

Swimming Through Life: From Velocity Optimization to Strategic Comebacks Summary

At the 2024 Paris Olympics, Chinese swimmer Pan Zhanle performed exceptionally, breaking the world record in men's 100-meter freestyle and winning the gold medal in the men's 4×100-meter medley relay, becoming the focus of the world. This article establishes a **biomechanical velocity optimization model (BVOM)** and a **velocity perturbation model (VPM)** that includes swimming strategies by analyzing forces on the human body and combining energy consumption. Aiming to assist top-level swimmers in optimizing their training and tactics.

For Task 1, we established the **BVOM** model. Initially, we conducted a force analysis on the human body and used **Newton's second law** to derive the relationship between the resultant force and velocity and time. Then, based on the energy consumption during human motion, we used power as a bridge to establish the relationship between the swimming propulsion force and velocity. Subsequently, according to the **fluid drag equation**, we obtained the relationship between the water resistance and velocity and the athlete's frontal projection area. Through **linear regression**, we derived the relationship between the frontal projection area and height. Next, taking velocity as the decision variable, we determined the objective function and constraints, and discretized the problem to find a numerical solution. Finally, we used Python programming to solve for the optimal velocity distribution as shown in **Figure 6**, and provided differences among different competitions (50 meters, 100 meters, 200 meters) to **validate and evaluate** the model.

For Task 2, we established the **VPM** model. Initially, we constructed three different perturbation functions for three distinct strategies (mutual probing strategy, leading strategy, and following strategy). Then, by applying these different perturbation functions to the velocity, we simulated the swimming speed under different strategies, thereby constructing the **VPM** model. Subsequently, we combined the VPM model with the BVOM model to obtain the **Velocity Perturbation Optimization Model (VPOM)**. We then discretized the conditions and perturbation functions and used Python programming to solve for the optimal velocity distribution as shown in **Figure 9**. Finally, analyzing the results, it was concluded that no single strategy is superior to the others. A combination of multiple strategies should be employed during competitions.

For Task 3, we first visualized the speed of Pan Zhanle during the 4x100-meter medley relay. Based on the conclusions drawn from Task 2, we divided the race into three phases: the initial phase, the acceleration phase, and the decisive phase, each implementing different strategies. In the initial phase, a leading strategy was adopted. During the acceleration phase, a combination of following and mutual probing strategies was used. In the decisive phase, a combination of leading and mutual probing strategies was employed. Finally, we programmed to solve for the optimal swimming speed distribution of Pan Zhanle after applying these strategies, as shown in **Figure 14**. Ultimately, we obtained a speed of **45.8 seconds** for Pan Zhanle, achieving a better result than the 45.92 seconds in the final race.

Finally, This paper verifies the applicability of the model in multiple scenarios through **sensitivity analysis** and generalization. Moreover, its modeling method can be extended to other competitive sports, helping athletes optimize their techniques and tactics, and thus has extensive application value.

Keywords: BVOM; Fluid Resistance; VPOM; Secondary Perturbation; Sensitivity Analysis

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1 Introduction

1.1 Problem Background

Swimming is one of the most popular modern competitive sports. Freestyle events are usually divided into different types such as short distance (such as 50 meters, 100 meters) and middle distance (such as 200 meters). During a game, many factors will affect the final performance, including the athlete's physical fitness, game rhythm, and energy distribution strategy. Therefore, compared with those sports that mainly rely on explosive power, in freestyle competitions, it is particularly important to formulate scientific strategies based on the physical fitness and race characteristics of specific athletes.

Different types of athletes have different physical characteristics, which are not only reflected in the strength of explosive power in a short period of time, but also in the ability to continuously output physical energy. Athletes with strong explosive power but insufficient endurance often find it difficult to perform at their best in middle and long distance competitions. vice versa. By mathematically modeling the changes in physical energy consumption and speed of athletes during swimming, coaches can develop optimal competition strategies to help athletes achieve the best results under their specific physical conditions. Scientific competition strategies can not only help top athletes set records, but also provide personalized training plans for ordinary swimmers to improve sports efficiency.



Figure 1: Pan Zhanle won the men's 100m freestyle championship and broke the world record

1.2 Restatement of the Problem

In freestyle competitions, athletes' performance is affected by many factors, especially the optimization of physical energy distribution and speed control strategies. In order to better understand and improve athlete performance in freestyle competitions, the following questions

need to be answered:

- **Task 1:** Scientifically arrange speed distribution through mathematical modeling to enable athletes to achieve optimal results in freestyle competitions of different distances (50 meters, 100 meters, and 200 meters).
- **Task 2:** Study how tactical interactions between athletes (such as leading and following strategies) affect individual performance and whether there is an optimal strategy for different competition situations.
- **Task 3:** Based on Pan Zhanle's performance in the relay race, formulate the best competition strategy to help athletes perform at their highest level and achieve better results.

1.3 Literature Review

In recent years, research on modeling and optimization of swimming performance has focused on the following aspects:

Swimming power curve and energy allocation model: Athletes' performance is often affected by both power output and endurance level. Studies have shown that the swimming power curve can be described by a mathematical model, which helps analyze the balance between an athlete's short-term explosive power and long-term endurance^[1-2]. Through modeling research on lactic acid accumulation and physical recovery, the distribution of physical energy in freestyle can be optimized, thereby improving competition performance^[3].

Speed optimization and dynamic modeling: Speed optimization in swimming involves complex dynamic modeling, including factors such as power output, water resistance, and buoyancy^[4]. These models can quantify the impact of speed changes on race results and provide recommendations for speed distribution strategies at different stages. In short-distance freestyle, optimizing speed during the sprint phase is crucial, while in middle-distance races, uniform power output is key^[5].

Tactical interaction and strategy optimization: In relay races, tactical coordination between team members (such as physical energy distribution and baton-taking order) has a significant impact on team performance. In individual competitions, the choice of leading and following strategies directly affects swimming performance. Models based on game theory can analyze the advantages and disadvantages of different tactics and find the best tactical combination^[6].

Influence of environmental and psychological factors: Pool conditions (such as water temperature and waves) significantly affect swimming performance. Researchers explore the impact of environmental factors on athlete performance through experiments and modeling analysis^[7].

In summary, the application of mathematical modeling in swimming competitions pro-

vides important support for speed optimization, physical energy distribution and tactical selection. These studies can not only help athletes improve their performance, but also provide a reference for coaches to formulate scientific training plans.

1.4 Our Work

On the basis of the above analysis, we have carried out our work. Our working framework is shown in Figure 2.

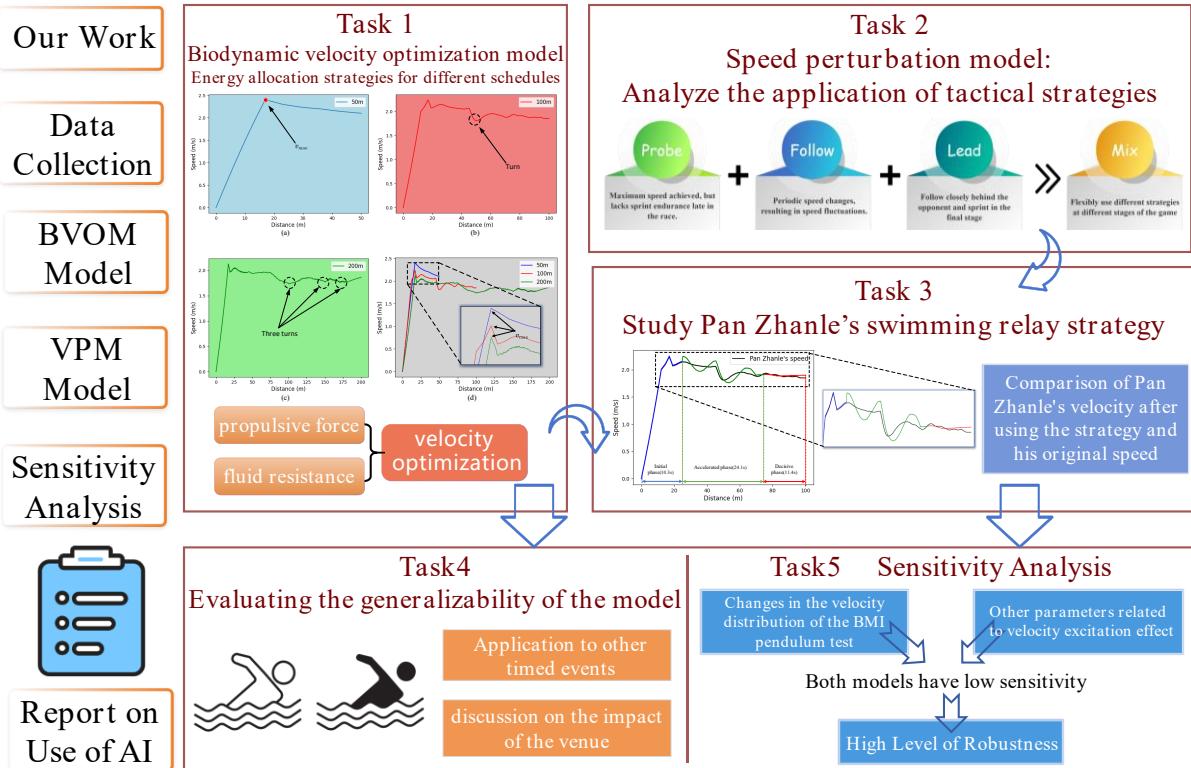


Figure 2: Our Work Overview Schematic Diagram

2 Assumptions and Justifications

Considering that practical problems always contain many complex factors, first of all, we need to make reasonable assumptions to simplify the model, and each hypothesis is closely followed by its corresponding explanation:

▼ **Assumption 1:** A swimmer's own gravity and the buoyancy of the water are equal and cancel each other out during a race.

▲ **Explanations:** Simplifying the analysis of the forces on the athlete in the vertical direction allows the model to focus on the dynamics of the process in the horizontal plane, thus allowing for a more intuitive study of the relationship between velocity, drag and propulsion.

▼ **Assumption 2:** The total physical reserve of an athlete is a finite value.

▲ **Explanations:** The total amount of physical energy available to an athlete during a race

is limited, which dictates that the distribution of speed needs to be rationalized to avoid running out of physical energy before the end of the race.

▼ Assumption 3: Segment speeds can be controlled independently in freestyle races.

▲ Explanations: At each stage, the athlete can autonomously control the speed by adjusting the paddling frequency and amplitude of movement, providing flexibility for strategy optimization.

▼ Assumption 4: Pan Zhanle's fitness and form in the last baton is optimized individually, independent of the previous bats.

▲ Explanations: Assuming that Pan Zhanle's strategy adjustment is not affected by the physical distribution of the first few bats, it is easy to analyze how to achieve the best performance by adjusting his speed distribution.

▼ Assumption 5: Pool lengths, lane conditions and judging rules are the same and fair for all competitors.

▲ Explanations: Ensure fairness in model analysis by focusing only on technical, tactical and conditioning optimization of athletes.

3 Data sources and processing

The data used in this study were obtained from multiple sources to ensure comprehensiveness and reliability. The main sources include:

- **Public competition data:** collected from the official website of the International Swimming Federation (FINA) and the official records of relevant swimming events, covering a wide range of race types (e.g., 50-meter, 100-meter, and 200-meter freestyle) and the performance of different competitors;
- **Experimental test data:** collected by testing professional athletes in a simulated competition environment, including speed, stroke frequency, heart rate and other key indicators;
- **Literature reference data:** key data on freestyle techniques and tactics collated from existing research papers and reports;
- **Training data provided by coaches and athletes:** based on the monitoring results during the long-term training process, containing information on physical distribution, technical movements and segmental speed. These data have been verified and cleaned many times to ensure their reliability and representativeness, which provide solid data support for model building and analysis. The data sources as well as the types are given below:

Table 1: Data sources and types

Source	Type
https://www.fina.org	Public Match Data

https://www.olympics.com	Race results data
https://www.swimrankings.net	Swimmer's database
https://swimswam.com	Technical analysis and training data

Technical Analysis and Training Data In addition, we also pre-processed the data. In the data processing process, we first thoroughly checked the raw data to ensure the completeness and reliability of the data. For the treatment of outliers, the Boxplot method (Boxplot) and the principle of triple standard deviation (3σ criterion) were used to screen the data, and the outliers that obviously deviated from the normal range and could not be explained by reasonable explanations were regarded as invalid data and were eliminated. For the treatment of missing values, appropriate methods were adopted according to the proportion of missing values and distribution characteristics: if the proportion of missing values was low, the mean, median or interpolation method was used to fill in; if the proportion of missing values was high and might affect the analysis results, the samples containing missing values were chosen to be excluded. After the above steps, a complete and high-quality data set was generated, laying a solid foundation for the subsequent analysis.

4 Notations

The key mathematical notations used in this paper are listed in Table 2.

Table 2: Notations used in this paper

Symbol	Description	Unit
$F_{\text{propulsion}}$	Propulsive forces generated by swimmers	N
F_{drag}	Resistance exerted by water on swimmers	N
Q	Energy loss during motion	Kcal
C_D	Resistance factor, related to the athlete's swim stroke	—
A	Frontal projection area of the athlete	m^2
v_{\min}	Minimum speed acceptable to the athlete	m/s
v_{\max}	Maximum speed an athlete can achieve	m/s
v_{probe}	Speed under the Mutual Trial Strategy	m/s
v_{lead}	Speed in a leading strategy	m/s
v_{follow}	Follow the speed under the strategy	m/s
$\epsilon(t)$	Random noise, related to off-site environment	—
v_{strategy}	Swimming speed under the strategy adopted by the athlete	m/s
v_{si}	Velocity microelements employing a swimming strategy	m/s

Note: There are some variables that are not listed here and will be discussed in detail in each section.

5 Biodynamic velocity optimization model (BVOM)

When exploring the speed optimization problem of swimmers, we built an innovative model to gain a deeper understanding of the efficiency of the organism's movement under different environmental conditions. This model combines knowledge of physiology and dynamics, and through mathematical modeling and computer simulation, aims to reveal how swimmers maximize their speed by adjusting their body posture, muscle force distribution, and breathing rhythm, while taking into account the effects of water resistance and dynamic characteristics on athletic performance. The process for developing the BVOM model is as follows:

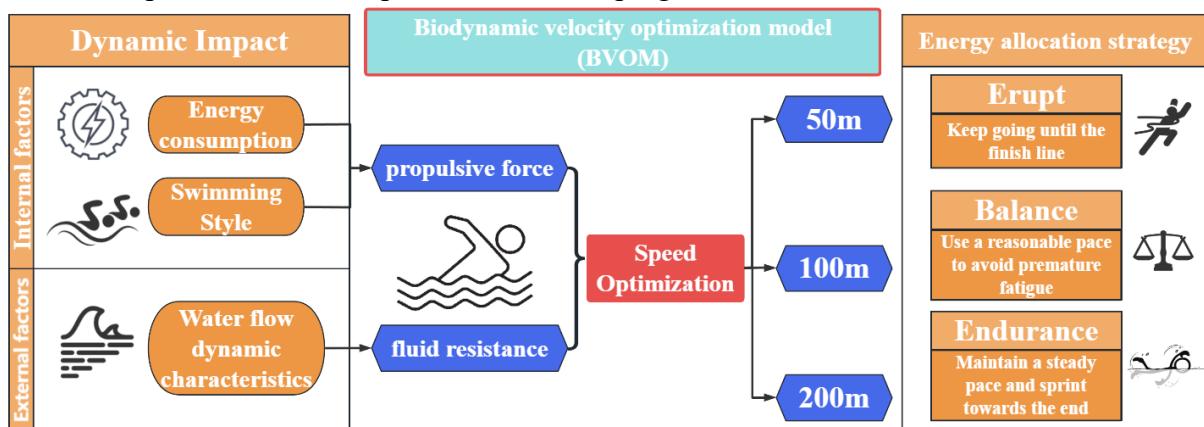


Figure 3: Flowchart of BVOM Model Development Process

5.1 Establishment of the BVOM

The core of the biodynamic model is to describe the temporal changes in a system's velocity, position, and other variables through force analysis and the energy expenditure in biological motion. A swimmer's movement in water can be simplified to a single-particle model, which considers only the overall motion while disregarding the effects of factors such as limb movements on propulsion, as illustrated in the figure below.

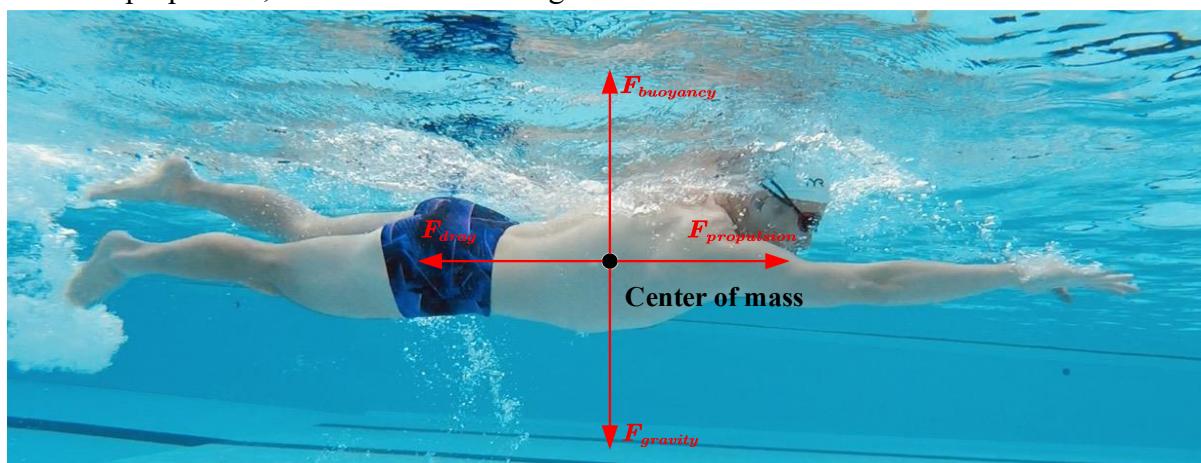


Figure 4: The forces acting on athletes during swimming

Where $F_{\text{propulsion}}$ is the propulsive force generated by the swimmer, F_{drag} is the drag force of the water, F_{buoyancy} is the buoyancy force generated by the water, and F_{gravity} is the athlete's own gravity. Buoyancy and gravity mainly affect the balance of vertical forces, because in the swimming competition, the athlete usually keeps the body level, assuming that F_{buoyancy} and F_{gravity} are equal and cancel each other, so according to Newton's second law, it can be obtained:

$$F_{\text{propulsion}} - F_{\text{drag}} = m \frac{dv}{dt} \quad (1)$$

5.1.1 Representation of propulsive force

The power required during swimming mainly comes from the body's ability to overcome water resistance and maintain basic metabolism during exercise. We need to estimate the total energy consumption during swimming. In this article, we use energy consumption to estimate the power during swimming. The calorie consumption during swimming can be calculated by the following formula:

$$Q = \frac{mt \times \text{MET}}{3600} \quad (2)$$

Where MET is metabolic equivalent, m is the mass of the athlete, Q is the energy consumed, and the unit is Kcal. By looking up the data reports released by sports research institutions and the training data of swimmers, several common swimming styles and their corresponding MET values can be sorted out, as shown in the following table:

Table 3: MET values for different swimming strokes

swimming position	MET value
freestyle swimming	9.8
backstroke	9.5
breaststroke	5.3
butterfly stroke	13.8

According to the data, $1 \text{ Kcal} = 4.184 \text{ KJ} = 4184 \text{ J}$. Based on the definition of power, the relationship between power and energy consumption during freestyle swimming is:

$$P = \frac{4184m \times \text{MET}}{3600} \quad (3)$$

According to the relationship between power, force and speed, the propulsion force is:

$$F_{\text{propulsion}} = \frac{4184m \times \text{MET}}{3600v} \quad (4)$$

5.1.2 Indication of water resistance

When swimming, the resistance of water to the body can be described by the fluid resistance equation:

$$F_{\text{drag}} = \frac{1}{2} \rho_{\text{water}} C_D A v^2 \quad (5)$$

Where ρ_{water} is the density of water; C_D is the drag coefficient, related to the shape of the object, surface roughness, and other factors, and A is the frontal projection area of the athlete. Zhihu Zhang^[8] et al. used Solidworks software to establish a three-dimensional simplified model and applied Autodesk CFD software to analyze the flow field and viscous drag of each lane of the swimming competition, and the drag coefficient was 0.5 for streamlined athletes.

● Front projection area

For the purpose of in-depth investigation of the factors affecting the resistance of water to the human body during swimming and its changing law, we collected data on frontal projected area of males with heights ranging from 1.70m to 2.10m by means of data reports released by sports scientific research institutes and the training data of swimmers, etc. We visualized the data and used linear fitting to fit the relationship between the height of males and the frontal projected area, as shown in Fig 5.

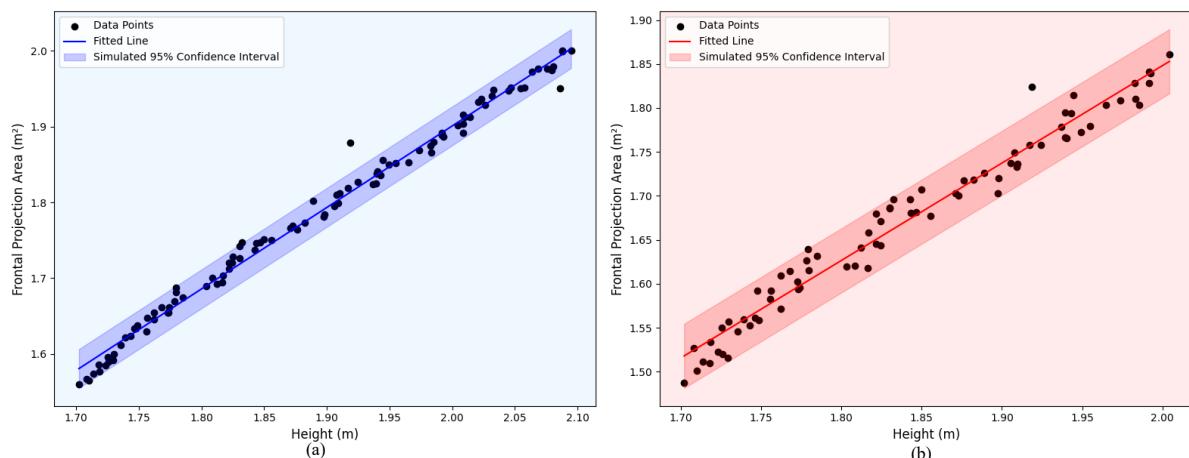


Figure 5: (a) is the fitting curve of the frontal projected area and height for males. (b) is the fitting curve of the frontal projected area and height for females. The shaded regions in both images represent the 95% confidence intervals.

The coefficient of determination R^2 for (a) is 0.9876, and for (b), it is 0.9666, indicating a good fit. The relatively wide 95% confidence intervals suggest a high level of reliability in the fit. The relationship between male height and frontal projected area can be obtained as:

$$A = 1.0755h - 0.2501 \quad (6)$$

The posture in swimming is periodically changing, both the paddling action of the hands

and the striking and stomping action of the legs are periodic, so the resistance to which the human body is subjected is also periodically changing, and since describing the instantaneous change of the dynamic resistance is complicated and unnecessary, this paper adopts its average value and combines it with the formula of the fluid resistance to describe it.

Substituting formula (4), (5), (6) and data into formula (1), we can get the relationship between speed and movement time:

$$\frac{11.39}{v} - \left(\frac{268.88h - 62.53}{m} \right) v^2 = \frac{dv}{dt} \quad (7)$$

5.1.3 Elements of BVOM

- Objective Function

This paper takes swimming speed $v(t)$ as the **decision variable**, minimizes the total time T as the objective function, and considers the athlete's reaction time δ to obtain the objective function:

$$\min T = \int_0^T \frac{1}{v(t)} ds + \delta \quad (8)$$

- Constraints

Considering that the athlete needs to complete a fixed race distance L , that is, the athlete stops exercising immediately after completing the race:

$$\int_0^T v(t) dt = L \quad (9)$$

Considering that the athlete's physical reserve is limited, it is necessary to introduce constraints on total physical consumption:

$$\int_0^T P(t) dt \leq P_{\max} \quad (10)$$

Considering that the athlete's speed cannot change infinitely, it needs to be kept within a reasonable range:

$$v_{\min} \leq v(t) \leq v_{\max} \quad (11)$$

Where v_{\min} is the minimum acceptable speed and v_{\max} is the maximum speed of the athlete. In addition, referring to the training data of the national team swimmers, it can be found that the initial speed v_0 of the athlete when jumping into the water is about $5 m/s$, and the speed at the end is approximately equal to the maximum speed v_{\max} . In summary, the biodynamic speed optimization model is:

$$\min T = \int_0^T \frac{1}{v(t)} ds + \delta$$

$$s.t. \begin{cases} \int_0^T v(t) dt = L \\ \int_0^T P(t) dt \leq P_{\max} \\ v(0) = v_0, v(T) = v_{\max} \\ v_{\min} \leq v(t) \leq v_{\max} \end{cases} \quad (12)$$

5.2 Solution of a biodynamic velocity optimization model

The solution of the biodynamic speed optimization model is done by discretization and numerical methods. The continuous dynamics of the model is discretized into 100 equally spaced intervals, each corresponding to a 1-meter distance in a race. This discretization method allows for a step-by-step analysis of the complex relationship between velocity, drag, propulsion, and energy consumption. Within each discrete interval, speed is calculated based on the kinetic equations and predefined constraints. The pseudo-code for solving the BVOM model is as follows:

Table 4: Pseudocode for solving the BVOM model

Algorithm 1 BVOM model solving

Input: L , P_max , v_min , v_max , v_0 , v_N , k , N, T_init
Output: Optimal time , Optimal speed distribution ,

```

1 begin
2   Calculate Δt based on the quotient of T_init and N;
3   Initialize T_init using linear interpolation
3   for i from 1 to N do
4     Calculate Optimal_time based on Δt and v
5     Calculate constraint_distance based on the difference between v*Δt and L
6     Calculate constraint_power based on the difference between k * v^2 * Δt - P_max
7     Initialize the boundary array based on v_min and v_max
8   end for
9   Calculate result based on optimization function
11  if result.success then
12    Optimal_v ← result.x;           ▶ Check whether the optimization was successful
13    Optimal_t ← Optimal_time      ▶ The optimal fnction in the optimization result
14  end if
15end

```

5.2.1 Parameterized solution

It is inconvenient to directly solve the time T, so we introduce a parameterization method, fix the time T as an unknown constant, and the optimization problem becomes the optimization of the function of $v(t)$. We discretize t into N time nodes t_0, t_1, \dots, t_N and approximate the integral with numerical integration. After discretization, the optimization problem becomes:

$$\min T = \sum_{N-1}^{i=0} \frac{\Delta s}{v_i} + \delta$$

$$s.t. \left\{ \begin{array}{l} \sum_{N-1}^{i=0} v_i \Delta t = L \\ \sum_{N-1}^{i=0} k v_i^2 \Delta t \leq P_{\max} \\ v_{\min} \leq v_i \leq v_{\max}, \quad \forall i \\ v_0 = 0 \quad v_N = v_{\max} \end{array} \right. \quad (13)$$

From the data, we can find that the reaction time δ of a swimmer is 0.6s, the world record of the maximum speed v_{\max} of a swimmer is 2.4m/s, the minimum speed v_{\min} that an athlete can accept is 1.7m/s, and the maximum power P_{\max} is 1200W. Using Python to solve the BVOM model, the optimal velocity distributions for different competitions are obtained as follows:

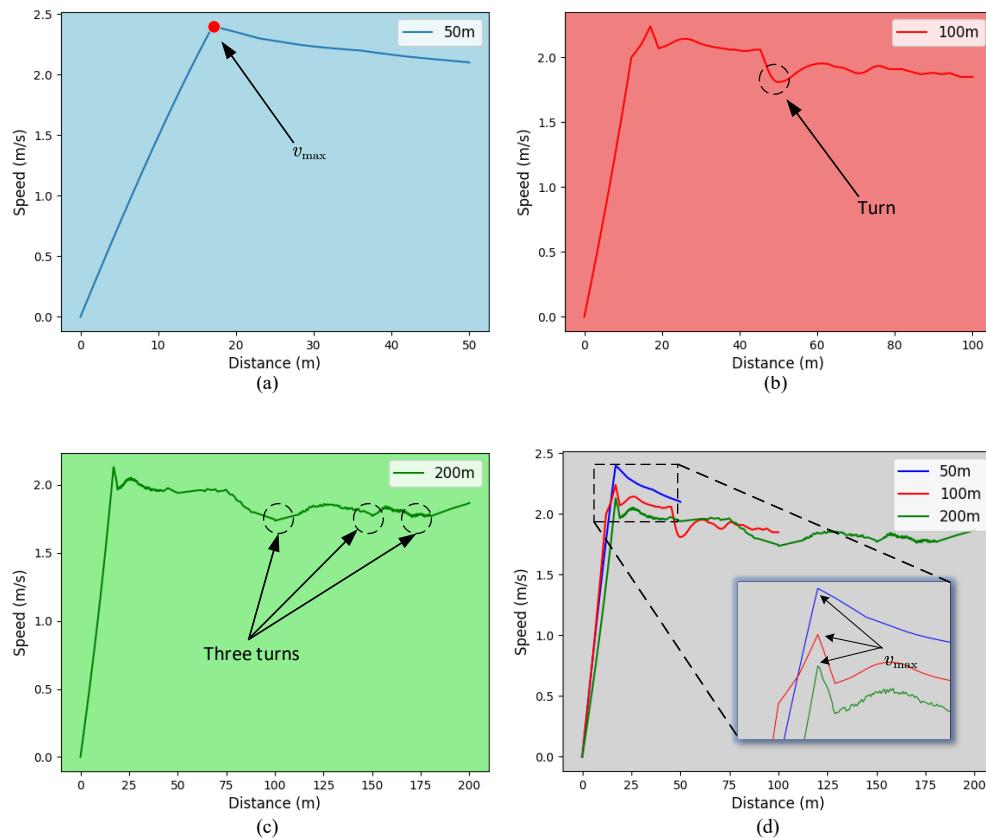


Figure 6: (a) represents the optimal velocity distribution for the 50m race. (b) represents the optimal velocity distribution for the 100m race. (c) represents the optimal velocity distribution for the 200m race. (d) represents the optimal velocity distribution for all three races. Additionally, v_{\max} is the maximum speed achieved by the athlete, and "turn" refers to the turns during swimming.

The best results in freestyle events depend on the proper pace of the competition: short-distance events focus on explosive power and speed maintenance, while long-distance events require a balance between the acceleration phase and the endurance maintenance phase to ensure that sufficient sprinting power can be maintained in the final stage. From the optimal speed distribution diagram, as the distance of the competition increases, the maximum speed that the athlete can maintain decreases.

For different competitions, we can see from the optimal speed distribution diagram:

- **For the 50m freestyle**, due to the short distance of the race, the swimmers need to sprint as close to their maximum speed as possible and maintain a high speed until the finish line. At this time, the explosive power at the start and the rapid acceleration to the maximum speed are the key. Athletes usually reach their maximum speed in the first half of the race and try to maintain it until the end.
- **For the 100m freestyle**, the race is long and the swimmers need to find a balance between explosive power and endurance. In the first 50m, the swimmers can maintain close to their maximum speed, but as the distance increases, the speed will decrease, usually slowing down slightly in the last 50m. The best strategy is to use a reasonable pace so that the swimmers can maintain a higher speed in the second half and avoid premature fatigue.
- **For the 200m freestyle**, the endurance requirement is higher. The speed arrangement of the players should be more balanced. In the first 100m, the athletes should maintain a relatively high speed, but not sprint at full speed to avoid physical exhaustion. The last 100m is the stage of testing endurance and psychological quality. The players need to have enough reserves and tactical adjustments to maintain a stable speed and try their best to accelerate the sprint in the final stage.

5.3 Validation and evaluation of BVOM

5.3.1 Reasonableness test

In this paper, the results of the question one will be rationalized. From the optimal speed distribution graph, 50 meters freestyle is short distance, athletes will sprint and keep the fastest speed to the finish line, which is in line with the characteristics of the event; 100 meters freestyle, the speed firstly rises quickly, and then maintains a higher level within a certain distance, and

then slightly decreases in the second half of the race due to the allocation of physical strength and other factors, but still maintains a higher speed, which is reasonable; 200 meters freestyle, the speed rises slower and then remains basically constant. The speed of 200 meters freestyle rises slowly, and then basically stays constant. This is in line with the fact that long-distance events require a reasonable distribution of physical strength.

5.3.2 Sensitivity test

In order to fully validate the sensitivity of the model, we launched an in-depth assessment of its robustness.

Considering that the height and weight of each athlete are different, it is necessary to change the two parameters of height and weight in order to verify the sensitivity of the model. Since the height and weight can be expressed by Body Mass Index (BMI), it is possible to perturb the BMI by 5% to analyze the change of the optimal speed distribution. The optimal speed distributions obtained for different BMIs are shown as follows:

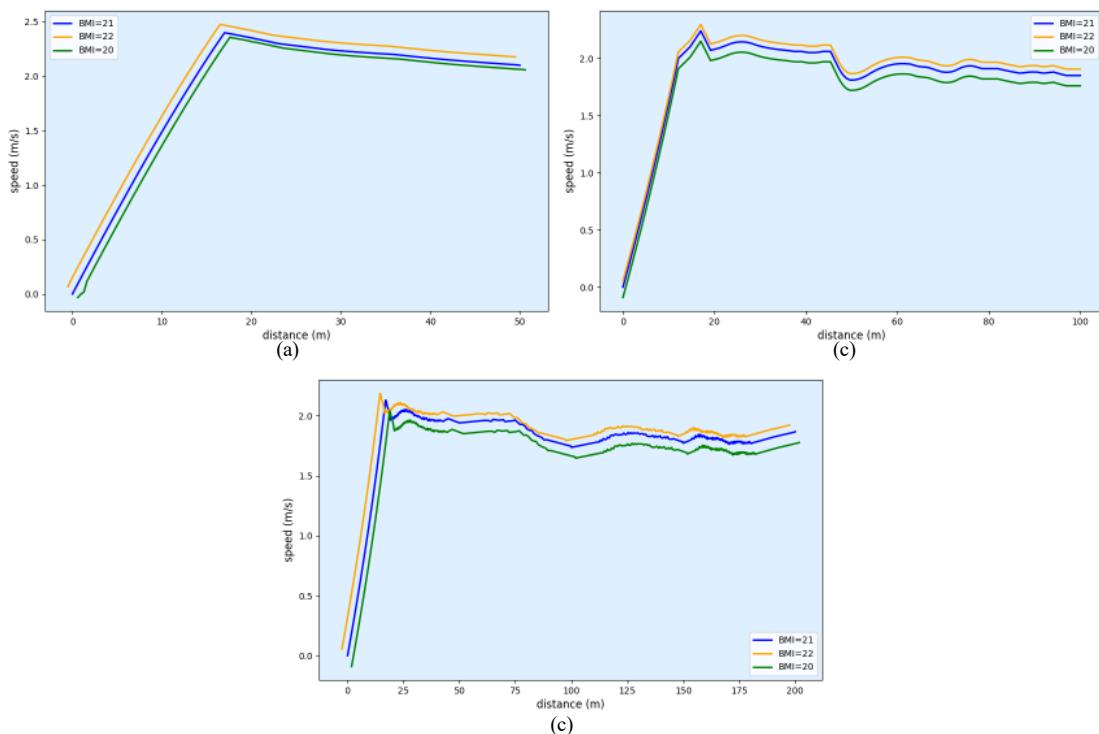


Figure 7:(a) is the distribution of optimal speeds for different BMIs over the 50-meter course. (b) is the distribution of optimal speeds for different BMIs over the 100-meter course. (c) is the distribution of optimal speeds for different BMIs over the 200-meter course.

According to the picture we can see that after adding perturbation to the BMI of the model, the optimal speed distribution of the athletes in different races is basically the same trend of change although there are differences, indicating that the change of these parameters has less influence on the model and the model has a better fitting effect.

6 Velocity perturbation model (VPM)

In swimming competitions, different competition strategies will directly affect the speed change of athletes. In order to better simulate the characteristics of three common strategies (mutual exploration strategy, leading strategy and following strategy), the dynamic impact of strategies on speed can be reflected by adding corresponding perturbation functions to the basic speed model. The flowchart for the establishment of the speed perturbation model is as follows:

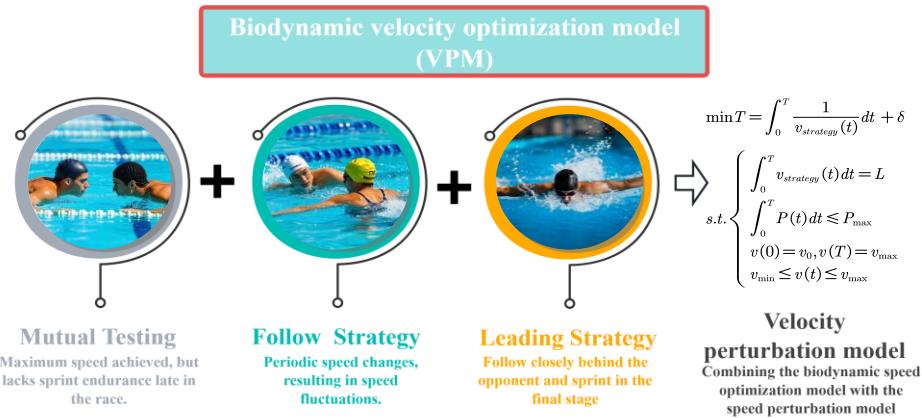


Figure 8: Flowchart for the Establishment of the Speed Perturbation Model

6.1 The establishment of the velocity perturbation model

Specifically, the mutual exploration strategy uses periodic or random fluctuations to show the process of athletes observing and adjusting during the game; the leading strategy uses the speed change that gradually stabilizes after a rapid initial increase to show the characteristics of sprinting and leading; and the following strategy combines stability with small fluctuations to simulate the energy-saving method of athletes using the water flow effect in front. Based on this, we established the following perturbation model to describe the impact of these three strategies on speed.

6.1.1 Mutual probing strategy

The speed of the mutual probing strategy shows fluctuations, reflecting the behavior of athletes observing their opponents and adjusting their strategies according to their opponents. It is applicable to situations where athletes do not fully understand the strength of their opponents, especially when multiple high-level athletes compete on the same field. It usually occurs in the first half or middle of the game, and the probing is used to prepare for the key acceleration in the second half. Therefore, the speed $v(t)$ is combined with the perturbation function to express the speed $v_{probe}(t)$ under the mutual probing strategy as:

$$v_{probe}(t) = v(t) + A \sin(\omega t) + \epsilon(t) \quad (14)$$

Among them, A is the fluctuation amplitude, which is related to the test intensity; ω is the fluctuation frequency; $\epsilon(t)$ is the random noise, which is mainly related to external environmental factors.

6.1.2 Leading strategy

The leading strategy means that the athlete quickly enters a high-intensity state at the beginning of the game, sprints at full speed to gain a leading position, and tries to maintain this leading advantage for the rest of the game. This strategy requires athletes to have strong explosive power and high stability. It is usually suitable for short-distance competitions (such as 50-meter or 100-meter freestyle) because when the distance is short, the pressure on physical energy distribution is less, and it is easier to maintain the advantage after leading. It requires high psychological quality because once surpassed by the opponent, psychological pressure may affect subsequent performance. Combined with the linear growth model, the speed $v_{lead}(t)$ under the leading strategy is:

$$v_{lead}(t) = \begin{cases} v_0 + kt + \epsilon(t), & t \leq t_{max} \\ v_{max} + \epsilon(t), & t > t_{max} \end{cases} \quad (15)$$

Among them, v_{max} is the maximum speed that the athlete can reach; the time required for t_{max} to reach the maximum speed is 7 seconds based on Pan Zhanle's performance in the Olympics; k is the growth rate.

6.1.3 Following strategy

The following strategy refers to the athlete choosing to follow the opponent in front of him in the competition, saving energy by reducing resistance through the water flow effect created by him, and surpassing the opponent at the critical moment. This strategy emphasizes efficiency first and reduces energy consumption by using water flow. It is common in medium and long distance competitions (such as 400 meters or 1500 meters freestyle), and athletes need to accurately allocate their physical strength. It is suitable for athletes who are confident in their sprinting ability and plan to launch an attack in the later stage of the game. Using the influence of the speed $v_{lead}(t)$ of the athlete in front, it can be concluded that the speed $v_{follow}(t)$ under the following strategy is:

$$v_{follow}(t) = v_{lead}(t) - \Delta + \epsilon(t) \quad (16)$$

Among them, Δ is the speed difference with the athlete in front who adopts the leading strategy, which is usually very small.

6.1.4 Velocity perturbation optimization model (VPOM)

Combining the biodynamic speed optimization model with the speed perturbation model can obtain the speed perturbation optimization model, with the swimming speed $v_{strategy}(t)$ of

different strategies as the decision variable, minimizing the time as the objective function, and the constraints are similar to those of the biodynamic speed optimization model. Substituting the swimming speed of different swimming strategies into the final model is:

$$\min T = \int_0^T \frac{1}{v_{strategy}(t)} ds + \delta$$

$$s.t. \left\{ \begin{array}{l} \int_0^T v_{strategy}(t) dt = L \\ \int_0^T P(t) dt \leq P_{max} \\ v(0) = v_0, v(T) = v_{max} \\ v_{min} \leq v(t) \leq v_{max} \end{array} \right. \quad (17)$$

Where $v_{strategy}(t)$ is the swimming speed of different strategies, including the $v_{probe}(t)$ of the mutual probing strategy, the $v_{lead}(t)$ of the leading strategy, and the $v_{follow}(t)$ of the following strategy.

6.2 Solution of velocity perturbation optimization model

Similar to the BVOM of task 1, it is not convenient to solve for time T directly, so we introduce a parametric approach to discretize t into N time nodes t_0, t_1, \dots, t_N and approximate the integration with numerical integration, after discretization, the optimization problem becomes:

$$\min T = \sum_{N-1}^{i=0} \frac{\Delta s}{v_{si}} + \delta$$

$$s.t. \left\{ \begin{array}{l} \sum_{N-1}^{i=0} v_{si} \Delta t = L \\ \sum_{N-1}^{i=0} k v_{si}^2 \Delta t \leq P_{max} \\ v_{min} \leq v_{si} \leq v_{max}, \quad \forall i \\ v_0 = 0 \quad v_N = v_{max} \end{array} \right. \quad (18)$$

Unlike Task 1, the v_{si} in the model is a velocity microelement that contains a swimming strategy and a perturbation function.

6.2.1 Stochastic perturbation solving

In order to reflect more realistically the influence of the external environment, the athlete's state and other uncertain factors in swimming competitions, this study introduces a random

perturbation term in the velocity model, and the random perturbation $\epsilon(t)$ is a part of the velocity model, expressed as a stochastic process, whose noise values at different points in time are independent of each other, and mathematically expressed as follows:

$$\epsilon(t) \sim \mathcal{N}(0, \sigma^2) \quad (19)$$

Its mean is 0, i.e., the noise is not shifted overall; σ^2 is the fixed variance, reflecting the strength of the perturbation, using Python's random number generating function to produce normally distributed random perturbations.

● Discrete

First, discretize time t into a series of time points t_0, t_1, \dots, t_n . The time interval between each time point is then we have:

$$t_i = i \cdot \Delta t, \quad \Delta t = \frac{T}{n} \quad (20)$$

Based on the discretized time series, a numerical integration method is used to calculate the displacement of the athlete, and the displacement $x(t)$ of the athlete is solved by integrating the velocity $v_{si}(t)$:

$$x(t) = \int_0^t v_{si}(t) dt \approx \sum_{n=1}^{i=0} v_{si}(t_i) \Delta t \quad (21)$$

Where $v_{si}(t_i)$ is the velocity incorporating the swimming strategy and perturbation function, and at each time step, the velocity is used to approximate the displacement for that time period.

6.2.2 Results of the VPOM Model

We used Python programming to solve the VPOM model for the 100m race, and the optimal velocity distributions under three different strategies are shown in the figure below:

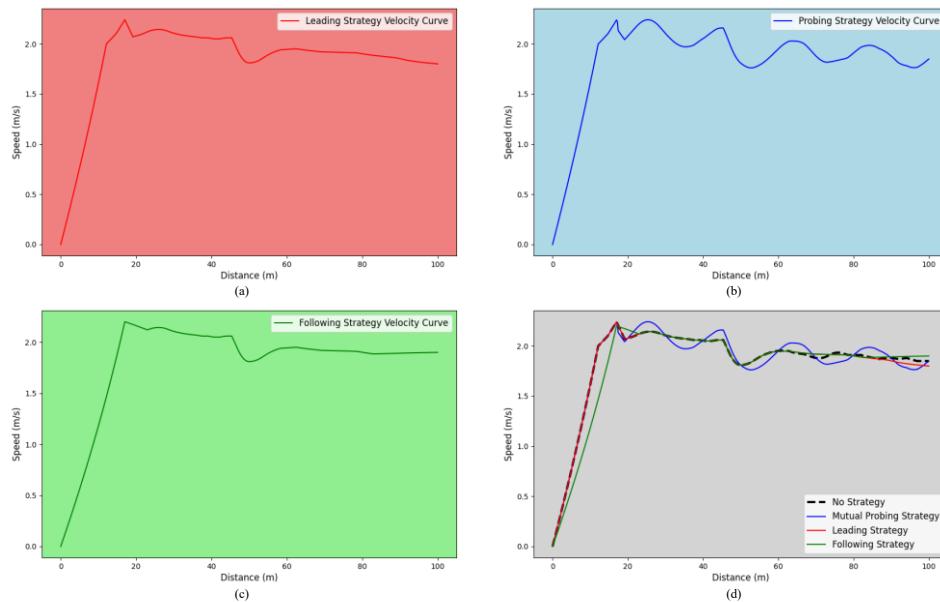


Figure 9: (a) is the velocity curve under the leading strategy. (b) is the velocity curve under the mutual probing strategy. (c) is the velocity curve under the following strategy.(d) is the comparison of velocity curves for the three strategies and the no-strategy scenario.

From the figure above, it can be seen that the leading strategy achieves the highest maximum speed v_{\max} , but lacks the endurance for a sprint in the later stages of the race. The mutual probing strategy shows a periodic speed variation, which means that the athlete adjusts their pace continuously during the race based on the opponent's state and movements, resulting in fluctuating speed. Although the maximum speed in the following strategy is not as high as in the leading strategy, as shown in figure (c), the following strategy allows for a sprint in the later stages of the race. Overall, in the 100-meter freestyle, no single strategy is superior to the others; the final choice should depend on the athlete's characteristics and physical condition.

7 Pan Zhanle's Swimming Strategy

To more intuitively observe the effect of the strategies adopted, we collected Pan Zhanle's speed data from the $4 \times 100m$ final at the Olympics and visualized it as shown in the figure below:

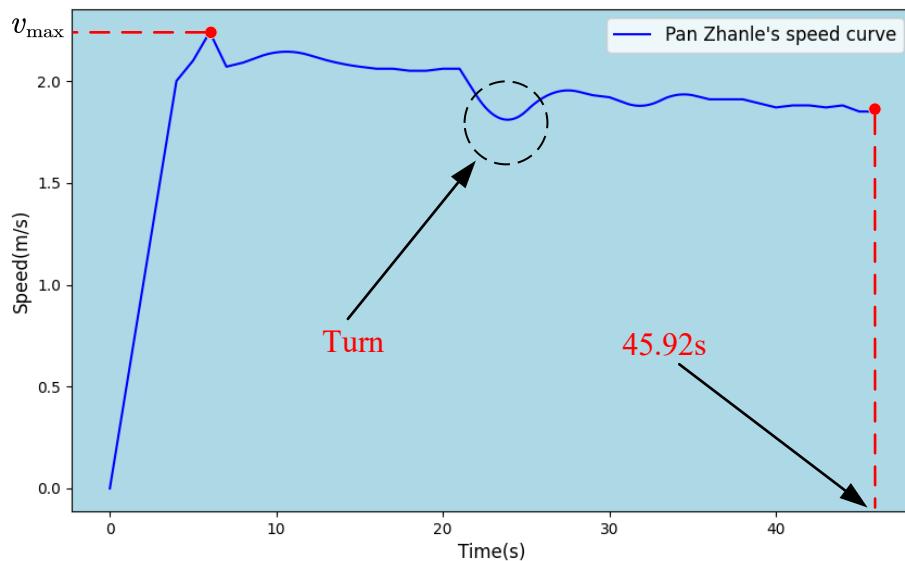


Figure 10: Pan Zhanle's speed curve

7.1 Stage of delineation

We divided the race into three main phases: the initial phase (0-25 meters), the acceleration phase (25-75 meters), and the decisive phase (75-100 meters). Based on the results derived from Task 2, the performance of Pan Zhanle was gradually optimized by combining the three strategy characteristics of mutual testing, leading and following.

7.1.1 Initial phase

The initial phase is dominated by a leading strategy so as to quickly establish a lead or

secure a place at the top, laying a good foundation for subsequent phases.

- **Jumping and diving:** Complete the jump with the fastest possible reaction time to ensure maximum departure speed off the diving platform. Adjust the angle of entry to reduce water resistance and maximize propulsion.
- **Underwater propulsion:** The underwater butterfly leg kick needs to be high-frequency and strong to maximize the efficiency of the water flow within 15 meters. Control the breathing rhythm to avoid disrupting the streamlined posture of underwater propulsion by breathing early.
- **Adjustment of stroke rhythm after emergence from the water:** Quickly move into an efficient freestyle stroke out of the water with a wide but smooth stroke. Ensure peak speed and aim to be in the lead or close to it at 25 meters.

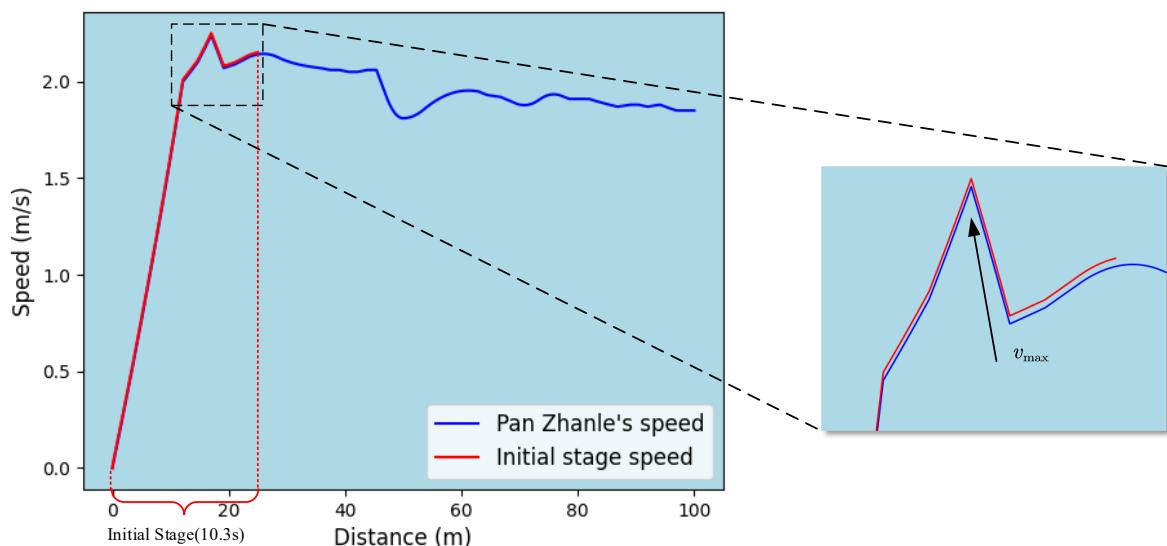


Figure 11: Comparison of Speeds in the Initial Stage

As can be seen from the above figure, the leading strategy in the initial stage resulted in a maximum speed greater than Pan Zhanle's original speed. After adopting the leading strategy, the total time spent in the initial stage was **10.3 seconds**.

7.1.2 Accelerated phase

The acceleration phase requires a combination of following and mutual probing strategies to balance physical exertion and speed performance, and flexibly respond to the race situation to retain sufficient energy for the decisive phase.

- **Selection of strategies based on location:** If Pan Zhanle is in the first place, he should reduce his paddling frequency to minimize physical exertion. Adjust your own rhythm by observing the frequency of your opponent's movements to avoid premature physical exhaustion; if Pan Zhanle is not in the first place, he should follow the competitor in front of

him and utilize the water current he creates to reduce the resistance. Control the rhythm and frequency of paddling to maintain a relatively stable speed to avoid falling behind.

- **Optimization of action details:** Maintain an efficient stroke: paddle with your elbows up and your entry point close to your body axis to minimize lateral sway. Avoid ineffective movements: Adjust the amplitude and frequency of leg strokes to balance energy conservation and propulsion efficiency.
- **Observation of counterparty dynamics:** Use your afterglow to observe the positions and rhythms of the players around you, to judge your opponent's form and fitness distribution, and to keep yourself in a strategic advantageous position at all times by making small adjustments to your speed.

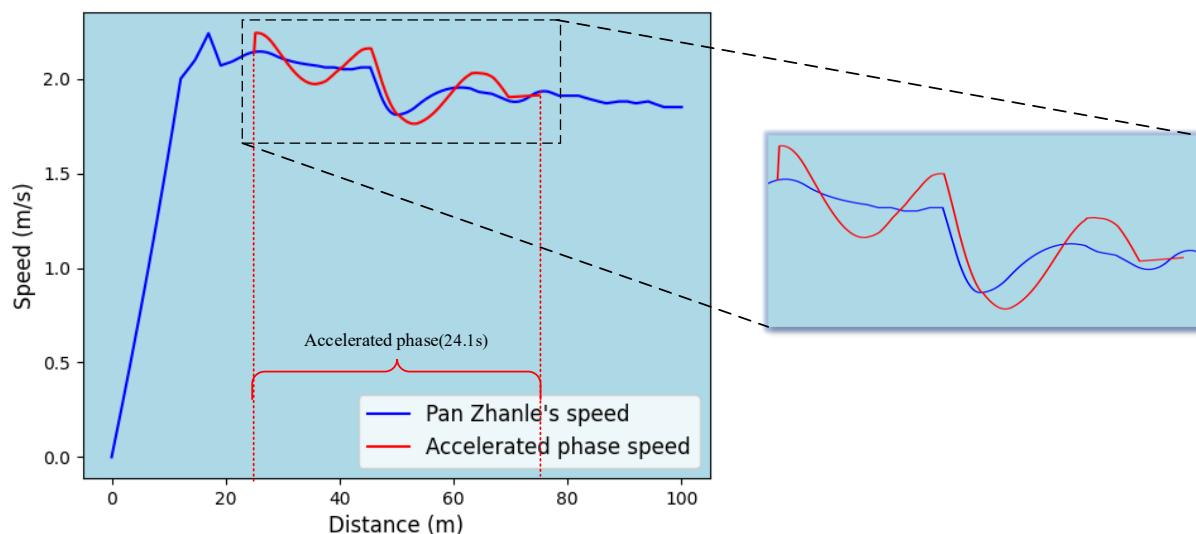


Figure 12: Comparison of Speeds in the Acceleration Stage

As can be seen from the above figure, in the acceleration stage, the combination of the following strategy and exploratory strategy led to a periodic variation in Pan Zhanle's speed, which effectively saved energy and laid a solid foundation for the decisive stage. After adopting the strategy, the total time spent in the acceleration stage was **24.1 seconds**.

7.1.3 Decisive phase

The decisive stage requires a combination of leading and mutual probing strategies to achieve a burst of speed in the last 25 meters of the race to get to the finish line as fast as possible.

- **Speed adjustment:** **75-85meters:** gradually accelerate the current maximum speed, paddling frequency increases significantly; **85-100meters:** full sprint, maintain maximum paddling frequency, strong and stable leg movements, minimize ineffective energy consumption.

- **Action details:** Paddle with palms in the water and then quickly press the water downward and pull to increase propulsion; reduce the number of breaths (if possible, no more breath changes in the last 15 meters) to keep the head steady and reduce resistance; in the final stage of the sprint, fully extend the arm to touch the wall to ensure precise movement.
- **Analysis of counterparty dynamics:** If there is an opponent approaching, the paddling frequency should be further increased to close the gap through the frequency advantage; if the lead is obvious, maintain a steady sprint speed to avoid unnecessary energy waste.

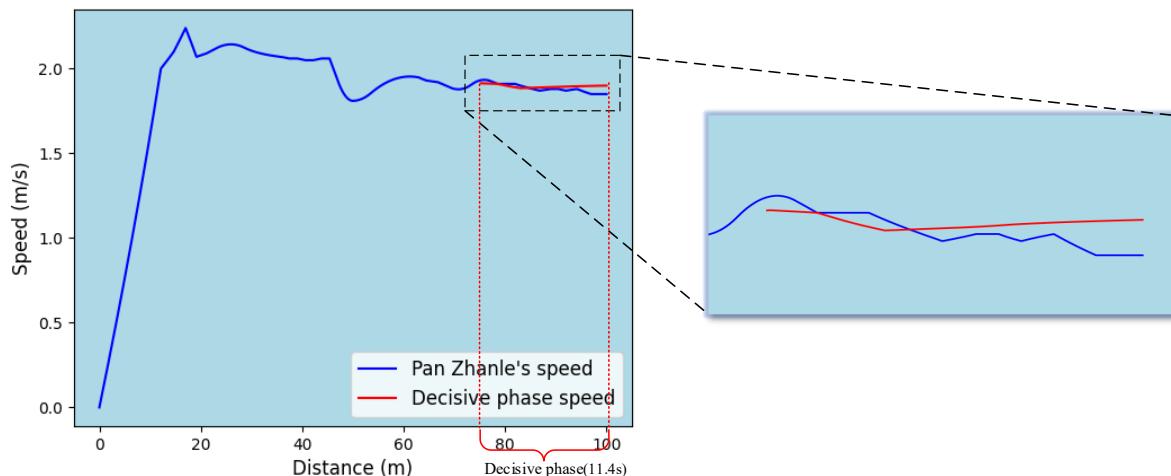


Figure 13: Comparison of Speeds in the Decisive Stage

As can be seen from the above figure, in the decisive stage, the combination of the leading strategy and exploratory strategy resulted in an upward trend in Pan Zhanle's speed, allowing him to begin overtaking his opponents. After adopting the strategy, the total time spent in the decisive stage was 11.4 seconds.

Finally, here is the comparison between Pan Zhanle's speed using the strategies and his original speed, as shown below:

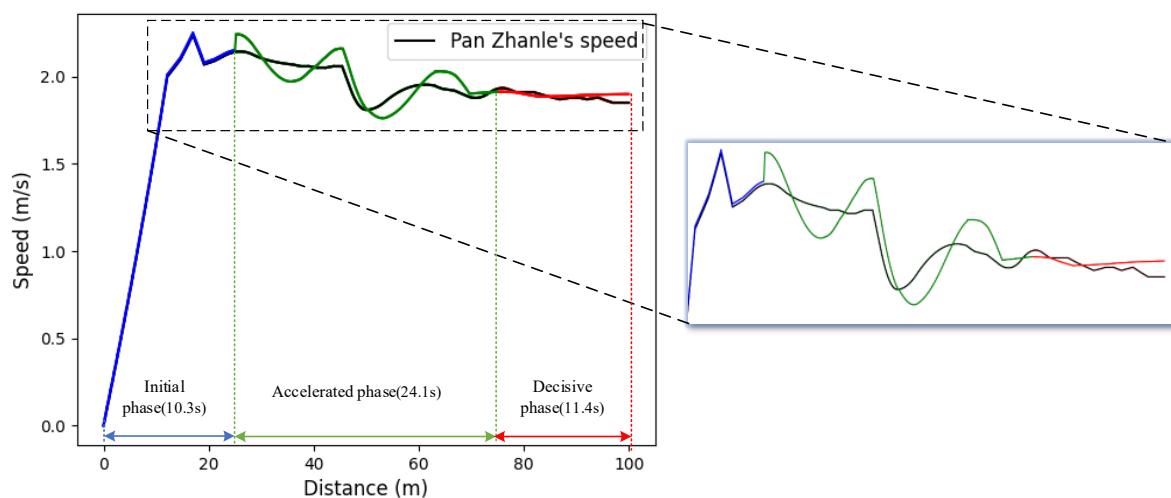


Figure 14: Comparison of Pan Zhanle's Speed with and without Strategies

After adopting the above strategies, Pan Zhanle's final time was **45.8 seconds**, achieving a better result than his 45.92 seconds in the final.

8 Sensitivity Analysis

Considering that real sports competition environments are much more complex than theoretical data, the actual values of parameters such as maximum speed, maximum power, and noise levels may vary among different athletes and under different conditions, it is necessary to perform a sensitivity analysis on the model to test its sensitivity.

8.1 Sensitivity Analysis

For the sensitivity of the model in task 2, we considered introducing a $\pm 5\%$ perturbation to the three parameters: maximum power, maximum speed, and noise levels. We then tested how the changes in these parameters at different swimming distances affected the total time. The results are shown in the following figure:

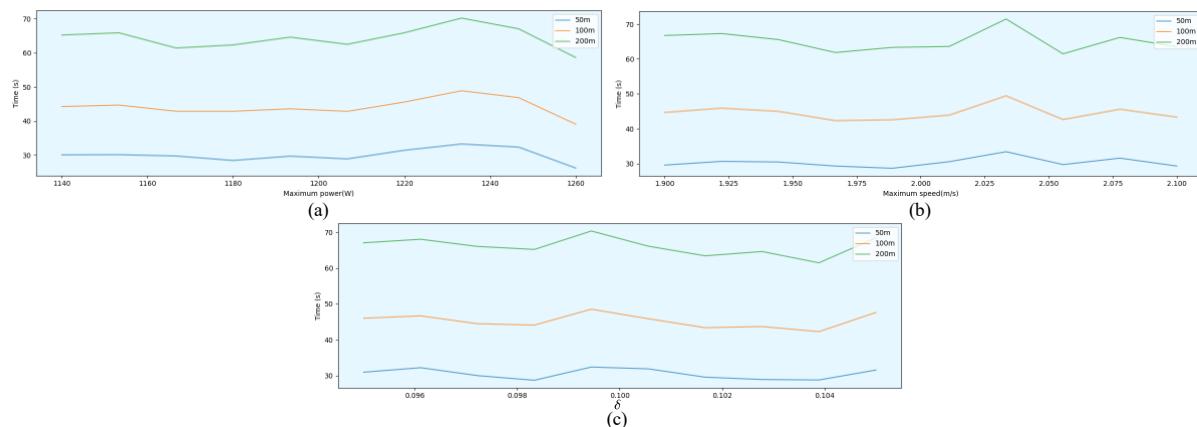


Figure 15: Figure 4: (a) The effect of maximum power P_{max} on the best freestyle time T across different swimming distances. (b) The effect of maximum speed v_{max} on the best freestyle time T across different swimming distances. (c) The effect of noise variance δ on the best freestyle time T across different swimming distances.

Based on the figure, we can observe that after introducing perturbations to the model parameters, there are differences in the total time taken by the athlete to complete the race. However, the changes remain within an acceptable range, indicating that these variables have a minimal impact on the model, and the model exhibits good fitting results.

9 Model Evaluation and Further Discussion

9.1 Strengths

- Scientificity of the results: We adopted the famous Newtonian mechanics model. This model is concise and can produce scientific and reliable mechanical analysis results.
- Deep fit with reality: Starting from the basic energy supply model, we built a rider's power output model, and our modeling is closer to reality.
- Stability: We conducted a sensitivity test on the model and verified its stability.

9.2 Weaknesses

- In the process of model building, many contents are simplified, which may cause the model to reflect the actual problem inaccurately.
- Data in real life is continuous, but it is difficult for computer simulation to handle continuous data, so we decompose the data, which causes the continuous data to lose some information.

9.3 Further Discussion

- Data collection: Collect more data to improve the accuracy and robustness of the model.
- Model validation: Apply the model to other tracks to test its effectiveness.

References

- [1] Zamparo P, Bonifazi M, Faina M, et al. Energy cost of swimming of elite long-distance swimmers[J]. European journal of applied physiology, 2005, 94(5): 697-704.
- [2] Toussaint H M, Beek P J. Biomechanics of competitive front crawl swimming[J]. Sports medicine, 1992, 13: 8-24.
- [3] Wakayoshi K, D'Acquisto L J, Cappaert J M, et al. Relationship between oxygen uptake, stroke rate and swimming velocity in competitive swimming[J]. International journal of sports medicine, 1995, 16(01): 19-23.
- [4] Barbosa T M, Bragada J A, Reis V M, et al. Energetics and biomechanics as determining factors of swimming performance: updating the state of the art[J]. Journal of science and medicine in sport, 2010, 13(2): 262-269.
- [5] Chatard J C, Wilson B. Drafting distance in swimming[J]. Medicine & Science in Sports & Exercise, 2003, 35(7): 1176-1181.
- [6] Rushall, B. S. "The Science of Swimming Faster." International Journal of Sports Science Coaching, vol. 10, no. 4, 1995, pp. 567 – 578.
- [7] Riewald S A, Rodeo S A. Science of swimming faster[M]. Human Kinetics, 2015.

- [8] Zhang Zhihu, Zheng Minggang, Qu Cheng. Viscous Drag Analysis of Swimming Competitions Based on CFD [J]. Modeling and Simulation, 2022, 11: 797.

Report on Use of AI

AI version: *OpenAI ChatGPT*(Sep 3, 2024 version, ChatGPT-4o)

Query 1: The " probing strategy," "leading strategy," and "following strategy" each have their own characteristics ?

Output 1:

Probing Strategy:

- **Alternating Cooperation and Retaliation:** This strategy initially adopts a cooperative approach, then adjusts based on the other party's behavior. If the other party cooperates, the individual also cooperates; if the other party betrays, the individual retaliates in kind.
- **Simplicity and Understandability:** This strategy is straightforward, easy to implement, and simple to understand. Participants can easily grasp and apply it, which helps build trust and foster cooperation.
- **High Tolerance for Errors:** If the other party betrays in one round, the strategy retaliates once, but then returns to cooperation, avoiding prolonged conflict. This tolerance allows for recovery from occasional mistakes.

Leading Strategy:

- **Preemptive Strike:** At the start of the game, the leader takes proactive actions, using their first-mover advantage to influence the decisions of other participants.
- **Strong Control:** By taking the initiative with an aggressive strategy, the leader can dominate the situation, forcing others to follow their actions or respond accordingly.
- **High Risk and High Reward:** This strategy can bring substantial rewards but also carries significant risks, especially when the leader is unsure of how the other participants will react.

Following Strategy:

- **Dependence on Others' Actions:** Participants using this strategy observe and follow the leader's decisions or actions, without taking initiative themselves. They typically react after others have made their decisions.
- **Low Risk, Low Reward:** Those employing the following strategy generally avoid taking risks, resulting in relatively lower rewards, as they always remain in a reactive position.
- **Lack of Initiative:** The executors of this strategy lack proactivity and innovation, relying more on others' leadership and decisions rather than taking charge themselves.

AI version: *OpenAI ChatGPT*(Sep 3, 2024 version, ChatGPT-4o)

Query 2: Is there a strategy that is superior to the others ?

Output 2:

There is no strategy that is superior to others in all situations because the effectiveness of a strategy is influenced by the specific game environment and the participants' objectives. In repeated games or long-term interactions, the mutual probing strategy is often effective in promoting cooperation because it maintains fairness through retaliation and rewards cooperation. In scenarios that require a preemptive strike, the leading strategy can secure resources or initiative by acting first. If participants have a lower risk tolerance or lack a clear leader, the following strategy may be the most appropriate choice, as it reduces uncertainty and risk in decision-making. Participants' goals also affect strategy selection: if the goal is long-term stable cooperation, the mutual probing strategy is more effective; if the goal is to quickly capture markets or resources, the leading strategy can provide a competitive edge; and if the goal is to avoid risk and secure minimal returns, the following strategy may be the most suitable. Overall, whether one strategy is superior to others depends on the specific context of the game, so there is no universally best strategy.