- 教材讨论
  - JH第3章第4节

问题1：Lowering Worst Case Complexity of Exponential Algorithms

- 3SAT
  - 用divide-and-conquer解决这个问题的方法是什么？

$$F = (x_1 \vee \overline{x}_2 \vee x_4) \wedge (\overline{x}_2) \wedge (\overline{x}_2 \vee \overline{x}_3 \vee x_5) \wedge (x_1 \vee \overline{x}_5) \wedge (\overline{x}_1 \vee x_2 \vee x_3)$$

$$F(\overline{x}_2 = 1) = (x_1 \vee \overline{x}_5) \wedge (\overline{x}_1 \vee x_3)$$

F is satisfiable $\Longleftrightarrow$ at least one of the formulae $F(l_1 = 1)$, $F(l_1 = 0, l_2 = 1), F(l_1 = 0, l_2 = 0, l_3 = 1)$ is satisfiable.

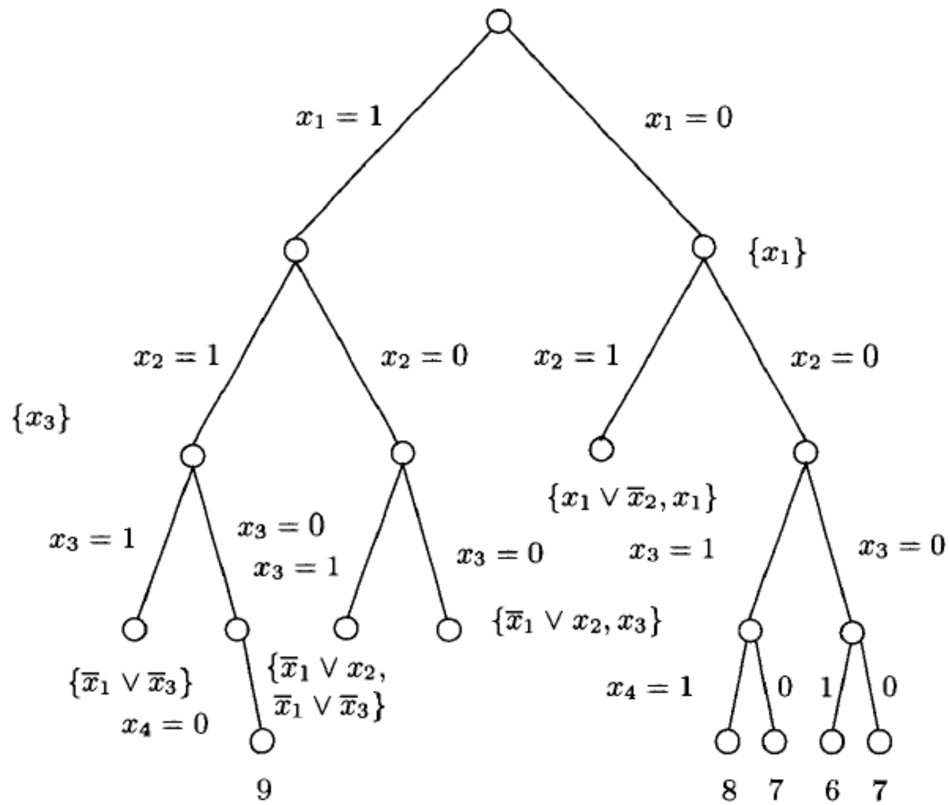
$$F(l_1 = 1) \in 3\mathrm{CNF}(n-1, r-1),$$
$$F(l_1 = 0, l_2 = 1) \in 3\mathrm{CNF}(n-2, r-1),$$
$$F(l_1 = 0, l_2 = 0, l_3 = 1) \in 3\mathrm{CNF}(n-3, r-1).$$

  - 这样做能够显著降低复杂度吗？

Complexity	$n = 10$	$n = 50$	$n = 100$	$n = 300$
2^n	1024	(16 digits)	(31 digits)	(91 digits)
$2^{n/2}$	32	$\sim 33 \cdot 10^6$	(16 digits)	(46 digits)
$(1.2)^n$	7	9100	$\sim 29 \cdot 10^6$	(24 digits)
$10 \cdot 2^{\sqrt{n}}$	89	1350	10240	$\sim 1.64 \cdot 10^6$
$n^2 \cdot 2^{\sqrt{n}}$	894	~ 336000	$\sim 10.24 \cdot 10^6$	$\sim 14.8 \cdot 10^9$

问题2：branch-and-bound

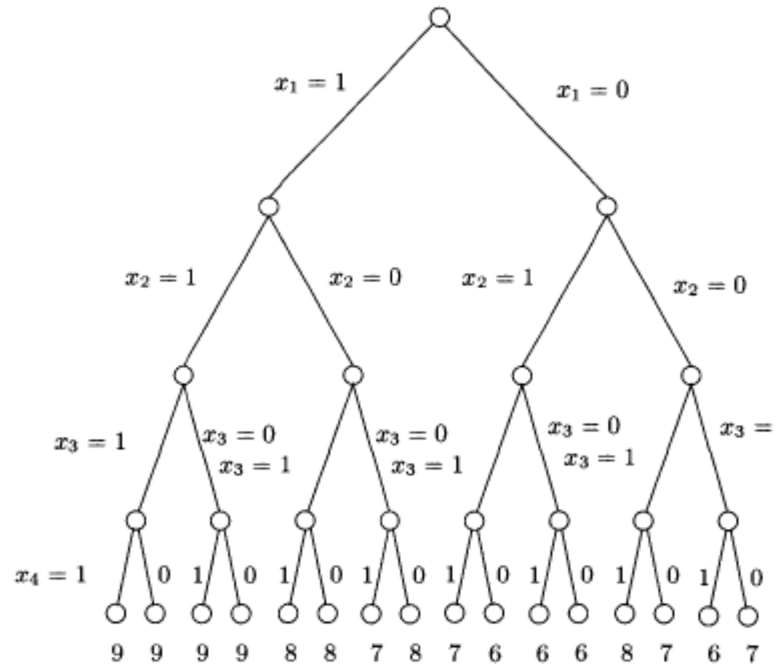
- branch-and-bound能够提高效率的基本原理是什么？
- branch-and-bound算法的效率与哪些因素有关？
  - 树的构造方法
  - 树的搜索策略
  - 子树中解的范围估计



问题2：branch-and-bound (续)

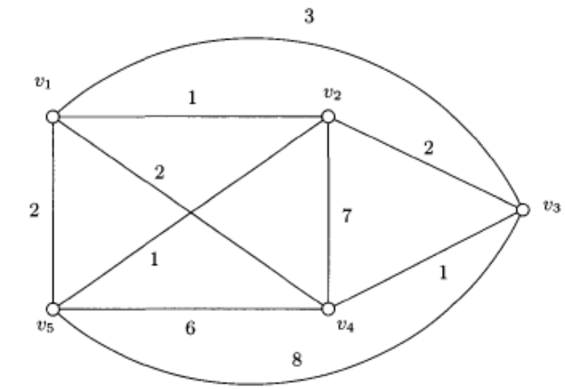
- MAX-SAT
  - 树的构造方法
  - 树的搜索策略
  - 子树中解的范围估计

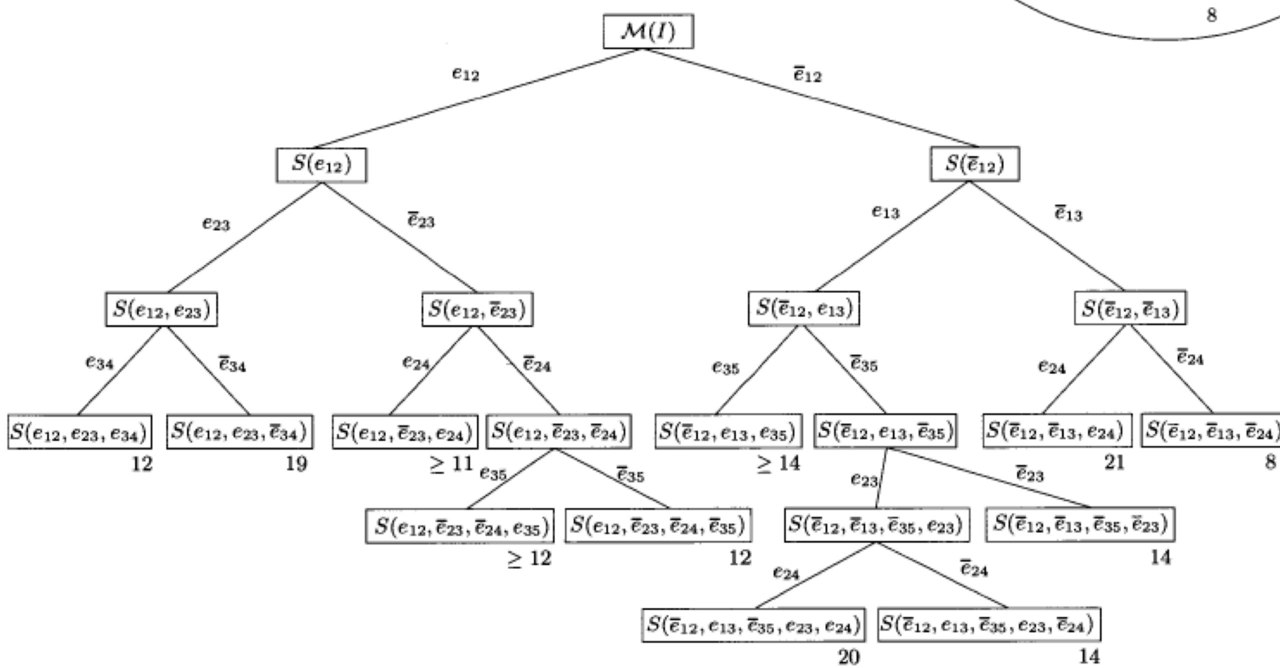
- 你能给出和书上不同的算法吗？
  - 例如：改变上述要素之一



问题2：branch-and-bound (续)

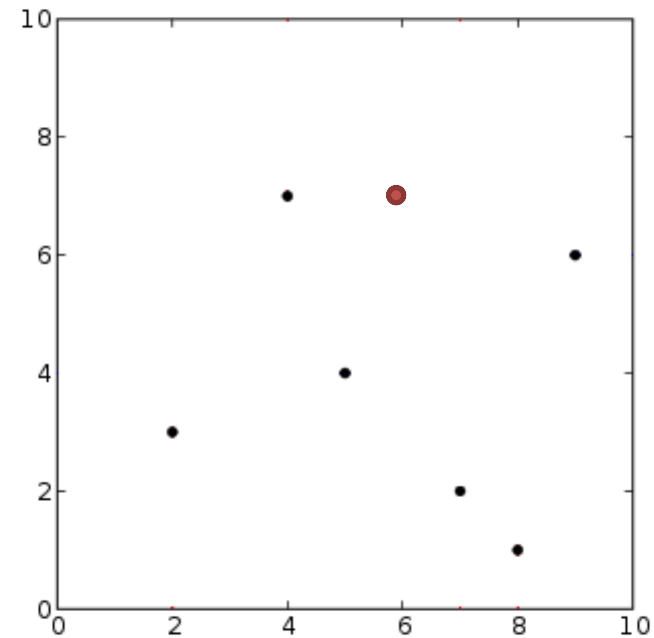
- TSP
  - 树的构造方法
  - 树的搜索策略
  - 子树中解的范围估计





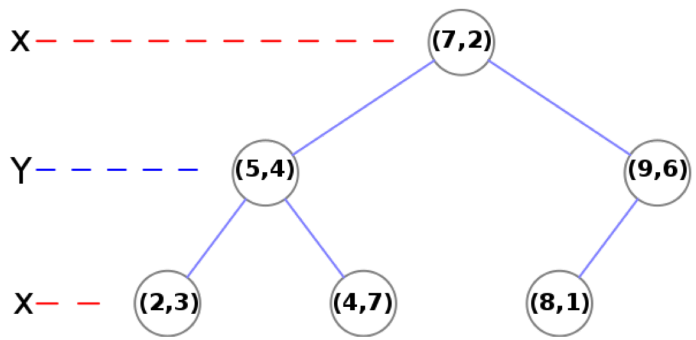
问题2： branch-and-bound (续)

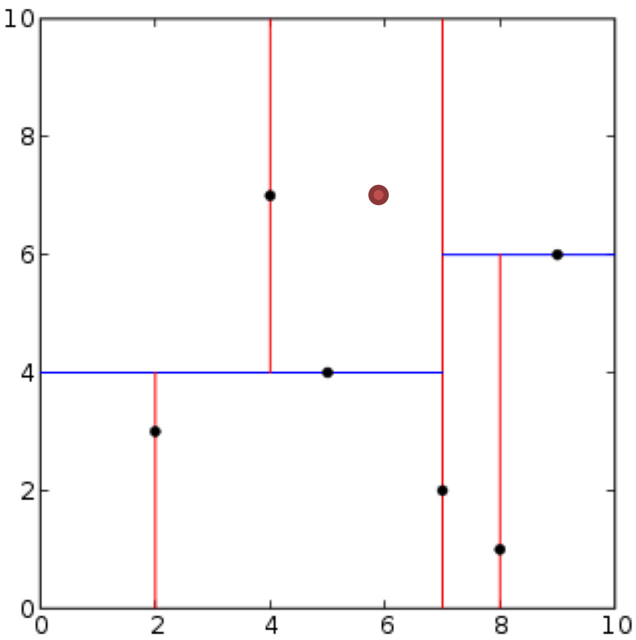
- NNS (nearest neighbor search)
  - 树的构造方法
  - 树的搜索策略
  - 子树中解的范围估计



问题2： branch-and-bound (续)

- k-d tree





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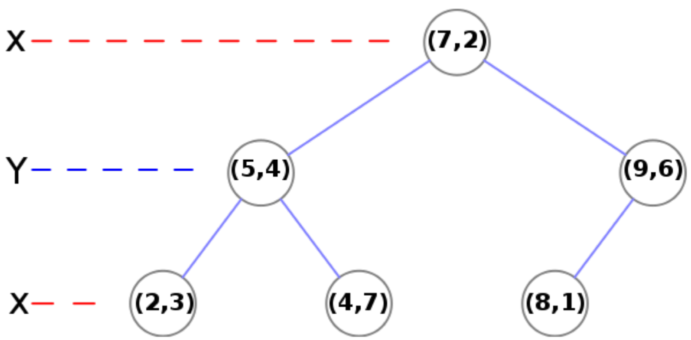

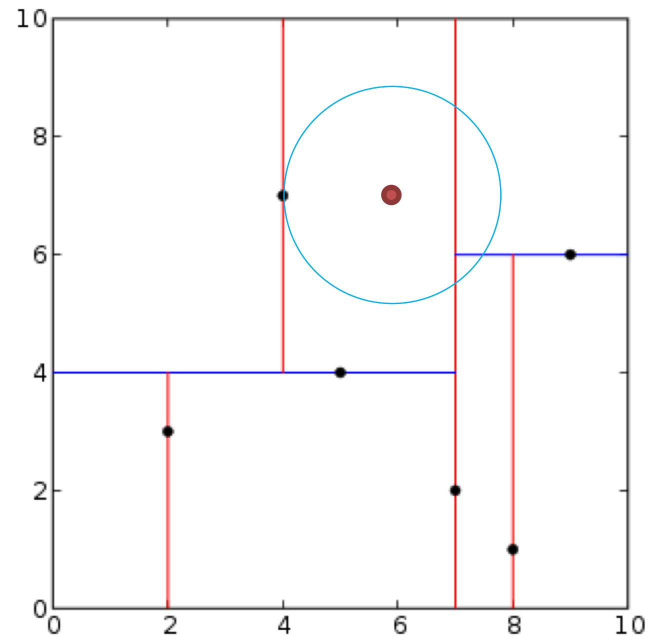

问题2： branch-and-bound (续)

- ILP
  - 树的构造方法
  - 树的搜索策略
  - 子树中解的范围估计

$$
\begin{aligned}
&\max z = 5x_1 + 8x_2, \\
&\text{subject to:} \quad x_1 + x_2 \le 6, \\
&\qquad\qquad\quad 5x_1 + 9x_2 \le 45, \\
&\qquad\qquad\quad x_1, x_2 \ge 0 \quad \text{and} \quad \text{integer.}
\end{aligned}
$$

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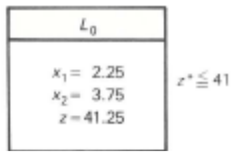

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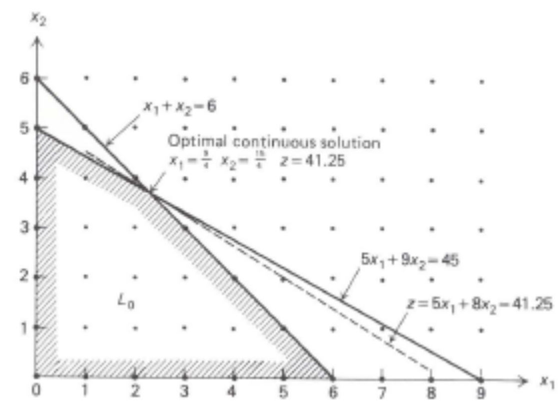

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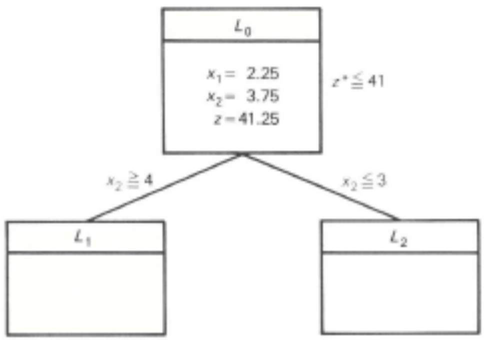

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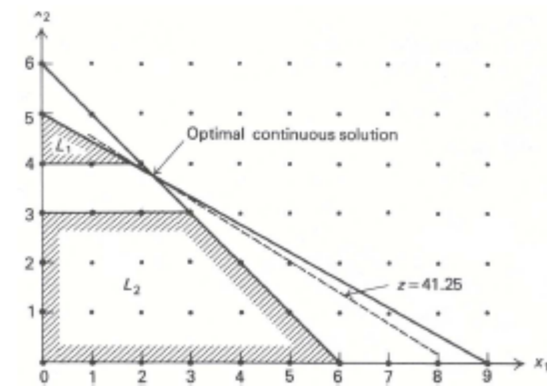

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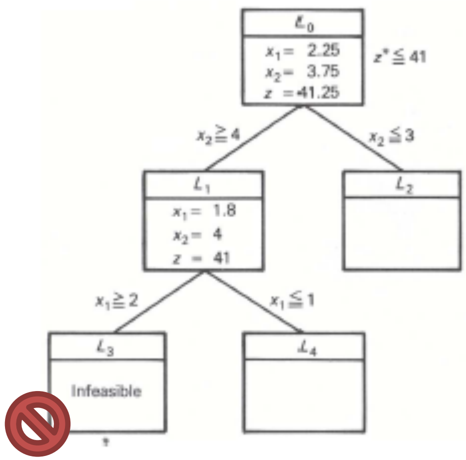

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$$\text{subject to:} \quad x_1 + x_2 \leq 6,$$

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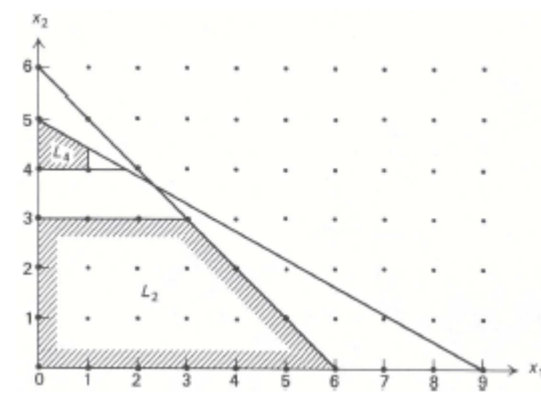

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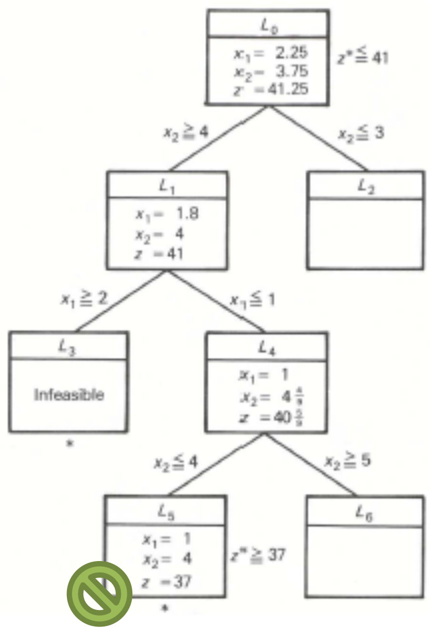

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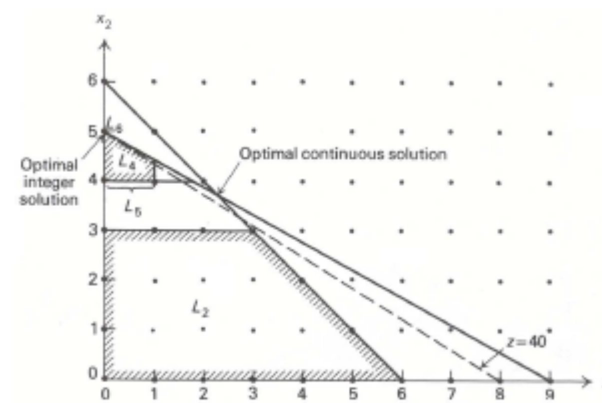

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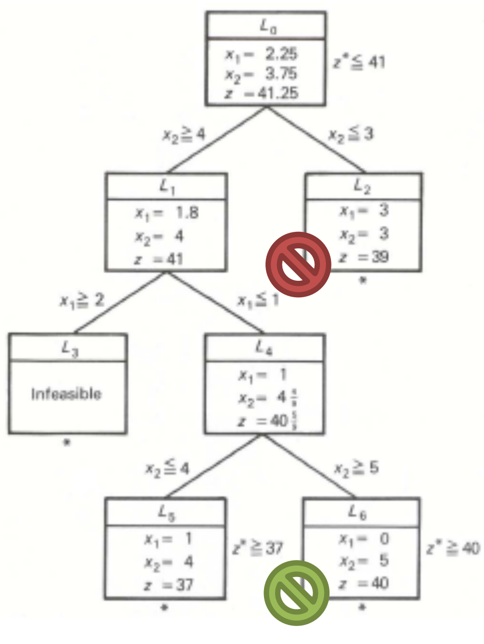

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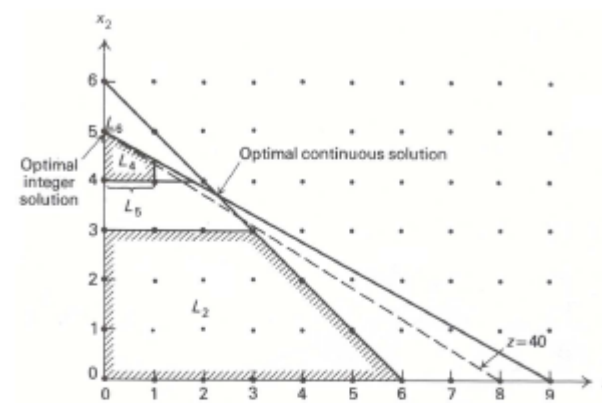
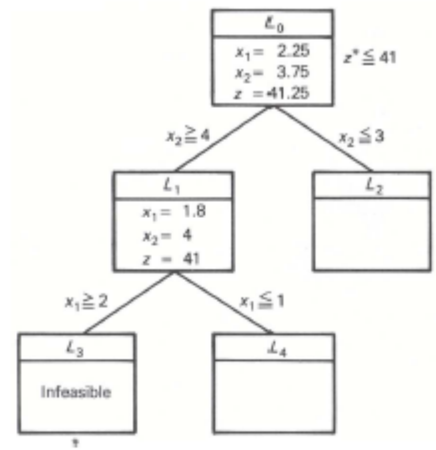


问题2：branch-and-bound (续)

- 如何选择下一个被计算的顶点？
- 如何选择用来细分的变量？



问题2： branch-and-bound (续)

- QAP (quadratic assignment problem)
  - There are a set of *n* facilities and a set of *n* locations. For each pair of locations, a distance is specified and for each pair of facilities a weight or flow is specified (e.g., the amount of supplies transported between the two facilities). The problem is to assign all facilities to different locations with the goal of minimizing the sum of the distances multiplied by the corresponding flows.

Given two sets, P ("facilities") and L ("locations"), of equal size, together with a weight function $w : P \times P \rightarrow \mathbf{R}$ and a distance function $d : L \times L \rightarrow \mathbf{R}$. Find the bijection $f : P \rightarrow L$ ("assignment") such that the cost function:

$$\sum_{a,b \in P} w(a,b) \cdot d(f(a), f(b))$$

is minimized.