Fourier Transform Spectrometry

主要讲授：傅里叶变换光谱测量原理和傅里变换红外波段测量法和可见光傅里叶变换测量方法。

研究重点对象：可见光傅里叶变换光谱测量方法。

邮箱： Fts_ouc@126.com 密码：**ftsouc**

参考书： *Fourier-Transform spectroscopy Instrumentation Engineering*（作者：Vidi Saptari）.

Chapter 1
General Introduction(概述)

Principal Contents

- **1.1 Background of Fourier Transform Spectrometry**
- **1.2** Review of mathematical background knowledge **(Fourier Transform, DTFT, DFT, FFT)**
- **1.3 Review of superposition of light waves**

1.1 Background of Fourier Transform Spectrometry

FTSs have been used in many infrared and near-infrared applications over the last couple of decades.

multiplex (Fellgett's), throughput (Jacquinot's) and wavenumber (Connes') advantages

The ultraviolet-visible FTS is more sensitive to the position-tracking accuracy and the quality of the interferometer alignment, the translation stage system etc. because of their short wavelengths.

1.1 Background of Fourier Transform Spectrometry

1.1.1History of Fourier Transform Spectrometry

(1)Michelson in 1891, Ruben and wood interferogram in 1911，but no computer is available.
(2)1950s, FFT 1965(N^2 to $N\ln N$)
(3) High-resolution FTS is applied in space shuttle in 1985
(4) In recent years, Imaging Fourier Transform Spectrometer (IFTS), Nonsanning FTS

1.1 Background of Fourier Transform Spectrometry

1.1.2 Current position-tracking methods of FTS

(1) Spectral folding technique.

(2) Uniform time-sampling method

(3) Subdivision of the He-Ne laser fringes by electronic or optical methods

返回子目录

1.2 Review of mathematical background knowledge

• 1.2.1 Fourier Transform

A continuous time signal *x*(*t*) with finite energy

$$E_N = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

Can be represented in the frequency domain

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \qquad \omega = 2\pi f$$

X(ω) is Fourier Transform expression of x(t)

1.2.2 Discrete Time Fourier Transform(DTFT)

1. Sampling Process（采样过程）

$$x^*(t) = x(t) \bullet p(t) \qquad p(t) = \frac{1}{\tau} \quad \left(-\frac{1}{2}\tau + kT \leqslant t \leqslant \frac{1}{2}\tau + kT\right)$$

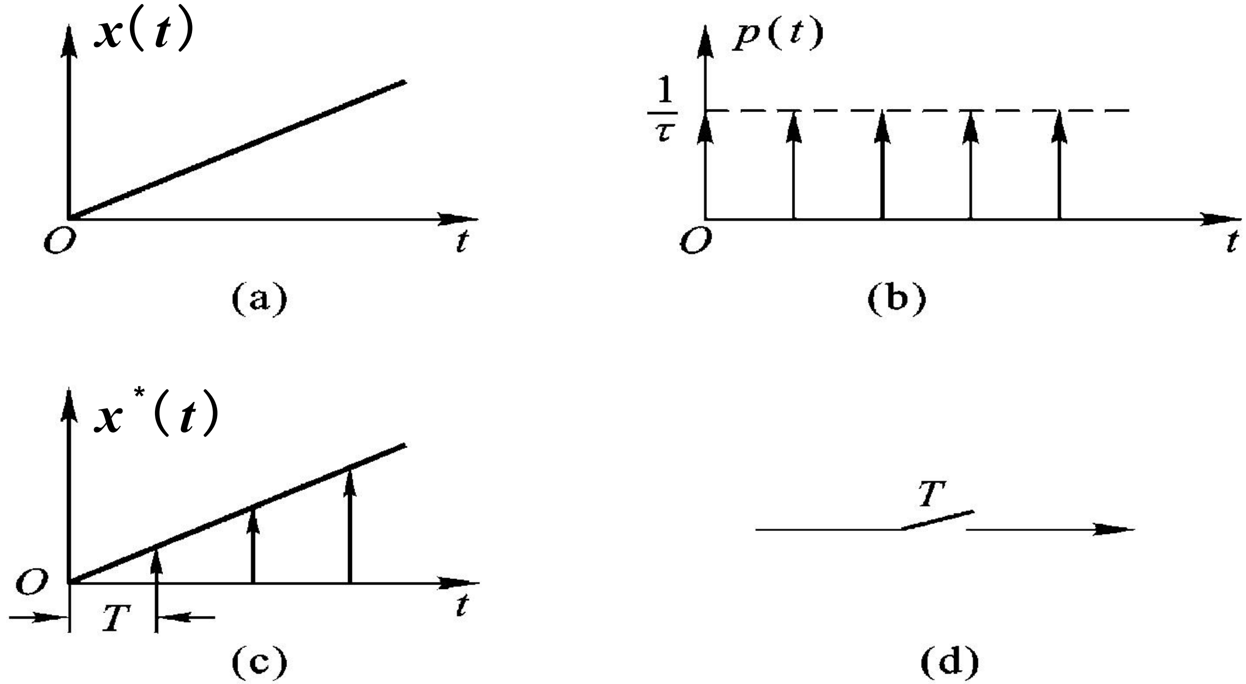

Fig.1.1 Sampling Process

2. Ideal Sampling Process (理想采样过程)

- To simplify the description of Sampling Process, Ideal Sampling Switch(**理想采样开关**)is introduced
- Carrier signal $p(t)$ can approximately be equal to the following ideal pulse sequence when $\tau \to 0$
- $$\delta_T(t) = \sum_{k=0}^{\infty} \delta(t - kT) \qquad (1\text{-}1)$$

Suppose $x(t) = 0$ when $t < 0$

Then the sampling signal can be described as

$$x^*(t) = x(t) \bullet \delta_T(t) = \sum_{k=0}^{+\infty} x(t) \bullet \delta(t\text{-}kT) \qquad (1\text{-}2)$$

此时，采样过程如图*Fig*.1.1(c)所示。

理想采样开关的输出是一个***理想***脉冲序列。

3. Discrete Time Fourier Transform (DTFT)

$$x^*(t) = x(t) \bullet \delta_T(t) = \sum_{k=0}^{+\infty} x(t) \bullet \delta(t-kT) \qquad (1\text{-}3)$$

Considering $x(t)=0$ when t is bigger enough($\geq NT$), then

$$\therefore X^*(\omega) = \int_{-\infty}^{\infty} x^*(t) e^{-j\omega t} dt \approx \sum_{n=0}^{N-1} x(nT) e^{-jn\omega T}$$

4. Shannon sampling theorem

显然$\delta_T(t)$是周期函数，故可以展成如Fourier级数

$$\delta_T(t) = \sum_{n=-\infty}^{+\infty} C_n e^{jn\omega_s t} \quad (1\text{-}4)$$

其中 $C_n = \frac{1}{T}$

1.2 Review of mathematical background knowledge

则有 $x^*(t)=\frac{1}{T}\sum_{n=-\infty}^{+\infty}x(t)e^{jn\omega_s t}$ （1−5）

和 $$X^*(j\omega)=\frac{1}{T}\sum_{n=\text{ÙT}}^{\infty}X(j\omega+jn\omega_s)$$ （1-6）

1.2 Review of mathematical background knowledge

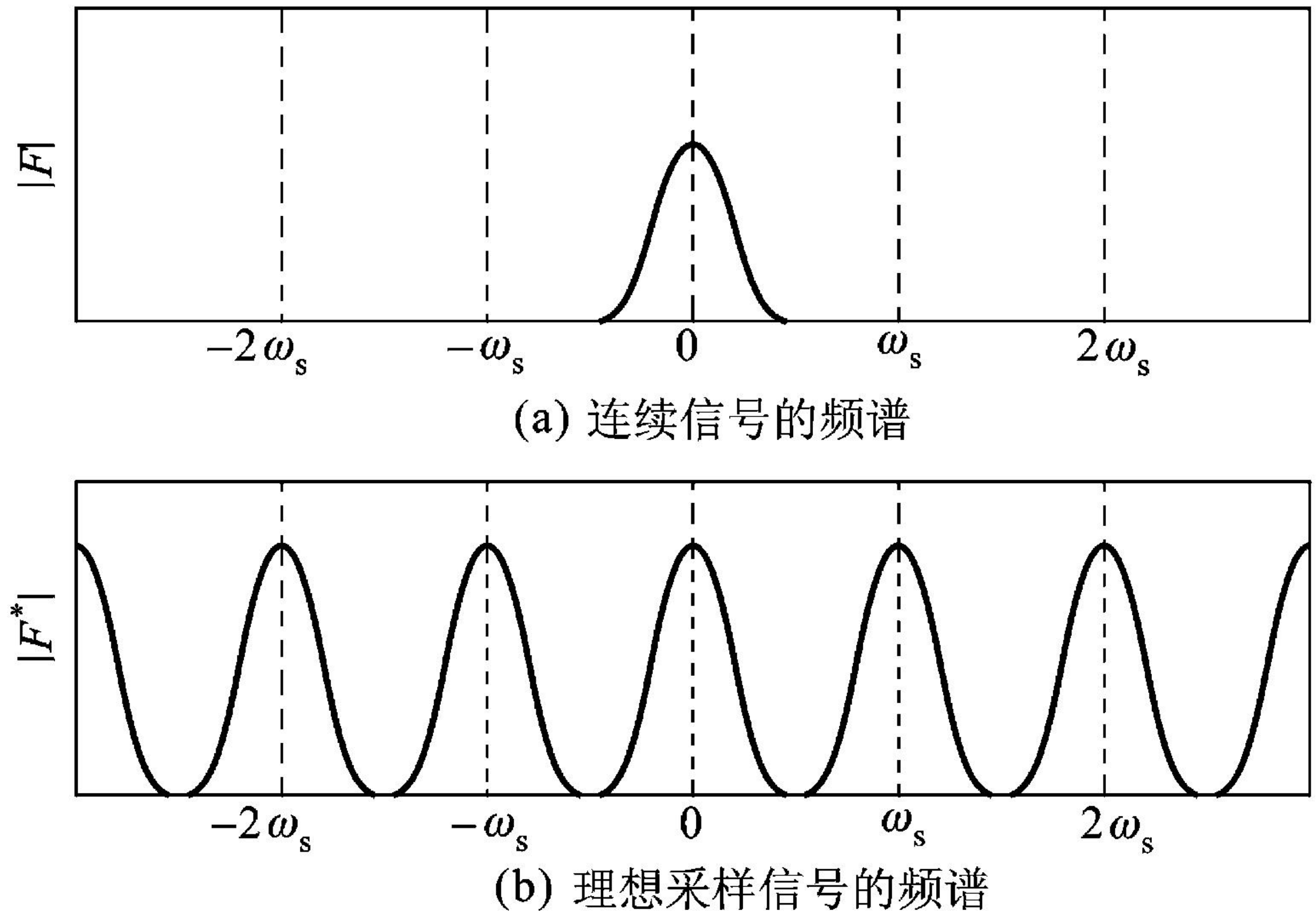

Fig.1.2 连续信号和采样信号的频谱

香农（Shannon）采样定理

- 若存在一个*理想*的*低通滤波器*，其频率特性如图Fig.1.3所示，便可以将采样信号完全恢复成原连续信号。由此可得如下著名的 香农（**Shannon**）采样定理 ：

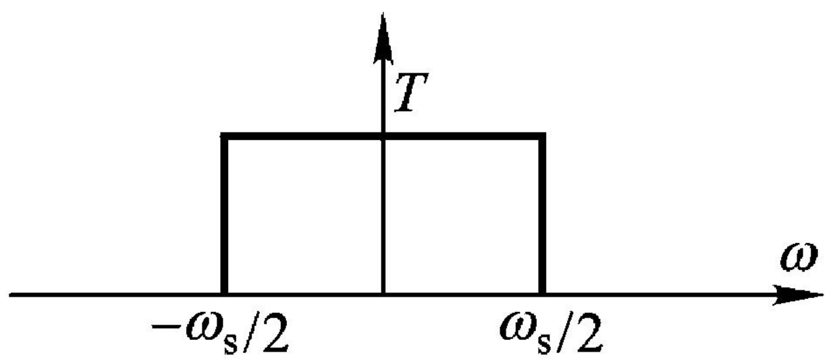

Fig.1.3

如果采样频率 ω_s 满足以下条件

$$\omega_s \geq \omega_{max} \quad （1\text{-}7）$$

式中 $\omega_{\max}$ 为连续信号频谱的上限频率

则经采样得到的脉冲序列可以无失真地恢复为原连续信号。

注 意：

上述香农采样定理要求满足以下两个条件：

① 频谱的上限频率是有限的；

② 存在一个理想的低通滤波器。但可以证明理想的低通滤波器在物理上是不可实现的，在实际应用中只能用非理想的低通滤波器来代替理想的低通滤波器；

•1.2.3 Discrete Fourier Transform (DFT)

For the sequence of data

$$x(n) = [x(0), x(1), x(2), \ldots\ldots\ldots x(N\text{-}1)]$$

DFT is Defined as follow.

$$X_s(\omega) = \sum_{n=0}^{N-1} x(t) e^{-jnT\omega}$$

1.2.3 Fast Fourier Transform Algorithm(FFT)

FFT是快速的离散傅里叶变换的计算方法，它要求采样数据长度为2^N

返回子目录

1.3 Review of superposition of light waves

1. Superposition of waves of the same frequency

A solution of the differential wave equation can be written in the form

$$E(x,t) = E_0 \sin[\omega t + kx + \varepsilon] = E_0 \sin[\omega t + \alpha(x,\varepsilon)] \qquad (1\text{-}9)$$

in which E_0 is the amplitude of the electric field, i.e. the of amplitude of harmonic disturbance propagation along the positive X-axis, ε *is the initial phase angle*

1.3 Review of superposition of light waves

Suppose then that there are two such waves

$$E_1(x,t) = E_{01}\sin[\omega t + \alpha_1] \qquad (1\text{-}10)$$

$$E_2(x,t) = E_{02}\sin[\omega t + \alpha_2] \qquad (1\text{-}11)$$

each with the same frequency, coexisting in space.

The resultant disturbance is the linear superposition of these waves

$$E = E_1 + E_2$$

1.3 Review of superposition of light waves

$$E = E_0 \cos\alpha \sin\omega t + E_0 \sin\alpha \cos\omega t$$

$$E = E_0 \sin(\omega t + \alpha) \qquad (1\text{-}12)$$

where

$$E_0^2 = E_{01}^2 + E_{02}^2 + 2E_{01}E_{02}\cos(\alpha_1 - \alpha_2) \qquad (1\text{-}13)$$

$$\tan\alpha = \frac{E_{01}\sin\alpha_{01} + E_{02}\sin\alpha_{02}}{E_{01}\cos\alpha_{01} + E_{02}\cos\alpha_{02}}$$

The superposition of Many Waves

$$E = E_0 \cos\alpha \sin\omega t + E_0 \sin\alpha \cos\omega t$$

$$E = E_0 \sin(\omega t + \alpha) \qquad (1\text{-}12)$$

where

$$E_0^2 = \sum_{i=1}^{N} E_{0i}^2 + 2\sum_{i=1}^{N}\sum_{j>i}^{N} E_{0i}E_{0j}\cos(\alpha_j - \alpha_i) \qquad (1\text{-}13)$$

$$\tan\alpha = \frac{\sum_{i=1}^{N} E_{0i}\sin\alpha_{0i}}{\sum_{i=1}^{N} E_{0i}\cos\alpha_{0i}}$$

1.3 Review of superposition of light waves

$$E_0^2 = \sum_{i=1}^{N} E_{0i}^2 + 2\sum_{i=1}^{N}\sum_{j>i}^{N} E_{0i}E_{0j}\cos(\alpha_j - \alpha_i) \qquad (1\text{-}13)$$

Random Sources:

$$E_0^2 = \sum_{i=1}^{N} E_{0i}^2 = NE_{01}^2 \qquad (1\text{-}14)$$

Coherent Sources:

$$E_0^2 = \sum_{i=1}^{N} E_{0i}^2 + 2\sum_{i=1}^{N}\sum_{j>i}^{N} E_{0i}E_{0j}$$

$$= \left(\sum_{i=1}^{N} E_{0i}\right)^2 = N^2 E_{01}^2 \qquad (1\text{-}15)$$

2. Addition of waves of different frequency

Suppose then that there are two such waves

$$E_1(x,t) = E_{01}\sin[\omega_1 t + \alpha_1] \qquad (1\text{-}16)$$

$$E_2(x,t) = E_{02}\sin[\omega_2 t + \alpha_2] \qquad (1\text{-}17)$$

each with the same frequency, coexisting in space.

The resultant disturbance is the linear superposition of these waves

$$E = E_1 + E_2$$