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T4 ________________	**A**	F4 ________________

2018
MCM/ICM
Summary Sheet

Analysis of Radio Propagation with Progressive Models

Summary

Nowadays, HF radio waves play a significant part in many aspects of wireless communication. However, HF wave attenuation makes the communication unstable under certain circumstances. We build two models to decide the range HF radio wave can cover under various of geographical environments.

First, we introduce Classical Geometrical Optics Model (CGO). The relative scale of wave length and distance allows us to use the basic rules of geometrical optics. We start with single hop attenuation, considering the reflections off ionosphere, reflection off the bottom plane and propagation, then iterate to get the result of multi-hop situation.

Secondly, we build Scaling Simulation of Radio Propagation Model (SSRP). We solve Maxwell's equations directly given boundary conditions numerically using Finite Element Method. Owing to the relative scale, we do a linear scaling to make the 'elements' few enough to compute. Then we visualize the power distribution of signal by simulation software COMSOL.

Thirdly, we analyze our models and choose a better one to carry on our further study. Different geographical environments are examined, including: smooth and turbulent oceans, smooth and rough terrains and mountainous areas. We also modify our transmitting antenna to simulate the time that the same multi-hop can last when a ship sails across the ocean. Sensitivity analysis is made.

Throughout our paper, our two models precisely coincide with each other on the attenuation problem. We provide a new method to tackle the great difference in relative scale and reasonably simplify the complicated problem. This makes our model more adaptable to a wider range of conditions. In addition, our figure and visualization offers an intuitive impression, helping us further understand the law of radio propagation.

Keywords: Geometrical Optics; Maxwell's Equations; Finite Element Method; COMSOL;

Analysis of Radio Propagation with Progressive Models

November 24, 2018

Contents

1 Introduction

1.1 Problem Statement

Date back to 1867, Scottish mathematical physicist James Clerk Maxwell first noticed wavelike properties of light and similarities in electrical and magnetic observations. Since then the study of radio wave has been strongly emphasized. Radio waves are a type of electromagnetic radiation with frequencies as high as 300 GHz to as low as 3 kHz and traveling at the speed of light. Nowadays radio waves have been widely used in aspect of wireless communication and make a great difference to our life. Figure 1 shows the propagation of radio wave.

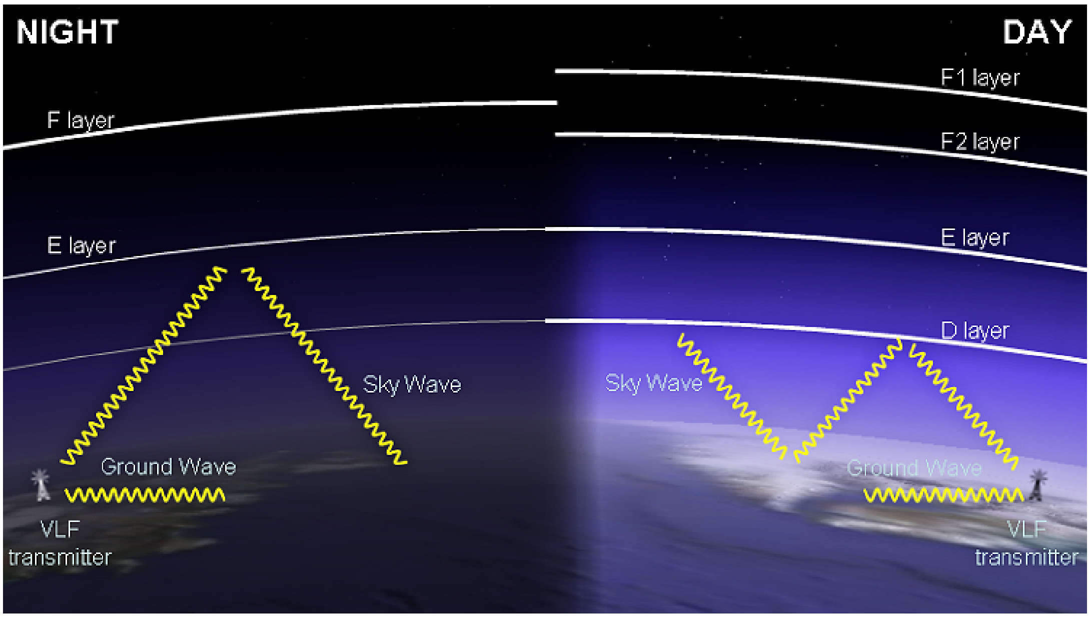


Figure 1: Propagation of Radio Wave[1]

The study of radio waves propagation, how radio waves move in free space and over the surface of the Earth, is vitally significant in the design of practical radio systems. Radio waves passing through different environments may experience reflection, polarization, refraction, diffraction and absorption. Radio waves of different frequencies experience different phenomenon in the atmosphere of Earth, making certain radio bands much more useful for specific purposes than other radio bands.

Due to the attribute of traveling long distance, high frequencies radio waves (HF) are adopted in the wireless communication across the ocean. For the frequencies below maximum usable frequency (MUF), HF radio waves not only reflect off the sea surface, but reflect off the ionosphere back to the earth as well. However, the reflection process of the radio wave will bring in loss and the radio wave may not maintain useful signal integrity. Hence, a quantitative model for this wave reflection off the ocean should be put forward to research the regular pattern of the transmission.

[1] https://hummap.wordpress.com/2013/08/10/the-physics-of-the-hum-a-primer-on-propagation-of-very-low-frequency-vlf-radio-waves-for-the-layperson-part-one/

We are going to first propose mathematical models to simulate the reflection and propagation process under different circumstance. For the reflection off ocean surface, how many hops it will take for the strength of radio wave to fall down to a threshold are supposed to be given. We will also provide the difference between propagation of mountainous or rugged terrain with smooth terrain. Then we are going to alter our model to better fit the situation of shipboard receiver and give the range of area remain in communication.

1.2 Related Work

Ahead of us, quite a few researches has been conducted on the theory of propagation of radio waves and the reflection over the ocean surface. The propagation process and how the attributes of radio waves change in different environments [1, 2, 3] are already fully discussed.

For reflection off the ocean, scientists have researched on the plain surface [4] and the rough surface [5] and given their numerical solution based on theory of classical electromagnetism in the same year. Recently, physicists and oceanographers have been working together to simulate the reflection and propagation process more precisely. A majority of state-of-the-art findings focus on more complex and accurate representation of the sea surface [6, 7] and use advanced simulation technology to get a more precise numerical solution [8].

1.3 Our Work

We discussed HF radio propagation with two progressive models.

In CGO model, the whole process of the radio propagation process can be divided into a series of reflections and propagation in straight line in the air. Hence, we first investigate a single reflection thoroughly and then combine them up to implement our model for multi-hop. For every single process of reflection, law of classical geometrical optics are applied.

In SSRP model, actually, we do not care the characteristic of radio wave after every reflection. Instead, we can acquire the whole distribution in the space based on Maxwell's equations together with boundary conditions. The problem then transforms into solving a partial differential equation. We tackle this with Finite Element Method and state-of-the-art simulation software COMSOL[2].

After discussion based on two different models, we evaluate them carefully and pick out the better one to carry on our work based on the simulation results. We then apply the model to different situations including calm ocean surface, turbulent ocean surface, smooth terrain, mountainous and rugged terrain. A distribution of power of signal above the sea will be given to find out how long the ship can remain in communication. And we will make full assessment of our model in these different situations. In the end, we'll analyze our model, giving the strengths and weakness of our model. Figure 2 shows the framework of our work.

[2]https://www.comsol.com/

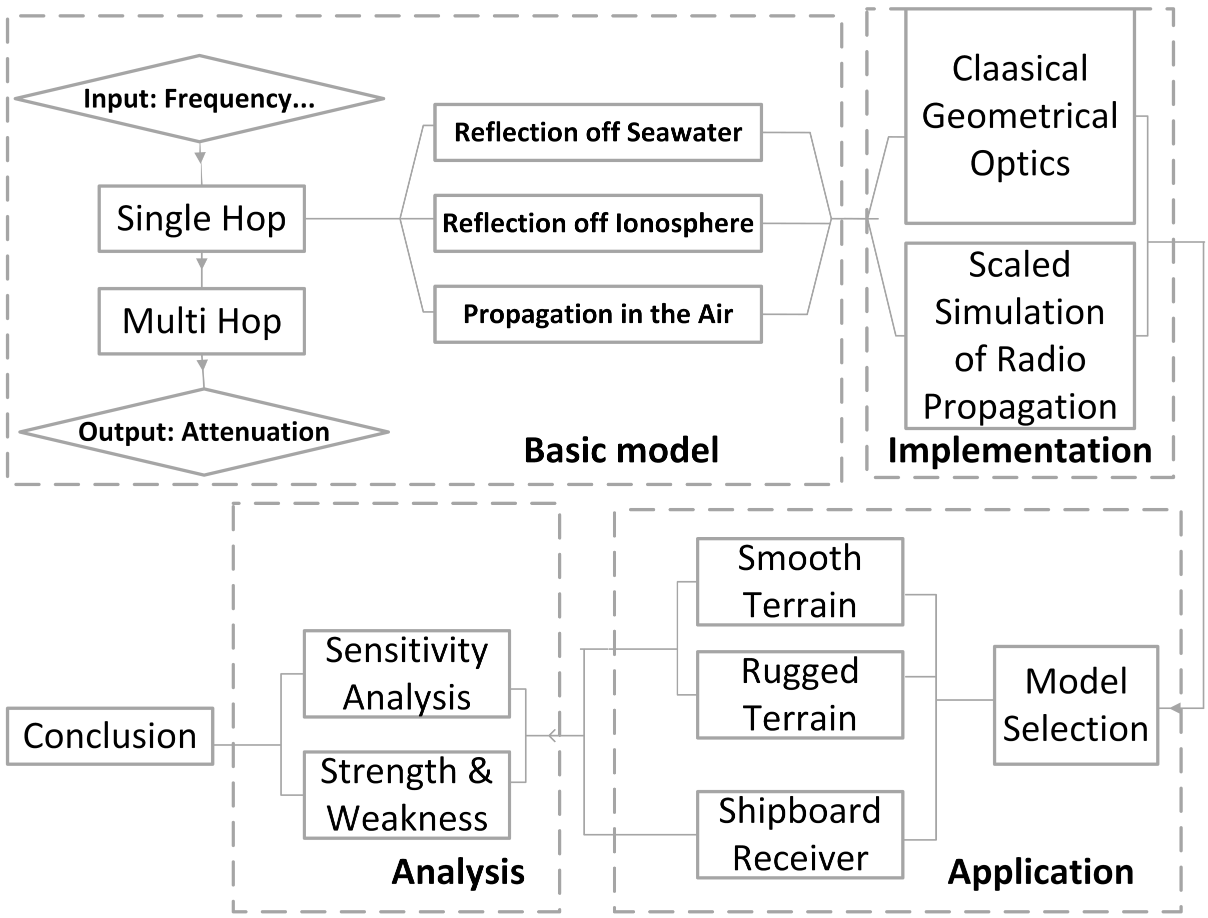


Figure 2: Framework of Our Work

2 Assumptions and Notations

2.1 Assumptions

1. We consider the propagation of radio wave is in a straight line in CGO model rather than a sphere as most signal source have a fixed launch direction.

2. When analyzing the reflection above the sea, we consider the sea to be motionless as the velocity of wave is negligible to the light speed.

3. The curvature of the earth is ignored because this is not going to influence the core of both of our models.

4. The surface of the sea and the ionosphere are parallel regardless of the motion of water or air in ionosphere.

5. The temperature and humidity do not change in the propagation path, so the maximum usable frequency (MUF) and loss constant β do not change.

2.2 Notations

Table 1 lists the symbols and notations of this paper, some of which will be defined latter in the following sections.

Table 1: Notations.

Symbol	Definition	Unit
$\mathbf{E}$	Electric field	N/C
$\mathbf{B}$	Magnetic B-field	T
$\mathbf{H}$	Magnetic H-field	A/m
$\mathbf{A}$	Magnetic potential	V·s/m
ϵ_0	Vacuum permittivity	F/m
μ_0	Vacuum permeability	H/m
μ_r	Equivalent relative permittivity	1
N	Density of electronic	C/m^3
f	Frequency of the radio wave	Hz
n_n	Refractive index of layer n in the ionosphere	1
λ	Wavelength of the radio wave	m
σ	Conductivity of ionosphere	S/m
γ	The number of collisions	1
L	Attenuation	dB/km
i	The number of hops	1
S	Salinity of seawater	g/kg
R_a	Roughness of a surface	m
β	Attenuation coefficient of a certain material	m^{-1}

3 Classical Geometrical Optics Model (CGO)

3.1 Theory of HF Communication

3.1.1 HF Communication

Generally speaking, HF communication consists of skywave propagation and ground wave propagation. However, only skywave can travels a long distance. Since we are now dealing with the problem mainly concerning about HF propagation at a great distance, we will only discuss about skywave.The skywave propagation relies on ionosphere, so we will deal with ionosphere more precisely in the latter part of our paper.

To determine when the propagation diminishes, we need to calculate signal to noise ratio (SNR).The definition of SNR is as follows:

$$\mathrm{SNR_{dB}} = 10\log_{10}\left(\frac{P_{\mathrm{signal}}}{P_{\mathrm{noise}}}\right),$$

where:

- P_{signal} is the power of a signal;
- P_{noise} is the power of radio noise.

Therefore,we need to determine the power of radio noise.

Owing to the fact that frequencies of most shortwaves are below 20 MHz, when it comes to radio noise, cosmic noise and industry disturbance can be neglected. Hence we can simplify the radio noise to atmospheric noise, which is primarily lightning discharges in thunderstorms. The atmospheric noise varies according to the change of seasons and diurnal variation. There is more disturbance in summer than in winter. During the day, the disturbance is more severe than night. Because of its sensitivity, we will make careful assumptions on the time and latitude our experiments carry out.

3.1.2 Ionosphere

The ionosphere is the ionized part of Earth's upper atmosphere from about 60 km (37 mi) to 1,000 km (620 mi) altitude [9]. The ionosphere is ionized by solar radiation. It plays a significant role in atmospheric electricity and forms the inner edge of the magnetosphere. It is rather important because it influences radio propagation to distant places on the Earth a lot. The ionosphere consists of four distinct layers during the day and two distinct layers at night. The layers are shown in 3

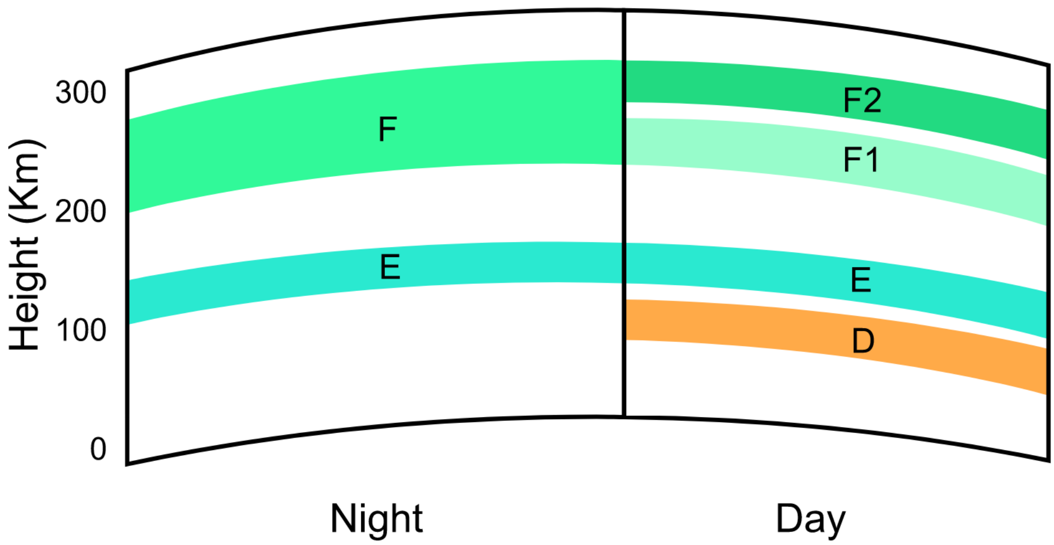


Figure 3: Layers of Ionosphere[3]

At night the F layer is the only layer of significant ionization present, while the ionization in the E and D layers is extremely low. During the day, the D and E layers become much more heavily ionized, as does the F layer, which develops an additional,

[3]https://en.wikipedia.org/wiki/Ionosphere

weaker region of ionization known as the F1 layer. The F2 layer persists by day and night and is the main region responsible for the refractions and reflections of radio waves.

3.2 Model of Multi-Hop HF Radio

Due to many electrically charged atoms and molecules in the ionosphere, the light will be refracted and reflected, the path of the radio wave resembles parabola curve. However, the thickness of the D layer in the day can be ignored, so we can analyze how electromagnetic waves propagate through the ionosphere by geometric optics. Since the ionosphere is a plasma, it can be shown that the refractive index is less than unity. Hence, the electromagnetic "ray" is bent away from the normal rather than toward the normal as would be indicated when the refractive index is greater than unity. It can also be shown that the refractive index of a plasma, and hence the ionosphere, is frequency-dependent. The reflections of radio wave in during the day and at night are shown in the Figure 4.

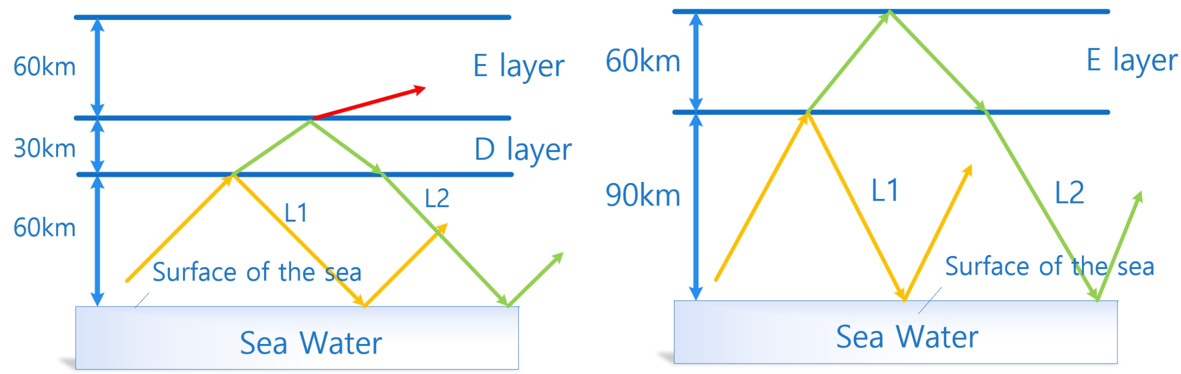


Figure 4: Propagation of Radio Wave

As most communication by HF radio above the sea is during the day and it is actually more complicated to research the radio wave during the day. We take the HF radio during the day to carry on our work. Because the characteristics of the ionosphere are determined by time in a day and the season, we make additional assumptions as follow:

- The time we research is 12 am in the summer with a temperature of $25°C$.
- The location we research in is 45 degrees north latitude.
- No sudden ionospheric disturbance (SID), geomagnetic storm or lightning occurrence.

3.2.1 Single Hop of Calm Ocean

Every single hop consists of 3 parts: propagation in the open air, reflection off the ocean, reflection off the ionosphere. In fact, the transmission loss in reflection originates from the refraction process. First let us research the reflection part.

the equivalent relative permittivity of ionosphere is

$$\varepsilon_{\mathrm{r}} = 1 - \frac{Ne^2}{m\varepsilon_0\omega^2}. \tag{1}$$

Apply the value in Equation 1, we have

$$n = \sqrt{\varepsilon_{\mathrm{r}}} = \sqrt{1 - 80.8\frac{N}{f^2}}, \tag{2}$$

Although we assume that the radio wave does not refract into the layer of ionosphere when we research the whole propagation process, we take it into consideration when calculating the propagation loss. A layer of ionosphere can be cut into infinite slice of charged particles, for the angle of incidence θ_n of slice n, we have $\theta_0 \sin\theta_0 = \theta_1 \sin\theta_1 = ... = \theta_n \sin\theta_n$, thus angle of incidence follow Equation 3

$$\sin\theta_n = n_n = \sqrt{1 - 80.8\frac{N_n}{f^2}}. \tag{3}$$

In the propagation process, the total absorption L can be calculated by Equation 4

$$L = \int_l \beta \cdot dl \tag{4}$$

where the loss constant β can be defined as

$$\beta = \frac{2\pi}{\lambda}\sqrt{\frac{1}{2}\left[\sqrt{\epsilon_r^2 + (60\lambda\sigma)^2} - \epsilon_r\right]}. \tag{5}$$

The equivalent relative permittivity and conductivity of ionosphere can be calculated by

$$\epsilon_r = 1 - \frac{Ne^2}{\epsilon_0 m(\omega^2 + \gamma^2)},\ \sigma = \frac{Ne^2\gamma}{m(\omega^2 + \gamma^2)}.$$

For $\omega^2 \gg \gamma^2, \epsilon_r^2 \gg (60\lambda\sigma)^2$, then

$$\beta \approx \frac{60\pi\sigma}{\sqrt{\epsilon_r}} = \frac{60\pi Ne^2\gamma}{\sqrt{\epsilon_r}m(\omega^2 + \gamma^2)} \approx \frac{60\pi Ne^2\gamma}{\sqrt{\epsilon_r}m\omega^2}, \tag{6}$$

then the loss constant L equals to

$$L = \int_l \frac{60\pi Ne^2\gamma}{\sqrt{\epsilon_r}m\omega^2} dl. \tag{7}$$

Considering our problem, the attenuation coefficient of the ionosphere, air and sea water is different. Assume the path of radio wave in the air, ionosphere and sea water are l_0, l_1 and l_2.For a whole single hop, the total attenuation can be defined as

$$L_{total} = \int_{l_1} \frac{60\pi N_1 e^2\gamma_1}{\sqrt{\epsilon_r}m\omega^2} dl_1 + \int_{l_2} \frac{60\pi N_2 e^2\gamma_2}{\sqrt{\epsilon_r}m\omega^2} dl_2 + 2\int_{l_0} \beta \cdot dl_0. \tag{8}$$

Referring to the data conducted by researchers [10], we make further calculation with radio wave of 10 MHz and launch angle of $5.7°$ above the surface of a calm ocean. According the assumptions we made before, the background noise equals to 30dB. Figure 5 shows the absorption constant calculated by different researchers [11, 12], among which we choose the results of Sen Wyller as it is more widely adopted by other researchers.

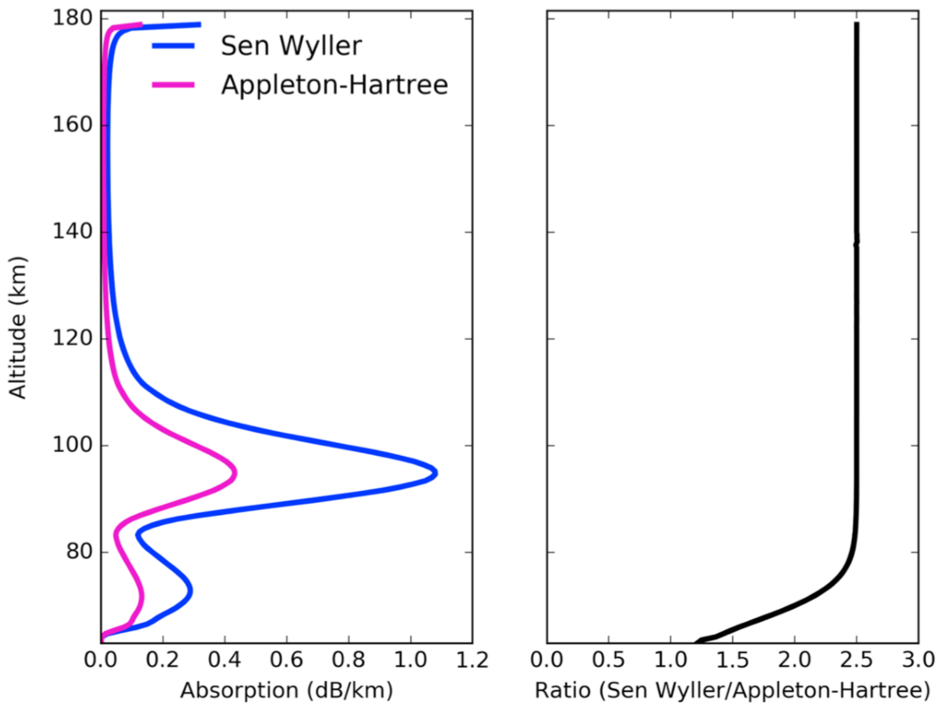


Figure 5: Absorption Constant of Different Research

For the parameter of sea water, we search the calculation formula and we get the value of them with a temperature of $25°C$, note that

$$\epsilon_0 = 76.54,\ \mu_0 = 1,\ \sigma_{sw} = 4.8\mathrm{S}\cdot\mathrm{m}^{-1}.$$

Substitute all values into Equation 8, we can get the total attenuation in a single hop is

$$L_{total} = 130.264\ dB.$$

3.2.2 Single Hop of Turbulent Ocean

When the ocean becomes turbulent, the change of parameter of classical optics model lies in the change of physical and chemical properties of the ocean surface. Concretely, the change of electrical conductivity and the change of dielectric constant owing to the change of salinity of sea water which is caused by turbulence. The changes will influence the propagation attenuation.

The dielectric constant of seawater follows Debye relaxation law [13]

$$\varepsilon(S,T,\omega) = \varepsilon_\infty(S,T) + \frac{\varepsilon_1(S,T) - \varepsilon_\infty}{1 - i\omega\tau(S,T)} - \frac{i\sigma(S,T)}{\omega\varepsilon_0}$$

where:

- ε_∞ is the dielectric constant at infinite frequencies;
- $\sigma(S, T)$ is the conductivity of water [in S/m].

When the sea becomes turbulent, we use the average salinity $\bar{S}$ of seawater to substitute the salinity of surface $S_{surface}$, which will lead to the change of propagation attenuation. The parameter of turbulent seawater is

$$\epsilon_0 = 76.54,\ \mu_0 = 1,\ \sigma_{sw} = 5.3\mathrm{S} \cdot \mathrm{m}^{-1}.$$

Substitute all new values of reflection off surface of a turbulent ocean into Equation (8) we can get the total attenuation in a single hop is

$$L'_{total} = 142.107\ dB.$$

3.2.3 Multi-hop of Calm Ocean

Multi-hop situation is the iteration of single-hop model. Because the height of the source point can be ignored comparing to the height of ionosphere, every single hop have the same starting height and the same launch angle. The difference between every single hop is the starting power of radio wave. To maintain a usable signal-to-noise ratio higher than the threshold of 10 dB. The power of the radio wave must follow: $P_{signal} \geq 10P_{noise}$. As the starting power of the HF constant-carrier signal is 100 watt, also can be described as 50 dBm. The power of HF radio wave follows

$$L_i = 10\log_{10} P_{signal_i} - iL_{total}.$$

To maintain the signal-to-noise ratio(SNR) when additional reflection take place off calm oceans, the maximum the number of hops

$$i_{max} = 2\ ,$$

that is to say, with in first two hops, the signal can still maintain a usable SNR. However, signal fails to convey accurate information when there is three hops or more.

Actually, researchers have already put forward an estimation formula[14] to calculate the field strength of a radio signal, and the field strength can take a conversion to power. The formula can be expressed as

$$E = 136.6 + P_t + G_t + 20\log f - L_{total}. \tag{9}$$

where:

- E is the field strength in dB above 1 muV/m
- P_t is the transmitter power in dB relative to 1 kW
- G_t is the antenna gain of the transmitting antenna in dB

The formula take antenna gain into consideration, whose influence is minor in the whole propagation attenuation calculation process. Based on the estimation, we can also get our maximum times of hop is $i_{max} = 2$.

4 Scaling Simulation of Radio Propagation Model (SSRP)

4.1 Theory of Maxwell's Equations

Radio waves are a type of electromagnetic radiation with wavelengths in the electromagnetic spectrum longer than infrared light. So radio waves obey every law an electromagnetic wave obeys. To fully understand the behavior of radio waves, we first need to dig into Maxwell's theory.

4.1.1 Electromagnetic Waves in Vacuum

In regions of spaces where there is no charge or current, Maxwell's equations is shown in Equation (10)

$$\begin{cases} \nabla \cdot \mathbf{E} = 0, \quad \nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}, \\ \nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{B} = \mu_0 \epsilon_0 \dfrac{\partial \mathbf{E}}{\partial t}. \end{cases} \tag{10}$$

They constitute four first-order partial equations for $\mathbf{E}$ and $\mathbf{B}$. They can be decoupled by applying curl to the two equation on the right.

$$\nabla \times (\nabla \times \mathbf{E}) = -\mu_0 \epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2},$$
$$\nabla \times (\nabla \times \mathbf{B}) = -\mu_0 \epsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2}.$$

In the meantime, the first equality in both two equations together with the relation $\nabla \cdot \mathbf{E} = 0$ and $\nabla \cdot \mathbf{B} = 0$ implies that

$$\nabla^2 \mathbf{E} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}, \nabla^2 \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2} \tag{11}$$

Notice that Equation (11) satisfies the three-dimensional wave equation,

$$\nabla^2 f = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2},$$

which implies that empty space supports the propagation of electromagnetic waves, traveling at a speed

$$v = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3.00 \times 10^8 \mathrm{m/s}.$$

Then it is trivial to derive the expression of speed in a more general linear medium by substituting $\epsilon_0 = \epsilon$ and $\mu_0 = \mu$.

Furthermore, we can derive the last two equations we need in simulation,

$$\frac{\partial \mathbf{A}}{\partial t} + \epsilon_0 \epsilon_r \frac{\partial^2 \mathbf{A}}{\partial t^2} + \frac{1}{\sigma \mu_0 \mu_r} \nabla \times (\nabla \times \mathbf{A}) = 0, \tag{12}$$

$$\nabla \times (\nabla \times \mathbf{E}) - k_0^2 (\epsilon_r \mu_r - \frac{j \sigma \mu_r}{\omega \epsilon_0}) = 0. \tag{13}$$

In the equations above, ϵ_r and μ_r represents relative permittivity and permeability. Once given the boundary conditions, the transient state is analyzed by the Equation (12) while steady state is analyzed by the Equation (13). And we are going to solve these two equations using simulation software COMSOL, electric field norm distribution of the whole space will be shown and attenuation calculated.

4.1.2 Electromagnetic Waves in Conductors

In conductors, Equation (11) changes into Equation (14)

$$\nabla^2 \mathbf{E} = \mu \epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} + \mu \sigma \frac{\partial \mathbf{E}}{\partial t}, \ \nabla^2 \mathbf{B} = \mu \epsilon \frac{\partial^2 \mathbf{B}}{\partial t^2} + \mu \sigma \frac{\partial \mathbf{B}}{\partial t}, \tag{14}$$

which gives admit plane-wave solutions

$$\tilde{E}(z,t) = \tilde{E}_0 e^{i(\tilde{k}z - \omega t)}, \ \tilde{E}(z,t) = \tilde{E}_0 e^{i(\tilde{k}z - \omega t)}, \tag{15}$$

the wave number $\tilde{k}$ is then complex $\tilde{k} = k + i\kappa$, together with $\tilde{k}^2 = \mu \epsilon \omega^2 + i \mu \sigma \omega$, which is solved to give

$$k = \omega \sqrt{\frac{\epsilon \mu}{2} \left[\sqrt{1 + (\frac{\sigma}{\epsilon \omega})^2} + 1 \right]}, \ \kappa = \omega \sqrt{\frac{\epsilon \mu}{2} \left[\sqrt{1 + (\frac{\sigma}{\epsilon \omega})^2} - 1 \right]}. \tag{16}$$

Substitute equations above into Equation (15), we get the wave function in Equation (17) and we can see that the amplitude is decreasing with increasing z.

$$\tilde{E}(z,t) = \tilde{E}_0 e^{\kappa z} e^{i(kz - \omega t)}, \ \tilde{B}(z,t) = \tilde{B}_0 e^{\kappa z} e^{i(kz - \omega t)}. \tag{17}$$

Considering the real value, the wave is shown in Figure 6.

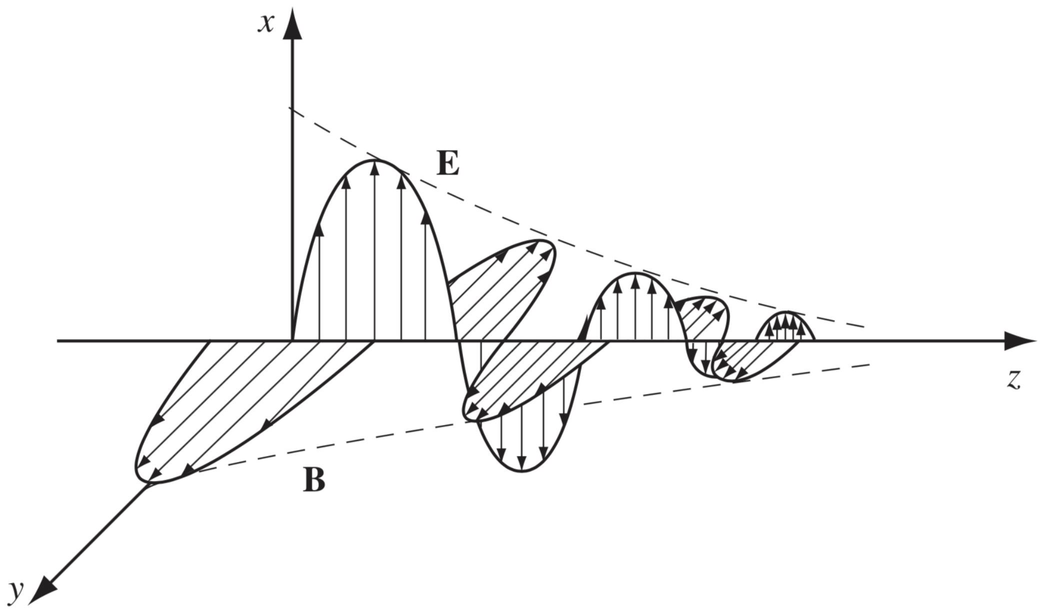


Figure 6: Attenuation of Electromagnetic Wave in Medium

4.1.3 Reflection at a Conducting Surface

To dig out the rule of reflection and refraction we are going to deal with the boundary conditions.

$$\begin{cases} \epsilon_1 E_1^{\perp} - \epsilon_2 E_2^{\perp} = \sigma_f, \mathbf{E}_1^{\parallel} - \mathbf{E}_2^{\parallel} = \mathbf{0}, \\ B_1^{\perp} - B_2^{\perp} = 0, \dfrac{1}{\mu_1}\mathbf{B}_1^{\parallel} - \dfrac{1}{\mu_2}\mathbf{B}_2^{\parallel} = \mathbf{K}_f \times \hat{\mathbf{n}}, \end{cases} \tag{18}$$

where σ_f is the free surface charge, $\mathbf{K}_f$ is the free surface current, and $\hat{\mathbf{n}}$ is a unit vector perpendicular to the surface, pointing from medium (2) to medium (1). Suppose now that the xy plane forms the boundary between a nonconducting linear medium (1) and a conductor (2). A monochromatic plane wave, traveling in the z direction and polarized in the x direction. Let the wave function be

$$\tilde{\mathbf{E}}_{\mathbf{I}} = \tilde{E}_{0I} e^{i(k_1 z - \omega t)} \hat{\mathbf{x}},\ \tilde{\mathbf{B}}_{\mathbf{I}} = \frac{1}{v_1} \tilde{E}_{0I} e^{i(k_1 z - \omega t)} \hat{\mathbf{y}}.$$

This gives rise to a reflected wave,

$$\tilde{\mathbf{E}}_{\mathbf{R}} = \tilde{E}_{0R} e^{i(-k_1 z - \omega t)} \hat{\mathbf{x}},\ \tilde{\mathbf{B}}_{\mathbf{I}} = -\frac{1}{v_1} \tilde{E}_{0R} e^{i(-k_1 z - \omega t)} \hat{\mathbf{y}},$$

propagating back to the left in medium (1) and a transmitted wave

$$\tilde{\mathbf{E}}_{\mathbf{T}} = \tilde{E}_{0T} e^{i(\tilde{k_2} z - \omega t)} \hat{\mathbf{x}},\ \tilde{\mathbf{B}}_{\mathbf{T}} = -\frac{\tilde{k_2}}{\omega} \tilde{E}_{0T} e^{i(\tilde{k_2} z - \omega t)} \hat{\mathbf{y}},$$

Since $E^{\perp} = 0$ on both sides, boundary conditions Equation 18, yields $\sigma_f = 0,\ \tilde{E}_{0I} +$

$\tilde{E}_{0R} = \tilde{E}_{0T},\ \tilde{E}_{0I} - \tilde{E}_{0R} = \tilde{\beta}\tilde{E}_{0T}$, in which

$$\tilde{\beta} \equiv \frac{\mu_1 v_1}{\mu_2 \omega}\tilde{k}_2. \tag{19}$$

It follows that

$$\tilde{E}_{0R} = \left(\frac{1-\tilde{\beta}}{1+\tilde{\beta}}\right)\tilde{E}_{0I},\ \tilde{E}_{0T} = \left(\frac{2}{1+\tilde{\beta}}\right)\tilde{E}_{0I}. \tag{20}$$

For a perfect conductor ($\sigma = \infty$), according to Equation (16, $k = \infty$, so $\beta = \infty$, which yields $\tilde{E}_{0R} = -\tilde{E}_{0I}, \tilde{E}_{0T} = 0$ according to Equation 20. In this case the wave is totally reflected and the larger the electrical conductance is, the more a electromagnetic wave is reflected.

4.2 Implementation

4.2.1 Scaling

Actually the wavelength of HF radio is completely negligible to the height of ionosphere, which is unacceptable if we use finite element method because tiny wavelength means a large number of "elements" and unavailable for numerical solving Maxwell's Equation.

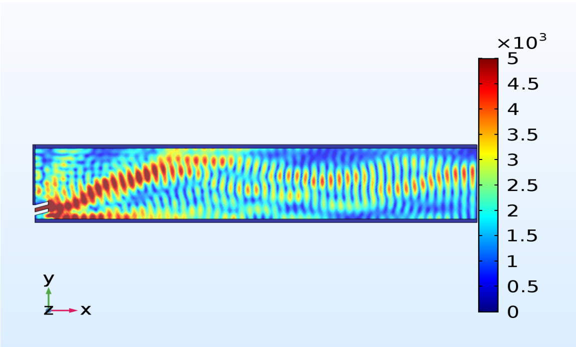
To tackle this problem, we notice that the most important quantity is the reflectivity of certain medium, which tells both attenuation inside a medium and absorption on the boundary of two medium. So we are going to do a linear scaling so that wavelength is no more negligible to height of ionosphere with reflectivity unchanged. Take a look at Equation (19) and Equation (20) we know that reflectivity is only dependent of $\tilde{\beta}$, which is inverse proportional to frequency ω. So we can adjust frequency 3 MHz to 3 kHz with $\tilde{\beta}$ changed correspondingly so that $\dfrac{1-\tilde{\beta}}{1+\tilde{\beta}}$ remains unchanged. This means we can do our simulation in a box of height is only 100 times as wavelength.

4.2.2 Simulation

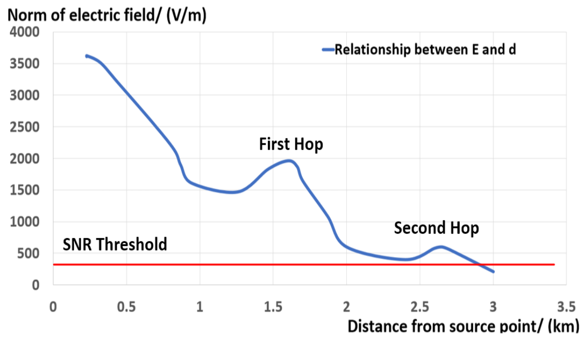
We simulate the calm water condition in order to compare with CGO calculation. We made a cuboid box representing air together with with two layers on and under the box representing ionosphere and ocean respectively. This makes perfect sense because we can set periodic boundary condition, which suffices for us to make a deep insight.

Then a microwave emission port is set and electric field norm is calculated by Equation 21 and shown in Figure 7a.

$$|\mathbf{E}| = \sqrt{E_x^2 + E_y^2 + E_z^2} \tag{21}$$



(a) Distribution of Signal



(b) Relation between E and d

Figure 7: Reflection between ionosphere and water

As Figure 7b shows, the power of the radio wave which is represented by the norm of electric field has two local maximum. The local maximum value of power can be viewed as the results of reflection. Hence, we can see clearly from the curve that there has been two hop before the strength falls below a usable SNR threshold. This perfectly fits the result of our first CGO model.

5 Evaluation of Models

We use the two different models to simulate the propagation of HF radio waves on a calm sea, using the basic assumptions we made above. For simplicity, we use 30 MHz as our shortwave frequency value and choose the atmospheric noise as 30 dB. Our first model come up with the result that the maximum hops of a 100 W HF is 2. The result of our second model is also 2 during the day. By consulting our real life experience that most HF band at 30 MHz only open in a relatively restricted area at day time and open nearly worldwide at night, it is confident to believe that the number of actual maximum hops during the day is 2. In this case, both two models coincide with the reality.

To compare, CGO is based on the classical geometrical optics and we regard the propagation of HF as the reflection, refraction and transmission of rays using the similar rules of geometrical optics while SSRP is based on Maxwell's theory. In fact the law of reflection and refraction is already contained in Maxwell's Equations. The only probable weak point of SSRP compared to CGO is that we use scaling method in SSRP. Due to the fact that we care about the reflectivity most as discussed before and the more complete distribution of whole space is preferred, we considered that SSRP is much better than CGO model. In addition, the visualization generated by COMSOL gives us a much more intuitive impression. Therefore, SSRP is adopted for further study.

6 Application and Analysis

6.1 Smooth Terrain

We have already fully discussed the power distribution when reflecting off surface of ocean. The same finite element method based on Maxwell's Equation and simulation software COMSOL can be applied to smooth terrain. For the three distinct stages: propagation in the air, reflection off the ionosphere, reflection off the surface. The first two stages remain the same and only thing we need to do is to change the surface from surface to seawater to different terrain.

For smooth terrain, we assume the whole surface of terrain to be flat. In the simulation process, we keep the material of ionosphere and air unchanging. And change the attributes of bottom plane from seawater to normal soil.

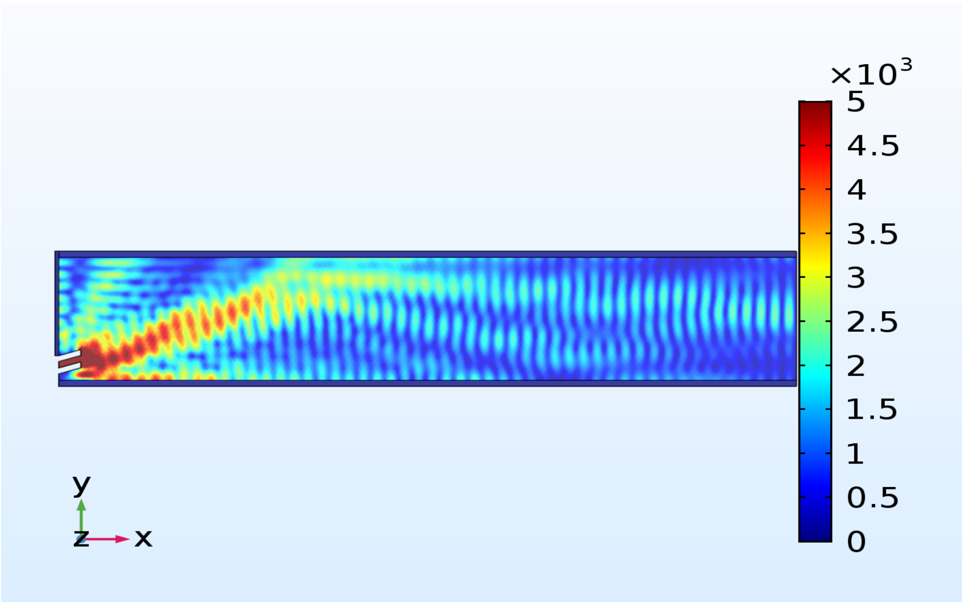


Figure 8: Signal distribution of smooth terrain

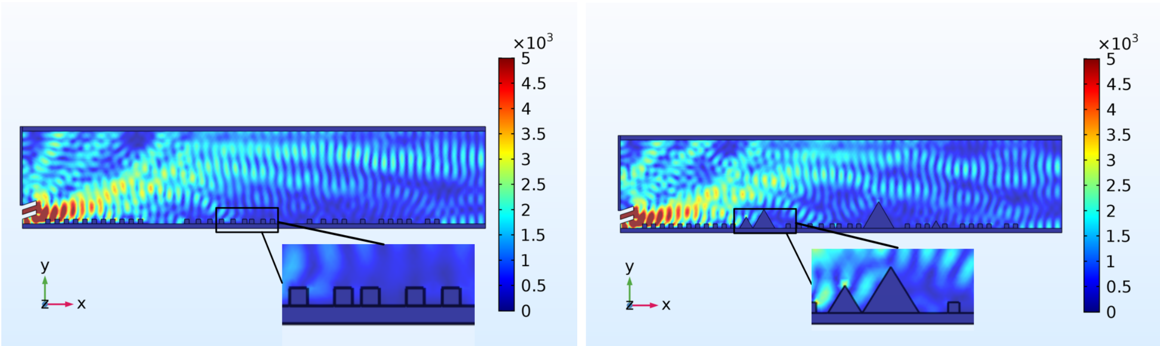
As Figure 8 shows, for the reflection off a smooth terrain, the attenuation of radio wave become much more significant. Compared to 7a, we can see that propagation on the land is more difficult than propagation above the sea.

6.2 Mountainous and Rugged Terrain

The research of mountainous and rugged terrain is also a problem of changing parameter in our simulation process. Mountainous terrain, to some extent, is an extreme condition of rugged terrain. Considering the rugged terrain, the whole bottom plane can no longer be viewed as smooth and the material will also change from soil to a mixture of soil and rock. As the material change, the physical property such as conductivity and relative permeability will also change, leading to the change of attenuation in the reflection off the bottom plane. For a better description of how rugged the terrain is, we introduce roughness of a surface R_a to simulate different situation, the roughness of a surface can be defined as standard deviation of a series of discrete height of the terrain

$$R_a = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(h_i - \bar{h})^2}$$

To simulate different terrain in our simulation process, we add rectangle and triangle of different size close to and above the surface of the ground. The material are no longer set to be soil but to be a mixture of soil and rock which have different permittivity and permeability. Figure 9a and Figure 9b shows the distribution of different situation



(a) Distribution of rugged terrain (b) Distribution of mountainous terrain

Figure 9: Reflection between ionosphere and different terrain

From Figure 9, it is obvious that the change of terrain do have influence on the distribution of signal, especially on the distribution near the mountain and the hill. However, the influence of the distribution of whole space is minor and can be neglected.

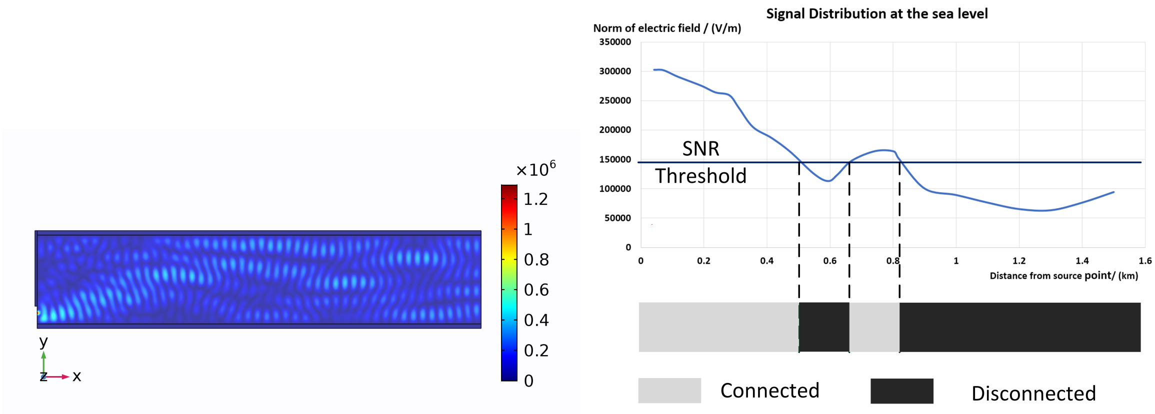
6.3 Analysis of an Shipboard Receiver

When we sail across an ocean, skywave communication is a cost-effective but unstable way. In dealing with the communication problem on a turbulent sea, we change the problem into deciding the time our ship stays in the area where SNR is above 10 dB. In this case, the ship is capable of resolving and processing the information that HF radio waves carry.

We assume that the initial location of our ship is just at the edge of the skip zone of the antenna and our ship goes in the direction of leaving the antenna which launches signals carrying information about weather and traffic. In addition, we choose the speed of our ship to be 20 knots, namely 37.04 km/h.

In this part, we do not consider Doppler Effect, because we only care about the power of signal we receive on our sea lane and it does not change according to the frequency.

We modify our second model by changing the direction of our transmitting antenna from a specific direction to all directions of space. Since our ship travels on the sea, its position is dynamic, so we do not know the exact distance between ship and transmitting antenna. Therefore, in this case, the antenna we use should not be a directional antenna, but an omnidirectional one. Then, we use this modified SSRP model to simulate the actual power distribution on our ship route. The result is shown in the Figure 10:



(a) Signal distribution of turbulent ocean (b) Range of Communication

Figure 10: Reflection between ionosphere and water for shipboard receiver

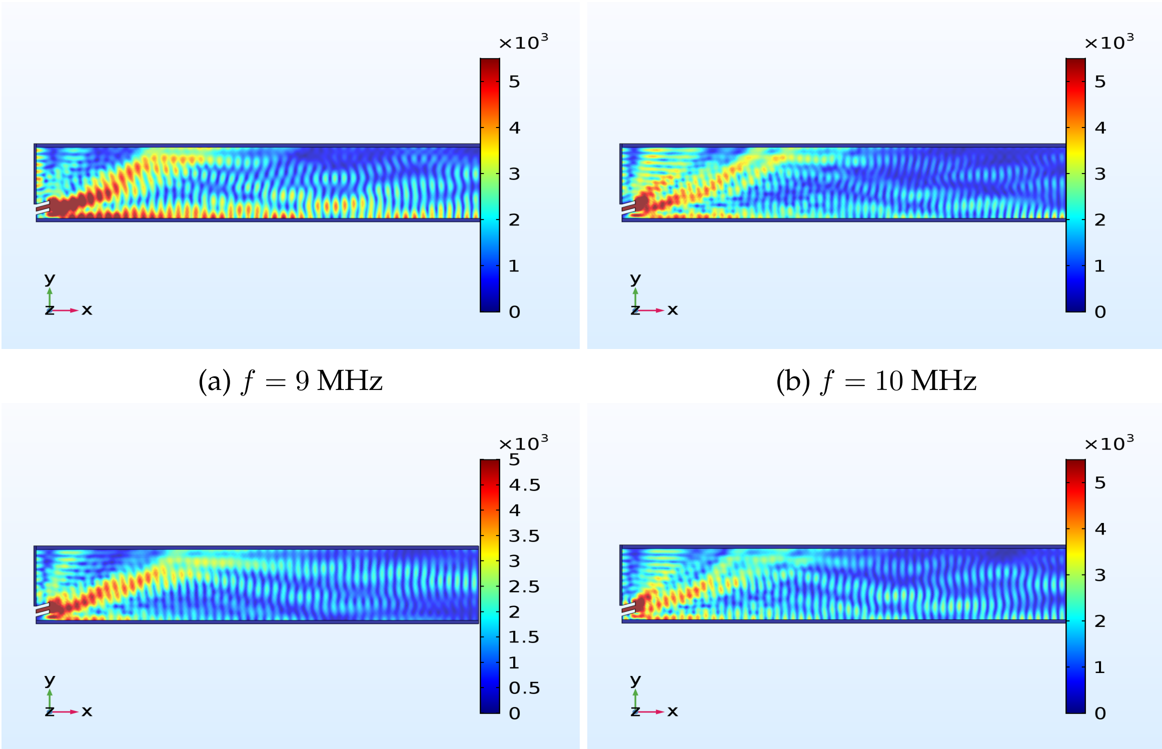
From the figure we find that there is an area whose signal power is under threshold and it is between two connected area. Actually, this make perfect sense as researchers[15] have discovered the phenomena which is caused by the difference of propagation scale of sky ware and ground ware. In the near area of signal transmitter the signal is available due to the ground ware and the signal of further area is available owing to the sky ware.

In this way we get the furthest distance we can travel before the SNR falls below the threshold value. The time the same multi-hop can maintain communication is 12.8 hours. According to the physical properties of ionosphere, in this timescale it is highly likely that it would not change much so our assumptions above are indeed reasonable.

7 Sensitivity Analysis

7.1 Impact of Frequency

Frequency is a vital factor in our whole model, the attenuation change dramatically when frequency change. And for Maxwell's equation, as Equation13 shows, ω representing frequency is also an important variable.We investigated frequency from 9 MHz to 12 MHz with the same power on the sea we have simulated, each condition is shown below in Figure 11a to Figure 11d.



(a) $f = 9$ MHz (b) $f = 10$ MHz

(c) $f = 11$ MHz (d) $f = 12$ MHz

Figure 11: The electric field norm distribution of different frequency

As the figure shows, a minor change of frequency will lead to the dramatical change of propagation. The higher the frequency, the less the attenuation is. Figure 12 may give a better intuition of the change pattern.

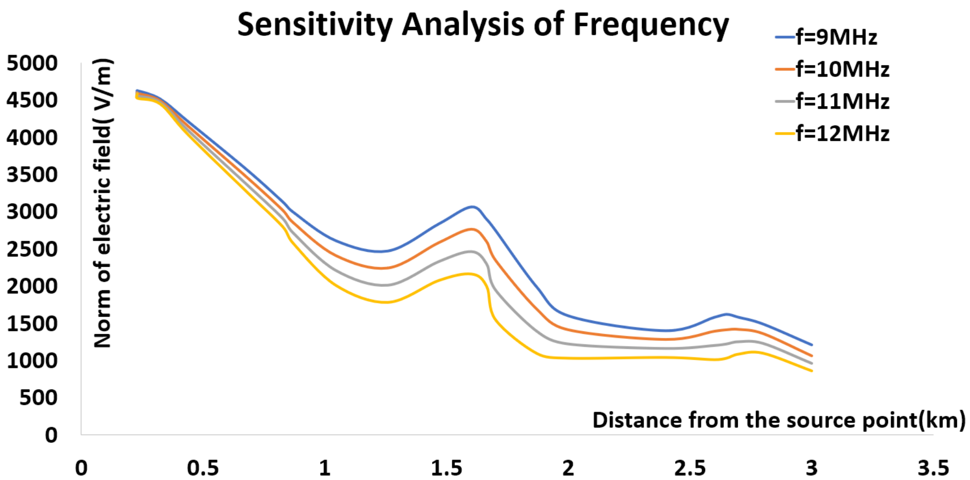


Figure 12: Comparison of the signal distribution of different frequency

7.2 Other Impacts

Apart from all that has been discussed, some natural phenomenon also play a part in influencing HF propagation. Some phenomenon, such as geomagnetic storm, have a great impact on the ionosphere. Thus these phenomenon undeniably affect the propagation of HF. Other extreme condition has already been discussed in our assumptions.

8 Strengths and Weaknesses

8.1 Strengths

- We not only work on specific formulas to get theoretical results, but provides a strong function to simulate the multi-hop of HF. According to the simulation results and sensitivity analysis, both of the two models coincide precisely with the real circumstance.
- In both models we discuss, we provide simple and clear visualization for our models. SSRP model simulation using finite element method by simulation software COMSOL returns very specific power distribution.
- We reasonably simplify the complicated problem and made it much more practical. Therefore, we can simulate skywave propagation under various geographical conditions.

8.2 Weaknesses

- Owing to the fact that atmospheric noise varies dramatically when the latitude and time changes, the constant we choose is only an approximate corresponding value on the corresponding circumstance. Yet our latter calculation proved that this assumption is valid on the scale we consider about in this problem.
- We assume that industry noise can be neglected, however,when HF travels on land, there are certain situations that industry noise also plays a vital part in deciding SNR. Nevertheless, most of our problem deals with places where industry noise is minor.
- Some parameters we use during our simulation are based on semi-educated guesses because we can only obtain the range of the parameters and few data are available, so we choose the mean value of the whole range.

9 Conclusions

Our paper provides a detailed analysis of propagation of radio wave. Before introducing the model, we give a brief but comprehensive introduction of ionosphere and the theory of HF communication. Then two different models including model of classical geometrical optics and scaling simulation of radio propagation model are proposed. We tackle the problem of multi-hop propagation above calm ocean and turbulent ocean by first solving the situation of single hop. Results of our two model fit perfectly: the maximum times of hop is two. We evaluate the models and take model based on Maxwell's theory to carry on our work.

Then analysis of the propagation over different terrain are provided and the more rugged terrain is, the harder it can propagate. The distribution off signal are significantly influenced in the area near the "obstacle". For a ship sailing on the surface, we optimize our model by changing the transmitted pattern of transmitter and give the distribution of area which is in connection. From the results, phenomena of skip zone are introduced.

We use real data from reliable source and all the situation are fully discussed. A detailed analysis including sensitivity analysis, strength and weakness are given.

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