The Clockwise Tower of Hanoi

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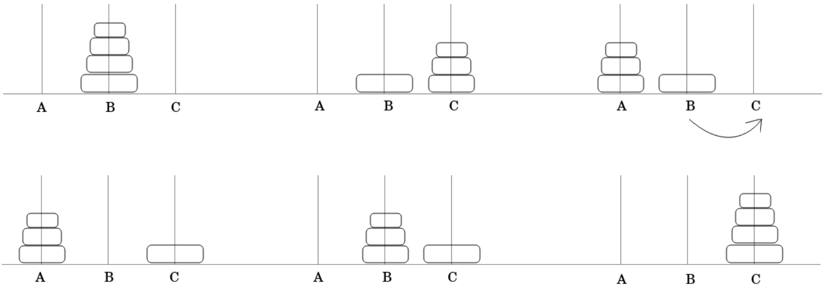

```
function move(n, B, C, A){
  if(n == 0) return;
  move(n-1, B, C, A);
  move(n-1, C, A, B)

  move(B, C);

  move(n-1, A, B, C);
  move(n-1, B, C, A);
}
```

Math

$$P_n = \begin{cases} 0, & \text{if } n = 0; \\ 4P_{n-1} + 1 & \text{if } n > 0; \end{cases}$$

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$$P_n = \frac{1}{3}(4^n - 1)$$

Math

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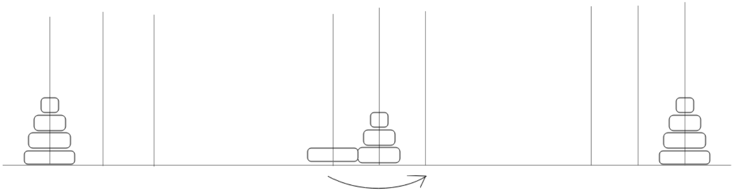
$$P_n = \frac{1}{3}(4^n - 1)$$

$$\lim_{n \to \infty} \frac{P_n}{3^n} = \infty$$

The Tower of Hanoi



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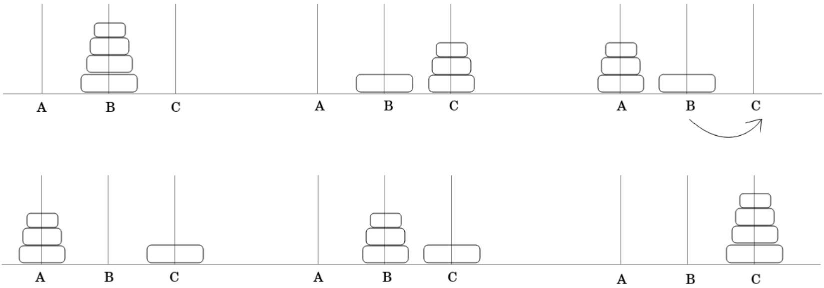

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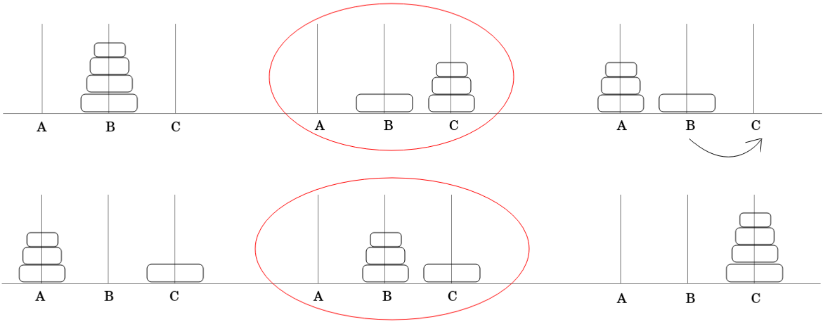
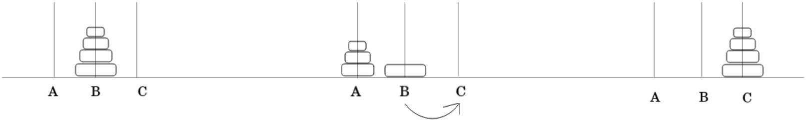


Clockwise

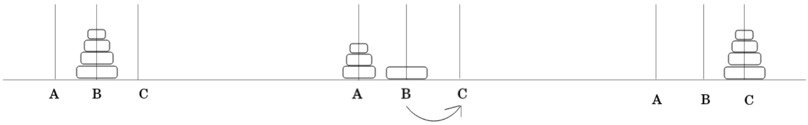



```
function move-clockwise(n, B, C, A){
  if(n == 0) return;
  move-anti-clockwise(n-1, B, A, C);
  move(B, C);
  move-anti-clockwise(n-1, A, C, B);
}
```

Anti-clockwise


```
function move-anti-clockwise(n, B, A, C){
  if(n == 0) return;
  move-anti-clockwise(n-1, B, A, C);
  move(B, C);
  move-clockwise(n-1, A, B, C);
  move(C, A);
  move-anti-clockwise(n-1, B, A, C);
}
```

Math



$$Q_n = \begin{cases} 0, & \text{if } n = 0; \\ 2R_{n-1} + 1 & \text{if } n > 0; \end{cases}$$

Math

$$R_n = \begin{cases} 0, & \text{if } n = 0; \\ R_{n-1} + 1 + Q_{n-1} + 1 + R_{n-1} & \text{if } n > 0; \end{cases}$$

Math

$$R_n = \begin{cases} 0, & \text{if } n = 0; \\ R_{n-1} + 1 + Q_{n-1} + 1 + R_{n-1} & \text{if } n > 0; \end{cases}$$

$$R_n = \begin{cases} 0, & \text{if } n = 0; \\ Q_n + Q_{n-1} + 1 & \text{if } n > 0; \end{cases}$$

Math

$$
\begin{aligned}
(n \geq 2) Q_n &= 2R_{n-1} + 1 \\
&= 2(Q_{n-1} + Q_{n-2} + 1) + 1 \\
&= 2Q_{n-1} + 2Q_{n-2} + 3
\end{aligned}
$$

Math

$$
\begin{aligned}
(n \geq 2) Q_n &= 2R_{n-1} + 1 \\
&= 2(Q_{n-1} + Q_{n-2} + 1) + 1 \\
&= 2Q_{n-1} + 2Q_{n-2} + 3
\end{aligned}
$$

$$Q_n = \frac{3-\sqrt{3}}{6}(1-\sqrt{3})^n + \frac{3+\sqrt{3}}{6}(1+\sqrt{3})^n - 1$$

Reference

- Ronald L. Graham, Donald E. Knuth and Oren Patashnik, "Exercise 1.10", *Concrete mathematics : a foundation for computer science* (1994), 17-18.