3-2 Greedy Algorithms

(How to justify your greed?)

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"GREED IS GOOD."

"BUT"

"Greedy algorithms without proofs are considered wrong."

Proof: Inductive Exchange Argument

Theorem (Greedy-Choice Property)

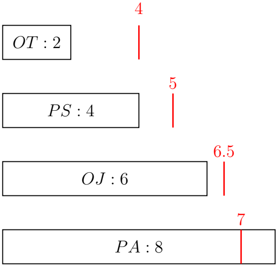
There is an optimal solution that makes the greedy choice.

Theorem (Optimal Substructure Property)

A problem exhibits optimal substructure if an optimal solution to the problem contains within it optimal solutions to subproblems.

Live with Your Deadlines (Additional Problem)

$$HW_i : (\textit{Length} : t_i, \ \textit{Deadline} : d_i)$$

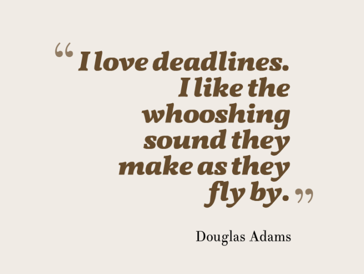


$$\textit{Finish time} : f_i = s_i + t_i$$

$$\textit{Lateness} : l_i = f_i - d_i \quad (0 \text{ if } f_i \le d_i)$$

$$\min \left(L \triangleq \max_i l_i\right)$$

What are your greedy strategies?



Shortest Job First: $\min t_i$

$t_1 = 1,\ d_1 = 100$

$t_2 = 10,\ d_2 = 10$

Urgentest Job First: $\min(d_i - t_i)$

$t_1 = 1,\ d_1 = 2$

$t_2 = 10,\ d_2 = 10$

Earliest Deadline First

Proof: Inductive Exchange Argument.

$$\mathcal{O} : J_1^*,\ J_2^*,\ \cdots,\ J_n^*$$

$$\mathcal{G} : J_1,\ J_2,\ \cdots,\ J_n, \quad (d_i \le d_j,\ \forall i \le j)$$

J_i : *a job with the earliest deadline*

$$d_{J_i} < d_{J_1^*}$$

$$d_{J_i} \le d_{J_i^*}$$

$\mathcal{O}$:	J_1^*		J_i^*		J_i		

$\mathcal{O}'$:	J_i		J_i^*		J_1^*		

$$l'_{J_i^*} = f'_{J_i^*} - d_{J_i^*} \le f_{J_i} - d_{J_i^*} \le f_{J_i} - d_{J_i} = l_{J_i}$$

Theorem

All schedules with no idle time and no inversions have the same maximum lateness.

Theorem (Optimal Substructure Property)

A problem exhibits optimal substructure if an optimal solution to the problem contains within it optimal solutions to subproblems.

$$L(s, S)$$

The optimal lateness obtainable from scheduling the jobs in S

which are available to start at the common start time s.

What is the first job to schedule?

$$L(s, S) = \min_{1 \le i \le n} \Big(\max \left(L(0, \{J_i\}), L(t_i, J \setminus \{J_i\}) \right) \Big)$$

$$J = \{J_1, J_2, J_3\}$$

$$t_1 = 6, t_2 = 2, t_3 = 2$$

$$d_1 = 2, d_2 = 7, d_3 = 8$$

$\mathcal{O} : J_1, J_3, J_2$

$\mathcal{O}' : J_2, J_3$

$L(0, \{J_1, J_2, J_3\}) = 4$

$L(6, \{J_2, J_3\}) = 2$

$$\mathcal{O}' \not\subset \mathcal{O}$$

$$\exists\ \mathcal{O} \triangleq J_1, J_2, J_3 : \mathcal{O}' \subset \mathcal{O}$$

Theorem

The greedy solution has no idle time and no inversions.

Theorem

There is an optimal schedule with no idle time.

Theorem

There is an optimal schedule with no idle time and no inversions.

By Exchange Argument.

Theorem

All schedules with no idle time and no inversions have the same maximum lateness.

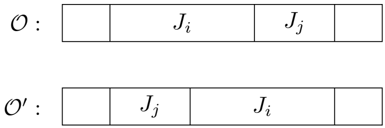
Theorem

There is an optimal schedule with no idle time and no inversions.

Immediate Inversion

$$d_j < d_i$$

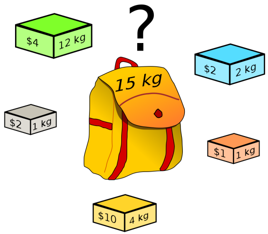
$$l'_j \leq l_j$$



$$l'_i = f_j - d_i < f_j - d_j = l_j$$

Fractional Knapsack Problem (Problem 16.2-1)

Prove that the fractional knapsack problem has the greedy-choice property.



$\exists$ *an optimal solution which contains the greedy choice.*

$$\frac{v_1}{w_1} \geq \frac{v_2}{w_2} \geq \cdots \geq \frac{v_n}{w_n}$$

As most item of $\dfrac{v_1}{w_1}$ as possible.

Change-Making Problem (Problem 16-1 (a))

25

10

5

1

Making change for v cents using the fewest number of coins.

$$\text{OPT}: v = 25 \times A^* + 10 \times B^* + 5 \times C^* + 1 \times D^*$$

$$\text{G}: v = 25 \times A + 10 \times B + 5 \times C + 1 \times D$$

$$(A^*, B^*, C^*, D^*) \textit{ vs. } (A, B, C, D)$$

$$A^* = A$$

$$\text{OPT}: v = 25 \times A^* + 10 \times B^* + 5 \times C^* + 1 \times D^*$$

Lemma (Properties of OPT)

(I) $B^* \leq 2$

(II) $C^* \leq 1$

(III) $D^* \leq 4$

(IV) $\neg(B^* = 2 \land C^* = 1)$

Theorem

$$10 \times B^* + 5 \times C^* + 1 \times D^* \leq 24$$

$$5 \times C^* + 1 \times D^* \leq 9$$

$$1 \times D^* \leq 4$$

$$(A^*, B^*, C^*, D^*) = (A, B, C, D)$$

Change-Making Problem (Problem 16-1 (c))

$$\{1, 3, 4\}, \qquad v = 6$$

G : $4, 1, 1$ *vs.* OPT : $3, 3$

Why does the previous proof fail here?

Change-Making Problem (Problem 16-1 (b))

$$c^0, c^1, c^2, \cdots, c^k, \quad c > 1, k \geq 1$$

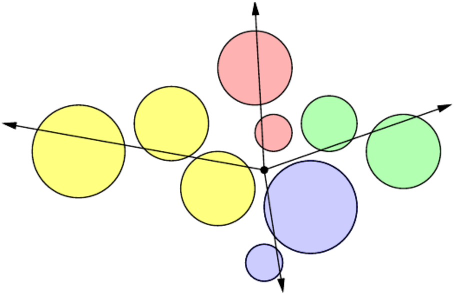
$$\mathsf{OPT}: v = \sum_{i=0}^{i=k} c^i a_i^*$$

$$\mathsf{G}: v = \sum_{i=0}^{i=k} c^i a_i$$

$$a_i^* = a_i$$

Canonical Coin Systems

Minimum Shots Problem (Additional Problem)



Assumption:

Possible to shoot a ray that does not intersect any balloons.


KEEP
CALM
MAKE
NO
ASSUMPTION


THANK YOU

Your opinon Matters

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