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Advanced Discretization Methods
Spring Semester 2020
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Exercise Sheet 1

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Exercise 1

Show that the 6th order accurate central finite difference approximation is given as

$$\frac{du}{dx}\Big|_{x_j} = \frac{-u_{j-3} + 9u_{j-2} - 45u_{j-1} + 45u_{j+1} - 9u_{j+2} + u_{j+3}}{60\Delta x}.$$

Exercise 2

Consider the linear wave problem

$$\frac{\partial u}{\partial t} = -c\frac{\partial u}{\partial x},$$
$$u(0,t) = u(2\pi, t),$$
$$u(x,0) = \exp(ikx).$$

For the 6th order approximation, show that the numerical wave speed is given as

$$c_3(k) = c\frac{45\sin(k\Delta x) - 9\sin(2k\Delta x) + \sin(3k\Delta x)}{30k\Delta x},$$

and that the leading order phase error is given as

$$e_3(p,\nu) = \frac{\pi\nu}{70}\left(\frac{2\pi}{p}\right)^6.$$

Based on this, show that

$$p_3(\varepsilon_p, \nu) \geq 2\pi\sqrt[6]{\frac{\pi\nu}{70\varepsilon_p}},$$

and compute the number of points required to ensure $\varepsilon_p = 0.1$ and $\varepsilon_p = 0.01$. Compare the results with 2nd and 4th order schemes discussed in the book. When it is advantageous to use a 6th order scheme?

Exercise 3

Write a short program using language of your choice (C, C++, Matlab, python, ..) to test the accuracy of the Fourier differentiation matrix

$$\tilde{D}_{ji} = \begin{cases} \frac{(-1)^{j+i}}{2}\left[\sin\left(\frac{(j-i)\pi}{N+1}\right)\right]^{-1} & \text{if } i \neq j \\ 0 & \text{if } i = j \end{cases}$$

on the function

$$u(x) = \exp(k\sin x),$$

in the interval $x \in [0, 2\pi]$. Take $k = 2, 4, 6, 8, 10, 12$, and measure the relative pointwise error. Determine the minimum N for each value of k that ensures a maximum error less than 10^{-5}.