- 教材讨论
 - -TC第25章

问题1: 简单的动态规划法

- 循环不变量是什么?
- 最优子结构是什么?
- 如何记录最短路?

EXTEND-SHORTEST-PATHS (L, W)

```
 \begin{array}{ll} 1 & n = L.rows \\ 2 & \mathrm{let}\ L' = \left(l'_{ij}\right) \ \mathrm{be}\ \mathrm{a}\ \mathrm{new}\ n \times n \ \mathrm{matrix} \\ 3 & \mathrm{for}\ i = 1\ \mathrm{to}\ n \\ 4 & \mathrm{for}\ j = 1\ \mathrm{to}\ n \\ 5 & l'_{ij} = \infty \\ 6 & \mathrm{for}\ k = 1\ \mathrm{to}\ n \\ 7 & l'_{ij} = \min(l'_{ij}, l_{ik} + w_{kj}) \\ 8 & \mathrm{return}\ L' \end{array}
```

问题2: Floyd-Warshall算法

- 循环不变量是什么?
- 最优子结构是什么?
- 如何记录最短路?

```
FLOYD-WARSHALL (W)

1  n = W.rows

2  D^{(0)} = W

3  for k = 1 to n

4  let D^{(k)} = (d_{ij}^{(k)}) be a new n \times n matrix

5  for i = 1 to n

6  for j = 1 to n

7  d_{ij}^{(k)} = \min(d_{ij}^{(k-1)}, d_{ik}^{(k-1)} + d_{kj}^{(k-1)})

8  return D^{(n)}
```

问题3: Johnson算法

• 这个算法的基本思路是什么?

```
JOHNSON(G, w)
 1 compute G', where G' \cdot V = G \cdot V \cup \{s\},
          G'.E = G.E \cup \{(s, v) : v \in G.V\}, \text{ and }
          w(s, v) = 0 for all v \in G.V
 2 if Bellman-Ford (G', w, s) == FALSE
          print "the input graph contains a negative-weight cycle"
     else for each vertex v \in G'. V
 5
               set h(v) to the value of \delta(s, v)
                    computed by the Bellman-Ford algorithm
          for each edge (u, v) \in G'.E
 6
               \widehat{w}(u,v) = w(u,v) + h(u) - h(v)
 8
          let D = (d_{uv}) be a new n \times n matrix
          for each vertex u \in G.V
 9
               run DIJKSTRA (G, \hat{w}, u) to compute \hat{\delta}(u, v) for all v \in G.V
10
               for each vertex v \in G, V
11
                    d_{uv} = \hat{\delta}(u, v) + h(v) - h(u)
12
13
          return D
```

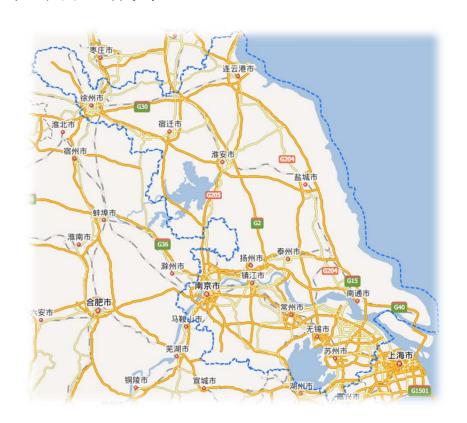
问题4: 炼钢厂选址

• 四川计划投资新建一个炼钢厂,集中冶炼从省内各城市开采的铁矿石。从降低生产成本的角度考虑,你认为炼钢厂应选址哪座城市?



问题5: 救援机库选址

• 江苏计划采购一架救援直升机,承担省内各城市突发灾害的救援任务。从缩短救援时间的角度考虑,你认为直升机日常应停放在哪座城市?



问题6: Schulze投票法

- 谁该被选为总统?
 - 1. 每个选民对所有候选人进行排序
 - 2. 将每对候选人之间的相对支持数表示成矩阵/图
 - 3. 图中每条路径的强度定义为最小边权
 - 4. p[A,B]: 从A到B的最大强度
 - 5. X当选: p[X,Y]≥p[Y,X] for every other Y
- 如何编程实现?关键步骤是什么?

Rank any number of options in your order of preference.

- Joe Smith
- 1 John Citizen
- 3 Jane Doe
- Fred Rubble
- 2 Mary Hill

Matrix of pairwise preferences

| | d[*, A] | d[*,B] | d[*,C] | d[*,D] | d[*,E] |
|---------|---------|--------|--------|--------|--------|
| d[A, *] | | 20 | 26 | 30 | 22 |
| d[B,*] | 25 | | 16 | 33 | 18 |
| d[C,*] | 19 | 29 | | 17 | 24 |
| d[D,*] | 15 | 12 | 28 | | 14 |
| d[E,*] | 23 | 27 | 21 | 31 | |

问题6: Schulze投票法(续)

```
FLOYD-WARSHALL (W)

1  n = W.rows

2  D^{(0)} = W

3  for k = 1 to n

4  let D^{(k)} = (d_{ij}^{(k)}) be a new n \times n matrix

5  for i = 1 to n

6  for j = 1 to n

7  d_{ij}^{(k)} = \min(d_{ij}^{(k-1)}, d_{ik}^{(k-1)} + d_{kj}^{(k-1)})

8  return D^{(n)}
```

$$d_{ij}^{(k)} = max\left(d_{ij}^{(k-1)}, min\left(d_{ik}^{(k-1)}, d_{kj}^{(k-1)}\right)\right)$$