习题2-8

CS 5.6-4，5.7-12、18

TC 5.2-2

TC Problem 5.2

5.6-4 In a card game, you remove the jacks, queens, kings, and aces from an ordinary deck of cards and shuffle them. You draw a card. If it is an ace, you are paid $1.00, and the game is repeated. If it is a jack, you are paid $2.00, and the game ends. If it is a queen, you are paid $3.00, and the game ends. If it is a king, you are paid $4.00, and the game ends. What is the maximum amount of money a rational person would pay to play this game?

设 随机变量 X 为最终所得的钱

如果放回

$$E(X) = \frac{1}{4}\left(1 + E(X)\right) + \frac{1}{4} * 2 + \frac{1}{4} * 3 + \frac{1}{4} * 4$$

$$E(X) = \frac{10}{3}$$

设 随机变量 x 为最终所得的钱

$$P(X=2)=\frac{1}{4}$$

$$P(X=3)=\frac{1}{4}\cdot\frac{4}{15}+\frac{1}{4}=\frac{19}{60}$$

$$P(X=4)=\frac{1}{4}\cdot\frac{3}{15}\cdot\frac{2}{7}+\frac{1}{4}\cdot\frac{4}{15}+\frac{1}{4}=\frac{6+28+105}{420}=\frac{139}{420}$$

$$P(X=5)=\frac{1}{4}\cdot\frac{3}{15}\cdot\frac{1}{7}\cdot\frac{4}{13}+\frac{1}{4}\cdot\frac{4}{15}\cdot\frac{2}{7}+\frac{1}{4}\cdot\frac{4}{15}=\frac{12+104+364}{5460}=\frac{480}{5460}$$

$$P(X=6)=\frac{1}{4}\cdot\frac{3}{15}\cdot\frac{1}{7}\cdot\frac{1}{13}\cdot\frac{1}{3}+\frac{1}{4}\cdot\frac{3}{15}\cdot\frac{1}{7}\cdot\frac{4}{13}+\frac{1}{4}\cdot\frac{4}{15}\cdot\frac{2}{7}=\frac{1+12+104}{5460}=\frac{117}{5460}$$

$$P(X=7)=\frac{1}{4}\cdot\frac{3}{15}\cdot\frac{1}{7}\cdot\frac{1}{13}\cdot\frac{1}{3}+\frac{1}{4}\cdot\frac{3}{15}\cdot\frac{1}{7}\cdot\frac{4}{13}=\frac{1+12}{5460}=\frac{13}{5460}$$

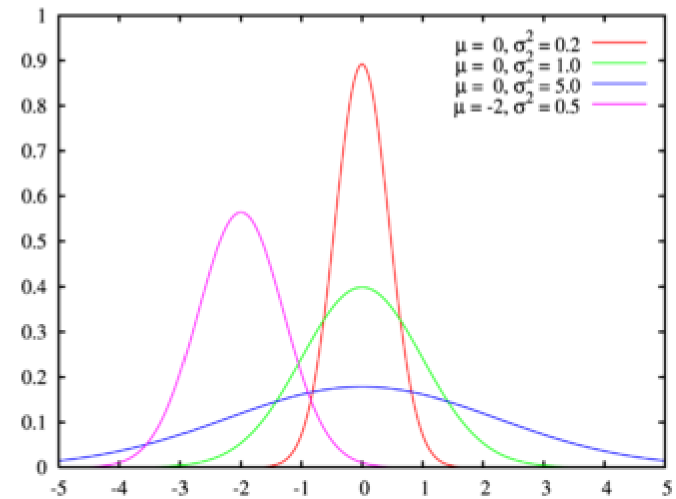
$$P(X=8)=\frac{1}{5460}$$

$$E(X)\approx 3.36$$

5.7-12. How many questions need to be on a short-answer test for you to be 95% sure that someone who knows 80% of the course material gets a grade between 75% and 85%?

正态分布

$$f(x;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$



5.7-12. How many questions need to be on a short-answer test for you to be 95% sure that someone who knows 80% of the course material gets a grade between 75% and 85%?

$$E(X) = 0.8n$$

$$\begin{aligned} Var(X) &= n \cdot p \cdot (1-p) \\ &= n \cdot 0.8 \cdot (1-0.8) \\ &= 0.16n \end{aligned}$$

$$\sigma(X) = \sqrt{Var(X)} = 0.4\sqrt{n}$$

$$2\sigma(X) \le 5\%\, n$$

$$0.8\sqrt{n} \le 5\%\, n$$

$$256 \le n$$

正态分布

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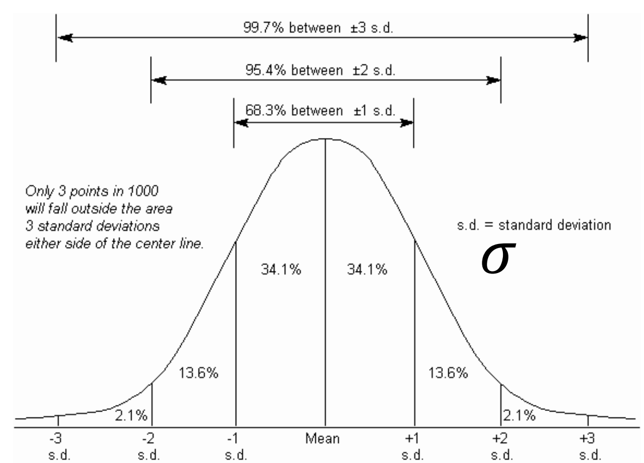

5.7- 18. This problem derives an intuitive law of probability known as the law of large numbers from Chebyshev's law. Informally, the **law of large numbers** says that if you repeat an experiment many times, the fraction of the time that an event occurs is very likely to be close to the probability of the event. The law applies to independent trials with probability p of success. It states that for any positive number s, no matter how small, you can make the probability of the number X of successes being between $np - ns$ and $np + ns$ as close to 1 as you choose by making the number n of trials large enough. For example, you can make the probability of the number of successes being within 1% (or 0.1%) of the expected number as close to 1 as you wish.

a. Show that the probability of $|X(x) - np| \geq sn$ is no more than $p(1-p)/s^2 n$.

b. Explain why part "a" means you can make the probability of $X(x)$ being between $np - sn$ and $np + sn$ as close to 1 as you want by making n large.

$$\because \frac{V(X)}{r^2} \geq P(|X(x) - E(X)| \geq r)$$

$$\therefore \frac{np(1-p)}{s^2 n^2} \geq P(|X(x) - np| \geq sn)$$

$$\therefore \frac{p(1-p)}{s^2 n} \geq P(|X(x) - np| \geq sn)$$

5.2-2

In HIRE-ASSISTANT, assuming that the candidates are presented in a random order, what is the probability that you hire exactly twice?

If I hire exactly twice that means I hire only the first one and the best one

We suppose the candidate is a set {1,2,3…n} and numbers represent their ability (the larger number, the higher ability)

Then we have two sets S1,S2: S1∩S2=Ø and S1∪S2={1,2,…,n-1}

The first element of S1 is the largest element in S1 and S1≠Ø

So S1nS2 is a legal permutation

Num of S1nS2= $\sum_{k=1}^{n-1}\binom{n-1}{k}(\mathrm{k}-1)!(\mathrm{n}-\mathrm{k}-1)!$

So P= $\frac{1}{n}\sum_{k=1}^{n-1}\frac{1}{k}=\frac{\ln(\mathrm{n}-1)}{n}$

5-2 *Searching an unsorted array*

Which of the three searching algorithms would you use? Explain your answer.

This problem examines three algorith[ms] ... array A consisting of n elements.

Consider the following randomized strategy: pick a random index i into A. If $A[i] = x$, then we terminate; otherwise, we continue the search by picking a new random index into A. We continue picking random indices into A until we find an index j such that $A[j] = x$ or until we have checked every element of A. Note that we pick from the whole set of indices each time, so that we may examine a given element more than once.

Now consider a deterministic linear search algorithm, which we refer to as DETERMINISTIC-SEARCH. Specifically, the algorithm searches A for x in order, considering $A[1], A[2], A[3], \ldots, A[n]$ until either it finds $A[i] = x$ or it reaches the end of the array. Assume that all possible permutations of the input array are equally likely.

Finally, consider a randomized algorithm SCRAMBLE-SEARCH that works by first randomly permuting the input array and then running the deterministic linear search given above on the resulting permuted array.