- 书面作业讲解
  - TC第24.1节练习2、3、4
  - TC第24.2节练习2
  - TC第24.3节练习2、4、7
  - TC第24.5节练习2、5
  - TC第24章问题2、3

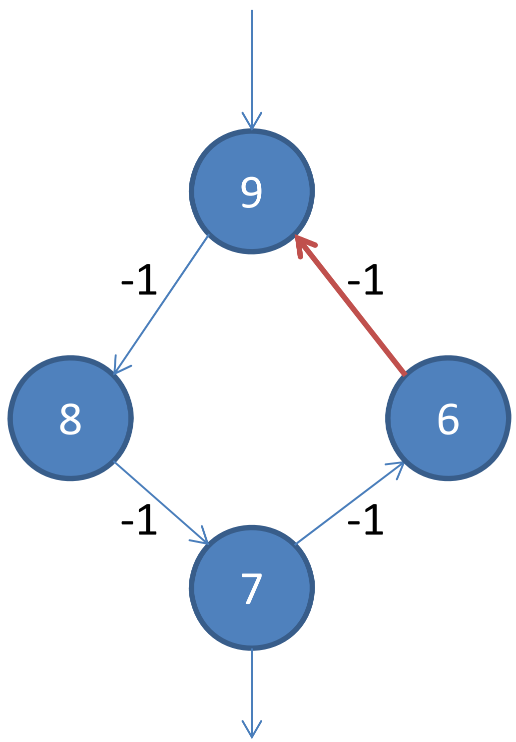
TC第24.1节练习3

24.1-3

Given a weighted, directed graph $G = (V, E)$ with no negative-weight cycles, let m be the maximum over all vertices $v \in V$ of the minimum number of edges in a shortest path from the source s to v. (Here, the shortest path is by weight, not the number of edges.) Suggest a simple change to the Bellman-Ford algorithm that allows it to terminate in $m + 1$ passes, even if m is not known in advance.

TC第24.1节练习4

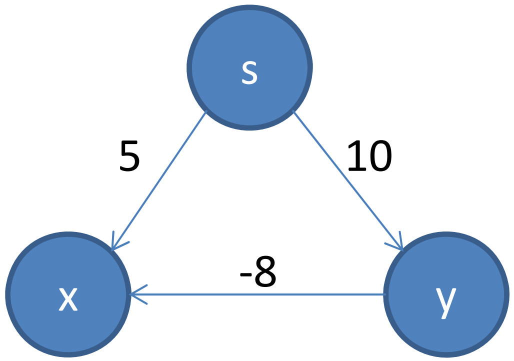
for each edge (u,v)∈G.E
    if v.d>u.d+w(u,v)
        v.mark=true
for each vertex v∈G.V
    if v.mark==true
        v.d=-∞
这样可以吗？



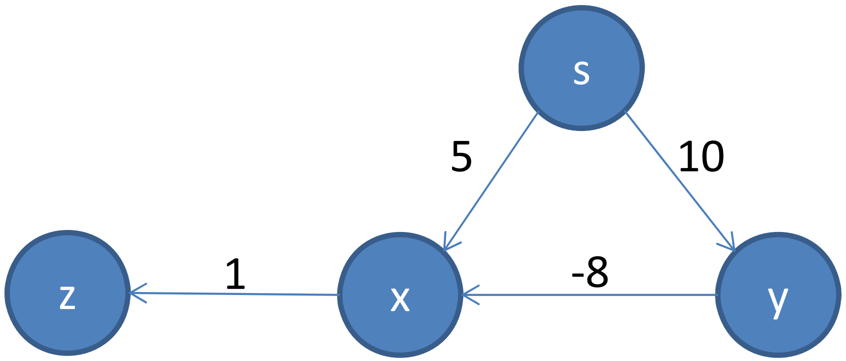
for i=1 to |G.V|-1
    for each edge (u,v)∈G.E
        if v.d>u.d+w(u,v)
            v.d=-∞

TC第24.3节练习2

- 这是反例吗？
- Dijkstra算法的结果是什么？



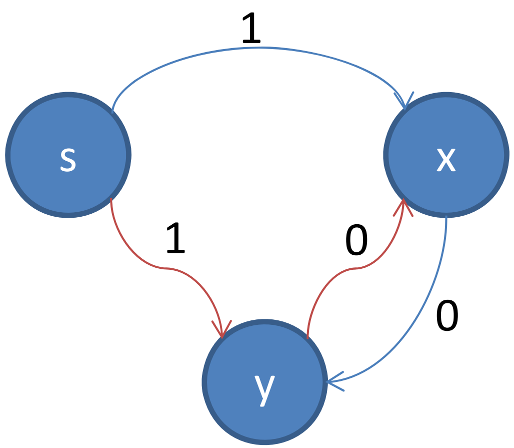
- 怎么将它扩展成反例？
- （不一定要有负圈）



TC第24.5节练习2

24.5-2

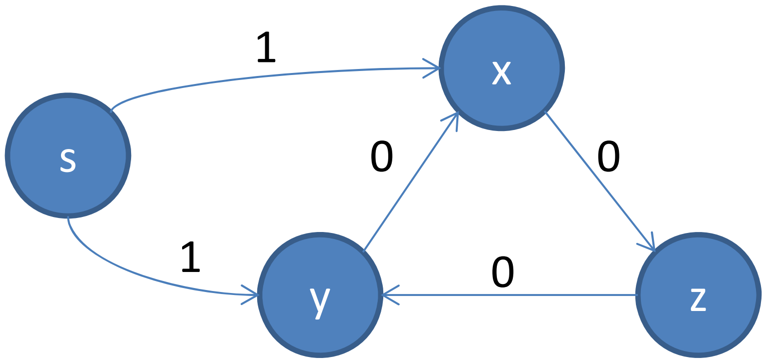
Give an example of a weighted, directed graph $G = (V, E)$ with weight function $w : E \to \mathbb{R}$ and source vertex s such that G satisfies the following property: For every edge $(u, v) \in E$, there is a shortest-paths tree rooted at s that contains (u, v) and another shortest-paths tree rooted at s that does not contain (u, v).



TC第24.5节练习5

24.5-5

Let $G = (V, E)$ be a weighted, directed graph with no negative-weight edges. Let $s \in V$ be the source vertex, and suppose that we allow $v.\pi$ to be the predecessor of v on *any* shortest path to v from source s if $v \in V - \{s\}$ is reachable from s, and NIL otherwise. Give an example of such a graph G and an assignment of π values that produces a cycle in G_π. (By Lemma 24.16, such an assignment cannot be produced by a sequence of relaxation steps.)



TC第24章问题2

24-2 Nesting boxes

A d-dimensional box with dimensions $(x_1, x_2, \ldots, x_d)$ ***nests*** within another box with dimensions $(y_1, y_2, \ldots, y_d)$ if there exists a permutation π on $\{1, 2, \ldots, d\}$ such that $x_{\pi(1)} < y_1, x_{\pi(2)} < y_2, \ldots, x_{\pi(d)} < y_d$.

a. Argue that the nesting relation is transitive.

- 问什么，就证什么
  - 已知Π_{xy}和Π_{yz}，构造Π_{xz}

TC第24章问题2 (续)

c. Suppose that you are given a set of n d-dimensional boxes $\{B_1, B_2, \ldots, B_n\}$. Give an efficient algorithm to find the longest sequence $\langle B_{i_1}, B_{i_2}, \ldots, B_{i_k} \rangle$ of boxes such that B_{i_j} nests within $B_{i_{j+1}}$ for $j = 1, 2, \ldots, k-1$. Express the running time of your algorithm in terms of n and d.

- 构建表示nest关系的有向图（边权设为-1）
- 求最短路（到哪个点的最短路？）

TC第24章问题3

24-3 Arbitrage

Arbitrage is the use of discrepancies in currency exchange rates to transform one unit of a currency into more than one unit of the same currency. For example, suppose that 1 U.S. dollar buys 49 Indian rupees, 1 Indian rupee buys 2 Japanese yen, and 1 Japanese yen buys 0.0107 U.S. dollars. Then, by converting currencies, a trader can start with 1 U.S. dollar and buy $49 \times 2 \times 0.0107 = 1.0486$ U.S. dollars, thus turning a profit of 4.86 percent.

Suppose that we are given n currencies $c_1, c_2, \ldots, c_n$ and an $n \times n$ table R of exchange rates, such that one unit of currency c_i buys $R[i, j]$ units of currency c_j.

a. Give an efficient algorithm to determine whether or not there exists a sequence of currencies $\langle c_{i_1}, c_{i_2}, \ldots, c_{i_k} \rangle$ such that

$$R[i_1, i_2] \cdot R[i_2, i_3] \cdots R[i_{k-1}, i_k] \cdot R[i_k, i_1] > 1 .$$

Analyze the running time of your algorithm.

- 兑换是乘法，路长是加法，如何转换？
  - 取对数

- 教材讨论
  - DW第3章

问题1：基本概念

- 这四个符号分别是什么含义？

	点	边
独立	α	α'
覆盖	β	β’

- 你能解释这些式子为什么成立吗？
  - α’≤β’
    - 最大边独立集有α’条边 ⇒ 图中至少有2α’个顶点 ⇒ 覆盖这些顶点至少需要α’条边 ⇒ α’≤β’
  - α≤β’
    - 最大点独立集中顶点互不相邻 ⇒ 至少要α条边才能覆盖其中所有顶点 ⇒ α≤β’
  - α’≤β
    - 最大边独立集中的每条边至少有一个端点在最小点覆盖集中，且所有这些端点互不相同 ⇒ α’≤β

问题1：基本概念 (续)

- 为什么点独立集和点覆盖集互补？
  - 为什么$\alpha+\beta=n$？

- 边独立集和边覆盖集互补吗？（假设无孤立顶点）
  - 你能举出反例吗？ 
  - 为什么仍然有$\alpha'+\beta'=n$？你理解其证明思路了吗？
    - 如何用构造法来证明$\beta'\le n-\alpha'$？
    - 如何用构造法来证明$\alpha'\ge n-\beta'$？

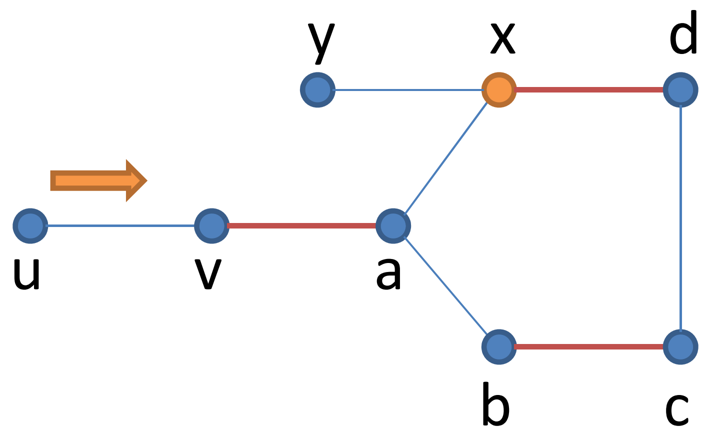
问题2：基本算法

- 面向二部图的增广路算法的基本思路是什么？

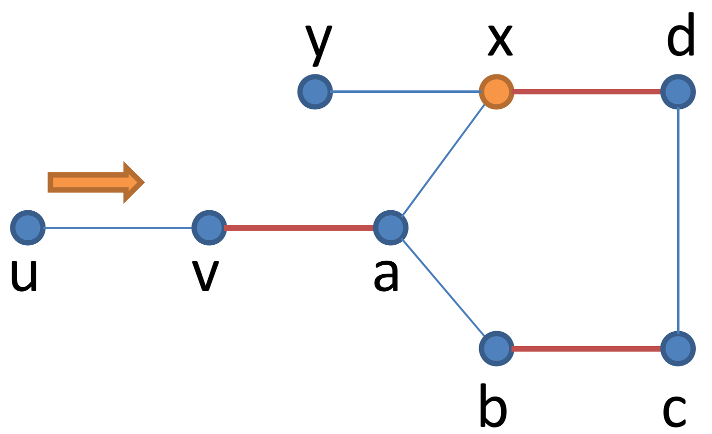
- 这个过程可以优化吗？
- Hopcroft-Karp算法的基本思路：
  - 每轮总是选最短的增广路
  - 每轮同时选取多条增广路

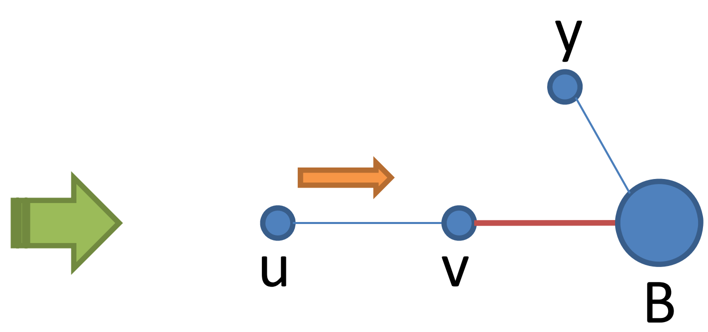
问题2：基本算法 (续)

- 为什么增广路算法在非二部图中行不通？
  - 奇圈让搜索变得复杂



- Edmonds算法的基本思路：
  - 将遇到的奇圈暂时收缩为一个顶点





问题3：实际应用中你会建模吗

- 学校教务员是如何为各门课程分配教室的？
- 如何选址街心公园，能以最小的成本实现市区道路全覆盖？
- 能不能用剪刀剪成若干1x2的矩形？

	1	2	3
4	5	6	7
8	9	10	11
12	13	14	