计算机问题求解 – 论题3–15 - 线性规划

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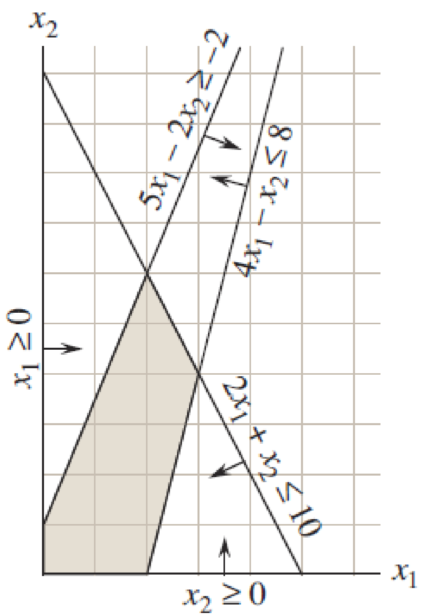
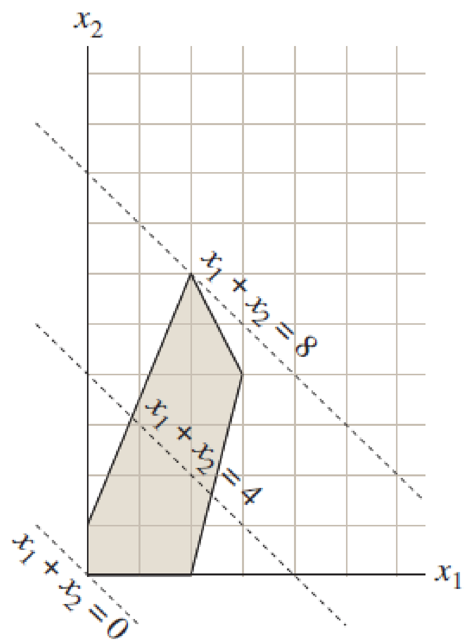
自学检查

When converting one linear program L into another linear program L', we would like the property that an optimal solution to L' yields an optimal solution to L. To capture this idea, we say that two maximization linear programs L and L' are ***equivalent*** if for each feasible solution $\bar{x}$ to L with objective value z, there is a corresponding feasible solution $\bar{x}'$ to L' with objective value z, and for each feasible solution $\bar{x}'$ to L' with objective value z, there is a corresponding feasible solution $\bar{x}$ to L with objective value z. (This definition does not imply a one-to-

$$
\begin{array}{llcrcr}
\text{maximize} & x_1 & + & x_2 & & \\
\text{subject to} & & & & & \\
& 4x_1 & - & x_2 & \leq & 8 \\
& 2x_1 & + & x_2 & \leq & 10 \\
& 5x_1 & - & 2x_2 & \geq & -2 \\
& & x_1, x_2 & & \geq & 0
\end{array}
$$





问题1：

你能否利用左边的式子和图解释：目标函数、约束条件、可行解、目标值、目标值的可行解、线性规划问题的解、线性规划？

问题2：

如何理解下列语句：

policy	urban	suburban	rural
build roads	−2	5	3
gun control	8	2	−5
farm subsidies	0	0	10
gasoline tax	10	0	−2

Although we cannot easily graph linear programs with more than two variables, the same intuition holds. If we have three variables, then each constraint corresponds to a half-space in three-dimensional space. The intersection of these half-spaces forms the feasible region.

$$
\begin{array}{lrcrcrcrcr}
\text{minimize} & x_1 & + & x_2 & + & x_3 & + & x_4 & & \\
\text{subject to} & & & & & & & & & \\
& -2x_1 & + & 8x_2 & + & 0x_3 & + & 10x_4 & \geq & 50 \\
& 5x_1 & + & 2x_2 & + & 0x_3 & + & 0x_4 & \geq & 100 \\
& 3x_1 & - & 5x_2 & + & 10x_3 & - & 2x_4 & \geq & 25 \\
& & & x_1, x_2, x_3, x_4 & & & & & \geq & 0
\end{array}
$$

问题3：

线性规划问题中的不等式能不能用严格的大于或小于？

一个“现实”问题：Product-Mix

This corporation has 10 plants in various parts of the world. Each of these plants produces the same 10 products and then sells them within its region. The *demand* (sales potential) for each of these products from each plant is known for each of the next 10 months. Although the amount of a product sold by a plant in a given month cannot exceed the demand, the amount produced can be larger, where the excess amount would be stored in inventory (at some unit cost per month) for sale in a later month

每个厂有10条生产线，可以安排任何产品，但效率和成本可能不同。必要时也可以考虑在厂间运输成品，特定两个厂之间单位产品运输成本是固定的。每个厂存储能力有上限，单位成本相同。

Management now needs to determine how much of each product should be produced by each machine in each plant during each month, as well as how much each plant should sell of each product in each month and how much each plant should ship of each product in each month to each of the other plants. Considering the worldwide price for each product, the objective is to find the feasible plan that maximizes the total profit (total sales revenue *minus* the sum of the total production costs, inventory costs, and shipping costs).

问题4：你能否估计一下这个问题“规模”有多大？

10,000 production variables: one for each combination of a plant, machine, product, and month

1,000 inventory variables: one for each combination of a plant, product, and month

1,000 sales variables: one for each combination of a plant, product, and month

9,000 shipping variables: one for each combination of a product, month, plant (the fromplant), and another plant (the toplant)

线性规划问题的标准形式

$$
\begin{aligned}
\text{maximize} \quad & \sum_{j=1}^{n} c_j x_j \\
\text{subject to} \quad & \\
& \sum_{j=1}^{n} a_{ij} x_j \le b_i \quad \text{for } i = 1, 2, \ldots, m \\
& x_j \ge 0 \quad \text{for } j = 1, 2, \ldots, n
\end{aligned}
$$

$$
\begin{aligned}
\text{maximize} \quad & c^{\mathrm{T}} x \\
\text{subject to} \quad & \\
& Ax \le b \\
& x \ge 0
\end{aligned}
$$

$$
\begin{array}{llcll}
\text{minimize} & -2x_1 & + & 3x_2 & \\
\text{subject to} & & & & \\
& x_1 & + & x_2 & = 7 \\
& x_1 & - & 2x_2 & \leq 4 \\
& x_1 & & & \geq 0
\end{array}
$$

问题5：

为什么说这不是“标准形式”

，

mizing a linear function subject to linear constraints, into standard form. A linear program might not be in standard form for any of four possible reasons:

1. The objective function might be a minimization rather than a maximization.
2. There might be variables without nonnegativity constraints.
3. There might be ***equality constraints***, which have an equal sign rather than a less-than-or-equal-to sign.
4. There might be ***inequality constraints***, but instead of having a less-than-or-equal-to sign, they have a greater-than-or-equal-to sign.

$$\begin{array}{lrcrcl} \text{minimize} & -2x_1 & + & 3x_2 & & \\ \text{subject to} & & & & & \\ & x_1 & + & x_2 & = & 7 \\ & x_1 & - & 2x_2 & \le & 4 \\ & x_1 & & & \ge & 0 \end{array} \Rightarrow \begin{array}{lrcrcl} \text{maximize} & 2x_1 & - & 3x_2 & & \\ \text{subject to} & & & & & \\ & x_1 & + & x_2 & = & 7 \\ & x_1 & - & 2x_2 & \le & 4 \\ & x_1 & & & \ge & 0\,. \end{array}$$

问题6：

如何将它转化为标准形式？这两个线性规划“一样”吗？

问题7：

我们说两个线性规划“等价”(equivalent) 是什么意思？

$$\begin{array}{llllllll}
\text{maximize} & 2x_1 & - & 3x_2 & & \\
\text{subject to} & & & & & \\
& x_1 & + & x_2 & = & 7 \\
& x_1 & - & 2x_2 & \leq & 4 \\
& x_1 & & & \geq & 0\,.
\end{array}
\Rightarrow
\begin{array}{llllllll}
\text{maximize} & 2x_1 & - & 3x'_2 & + & 3x''_2 & & \\
\text{subject to} & & & & & & & \\
& x_1 & + & x'_2 & - & x''_2 & = & 7 \\
& x_1 & - & 2x'_2 & + & 2x''_2 & \leq & 4 \\
& & x_1, x'_2, x''_2 & & & & \geq & 0\,.
\end{array}$$

问题8：

这两个线性规划是如何等价的？

$$
\begin{array}{lrcrcrcl}
\text{maximize} & 2x_1 & - & 3x_2' & + & 3x_2'' & & \\
\text{subject to} & & & & & & & \\
& x_1 & + & x_2' & - & x_2'' & = & 7 \\
& x_1 & - & 2x_2' & + & 2x_2'' & \leq & 4 \\
& & & x_1, x_2', x_2'' & & & \geq & 0\,.
\end{array}
$$

$$
\begin{array}{lrcrcrcl}
\text{maximize} & 2x_1 & - & 3x_2' & + & 3x_2'' & & \\
\text{subject to} & & & & & & & \\
& x_1 & + & x_2' & - & x_2'' & \leq & 7 \\
& x_1 & + & x_2' & - & x_2'' & \geq & 7 \\
& x_1 & - & 2x_2' & + & 2x_2'' & \leq & 4 \\
& & & x_1, x_2', x_2'' & & & \geq & 0
\end{array}
$$

$$
\boxed{
\begin{array}{lrcl}
\text{maximize} & c^{\mathrm{T}}x & & \\
\text{subject to} & & & \\
& Ax & \leq & b \\
& x & \geq & 0\,.
\end{array}
}
$$

$$
\begin{array}{lrcrcrcr}
\text{maximize} & 2x_1 & - & 3x_2 & + & 3x_3 & & \\
\text{subject to} & & & & & & & \\
& x_1 & + & x_2 & - & x_3 & \leq & 7 \\
& -x_1 & - & x_2 & + & x_3 & \leq & -7 \\
& x_1 & - & 2x_2 & + & 2x_3 & \leq & 4 \\
& & & x_1, x_2, x_3 & & & \geq & 0\,.
\end{array}
$$

问题9：

怎么样就可以将不等式改写为等式形式？

$$\sum_{j=1}^{n} a_{ij} x_j \le b_i \quad \Longrightarrow \quad \begin{aligned} s &= b_i - \sum_{j=1}^{n} a_{ij} x_j \\ s &\ge 0. \end{aligned}$$

线性规划：Slack Form

$$
\begin{array}{lrcrcrcl}
\text{maximize} & 2x_1 & - & 3x_2 & + & 3x_3 & & \\
\text{subject to} & & & & & & & \\
& x_1 & + & x_2 & - & x_3 & \leq & 7 \\
& -x_1 & - & x_2 & + & x_3 & \leq & -7 \\
& x_1 & - & 2x_2 & + & 2x_3 & \leq & 4 \\
& & x_1, x_2, x_3 & & & & \geq & 0 .
\end{array}
$$

$$
\begin{array}{rcrcrcrcr}
z & = & & & 2x_1 & - & 3x_2 & + & 3x_3 \\
x_4 & = & 7 & - & x_1 & - & x_2 & + & x_3 \\
x_5 & = & -7 & + & x_1 & + & x_2 & - & x_3 \\
x_6 & = & 4 & - & x_1 & + & 2x_2 & - & 2x_3
\end{array}
$$

线性规划：Slack Form

$$\begin{aligned}
z &= 0+ & 2x_1 &- 3x_2 + 3x_3 \\
x_4 &= 7 &- x_1 &- x_2 + x_3 \\
x_5 &= -7 &+ x_1 &+ x_2 - x_3 \\
x_6 &= 4 &- x_1 &+ 2x_2 - 2x_3
\end{aligned}$$

注意：这是“省略”形式。
省了什么？

$$(N, B, A, b, c, v),$$

$$z = v + \sum_{j \in N} c_j x_j$$

$$x_i = b_i - \sum_{j \in N} a_{ij} x_j \quad \text{for } i \in B$$

Simplex: 基本思想

问题10：这个线性规划的解是什么？
是否很容易得到这个解？

$$
\begin{aligned}
z &= 28 - \frac{x_3}{6} - \frac{x_5}{6} - \frac{2x_6}{3}\\
x_1 &= 8 + \frac{x_3}{6} + \frac{x_5}{6} - \frac{x_6}{3}\\
x_2 &= 4 - \frac{8x_3}{3} - \frac{2x_5}{3} + \frac{x_6}{3}\\
x_4 &= 18 - \frac{x_3}{2} + \frac{x_5}{2}.
\end{aligned}
$$

Simplex: 基本思想

通过逐次交换"基本变量"和"非基本变量"获得一系列的"基本解"，每个基本解一定是可行解，并构成非递减序列，不断"逼近"最优解。

问题11：

你能回答以下"基本"问题吗？

- "基本解"是什么？
- 如何选择要"交换"的"非基本变量"？
- 如何确定被交换的"基本"变量？
- 为什么目标函数值会增加？
- 什么时候结束？

$$
\begin{array}{lrcrcrcr}
\text{maximize} & 3x_1 & + & x_2 & + & 2x_3 & & \\
\text{subject to} & & & & & & & \\
& x_1 & + & x_2 & + & 3x_3 & \le & 30 \\
& 2x_1 & + & 2x_2 & + & 5x_3 & \le & 24 \\
& 4x_1 & + & x_2 & + & 2x_3 & \le & 36 \\
& & & x_1, x_2, x_3 & & & \ge & 0
\end{array}
\quad \Rightarrow \quad
\begin{array}{rcrcrcrcr}
z & = & & & 3x_1 & + & x_2 & + & 2x_3 \\
x_4 & = & 30 & - & x_1 & - & x_2 & - & 3x_3 \\
x_5 & = & 24 & - & 2x_1 & - & 2x_2 & - & 5x_3 \\
x_6 & = & 36 & - & 4x_1 & - & x_2 & - & 2x_3
\end{array}
$$

基本解：<0,0,0,30,24,36>，目标值：0

问题11：松弛变量到底有什么含义？

让松弛变量tight起来！

$$
\begin{array}{lrcrcrcl}
\text{maximize} & 3x_1 & + & x_2 & + & 2x_3 & & \\
\text{subject to} & & & & & & & \\
& x_1 & + & x_2 & + & 3x_3 & \le & 30 \\
& 2x_1 & + & 2x_2 & + & 5x_3 & \le & 24 \\
& 4x_1 & + & x_2 & + & 2x_3 & \le & 36 \\
& & & x_1, x_2, x_3 & & & \ge & 0
\end{array}
\quad \Rightarrow \quad
\begin{array}{rcrcrcrcr}
z & = & & & 3x_1 & + & x_2 & + & 2x_3 \\
x_4 & = & 30 & - & x_1 & - & x_2 & - & 3x_3 \\
x_5 & = & 24 & - & 2x_1 & - & 2x_2 & - & 5x_3 \\
x_6 & = & 36 & - & 4x_1 & - & x_2 & - & 2x_3
\end{array}
$$

$$
\begin{array}{rcrcrcrcr}
z & = & 27 & + & \frac{x_2}{4} & + & \frac{x_3}{2} & - & \frac{3x_6}{4} \\
x_1 & = & 9 & - & \frac{x_2}{4} & - & \frac{x_3}{2} & - & \frac{x_6}{4} \\
x_4 & = & 21 & - & \frac{3x_2}{4} & - & 5 \ldots & & \\
x_5 & = & 6 & - & \frac{3x_2}{2} & - & \ldots & &
\end{array}
$$

经过一次pivoting

x6：tight到0，交换为非基本变量
x1：将x6”tight”到0后，和x6交换，成为基本变量（松弛变量）

基本解：<9,0,0,21,6,0>, 目标值：27

$$\begin{aligned} z &= & & 3x_1 &+& x_2 &+& 2x_3 \\ x_4 &= 30 &-& x_1 &-& x_2 &-& 3x_3 \\ x_5 &= 24 &-& 2x_1 &-& 2x_2 &-& 5x_3 \\ x_6 &= 36 &-& 4x_1 &-& x_2 &-& 2x_3 \end{aligned}$$

$$\Rightarrow$$

$$\begin{aligned} z &= 27 + \frac{x_2}{4} + \frac{x_3}{2} - \frac{3x_6}{4} \\ x_1 &= 9 - \frac{x_2}{4} - \frac{x_3}{2} - \frac{x_6}{4} \\ x_4 &= 21 - \frac{3x_2}{4} - \frac{5x_3}{2} + \frac{x_6}{4} \\ x_5 &= 6 - \frac{3x_2}{2} - 4x_3 + \frac{x_6}{2} \end{aligned}$$

如何选择哪个非基本变量去tight某个松弛变量？

目标函数中c_j为正数且最大的那个x_j！

如何选择哪个约束条件去tight那个松弛变量？

约束函数中取x_j上界最小的那条，对应的x_i！

$$\begin{array}{lrcrcrcl}
\text{maximize} & 3x_1 & + & x_2 & + & 2x_3 & & \\
\text{subject to} & & & & & & & \\
 & x_1 & + & x_2 & + & 3x_3 & \leq & 30 \\
 & 2x_1 & + & 2x_2 & + & 5x_3 & \leq & 24 \\
 & 4x_1 & + & x_2 & + & 2x_3 & \leq & 36 \\
 & & x_1, x_2, x_3 & & & & \geq & 0
\end{array}$$

$\Rightarrow$

$$\begin{array}{rcrcrcrcr}
z & = & & & 3x_1 & + & x_2 & + & 2x_3 \\
x_4 & = & 30 & - & x_1 & - & x_2 & - & 3x_3 \\
x_5 & = & 24 & - & 2x_1 & - & 2x_2 & - & 5x_3 \\
x_6 & = & 36 & - & 4x_1 & - & x_2 & - & 2x_3
\end{array}$$

$$\begin{array}{rcrcrc}
z & = & \frac{111}{4} & + & \frac{x_2}{16} & - \\
x_1 & = & \frac{33}{4} & - & \frac{x_2}{16} & + \\
x_3 & = & \frac{3}{2} & - & \frac{3x_2}{8} & - \\
x_4 & = & \frac{69}{4} & + & \frac{3x_2}{16} & +
\end{array}$$

经过两次pivotir

经过一次pivoting

$$\begin{array}{rcrcrcrcr}
z & = & 28 & - & \frac{x_3}{6} & - & \frac{x_5}{6} & - & \frac{2x_6}{3} \\
x_1 & = & 8 & + & \frac{x_3}{6} & + & \frac{x_5}{6} & - & \frac{x_6}{3} \\
x_2 & = & 4 & - & \frac{8x_3}{3} & - & \frac{2x_5}{3} & + & \frac{x_6}{3} \\
x_4 & = & 18 & - & \frac{x_3}{2} & + & \frac{x_5}{2} & . &
\end{array}$$

三次pivoting后，无可替换了。

基本解：<8,4,0,18,0,0>, 目标值：28

PIVOT(N, B, A, b, c, v, l, e)

```
// Compute the coefficients of the equation for new basic variable x_e.
```

$$x_6 = 36 - 4x_1 - x_2 - 2x_3$$

$$x_1 = 9 - \frac{x_2}{4} - \frac{x_3}{2} - \frac{x_6}{4} .$$

```
// Compute the coefficients of the remaining constraints.
```

$$\begin{aligned} x_4 &= 30 - x_1 - x_2 - 3x_3 \\ &= 30 - \left(9 - \frac{x_2}{4} - \frac{x_3}{2} - \frac{x_6}{4}\right) - x_2 - 3x_3 \\ &= 21 - \frac{3x_2}{4} - \frac{5x_3}{2} + \frac{x_6}{4} . \end{aligned}$$

```
// Compute the objective function.
```

$$z = 3x_1 + x_2 + 2x_3$$

$$z = 27 + \frac{x_2}{4} + \frac{x_3}{2} - \frac{3x_6}{4}$$

```
// Compute new sets of basic and nonbasic variables.
```

return $(\widehat{N}, \widehat{B}, \widehat{A}, \widehat{b}, \widehat{c}, \widehat{v})$

每一轮Pivot得到的“新”基本解

Lemma 29.1

Consider a call to PIVOT(N, B, A, b, c, v, l, e) in which $a_{le} \neq 0$. Let the values returned from the call be $(\hat{N}, \hat{B}, \hat{A}, \hat{b}, \hat{c}, \hat{v})$, and let $\bar{x}$ denote the basic solution after the call. Then

1. $\bar{x}_j = 0$ for each $j \in \hat{N}$. 直接设置
2. $\bar{x}_e = b_l / a_{le}$.
3. $\bar{x}_i = b_i - a_{ie}\hat{b}_e$ for each $i \in \hat{B} - \{e\}$.

$$\boxed{x_i = \hat{b}_i - \sum_{j \in \hat{N}} \hat{a}_{ij} x_j \Longrightarrow \bar{x}_i = \hat{b}_i}$$

相应的算法步骤

3 $\quad \hat{b}_e = b_l / a_{le}$	8 $\quad$ **for** each $i \in B - \{l\}$
	9 $\quad\quad \hat{b}_i = b_i - a_{ie}\hat{b}_e$

SIMPLEX(A, b, c)

```
(N, B, A, b, c, ν) = INITIALIZE-SIMPLEX(A, b, c)
let Δ be a new vector of length n
while some index j ∈ N has c_j > 0
    choose an index e ∈ N for which c_e > 0
    for each index i ∈ B
        if a_ie > 0
            Δ_i = b_i / a_ie
        else Δ_i = ∞
    choose an index l ∈ B that minimizes Δ_i
    if Δ_l == ∞
        return "unbounded"
    else (N, B, A, b, c, ν) = PIVOT(N, B, A, b, c, ν, l, e)
for i = 1 to n
    if i ∈ B
        x̄_i = b_i
    else x̄_i = 0
return (x̄_1, x̄_2, ..., x̄_n)
```

如果有“初始可行解”……

Lemma 29.2

Given a linear program (A, b, c), suppose that the call to INITIALIZE-SIMPLEX in line 1 of SIMPLEX returns a slack form for which the basic solution is feasible. Then if SIMPLEX returns a solution in line 17, that solution is a feasible solution to the linear program. If SIMPLEX returns “unbounded” in line 11, the linear program is unbounded.

问题12:

Feasible就是每个变量有非负值，为什么？

循环不变量:

At the start of each iteration of the **while** loop of lines 3–12,

1. the slack form is equivalent to the slack form returned by the call of INITIALIZE-SIMPLEX,
2. for each $i \in B$, we have $b_i \geq 0$, and
3. the basic solution associated with the slack form is feasible.

问题13:

什么情况下Simplex算法不能终止?

“偶尔”一次pivot目标函数值不递增, 也不一定不终止。

问题14：

给定一个线性规划，为什么可以有很多不同的slack形式？又为什么只会有有限多种slack形式？这个问题与Simplex算法循环次数有什么关系？

线性规划的对偶

注意：此例$m=n$，“非典型”

$$
\begin{aligned}
\text{maximize} \quad & \sum_{j=1}^{n} c_j x_j \\
\text{subject to} \quad & \\
& \sum_{j=1}^{n} a_{ij} x_j \le b_i \quad \text{for } i = 1, 2, \ldots, m \\
& x_j \ge 0 \quad \text{for } j = 1, 2, \ldots, n
\end{aligned}
$$

$$
\begin{array}{rcrcrcl}
3x_1 & + & x_2 & + & 2x_3 & & \\
\\
x_1 & + & x_2 & + & 3x_3 & \le & 30 \\
2x_1 & + & 2x_2 & + & 5x_3 & \le & 24 \\
4x_1 & + & x_2 & + & 2x_3 & \le & 36 \\
& & x_1, x_2, x_3 & & & \ge & 0
\end{array}
$$

大小互换；
行列互换。

$$
\begin{aligned}
\text{minimize} \quad & \sum_{i=1}^{m} b_i y_i \\
\text{subject to} \quad & \\
& \sum_{i=1}^{m} a_{ij} y_i \ge c_j \quad \text{for } j = 1, 2, \ldots, n \\
& y_i \ge 0 \quad \text{for } i = 1, 2, \ldots, m
\end{aligned}
$$

$$
\begin{array}{rcrcrcl}
30y_1 & + & 24y_2 & + & 36y_3 & & \\
\\
y_1 & + & 2y_2 & + & 4y_3 & \ge & 3 \\
y_1 & + & 2y_2 & + & y_3 & \ge & 1 \\
3y_1 & + & 5y_2 & + & 2y_3 & \ge & 2 \\
& & y_1, y_2, y_3 & & & \ge & 0
\end{array}
$$

一个很强的对偶线性规划系统间的目标值特性：

Lemma 29.8 (Weak linear-programming duality)

Let $\bar{x}$ be any feasible solution to the primal linear program in (29.16)–(29.18) and let $\bar{y}$ be any feasible solution to the dual linear program in (29.83)–(29.85). Then, we have

$$\sum_{j=1}^{n} c_j \bar{x}_j \leq \sum_{i=1}^{m} b_i \bar{y}_i \ .$$

问题15：

对偶与线性规划的最优解有什么关系？

Corollary 29.9

Let $\bar{x}$ be a feasible solution to a primal linear program (A, b, c), and let $\bar{y}$ be a feasible solution to the corresponding dual linear program. If

$$\sum_{j=1}^{n} c_j \bar{x}_j = \sum_{i=1}^{m} b_i \bar{y}_i ,$$

then $\bar{x}$ and $\bar{y}$ are optimal solutions to the primal and dual linear programs, respectively.

关键问题是：等值的“对偶解”存在吗？

More precisely, suppose that the last slack form of the primal is

$$\begin{aligned} z &= v' + \sum_{j \in N} c'_j x_j \\ x_i &= b'_i - \sum_{j \in N} a'_{ij} x_j \quad \text{for } i \in B \; . \end{aligned}$$

Then, to produce an optimal dual solution, we set

$$\bar{y}_i = \begin{cases} -c'_{n+i} & \text{if } (n+i) \in N \; , \\ 0 & \text{otherwise} \; . \end{cases}$$

Theorem 29.10 (Linear-programming duality)
Suppose that SIMPLEX returns values $\bar{x} = (\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n)$ for the primal linear program (A, b, c). Let N and B denote the nonbasic and basic variables for the final slack form, let c' denote the coefficients in the final slack form, and let $\bar{y} = (\bar{y}_1, \bar{y}_2, \ldots, \bar{y}_m)$ be defined by equation (29.91). Then $\bar{x}$ is an optimal solution to the primal linear program, $\bar{y}$ is an optimal solution to the dual linear program, and

$$\sum_{j=1}^{n} c_j \bar{x}_j = \sum_{i=1}^{m} b_i \bar{y}_i \; . \tag{29.92}$$

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THE (DANTZIG) SIMPLEX METHOD FOR LINEAR PROGRAMMING

George Dantzig created a simplex algorithm to solve linear programs for planning and decision-making in large-scale enterprises. The algorithm's success led to a vast array of specializations and generalizations that have dominated practical operations research for half a century.

课外作业

- TC Ex.29.1: 4, 5, 6, 7, 9
- TC Ex.29.2: 2, 3, 6
- TC Ex.29.3: 2, 3, 5
- TC Ex.29.4: 2
- TC Prob.29: 1