–23	18	20	–7	12	–5	–22	15	–4	7

抛弃和为负数的前缀

TC第4.3节练习3

- 大胆缩放、正确缩放

$$T(n) \geq 2\left(c\left\lfloor\frac{n}{2}\right\rfloor \lg\left\lfloor\frac{n}{2}\right\rfloor\right) + n \geq 2c\frac{n-1}{2}\lg\frac{n}{4} + n$$

$$= cn\lg n - c\lg n - 2cn + 2c + n = cn\lg n - \frac{1}{4}\lg n + \frac{1}{2}n + \frac{1}{2} \geq cn\lg n$$

(c=1/4)

- 不要忘记boundary condition

TC第4.3节练习7

如果欲证$T(n) \le cn^{\log_3 4}$

$T(n) \le \ldots \le cn^{\log_3 4} + n \ge cn^{\log_3 4}$, fails

改为欲证$T(n) \le cn^{\log_3 4} - \frac{1}{4}n$

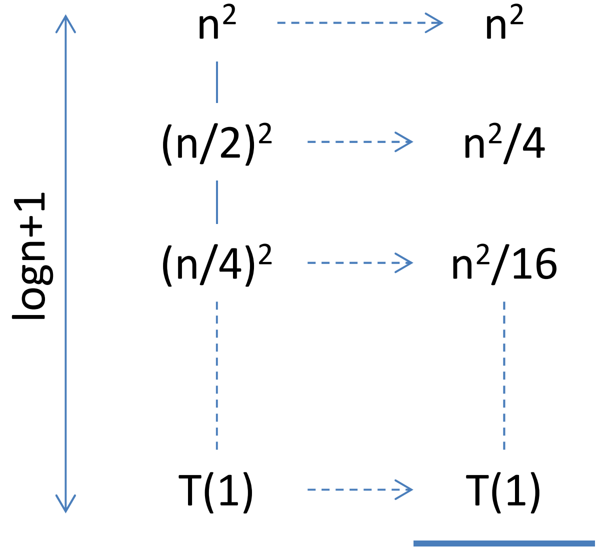
$T(n) \le \ldots \le cn^{\log_3 4} + n - n = cn^{\log_3 4}$, 这样对吗？

改为欲证$T(n) \le cn^{\log_3 4} + dn$

$T(n) \le \ldots \le cn^{\log_3 4} + \left(\frac{4}{3}d + 1\right)n \le cn^{\log_3 4} + dn$, 只要$\frac{4}{3}d + 1 \le d$

TC第4.4节练习2

- recursion tree的组成元素缺一不可



$T(n)=n^2+n^2/4+\ldots+T(1)=O(n^2)$

- 如何证明$T(n)\in O(f(n))$？
  - 对于$n>n_0$，$T(n)\leq c_1f(n)$
- 如何证明$T(n)\in \Theta(f(n))$？
  - 对于$n>n_0$，$c_1f(n)\leq T(n)\leq c_2f(n)$
- 什么是substitution method？
  - 数学归纳法

TC第4.5节练习4

- 因为$n^2 \le n^2 \lg n \le n^3$，所以$n^2 \lg n = n^{2+\varepsilon}$。这个逻辑对吗？

- $\lg n \in o(n^{\text{任意正数}})$

TC第4章问题3b

- 因为$n/\lg n<n$，所以$n/\lg n=n^{1-\varepsilon}$。这个逻辑对吗？
- $n/\lg n=n^{1-\varepsilon}$ → $1/\lg n=n^{-\varepsilon}$ → $\lg n=n^{\varepsilon}$ → $\lg n\in\Theta(n^{\varepsilon})$ → 与$\lg n\in o(n^{\text{任意正数}})$矛盾

TC第4章问题3j

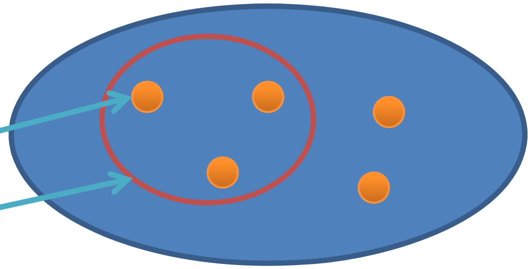
- $\Theta(n\lg\lg n)$
- 数学归纳法很容易证明

- 怎么猜出这个结果?

- 教材答疑和讨论
  - CS第5章第1、2、3、4节

问题1：probability

- 你是怎么理解这些概念的？
  - Sample space
  - Element
  - Event



  - Probability weight
  - Probability
- Probability distribution function的三个条件

1. $P(A) \geq 0$ for any $A \subseteq S$.
2. $P(S) = 1$.
3. $P(A \cup B) = P(A) + P(B)$ for any two disjoint events A and B.

问题1：probability (续)

- 在以下这些例子中，sample space、element和event分别是什么？
  - The probability of getting at least 1 head in 5 flips of a coin.
  - The probability of getting a total of 6 or 7 on the 2 dice.
  - The probability that all 3 keys hash to different locations (among 20).

- 在这些例子中，probability distribution function分别是怎样定义的？
- 在没有其它条件的情况下，你能给出它们的答案吗？

问题1：probability (续)

- Uniform probability distribution

Theorem 5.2 *Suppose P is the uniform probability measure defined on a sample space S. Then for any event E,*

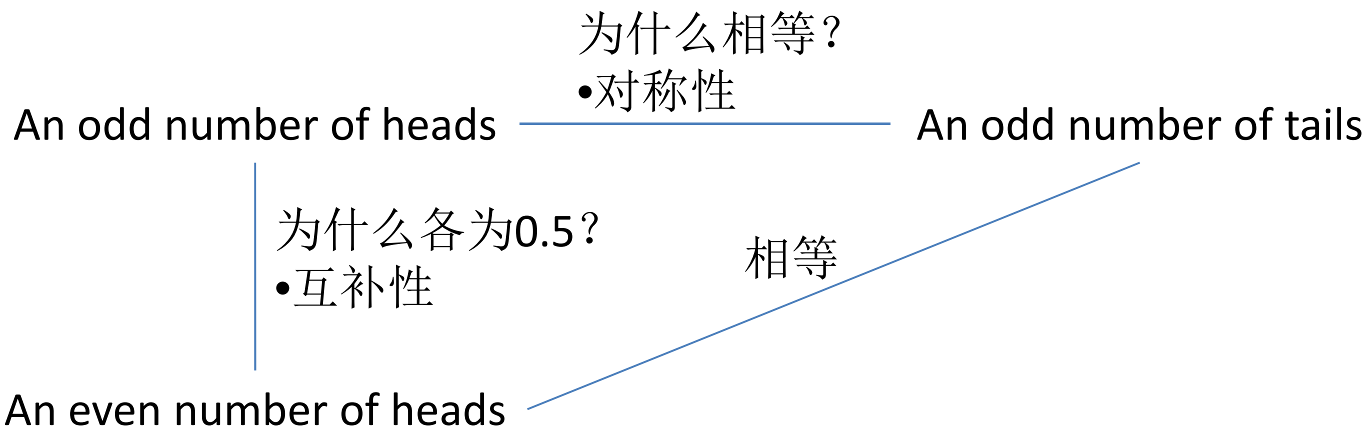
$$P(E) = |E|/|S|,$$

the size of E divided by the size of S.

- 现在你能给出之前几题的答案了吗？
- Uniform probability distribution的意义
  - Probability → Counting

问题1：probability (续)

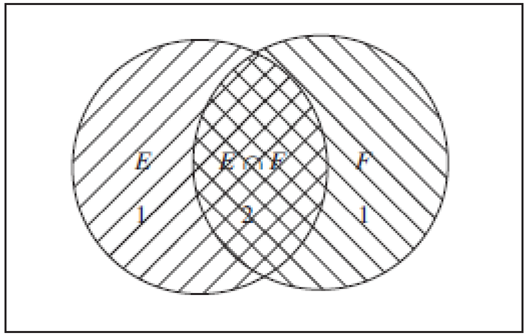
- What is the probability of an odd number of heads in three tosses of a coin?（假设是uniform probability distribution）



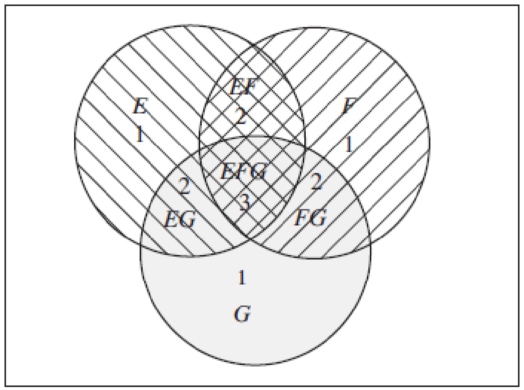
- 如果不是uniform probability distribution呢？

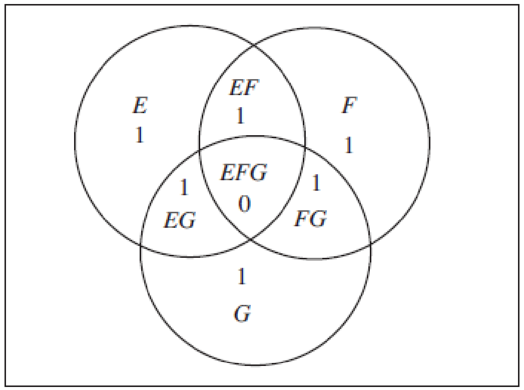
问题2：the principle of inclusion and exclusion

- 你能基于以下这些图来解释公式吗？
  - $P(E \cup F) = P(E) + P(F) - P(E \cap F)$



  - $P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(E \cap F) - P(E \cap G) - P(F \cap G) + P(E \cap F \cap G)$





问题2： the principle of inclusion and exclusion (续)

- 以下两个公式的直观含义是？
  - $P(\bigcup_{i=1}^{n} E_i) = \sum_{i=1}^{n} P(E_i) - \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} P(E_i \cap E_j) + \sum_{i=1}^{n-2} \sum_{j=i+1}^{n-1} \sum_{k=j+1}^{n} P(E_i \cap E_j \cap E_k) - \ldots.$
  - $P\left(\bigcup_{i=1}^{n} E_i\right) = \sum_{k=1}^{n} (-1)^{k+1} \sum_{\substack{i_1, i_2, \ldots, i_k: \\ 1 \le i_1 < i_2 < \cdots < i_k \le n}} P\left(E_{i_1} \cap E_{i_2} \cap \cdots \cap E_{i_k}\right)$
- 它们为什么相等？

问题2： the principle of inclusion and exclusion (续)

- What is the probability that at least 1 person (among 5) gets his or her own backpack?
  - Sample space?
  - Element?
  - Event?

$$P(E_1 \cup E_2 \cup E_3 \cup E_4 \cup E_5) = \sum_{k=1}^{5} (-1)^{k+1} \sum_{\substack{i_1, i_2, \ldots, i_k: \\ 1 \le i_1 < i_2 < \cdots < i_k \le 5}} P(E_{i_1} \cap E_{i_2} \cap \cdots \cap E_{i_k})$$

问题2：the principle of inclusion and exclusion (续)

- How many functions from an *m*-element set *M* to an *n*-element set *N* map nothing to at least one element of *N*?
  - Sample space?
  - Element?
  - Event?

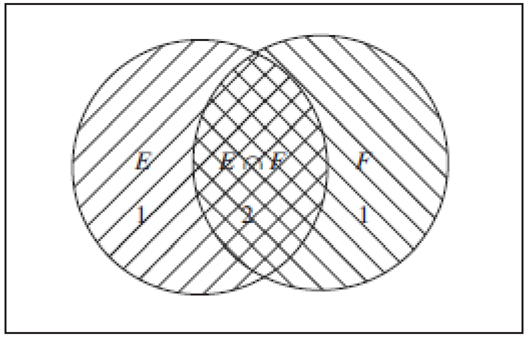
$$\left|\bigcup_{i=1}^{n} E_i\right| = \sum_{k=1}^{n}(-1)^{k+1} \underline{\sum_{\substack{i_1,i_2,\ldots,i_k:\\ 1\le i_1<i_2<\cdots<i_k\le n}} |E_{i_1}\cap E_{i_2}\cap\cdots\cap E_{i_k}|}$$

$$\binom{m}{k}(m-k)^n$$

问题3：conditional probability

- 你能结合Venn图解释条件概率的定义吗？

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$



- 独立性呢?

$$P(E|F) = P(E)$$

问题3：conditional probability (续)

- 你能自己推导出这些定理吗？

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}.$$

Theorem 5.4 *Suppose E and F are events in a sample space. Then E is independent of F if and only if $P(E \cap F) = P(E)P(F)$.*

问题3：conditional probability (续)

$P(x_i = a_i | x_1 = a_1, \ldots, x_{i-1} = a_{i-1}) = P(x_i = a_i)$

- 你理解independent trials process了吗？

Exercise 5.3-7 Suppose we draw a card from a standard deck of 52 cards, discard it (i.e. we do not replace it), draw another card and continue for a total of ten draws. Is this an independent trials process?

- 为什么这不是一个independent trials process？

- 为这个过程绘制tree diagram，并计算：第i张抽到梅花A的概率是多少？

- 如果是independent trails process，其tree diagram有什么特征？

问题4：random variables

- 你理解这些概念了吗？能自己举个例子吗？
  - Random variable
  - Expected value
  - $E(X+Y) = E(X) + E(Y)$

- 你能直观解释它们为什么相等吗？

$$E(X) = \sum_{i=1}^{k} x_i P(X = x_i)$$

$$E(X) = \sum_{s:s \in S} X(s) P(s)$$

问题4： random variables (续)

- What is the expected number of times we need to roll two dice until we get a 7?
  - 提示：根据期望的定义

问题4：random variables (续)

```
(1)  min=A[1]
(2)  for i=2 to n
(3)      if (A[i]<min)
(4)          min=A[i]
(5)  return min
```

- What is the expected number of times that min is assigned a value?
  - 提示：根据additivity of expectation