

1-9 关系及其基本性质 (II)

魏恒峰

hfwei@nju.edu.cn

2017 年 12 月 18 日



了呢？如果要详细地回答一

此只能举例一二，点到为止。

现在计算机专业的大学一、二年级学生，普遍不愿意学习逻辑演

算与集合论课程，认为相关内容与计算机专业没有什么用。那么我们
的教育管理部門和相关专业人士又是如何认知的呢？据我所知，南
京大学早年不仅要给计算机专业本科生开设这两门课程，而且还要
开设递归论和模型论课程。然而随着思维模式的不断转移，不仅递归
论和模型论早已停开，而且逻辑演算与集合论课程的学时数也在逐
步缩减。现在国内坚持开设这两门课的高校已经很少了，大部分高校
只在离散数学课程中，给学生讲很少一点逻辑演算与集合论知识。其
实，相关知识对于培养计算机专业的高科技人才来说是至关重要的，
即使不谈这是最起码的专业文化素养，难道不明白我们所学之程序
设计语言是靠逻辑设计出来的？而且柯特(E. P. Codd)博士创立关
系数据库，以及许华兹(J. T. Schwartz)教授开发的集合论程序设计
语言 SETL，可谓全都依靠数理逻辑与集合论知识的积累。但却很少
有专业教师能从历史的角度并依此为例去教育学生，甚至还有极个
别的专家教授，竟然主张把“计算机科学理论”这门硕士研究生学位
课取消，认为这门课相对于毕业后去公司就业的学生太空洞，这真是
令人瞠目结舌。特别是对于那些初涉高等学府的学子来说，甘愿重性
而在工他何以

The Relational Data Model

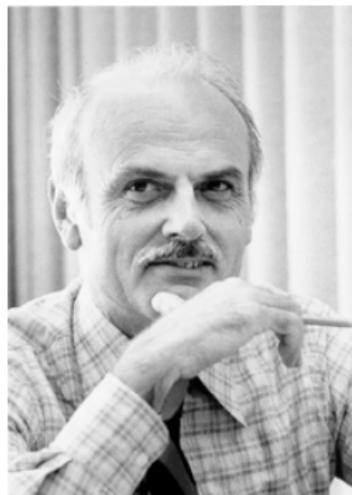
A Relational Model of Data for Large Shared Data Banks

E. F. CODD

IBM Research Laboratory, San Jose, California

Future users of large data banks must be protected from having to know how the data is organized in the machine (the internal representation). A prompting service which supplies such information is not a satisfactory solution. Activities of users at terminals and most application programs should remain unaffected when the internal representation of data is changed and even when some aspects of the external representation are changed. Changes in data representation will often be needed as a result of changes in query, update, and report traffic and natural growth in the types of stored information.

Existing noninferential, formatted data systems provide users with tree-structured files or slightly more general network models of the data. In Section 1, inadequacies of these models are discussed. A model based on n -ary relations, a normal form for data base relations, and the concept of a universal date sublanguage are introduced. In Section 2, certain operations on relations (other than logical inference) are discussed and applied to the problems of redundancy and consistency in the user's model.



Codd@CACM'1970

Edgar F. Codd (1923 – 2003)

The Relational Data Model

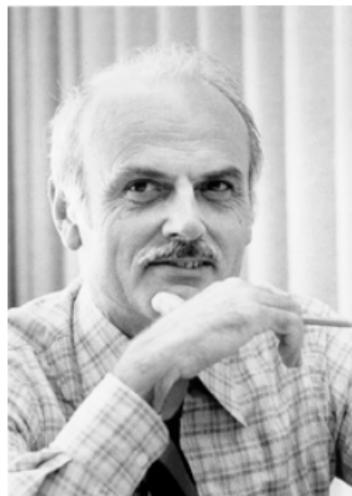
— 如何靠“关系”赢得图灵奖?

A Relational Model of Data for Large Shared Data Banks

E. F. CODD
IBM Research Laboratory, San Jose, California

Future users of large data banks must be protected from having to know how the data is organized in the machine (the internal representation). A prompting service which supplies such information is not a satisfactory solution. Activities of users at terminals and most application programs should remain unaffected when the internal representation of data is changed and even when some aspects of the external representation are changed. Changes in data representation will often be needed as a result of changes in query, update, and report traffic and natural growth in the types of stored information.

Existing noninferential, formatted data systems provide users with tree-structured files or slightly more general network models of the data. In Section 1, inadequacies of these models are discussed. A model based on n -ary relations, a normal form for data base relations, and the concept of a universal date sublanguage are introduced. In Section 2, certain operations on relations (other than logical inference) are discussed and applied to the problems of redundancy and consistency in the user's model.



Codd@CACM'1970

Edgar F. Codd (1923 – 2003)

1

2

3





A Relational View of Data

$$(a, b) = \{\{a\}, \{a, b\}\}$$

$$R \subseteq A \times B$$

$$(a, b) = \{\{a\}, \{a, b\}\}$$

$$R \subseteq A \times B$$

$$(a, b, c) = ((a, b), c)$$

$$(a, b) = \{\{a\}, \{a, b\}\}$$

$$R \subseteq A \times B$$

$$(a, b, c) = ((a, b), c)$$

$$(x_1, x_2, \dots, x_n) = ((x_1, x_2, \dots, x_{n-1}), x_n)$$

$$(a, b) = \{\{a\}, \{a, b\}\}$$

$$R \subseteq A \times B$$

$$(a, b, c) = ((a, b), c)$$

$$(x_1, x_2, \dots, x_n) = ((x_1, x_2, \dots, x_{n-1}), x_n)$$

$$R \subseteq X_1 \times X_2 \times \dots \times X_n$$

$\text{Course} = \{\text{CS101}, \text{EE200}, \text{PH100}\}$

$\text{StudentId} = \{12345, 67890, 22222, 33333\}$

$\text{Grade} = \{A, B, C, D, A^-, B^+, C^+\}$

$\text{Course} = \{\text{CS101}, \text{EE200}, \text{PH100}\}$

$\text{StudentId} = \{12345, 67890, 22222, 33333\}$

$\text{Grade} = \{A, B, C, D, A^-, B^+, C^+\}$

$\text{CSG} \subseteq \text{Course} \times \text{StudentId} \times \text{Grade}$

$\text{Course} = \{\text{CS101}, \text{EE200}, \text{PH100}\}$

$\text{StudentId} = \{12345, 67890, 22222, 33333\}$

$\text{Grade} = \{A, B, C, D, A^-, B^+, C^+\}$

$\text{CSG} \subseteq \text{Course} \times \text{StudentId} \times \text{Grade}$

$\text{CSG} = \{(\text{CS101}, 12345, A),$
 $(\text{CS101}, 67890, B),$
 $(\text{CS101}, 33333, C),$
 $(\text{EE200}, 12345, B^+),$
 $(\text{EE200}, 22222, A^-),$
 $(\text{PH100}, 67890, C^+)\}$

| Course | StudentId | Grade |
|--------|-----------|-------|
| CS101 | 12345 | A |
| CS101 | 67890 | B |
| EE200 | 12345 | C |
| EE200 | 22222 | B+ |
| CS101 | 33333 | A- |
| PH100 | 67890 | C+ |

| Course | StudentId | Grade |
|--------|-----------|-------|
| CS101 | 12345 | A |
| CS101 | 67890 | B |
| EE200 | 12345 | C |
| EE200 | 22222 | B+ |
| CS101 | 33333 | A- |
| PH100 | 67890 | C+ |

Row : tuple

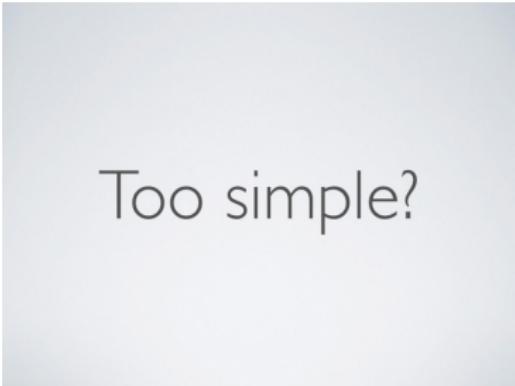
Column : attribute/component

| Course | StudentId | Grade |
|--------|-----------|-------|
| CS101 | 12345 | A |
| CS101 | 67890 | B |
| EE200 | 12345 | C |
| EE200 | 22222 | B+ |
| CS101 | 33333 | A- |
| PH100 | 67890 | C+ |

| StudentId | Name | Address | Phone |
|-----------|-------------|----------------|----------|
| 12345 | C. Brown | 12 Apple St. | 555-1234 |
| 67890 | L. Van Pelt | 34 Pear Ave. | 555-5678 |
| 22222 | P. Patty | 56 Grape Blvd. | 555-9999 |

| Course | Day | Hour |
|--------|-----|------|
| CS101 | M | 9AM |
| CS101 | W | 9AM |
| CS101 | F | 9AM |
| EE200 | Tu | 10AM |
| EE200 | W | 1PM |
| EE200 | Th | 10AM |

| Course | Room |
|--------|-------------|
| CS101 | Turing Aud. |
| EE200 | 25 Ohm Hall |
| PH100 | Newton Lab. |



Too simple?



2

Relational Algebra

Relational Algebra



Relational Algebra

Relational Algebra

A **set** of relations

Operations on relations

Laws of operations

\cup \cap \

Selection ($\sigma_C(R)$)

$\sigma_{\text{Course} = \text{"CS101"}}(\text{CSG})$

| Course | StudentId | Grade |
|--------|-----------|-------|
| CS101 | 12345 | A |
| CS101 | 67890 | B |
| EE200 | 12345 | C |
| EE200 | 22222 | B+ |
| CS101 | 33333 | A- |
| PH100 | 67890 | C+ |

| Course | StudentId | Grade |
|--------|-----------|-------|
| CS101 | 12345 | A |
| CS101 | 67890 | B |
| CS101 | 33333 | A- |

Projection ($\pi_L(R)$)

$\pi_{\text{StudentId}} \left(\sigma_{\text{Course} = \text{"CS101"}}(\text{CSG}) \right)$

| Course | StudentId | Grade |
|--------|-----------|-------|
| CS101 | 12345 | A |
| CS101 | 67890 | B |
| EE200 | 12345 | C |
| EE200 | 22222 | B+ |
| CS101 | 33333 | A- |
| PH100 | 67890 | C+ |

| Course | StudentId | Grade |
|--------|-----------|-------|
| CS101 | 12345 | A |
| CS101 | 67890 | B |
| CS101 | 33333 | A- |

| StudentId |
|-----------|
| 12345 |
| 67890 |
| 33333 |

Join ($R \underset{R.A_i=S.B_j}{\bowtie} S$)

$$R \subseteq A_1 \times \cdots \times A_n$$

$$S \subseteq B_1 \times \cdots \times B_m$$

$$R \underset{R.A_i=S.B_j}{\bowtie} S \subseteq A_1 \times \cdots \times A_n \times B_1 \times \cdots B_{j-1} \times B_{j+1} \times \cdots \times B_m$$

Join ($R \underset{R.A_i=S.B_j}{\bowtie} S$)

$R \subseteq A_1 \times \cdots \times A_n$

$S \subseteq B_1 \times \cdots \times B_m$

$R \underset{R.A_i=S.B_j}{\bowtie} S \subseteq A_1 \times \cdots \times A_n \times B_1 \times \cdots B_{j-1} \times B_{j+1} \times \cdots \times B_m$

Natural Join ($R \bowtie S$)

CR



CR.Course = CDH.Course

CDH

(CR \bowtie CDH)

| Course | Room |
|--------|-------------|
| CS101 | Turing Aud. |
| EE200 | 25 Ohm Hall |
| PH100 | Newton Lab. |

| Course | Day | Hour |
|--------|-----|------|
| CS101 | M | 9AM |
| CS101 | W | 9AM |
| CS101 | F | 9AM |
| EE200 | Tu | 10AM |
| EE200 | W | 1PM |
| EE200 | Th | 10AM |

| Course | Room | Day | Hour |
|--------|-------------|-----|------|
| CS101 | Turing Aud. | M | 9AM |
| CS101 | Turing Aud. | W | 9AM |
| CS101 | Turing Aud. | F | 9AM |
| EE200 | 25 Ohm Hall | Tu | 10AM |
| EE200 | 25 Ohm Hall | W | 1PM |
| EE200 | 25 Ohm Hall | Th | 10AM |



| Course | StudentId | Grade |
|--------|-----------|-------|
| CS101 | 12345 | A |
| CS101 | 67890 | B |
| EE200 | 12345 | C |
| EE200 | 22222 | B+ |
| CS101 | 33333 | A- |
| PH100 | 67890 | C+ |

| StudentId | Name | Address | Phone |
|-----------|-------------|----------------|----------|
| 12345 | C. Brown | 12 Apple St. | 555-1234 |
| 67890 | L. Van Pelt | 34 Pear Ave. | 555-5678 |
| 22222 | P. Patty | 56 Grape Blvd. | 555-9999 |

| Course | Day | Hour |
|--------|-----|------|
| CS101 | M | 9AM |
| CS101 | W | 9AM |
| CS101 | F | 9AM |
| EE200 | Tu | 10AM |
| EE200 | W | 1PM |
| EE200 | Th | 10AM |

| Course | Room |
|--------|-------------|
| CS101 | Turing Aud. |
| EE200 | 25 Ohm Hall |
| PH100 | Newton Lab. |

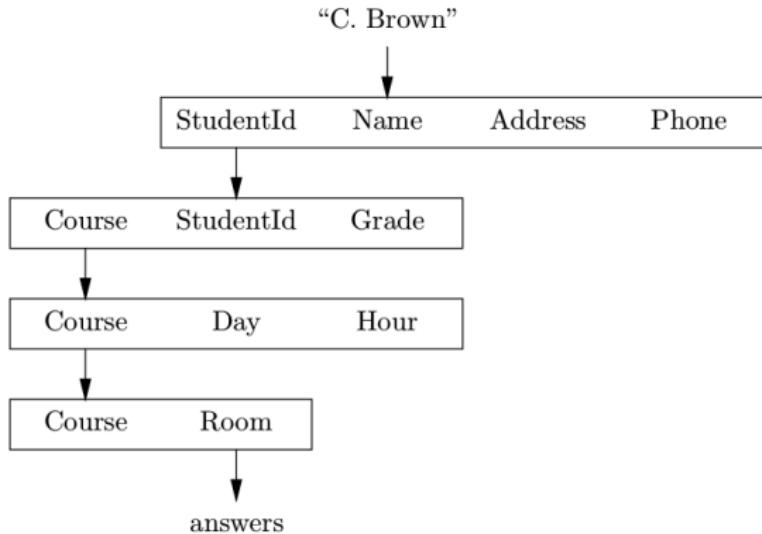
| Course | StudentId | Grade |
|--------|-----------|-------|
| CS101 | 12345 | A |
| CS101 | 67890 | B |
| EE200 | 12345 | C |
| EE200 | 22222 | B+ |
| CS101 | 33333 | A- |
| PH100 | 67890 | C+ |

| StudentId | Name | Address | Phone |
|-----------|-------------|----------------|----------|
| 12345 | C. Brown | 12 Apple St. | 555-1234 |
| 67890 | L. Van Pelt | 34 Pear Ave. | 555-5678 |
| 22222 | P. Patty | 56 Grape Blvd. | 555-9999 |

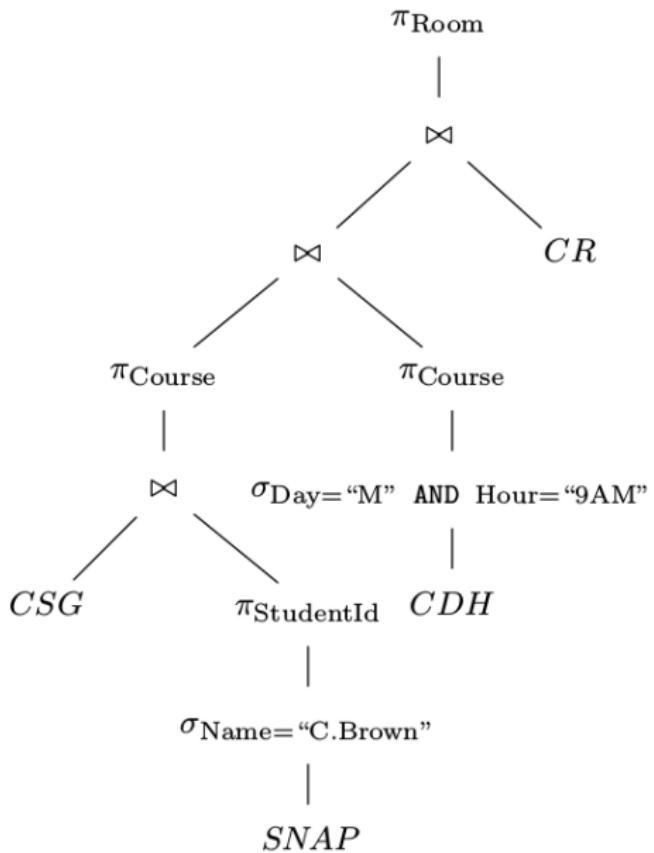
| Course | Day | Hour |
|--------|-----|------|
| CS101 | M | 9AM |
| CS101 | W | 9AM |
| CS101 | F | 9AM |
| EE200 | Tu | 10AM |
| EE200 | W | 1PM |
| EE200 | Th | 10AM |

| Course | Room |
|--------|-------------|
| CS101 | Turing Aud. |
| EE200 | 25 Ohm Hall |
| PH100 | Newton Lab. |

Q : Where is “C. Brown” 9 AM on Mondays?



```
1 π Room (
2   π Course (
3     π StudentId (
4       σ Name = 'C. Brown' (SNAP)
5     ) ⋈ (CSG)
6   ) ⋈ (
7     π Course (
8       σ Day = 'M' ∧ Hour = '9AM' (CDH)
9     )
10   ) ⋈ CR
11 )
```



**TOO
CLEVER IS
DUMB.**

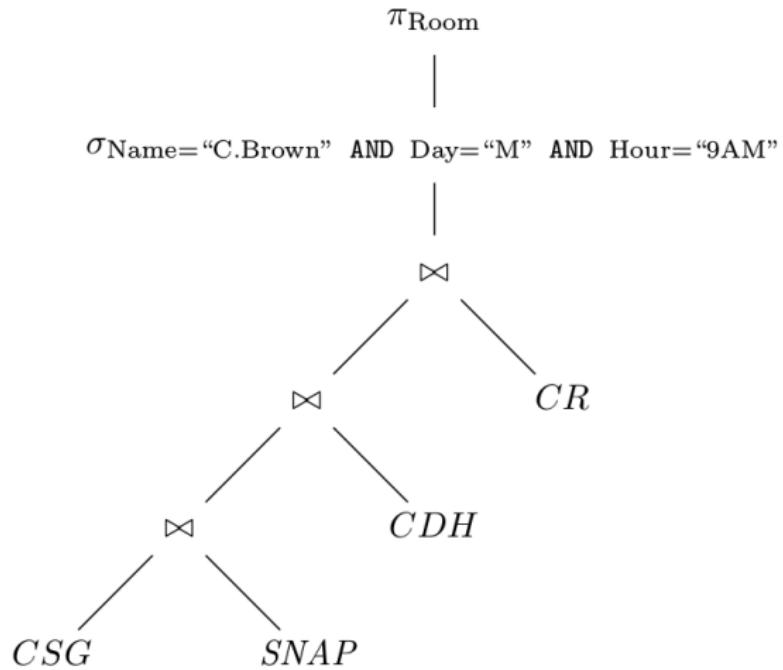


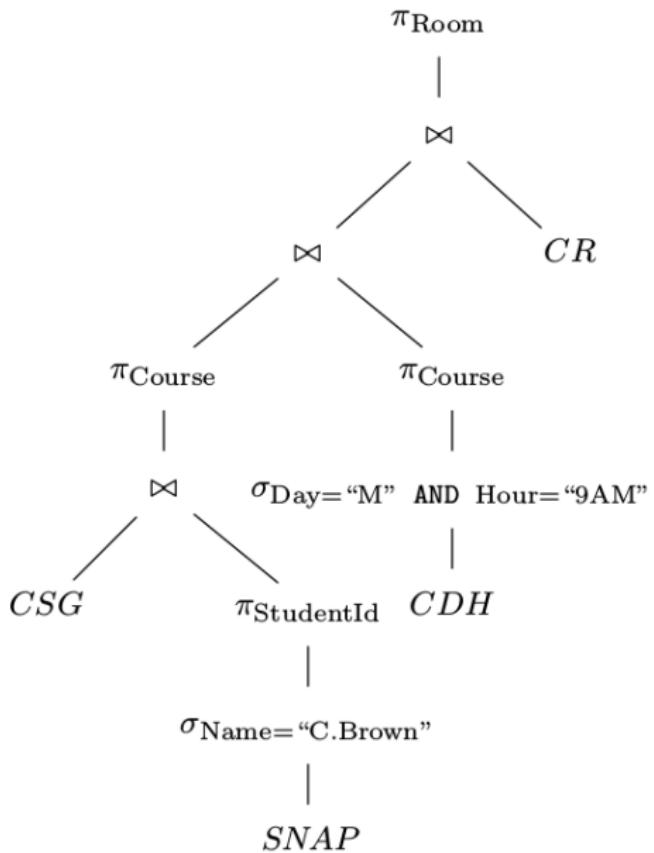
Ogden Nash
American Poet

1902 - 1971



Algebraic Laws for Relations





“In fact, the improvement in efficiency made possible by transforming expressions of relational algebra is arguably the most striking example of the power of algebra that we find in computer science.”

Laws Involving Selection ($\sigma_C(R)$)

Selection Splitting

$$\sigma_{C \wedge C'}(R) \equiv \sigma_C(\sigma_{C'}(R))$$

Commutativity

$$\sigma_C(\sigma_{C'}(R)) \equiv \sigma_{C'}(\sigma_C(R))$$

Laws Involving Selection ($\sigma_C(R)$)

Selection Splitting

$$\sigma_{C \wedge C'}(R) \equiv \sigma_C(\sigma_{C'}(R))$$

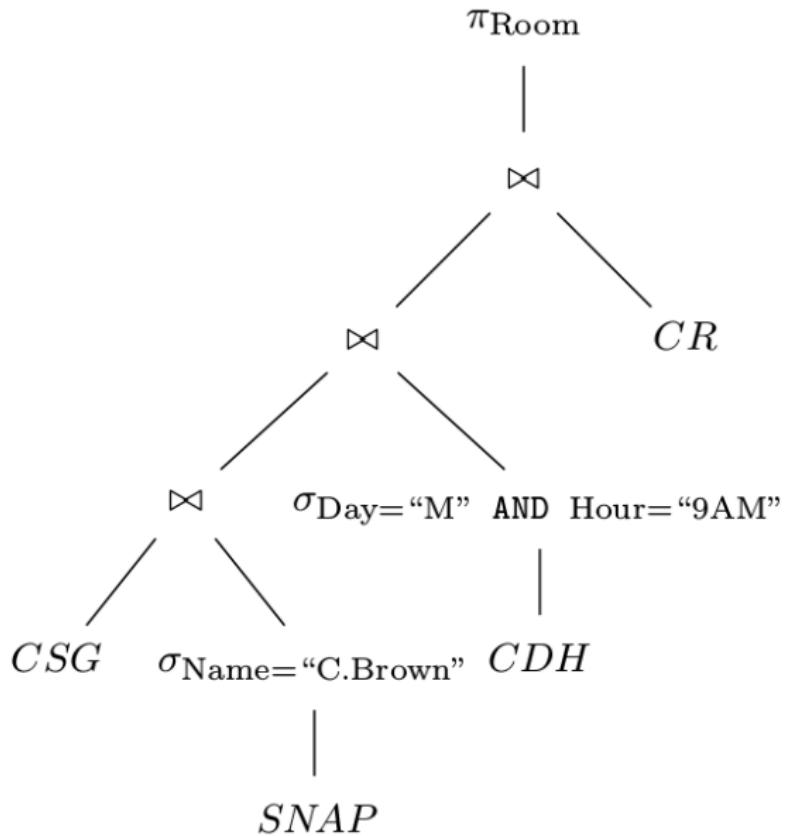
Commutativity

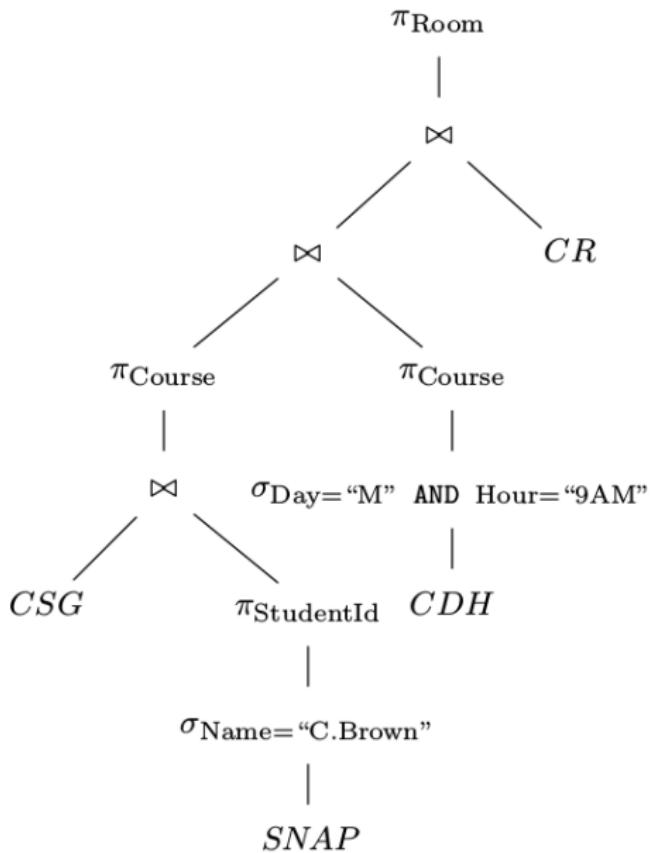
$$\sigma_C(\sigma_{C'}(R)) \equiv \sigma_{C'}(\sigma_C(R))$$

Selection Pushing

$$(\sigma_C(R \bowtie S)) \equiv (\sigma_C(R) \bowtie S)$$

$$(\sigma_C(R \bowtie S)) \equiv (R \bowtie \sigma_C(S))$$





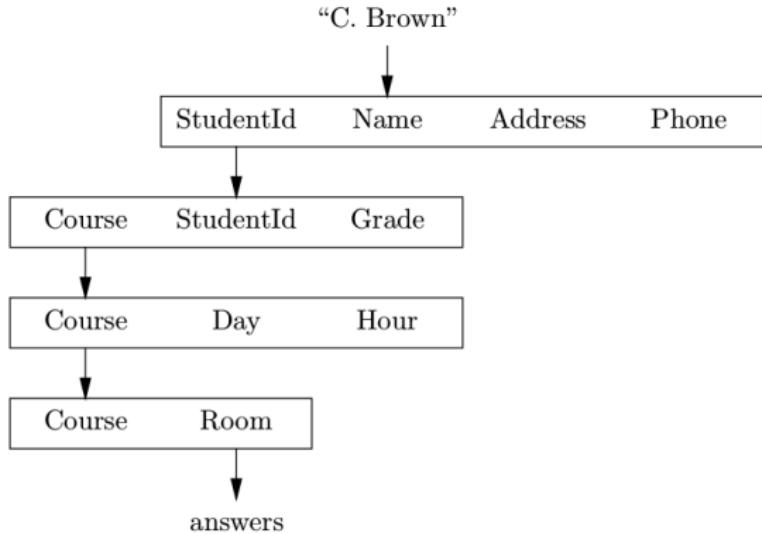
Laws Involving Projection ($\pi_L(R)$)

Projection Pushing

$$\left(\pi_L(R) \underset{A_i=B_j}{\bowtie} S \right) \equiv \left(\pi_L(\pi_M(R) \underset{A_i=B_j}{\bowtie} \pi_N(S)) \right)$$

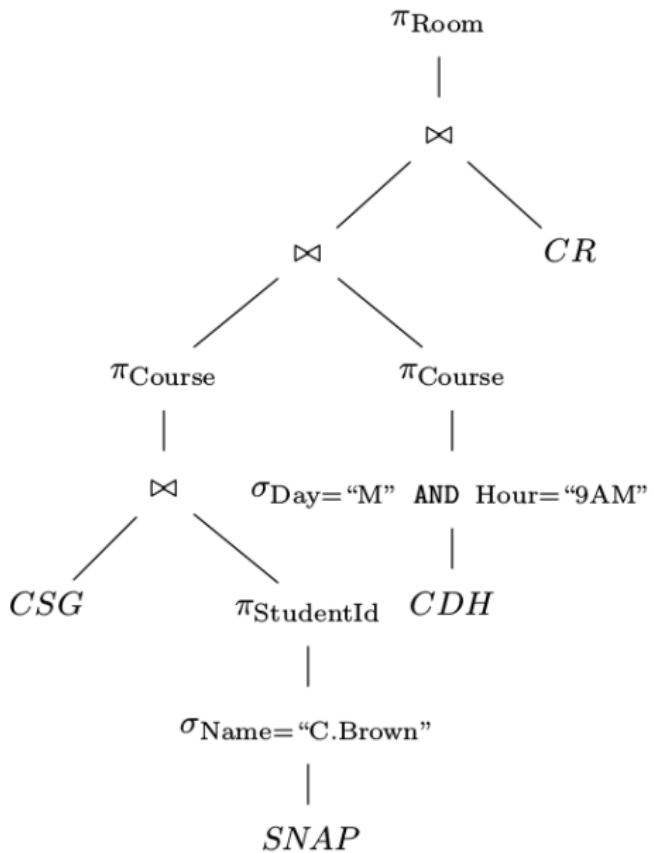
$$M = \left(L \cup \{A_i\} \right) \cap \{A_i\}_i$$

$$N = \left(L \cup \{B_j\} \right) \cap \{B_j\}_j$$



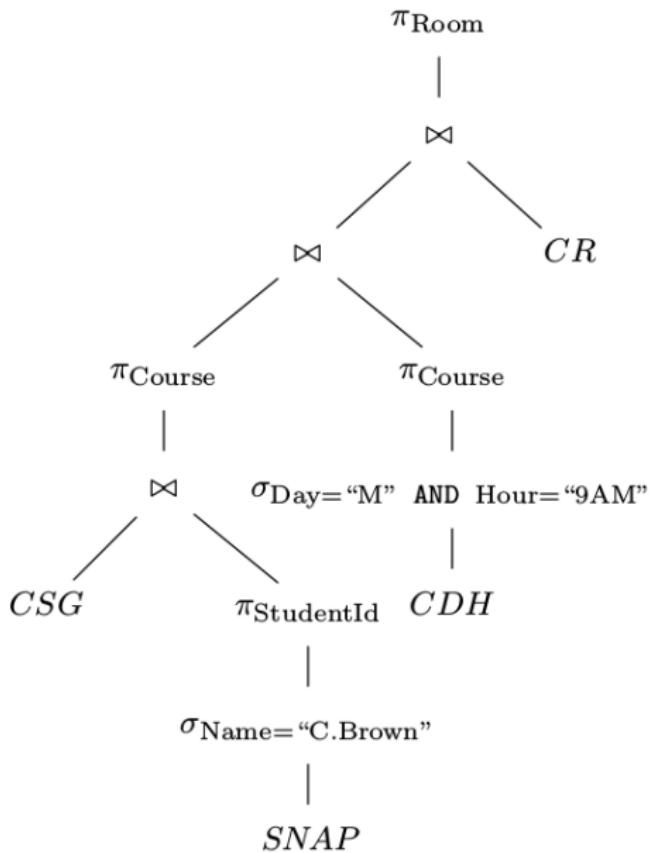
$$\pi_{\text{Course}} \left(\text{CSG} \underset{\text{StudentId}}{\bowtie} \text{SNAP} \right)$$

$$\equiv \pi_{\text{Course}} \left(\pi_{\text{Course}, \text{StudentId}}(\text{CSG}) \underset{\text{StudentId}}{\bowtie} \pi_{\text{StudentId}}(\text{SNAP}) \right)$$



3

Algebra as Query Language



CourseManage..▼

CSG

Course string
StudentId
number
Grade string

SNAP

StudentId
number
Name string
Address string
Phone string

CDH

Course string
Day string
Hour string

CR

Relational Algebra

SQL

π σ $\rho \leftarrow \tau \gamma$ \wedge \vee \neg $=$ \neq \geq \leq \cap \cup \div $-$ \times \bowtie

```
1 π Room (
2   π Course (
3     π StudentId (
4       σ Name = 'C. Brown' (SNAP)
5     ) ⋈ (CSG)
6   ) ⋈ (
7     π Course (
8       σ Day = 'M' ∧ Hour = '9AM' (CDH)
9     )
10   ) ⋈ CR
11 )
```

▶ execute query



Structured Query Language



Structured Query Language

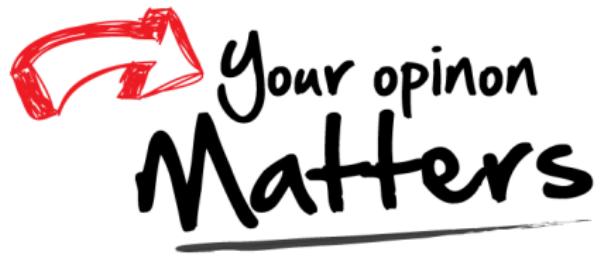
Declarative Programming

1

2

3

Thank You!



Office 302

Mailbox: H016

hfwei@nju.edu.cn