# HW01

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**Problem 1**

**(a)** The treatment $D$: the diet provided in the university dining halls.

The potential outcome $Y(1)$: the weight of the student treated by the university dining halls in June.

The potential outcome $Y(0)$: the weight of the student treated by himself in June.

**(b)** Suppose the university focuses average treatment effects, the casual parameters that the university hopes to know is

$$\mathbb{E}[Y(1) - Y(0)] \tag{1}$$

**(c)** Not right. There is no control treatment. So there might be confounders affect the weight which offset the dining hall diet but not be considered. That is, the treatment assignment mechanism is not known. Moreover, there is no stability assumptions, so the individuals might also affect each other which might offset the effect.

**(d)** Not right. The reason is similar to part(c), there is no control treatment and necessary assumptions.

**Problem 2**

**(a)** The proof can be shown as:

$$
\begin{aligned}
\tau_{\text{ATT}} &= \mathbb{E}[Y(1) - Y(0)|D = 1] \\
&= \mathbb{E}[Y(1)|D = 1] - \mathbb{E}[Y(0)|D = 1] \\
&= \mathbb{E}[Y|D = 1] - \mathbb{E}[\mathbb{E}[Y(0)|X]|D = 1] \\
&= \mathbb{E}[Y|D = 1] - \mathbb{E}[\mathbb{E}[Y(0)|D = 0, X]|D = 1](unconfoundedness) \\
&= \mathbb{E}[Y|D = 1] - \mathbb{E}[\mathbb{E}[Y|D = 0, X]|D = 1]
\end{aligned} \tag{2}
$$

□

**(b)** A weaker condition $e(X) = \mathbb{P}(D = 1|X) \in [0, 1)$ is possible to be used. That's because in our proof, what we need is $\mathbb{E}[Y(0)|D = 0, X]$ is meaningful which means the individuals surely assigned into the treated group can't exist, but it doesn't matter if some individuals are surely assigned into the control group.

**(c)** Proof:

$$
\begin{aligned}
\tau_{\text{ATT}} &= \mathbb{E}[Y|D = 1] - \mathbb{E}[\mathbb{E}[Y|D = 0, X]|D = 1] \\
&= \mathbb{E}[\mathbb{E}[Y(1)|X]|D = 1] - \mathbb{E}[\mathbb{E}[Y|D = 0, X]|D = 1] \\
&= \mathbb{E}[\mathbb{E}[Y(1)|D = 1, X]|D = 1] - \mathbb{E}[\mathbb{E}[Y|D = 0, X]|D = 1] \\
&= \mathbb{E}[\mathbb{E}[Y|D = 1, X] - \mathbb{E}[Y|D = 0, X]|D = 1]
\end{aligned}
$$

The difference between identification formula for ATE and ATT is that ATE is under no condition, but ATT is under the condition of $D = 1$. That is, $\tau_{\text{ATE}}$ directly compares the treated group and the control group, while $\tau_{\text{ATT}}$ only compares the different outcomes in treated group.

**(d)** The proof can be shown as:

$$
\begin{aligned}
\tau_{\text{CATE}}(x_1) &= \mathbb{E}[Y(1) - Y(0)|X_1 = x_1] \\
&= \mathbb{E}[Y(1)|X_1 = x_1] - \mathbb{E}[Y(0)|X_1 = x_1] \\
&= \mathbb{E}[Y|D = 1, X_1 = x_1] - \mathbb{E}[Y|D = 0, X_1 = x_1](unconfoundedness)
\end{aligned}
$$

We've known overlap assumption is valid, so $\mathbb{E}[Y|D=1, X_1=x_1]$ and $\mathbb{E}[Y|D=0, X_1=x_1]$ are meaningful and observable, so $\tau_{\text{CATE}}(x_1)$ are identifiable.

**Problem 3**

**(a)** No, since there exists $T \to M_1 \to M_2 \to Y$ which is a chain, and $T \leftarrow W_1 \leftarrow W_2 \to W_3 \to Y$ which is a fork.

**(b)** No, there exists a chain $T \to M_1 \to M_2 \to Y$.

**(c)** Yes, when $\{W_2, M_1\}$ is conditioned, both the chain and the fork would be blocked.

**(d)** Yes, when $\{W_1, M_2\}$ is conditioned, the result is same as in **(c)**.

**(e)** No, in path $T \to X_1 \to X_2 \leftarrow X_3 \leftarrow Y$, $X_2$ is a collider of $T, Y$. Conditioning on collider $X_2$ will lead to dependence between $T$ and $Y$.

**(f)** Yes, according to **(d)**, when $\{W_1, M_2\}$ is conditioned on, the chain and the fork are blocked. Moreover, $X_2 \leftarrow X_3 \leftarrow Y$ would be blocked if $\{X_2, X_3\}$ is conditioned on. Then $T$ and $Y$ would be independent.

**(g)** $p(y|\text{do}(t)) = \sum_{w_2} p(y|t, w_2)p(w_2)$

**Problem 4**

**(a)** Yes, since **(1)** $\{U_1, U_2\}$ does not have any descendants of $D$ and **(2)** the path between $D$ and $Y$ which contains an arrow to $D$ is blocked by $U_2$.

**(b)** $P(Y|\text{do}(d)) = \sum_x P(Y|\text{do}(d), x)P(x|\text{do}(d))$

**(c)** The backdoor path between $D$ and $Y$ is not blocked by $\varnothing$ as the path $Y \to U_1 \to X \to D$ exists. Controlling on $X$ is not sufficient to remove all non-causal association between $D$ and $Y$ since $X$ is a collider.

**(d)** Controlling on $\{X, Z_2\}$ or $\{X, Z_1\}$ should be sufficient to identify the average treatment effect.