

- 书面作业讲解
  - CS第1.1节问题9、13
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# CS第1.1节 问题13

- ... on Day  $i$  you **receive** twice as many pennies as you did on Day  $i-1$ ... How many will you **have** on Day  $n$ ?
  - $1+2^1+\dots+2^{n-1}=2^n-1$
  - 加法原理

# CS第1.2节问题15

- In how many ways can we pair up all the members of the club?

- 检查简单的情况：n=2时，有3种

- 一般的情况： $(2n)!/(n!2^n)$

- 另一种思路：
$$\frac{\binom{2n}{2}\binom{2n-2}{2}\cdots\binom{2}{2}}{n!}$$

- ... we also determine who serves first for each pairing...

- 检查简单的情况：n=2时，有12种

- 一般的情况： $(2n)!/(n!)$

- 另一种思路：
$$\frac{\binom{2n}{2}\binom{2n-2}{2}\cdots\binom{2}{2}}{n!} \cdot 2^n$$

# CS第1.3节问题6

- $\left( \frac{n!}{n_1!n_2!\cdots n_k!} \right)$  是  $(x_1 + x_2 + \cdots + x_k)^n$  中  $x_1^{n_1} x_2^{n_2} \cdots x_k^{n_k}$  的系数。
- 如何直观解释？
  - 方法1:  $\binom{n}{n_1} \binom{n-n_1}{n_2} \cdots = \left( \frac{n!}{n_1!n_2!\cdots n_k!} \right)$
  - 方法2: 你能直接解释吗？

# CS第1.5节 问题8b

- How can you relate the number of ways of placing  $k$  red checkers and  $n-1$  black checkers in a row to the number of  $k$ -element multisets of an  $n$ -element set?



- $m(1)=1$
- $m(2)=2$
- $m(3)=0$
- $m(4)=3$
- $m(5)=2$
- $m(6)=0$

# CS第1.5节 问题8c

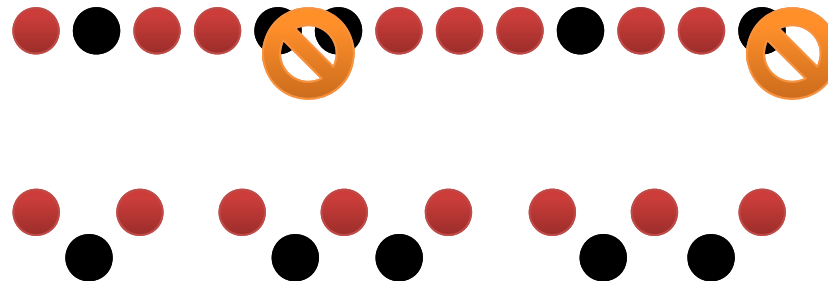
- How can you relate the choice of  $k$  items out of  $n+k-1$  items to the placement of red and black checkers?



$$\frac{(n+k-1)!}{(n-1)!k!} = \binom{n+k-1}{k}$$

# CS第1.5节 问题10

- How many solutions to the equation  $x_1 + x_2 + \dots + x_n = k$  are there with each  $x_i$  a **positive** integer?



# CS第1.5节 问题15

a.  $n^k$

b.  $n^{\underline{k}}$

c.  $\binom{n+k-1}{k}$

d.  $\binom{n}{k}$

e.  $n^{\underline{k}}$

f.  $n^k$

g.  $\binom{n}{k}$

h.  $\binom{n+k-1}{k}$

i.  $n^{\underline{k}}$

j.  $\binom{n+k-1}{k}$

k.  $n^{\underline{k}}$



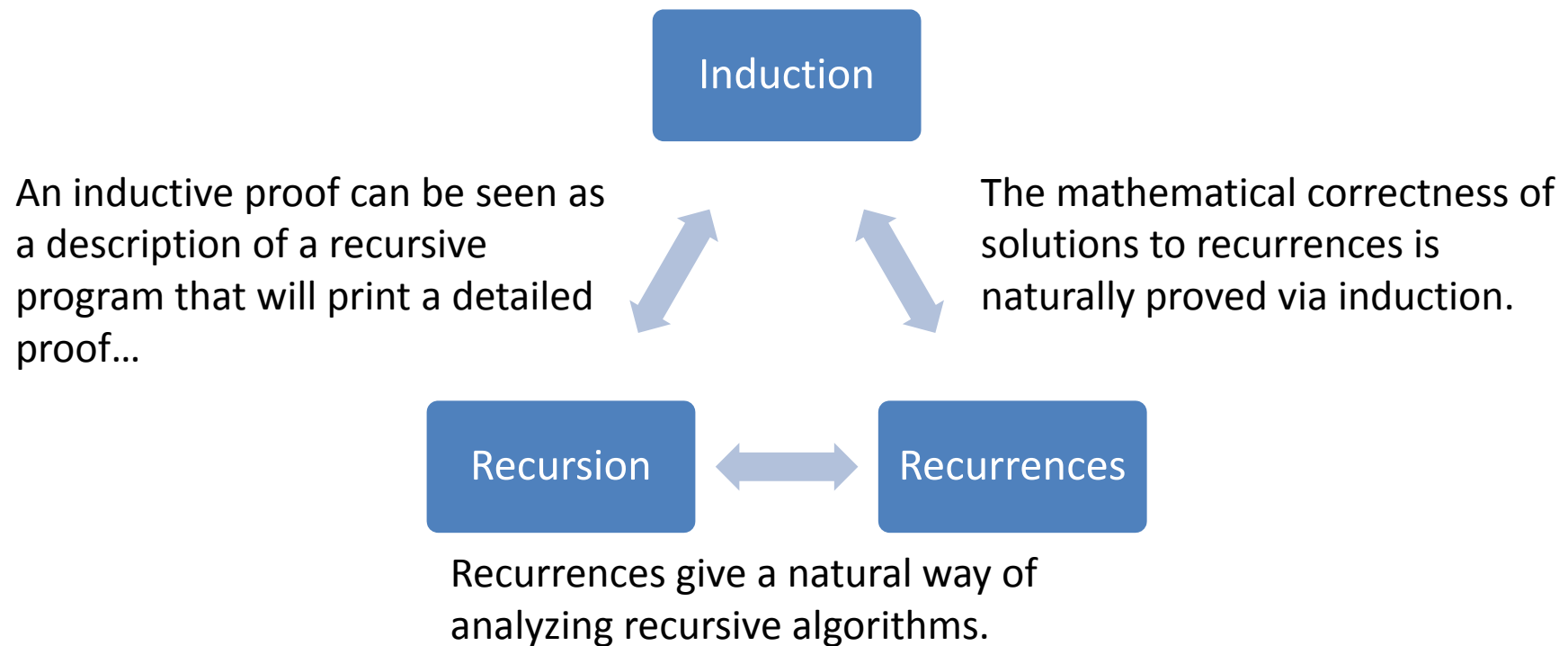
- 教材答疑和讨论  
– CS第4章

# 问题1: induction, recursion, recurrences

- 什么是induction?
- 什么是recursion?
- 什么是recurrences?

# 问题1: induction, recursion, recurrences (续)

- induction, recursion和recurrences之间分别有什么联系?



## 问题1: induction, recursion, recurrences (续)

- 什么是well-ordering principle?
- induction和well-ordering principle之间有什么联系?

## 问题1: induction, recursion, recurrences (续)

- 你怎么理解build a smaller case to a larger one (bottom-up) 和decompose the larger case into smaller ones (top-down)?
- 后者有哪些优势?
  - There are times when this way of thinking is clearly the best way to get a valid proof. Such examples occur throughout computer science.
  - “Building up” small cases into larger ones requires an additional step, namely showing that all larger cases can be created by using the construction given. It is often the case that a “building up” process constructs a proper subset of the possible cases.
  - With a top-down approach, there is no question about what our base case or base cases should be. The base cases are the ones where the recursive decomposition no longer works.

## 问题1: induction, recursion, recurrences (续)

- 你怎么理解structural induction?

## 问题2: first-order constant coefficient linear recurrence

$$T(n) = \begin{cases} rT(n-1) + g(n) & \text{if } n > 0 \\ a & \text{if } n = 0 \end{cases}$$

$$T(n) = r^n a + \sum_{i=1}^n r^{n-i} g(i).$$

- 如何猜测这个结论?
  - 递推展开
- 如何证明这个结论?
  - 数学归纳法

- 读完这一章以后，你发现哪个定理被用得最多？



# 问题3: the master theorem

- 请为这个引理绘制recursion tree

**Lemma 4.7** *Suppose that we have a recurrence of the form*

$$T(n) = aT(n/2) + n,$$

*where  $a$  is a positive integer and  $T(1)$  is nonnegative. Thus we have the following big-Theta bounds on the solution.*

1. *If  $a < 2$  then  $T(n) = \Theta(n)$ .*
2. *If  $a = 2$  then  $T(n) = \Theta(n \log n)$*
3. *If  $a > 2$  then  $T(n) = \Theta(n^{\log_2 a})$*

- 并证明其结论

# 问题3: the master theorem (续)

- 请为这个定理绘制recursion tree

**Theorem 4.9** *Let  $a$  be an integer greater than or equal to 1 and  $b$  be a real number greater than 1. Let  $c$  be a positive real number and  $d$  a nonnegative real number. Given a recurrence of the form*

$$T(n) = \begin{cases} aT(n/b) + n^c & \text{if } n > 1 \\ d & \text{if } n = 1 \end{cases}$$

*in which  $n$  is restricted to be a power of  $b$ ,*

1. *if  $\log_b a < c$ ,  $T(n) = \Theta(n^c)$ ,*
2. *if  $\log_b a = c$ ,  $T(n) = \Theta(n^c \log n)$ ,*
3. *if  $\log_b a > c$ ,  $T(n) = \Theta(n^{\log_b a})$ .*

- 并证明其结论

# 问题3: the master theorem (续)

- 如何从定理4.9推广到定理4.10?

**Theorem 4.9** Let  $a$  be an integer greater than or equal to 1 and  $b$  be a real number greater than 1. Let  $c$  be a positive real number and  $d$  a nonnegative real number. Given a recurrence of the form

$$T(n) = \begin{cases} aT(n/b) + n^c & \text{if } n > 1 \\ d & \text{if } n = 1 \end{cases}$$

in which  $n$  is restricted to be a power of  $b$ ,

1. if  $\log_b a < c$ ,  $T(n) = \Theta(n^c)$ ,
2. if  $\log_b a = c$ ,  $T(n) = \Theta(n^c \log n)$ ,
3. if  $\log_b a > c$ ,  $T(n) = \Theta(n^{\log_b a})$ .

**Theorem 4.10** Let  $a$  and  $b$  be positive real numbers with  $a \geq 1$  and  $b > 1$ . Let  $T(n)$  be defined for powers  $n$  of  $b$  by

$$T(n) = \begin{cases} aT(n/b) + f(n) & \text{if } n > 1 \\ d & \text{if } n = 1. \end{cases}$$

Then

1. if  $f(n) = \Theta(n^c)$  where  $\log_b a < c$ , then  $T(n) = \Theta(n^c) = \Theta(f(n))$ .
2. if  $f(n) = \Theta(n^c)$ , where  $\log_b a = c$ , then  $T(n) = \Theta(n^{\log_b a} \log_b n)$
3. if  $f(n) = \Theta(n^c)$ , where  $\log_b a > c$ , then  $T(n) = \Theta(n^{\log_b a})$ .

# 问题3: the master theorem (续)

- 为什么the master theorem中用 $n^c$ ?

$$a^i(n/b^i)^c = n^c \left(\frac{a^i}{b^{ci}}\right) = n^c \left(\frac{a}{b^c}\right)^i \quad \Rightarrow \quad \sum_{i=0}^{\log_b n} n^c \left(\frac{a}{b^c}\right)^i = n^c \sum_{i=0}^{\log_b n} \left(\frac{a}{b^c}\right)^i$$

- 如果是别的函数怎么办?

$$T(n) = \begin{cases} 2T(n/2) + n \log n & \text{if } n > 1 \\ 1 & \text{if } n = 1 \end{cases}$$

# 问题4: recurrences and selection

- 第4.6节与前几节有什么联系?

```
Select1(A,i,n)  
(selects the  $i$ th smallest element in set  $A$ , where  $n = |A|$  )  
(1)  if ( $n = 1$ )  
(2)      return the one item in  $A$   
(3)  else  
(4)       $p = \text{MagicMiddle}(A)$   
(5)      Let  $H$  be the set of elements greater than  $p$   
(6)      Let  $L$  be the set of elements less than or equal to  $p$   
(7)      if ( $i \leq |L|$ )  
(8)          Return  $\text{Select1}(L, i, |L|)$   
(9)      else  
(10)         Return  $\text{Select1}(H, i - |L|, |H|)$ .
```

- 你能解释MagicMiddle的基本思路吗?

```
MagicMiddle(A)  
(1)  Let  $n = |A|$   
(2)  if ( $n < 60$ )  
(3)      use sorting to return the median of  $A$   
(4)  else  
(5)      Break  $A$  into  $k = n/5$  groups of size 5,  $G_1, \dots, G_k$   
(6)      for  $i = 1$  to  $k$   
(7)          find  $m_i$ , the median of  $G_i$  (by sorting)  
(8)      Let  $M = \{m_1, \dots, m_k\}$   
(9)      return  $\text{Select1}(M, \lceil k/2 \rceil, k)$ .
```

