Chapter 11

Controller Parameterization

The basic configuration of the feedback systems considered in this chapter is an LFT , as shown in Figure 11.1, where G is the generalized plant with two sets of inputs: the exogenous inputs w, which include disturbances and commands, and control inputs u. The plant G also has two sets of outputs: the measured (or sensor) outputs y and the regulated outputs z. K is the controller to be designed. A control problem in this setup is either to analyze some specific properties (e.g., stability or performance) of the closed loop or to design the feedback control K such that the closed-loop system is stable in some appropriate sense and the error signal z is specified (i.e., some performance specifications are satisfied). In this chapter we are only concerned with the basic internal stabilization problems. We will see again that this setup is very convenient for other general control synthesis problems in the coming chapters.

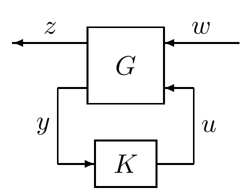


Figure 11.1: General system interconnection

Suppose that a given feedback system is feedback stabilizable. In this chapter, the problem we are mostly interested in is parameterizing all controllers that stabilize the system. The *parameterization* of all internally stabilizing controllers was first introduced by Youla et al. [1976a, 1976b] using the *coprime factorization* technique. We shall, however, focus on the state-space approach in this chapter.

11.1 Existence of Stabilizing Controllers

Consider a system described by the standard block diagram in Figure 11.1. Assume that $G(s)$ has a *stabilizable and detectable* realization of the form

$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right]. \tag{11.1}$$

The stabilization problem is to find feedback mapping K such that the closed-loop system is internally stable; the well-posedness is required for this interconnection.

Definition 11.1 A proper system G is said to be *stabilizable* through output feedback if there exists a proper controller K internally stabilizing G in Figure 11.1. Moreover, a proper controller $K(s)$ is said to be *admissible* if it internally stabilizes G.

The following result is standard and follows from Chapter 3.

Lemma 11.1 *There exists a proper K achieving internal stability iff (A, B_2) is stabilizable and (C_2, A) is detectable. Further, let F and L be such that $A + B_2F$ and $A + LC_2$ are stable; then an observer-based stabilizing controller is given by*

$$K(s) = \left[\begin{array}{c|c} A + B_2F + LC_2 + LD_{22}F & -L \\ \hline F & 0 \end{array}\right].$$

Proof. ($\Leftarrow$) By the stabilizability and detectability assumptions, there exist F and L such that $A + B_2F$ and $A + LC_2$ are stable. Now let $K(s)$ be the observer-based controller given in the lemma, then the closed-loop A matrix is given by

$$\tilde{A} = \begin{bmatrix} A & B_2F \\ -LC_2 & A + B_2F + LC_2 \end{bmatrix}.$$

It is easy to check that this matrix is similar to the matrix

$$\begin{bmatrix} A + LC_2 & 0 \\ -LC_2 & A + B_2F \end{bmatrix}.$$

Thus the spectrum of $\tilde{A}$ equals the union of the spectra of $A + LC_2$ and $A + B_2F$. In particular, $\tilde{A}$ is stable.

($\Rightarrow$) If (A, B_2) is not stabilizable or if (C_2, A) is not detectable, then there are some eigenvalues of $\tilde{A}$ that are fixed in the right-half plane, no matter what the compensator is. The details are left as an exercise. $\square$

The stabilizability and detectability conditions of (A, B_2, C_2) are assumed throughout the remainder of this chapter.[1] It follows that the realization for G_{22} is stabilizable and detectable, and these assumptions are enough to yield the following result.

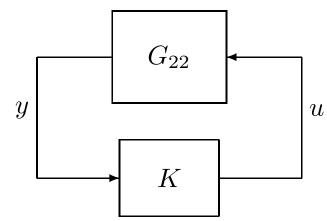


Figure 11.2: Equivalent stabilization diagram

Lemma 11.2 *Suppose the inherited realization* $\left[\begin{array}{c|c} A & B_2 \\ \hline C_2 & D_{22} \end{array}\right]$ *for* G_{22} *is stabilizable and detectable. Then the system in Figure 11.1 is internally stable iff the one in Figure 11.2 is internally stable.*

In other words, $K(s)$ internally stabilizes $G(s)$ if and only if it internally stabilizes G_{22} [provided that (A, B_2, C_2) is stabilizable and detectable].

Proof. The necessity follows from the definition. To show the sufficiency, it is sufficient to show that the system in Figure 11.1 and that in Figure 11.2 share the same A matrix, which is obvious. □

From Lemma 11.2, we see that the stabilizing controller for G depends only on G_{22}. Hence all stabilizing controllers for G can be obtained by using only G_{22}.

Remark 11.1 There should be no confusion between a given realization for a transfer matrix G_{22} and the inherited realization from G, where G_{22} is a submatrix. A given realization for G_{22} may be stabilizable and detectable while the inherited realization may not be. For instance,

$$G_{22} = \frac{1}{s+1} = \left[\begin{array}{c|c} -1 & 1 \\ \hline 1 & 0 \end{array}\right]$$

is a minimal realization but the inherited realization of G_{22} from

$$\left[\begin{array}{cc} G_{11} & G_{12} \\ G_{21} & G_{22} \end{array}\right] = \left[\begin{array}{cc|cc} -1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ \hline 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{array}\right]$$

[1] It should be clear that the stabilizability and detectability of a realization for G do not guarantee the stabilizability and/or detectability of the corresponding realization for G_{22}.

is

$$G_{22} = \left[\begin{array}{cc|c} -1 & 0 & 1 \\ 0 & 1 & 0 \\ \hline 1 & 0 & 0 \end{array}\right] \quad \left(= \frac{1}{s+1}\right),$$

which is neither stabilizable nor detectable. ◇

11.2 Parameterization of All Stabilizing Controllers

Consider again the standard system block diagram in Figure 11.1 with

$$G(s) = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right] = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix}.$$

Suppose (A, B_2) is stabilizable and (C_2, A) is detectable. In this section we discuss the following problem:

Given a plant G, parameterize all controllers K that internally stabilize G.

This parameterization for all stabilizing controllers is usually called Youla parameterization. The parameterization of all stabilizing controllers is easy when the plant itself is stable.

Theorem 11.3 *Suppose $G \in \mathcal{RH}_\infty$; then the set of all stabilizing controllers can be described as*

$$K = Q(I + G_{22}Q)^{-1} \tag{11.2}$$

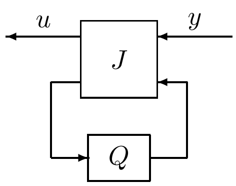
for any $Q \in \mathcal{RH}_\infty$ and $I + D_{22}Q(\infty)$ nonsingular.

Remark 11.2 This result is very natural considering Corollary 5.3, which says that a controller K stabilizes a stable plant G_{22} iff $K(I - G_{22}K)^{-1}$ is stable. Now suppose $Q = K(I - G_{22}K)^{-1}$ is a stable transfer matrix, then K can be solved from this equation which gives exactly the controller parameterization in the preceding theorem. ◇

Proof. Note that $G_{22}(s)$ is stable by the assumptions on G. Then it is straightforward to verify that the controllers given previously stabilize G_{22}. On the other hand, suppose K_0 is a stabilizing controller; then $Q_0 := K_0(I - G_{22}K_0)^{-1} \in \mathcal{RH}_\infty$, so K_0 can be expressed as $K_0 = Q_0(I + G_{22}Q_0)^{-1}$. Note that the invertibility in the last equation is guaranteed by the well-posedness condition of the interconnected system with controller K_0 since $I + D_{22}Q_0(\infty) = (I - D_{22}K_0(\infty))^{-1}$. □

However, if G is not stable, the parameterization is much more complicated. The results can be more conveniently stated using state-space representations.

Theorem 11.4 *Let F and L be such that $A + LC_2$ and $A + B_2F$ are stable; then all controllers that internally stabilize G can be parameterized as the transfer matrix from y to u:*



$$J = \left[\begin{array}{c|cc} A + B_2F + LC_2 + LD_{22}F & -L & B_2 + LD_{22} \\ \hline F & 0 & I \\ -(C_2 + D_{22}F) & I & -D_{22} \end{array}\right]$$

with any $Q \in \mathcal{RH}_\infty$ and $I + D_{22}Q(\infty)$ nonsingular. Furthermore, the set of all closed-loop transfer matrices from w to z achievable by an internally stabilizing proper controller is equal to

$$\mathcal{F}_\ell(T, Q) = \{T_{11} + T_{12}QT_{21} : \ Q \in \mathcal{RH}_\infty, \ I + D_{22}Q(\infty) \text{ invertible}\}$$

where T is given by

$$T = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \left[\begin{array}{cc|cc} A + B_2F & -B_2F & B_1 & B_2 \\ 0 & A + LC_2 & B_1 + LD_{21} & 0 \\ \hline C_1 + D_{12}F & -D_{12}F & D_{11} & D_{12} \\ 0 & C_2 & D_{21} & 0 \end{array}\right].$$

Proof. Let $K = \mathcal{F}_\ell(J, Q)$. Then it is straightforward to verify, by using the state-space star product formula and some tedious algebra, that $\mathcal{F}_\ell(G, K) = T_{11} + T_{12}QT_{21}$ with the T given in the theorem. Hence the controller $K = \mathcal{F}_\ell(J, Q)$ for any given $Q \in \mathcal{RH}_\infty$ does internally stabilize G. Now let K be any stabilizing controller for G; then $\mathcal{F}_\ell(\hat{J}, K) \in \mathcal{RH}_\infty$, where

$$\hat{J} = \left[\begin{array}{c|cc} A & -L & B_2 \\ \hline -F & 0 & I \\ C_2 & I & D_{22} \end{array}\right].$$

($\hat{J}$ is stabilized by K since it has the same G_{22} matrix as G.)

Let $Q_0 := \mathcal{F}_\ell(\hat{J}, K) \in \mathcal{RH}_\infty$; then $\mathcal{F}_\ell(J, Q_0) = \mathcal{F}_\ell(J, \mathcal{F}_\ell(\hat{J}, K)) =: \mathcal{F}_\ell(J_{tmp}, K)$, where J_{tmp} can be obtained by using the state-space star product formula given in Chapter 9:

$$J_{tmp} = \left[\begin{array}{cc|cc} A + LC_2 + B_2F + LD_{22}F & -(B_2 + LD_{22})F & -L & B_2 + LD_{22} \\ L(C_2 + D_{22}F) & A - LD_{22}F & -L & B_2 + LD_{22} \\ \hline F & -F & 0 & I \\ -(C_2 + D_{22}F) & C_2 + D_{22}F & I & 0 \end{array}\right]$$

$$= \left[\begin{array}{cc|cc} A+LC_2 & -(B_2+LD_{22})F & -L & B_2+LD_{22} \\ 0 & A+B_2F & 0 & 0 \\ \hline 0 & -F & 0 & I \\ 0 & C_2 & I & 0 \end{array}\right]$$

$$= \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}.$$

Hence $\mathcal{F}_\ell(J, Q_0) = \mathcal{F}_\ell(J_{tmp}, K) = K$. This shows that any stabilizing controller can be expressed in the form of $\mathcal{F}_\ell(J, Q_0)$ for some $Q_0 \in \mathcal{RH}_\infty$. □

An important point to note is that the closed-loop transfer matrix is simply an affine function of the controller parameter matrix Q. The proper K's achieving internal stability are precisely those represented in Figure 11.3.

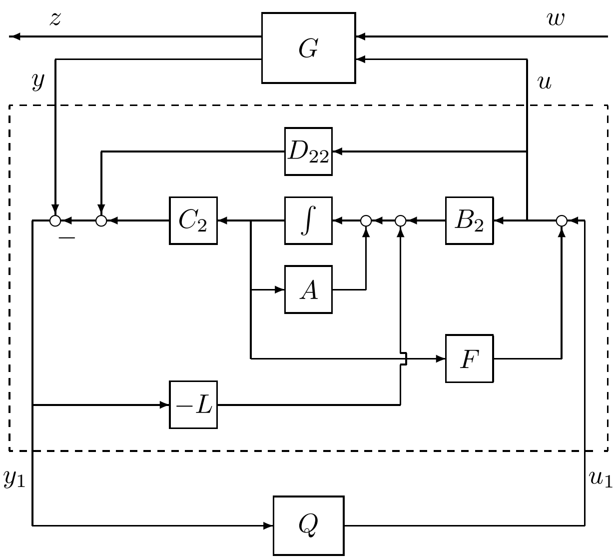


Figure 11.3: Structure of stabilizing controllers

It is interesting to note that the system in the dashed box is an observer-based stabilizing controller for G (or G_{22}). Furthermore, it is easy to show that the transfer function between (y, u_1) and (u, y_1) is J; that is,

$$\begin{bmatrix} u \\ y_1 \end{bmatrix} = J \begin{bmatrix} y \\ u_1 \end{bmatrix}.$$

It is also easy to show that the transfer matrix from u_1 to y_1 is $T_{22} = 0$.

This diagram of the parameterization of all stabilizing controllers also suggests an interesting interpretation: Every internal stabilization amounts to adding stable dynamics to the plant and then stabilizing the extended plant by means of an observer. The precise statement is as follows: For simplicity of the formulas, only the cases of strictly proper G_{22} and K are treated.

Theorem 11.5 *Assume that G_{22} and K are strictly proper and the system in Figure 11.1 is internally stable. Then G_{22} can be embedded in a system*

$$\left[\begin{array}{c|c} A_e & B_e \\ \hline C_e & 0 \end{array}\right]$$

where

$$A_e = \begin{bmatrix} A & 0 \\ 0 & A_a \end{bmatrix}, \; B_e = \begin{bmatrix} B_2 \\ 0 \end{bmatrix}, \; C_e = \begin{bmatrix} C_2 & 0 \end{bmatrix} \tag{11.3}$$

and where A_a is stable, such that K has the form

$$K = \left[\begin{array}{c|c} A_e + B_eF_e + L_eC_e & -L_e \\ \hline F_e & 0 \end{array}\right] \tag{11.4}$$

where $A_e + B_eF_e$ and $A_e + L_eC_e$ are stable.

Proof. K is representable as in Figure 11.3 for some Q in $\mathcal{RH}_\infty$. For K to be strictly proper, Q must be strictly proper. Take a minimal realization of Q:

$$Q = \left[\begin{array}{c|c} A_a & B_a \\ \hline C_a & 0 \end{array}\right].$$

Since $Q \in \mathcal{RH}_\infty$, A_a is stable. Let x and x_a denote state vectors for J and Q, respectively, and write the equations for the system in Figure 11.3:

$$\begin{aligned} \dot{x} &= (A + B_2F + LC_2)x - Ly + B_2u_1 \\ u &= Fx + u_1 \\ y_1 &= -C_2x + y \\ \dot{x}_a &= A_ax_a + B_ay_1 \\ u_1 &= C_ax_a \end{aligned}$$

These equations yield

$$\begin{aligned} \dot{x}_e &= (A_e + B_eF_e + L_eC_e)x_e - L_ey \\ u &= F_ex_e \end{aligned}$$

where

$$x_e := \begin{bmatrix} x \\ x_a \end{bmatrix}, \; F_e := \begin{bmatrix} F & C_a \end{bmatrix}, \; L_e := \begin{bmatrix} L \\ -B_a \end{bmatrix}$$

and where A_e, B_e, C_e are as in equation (11.3). □

Example 11.1 Consider a standard feedback system shown in Figure 5.1 with $P = \dfrac{1}{s-1}$. We shall find all stabilizing controllers for P such that the steady-state errors with respect to the step input and $\sin 2t$ are both zero. It is easy to see that the controller must provide poles at 0 and $\pm 2j$. Now let the set of stabilizing controllers for a modified plant $\dfrac{(s+1)^3}{(s-1)s(s^2+2^2)}$ be K_m. Then the desired set of controllers is given by $K = \dfrac{(s+1)^3}{s(s^2+2^2)} K_m$.

11.3 Coprime Factorization Approach

In this section, all stabilizing controller parameterization will be derived using the conventional coprime factorization approach. Readers should be familiar with the results presented in Section 5.4 of Chapter 5 before preceding further.

Theorem 11.6 *Let $G_{22} = NM^{-1} = \tilde{M}^{-1}\tilde{N}$ be the rcf and lcf of G_{22} over $\mathcal{RH}_\infty$, respectively. Then the set of all proper controllers achieving internal stability is parameterized either by*

$$K = (U_0 + MQ_r)(V_0 + NQ_r)^{-1}, \quad \det(I + V_0^{-1}NQ_r)(\infty) \neq 0 \tag{11.5}$$

for $Q_r \in \mathcal{RH}_\infty$ or by

$$K = (\tilde{V}_0 + Q_l\tilde{N})^{-1}(\tilde{U}_0 + Q_l\tilde{M}), \quad \det(I + Q_l\tilde{N}\tilde{V}_0^{-1})(\infty) \neq 0 \tag{11.6}$$

for $Q_l \in \mathcal{RH}_\infty$, where $U_0, V_0, \tilde{U}_0, \tilde{V}_0 \in \mathcal{RH}_\infty$ satisfy the Bezout identities:

$$\tilde{V}_0 M - \tilde{U}_0 N = I, \qquad \tilde{M}V_0 - \tilde{N}U_0 = I.$$

Moreover, if $U_0, V_0, \tilde{U}_0$, and $\tilde{V}_0$ are chosen such that $U_0V_0^{-1} = \tilde{V}_0^{-1}\tilde{U}_0$; that is,

$$\begin{bmatrix} \tilde{V}_0 & -\tilde{U}_0 \\ -\tilde{N} & \tilde{M} \end{bmatrix} \begin{bmatrix} M & U_0 \\ N & V_0 \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$$

then

$$\begin{aligned} K &= (U_0 + MQ_y)(V_0 + NQ_y)^{-1} \\ &= (\tilde{V}_0 + Q_y \tilde{N})^{-1}(\tilde{U}_0 + Q_y \tilde{M}) \\ &= \mathcal{F}_\ell(J_y, Q_y) \end{aligned} \tag{11.7}$$

where

$$J_y := \begin{bmatrix} U_0 V_0^{-1} & \tilde{V}_0^{-1} \\ V_0^{-1} & -V_0^{-1} N \end{bmatrix} \tag{11.8}$$

and where Q_y ranges over $\mathcal{RH}_\infty$ such that $(I + V_0^{-1} N Q_y)(\infty)$ is invertible.

Proof. We shall prove the parameterization given in equation (11.5) first. Assume that K has the form indicated, and define

$$U := U_0 + MQ_r, \; V := V_0 + NQ_r.$$

Then

$$\tilde{M}V - \tilde{N}U = \tilde{M}(V_0 + NQ_r) - \tilde{N}(U_0 + MQ_r) = \tilde{M}V_0 - \tilde{N}U_0 + (\tilde{M}N - \tilde{N}M)Q_r = I.$$

Thus K achieves internal stability by Lemma 5.7.

Conversely, suppose K is proper and achieves internal stability. Introduce an rcf of K over $\mathcal{RH}_\infty$ as $K = UV^{-1}$. Then by Lemma 5.7, $Z := \tilde{M}V - \tilde{N}U$ is invertible in $\mathcal{RH}_\infty$. Define Q_r by the equation

$$U_0 + MQ_r = UZ^{-1}, \tag{11.9}$$

so

$$Q_r = M^{-1}(UZ^{-1} - U_0).$$

Then, using the Bezout identity, we have

$$\begin{aligned} V_0 + NQ_r &= V_0 + NM^{-1}(UZ^{-1} - U_0) \\ &= V_0 + \tilde{M}^{-1}\tilde{N}(UZ^{-1} - U_0) \\ &= \tilde{M}^{-1}(\tilde{M}V_0 - \tilde{N}U_0 + \tilde{N}UZ^{-1}) \\ &= \tilde{M}^{-1}(I + \tilde{N}UZ^{-1}) \\ &= \tilde{M}^{-1}(Z + \tilde{N}U)Z^{-1} \\ &= \tilde{M}^{-1}\tilde{M}VZ^{-1} \\ &= VZ^{-1}. \end{aligned} \tag{11.10}$$

Thus,

$$\begin{aligned} K &= UV^{-1} \\ &= (U_0 + MQ_r)(V_0 + NQ_r)^{-1}. \end{aligned}$$

To see that Q_r belongs to $\mathcal{RH}_\infty$, observe first from equation (11.9) and then from equation (11.10) that both MQ_r and NQ_r belong to $\mathcal{RH}_\infty$. Then

$$Q_r = (\tilde{V}_0 M - \tilde{U}_0 N)Q_r = \tilde{V}_0(MQ_r) - \tilde{U}_0(NQ_r) \in \mathcal{RH}_\infty.$$

Finally, since V and Z evaluated at $s = \infty$ are both invertible, so is $V_0 + NQ_r$ from equation (11.10), and hence so is $I + V_0^{-1}NQ_r$.

Similarly, the parameterization given in equation (11.6) can be obtained.

To show that the controller can be written in the form of equation (11.7), note that

$$(U_0 + MQ_y)(V_0 + NQ_y)^{-1} = U_0V_0^{-1} + (M - U_0V_0^{-1}N)Q_y(I + V_0^{-1}NQ_y)^{-1}V_0^{-1}$$

and that $U_0V_0^{-1} = \tilde{V}_0^{-1}\tilde{U}_0$. We have

$$(M - U_0V_0^{-1}N) = (M - \tilde{V}_0^{-1}\tilde{U}_0N) = \tilde{V}_0^{-1}(\tilde{V}_0M - \tilde{U}_0N) = \tilde{V}_0^{-1}$$

and

$$K = U_0V_0^{-1} + \tilde{V}_0^{-1}Q_y(I + V_0^{-1}NQ_y)^{-1}V_0^{-1}.$$

□

Corollary 11.7 *Given an admissible controller K with coprime factorizations $K = UV^{-1} = \tilde{V}^{-1}\tilde{U}$, the free parameter $Q_y \in \mathcal{RH}_\infty$ in Youla parameterization is given by*

$$Q_y = M^{-1}(UZ^{-1} - U_0)$$

where $Z := \tilde{M}V - \tilde{N}U$.

Next, we shall establish the precise relationship between the preceding all stabilizing controller parameterization and the state-space parameterization in the last section. The following theorem follows from some algebraic manipulation.

Theorem 11.8 *Let the doubly coprime factorizations of G_{22} be chosen as*

$$\begin{bmatrix} M & U_0 \\ N & V_0 \end{bmatrix} = \left[\begin{array}{c|cc} A + B_2F & B_2 & -L \\ \hline F & I & 0 \\ C_2 + D_{22}F & D_{22} & I \end{array}\right]$$

$$\begin{bmatrix} \tilde{V}_0 & -\tilde{U}_0 \\ -\tilde{N} & \tilde{M} \end{bmatrix} = \left[\begin{array}{c|cc} A + LC_2 & -(B_2 + LD_{22}) & L \\ \hline F & I & 0 \\ C_2 & -D_{22} & I \end{array}\right]$$

where F and L are chosen such that $A + B_2F$ and $A + LC_2$ are both stable. Then J_y can be computed as

$$J_y = \left[\begin{array}{c|cc} A + B_2F + LC_2 + LD_{22}F & -L & B_2 + LD_{22} \\ \hline F & 0 & I \\ -(C_2 + D_{22}F) & I & -D_{22} \end{array}\right].$$

Remark 11.3 Note that J_y is exactly the same as the J in Theorem 11.4 and that $K_0 := U_0 V_0^{-1}$ is an observer-based stabilizing controller with

$$K_0 := \left[\begin{array}{c|c} A + B_2 F + LC_2 + LD_{22}F & -L \\ \hline F & 0 \end{array} \right].$$

◇

11.4 Notes and References

The conventional Youla parameterization can be found in Youla et al. [1976a, 1976b], Desoer et al. [1980], Doyle [1984], Vidyasagar [1985], and Francis [1987]. The state-space derivation of all stabilizing controllers was reported in Lu, Zhou, and Doyle [1996]. The paper by Moore et al. [1990] contains some other related interesting results. The parameterization of all two-degree-of-freedom stabilizing controllers is given in Youla and Bongiorno [1985] and Vidyasagar [1985].

11.5 Problems

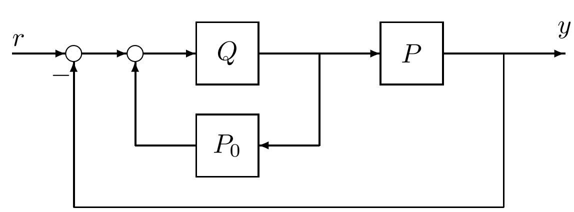
Problem 11.1 Let $P = \dfrac{1}{s-1}$. Find the set of all stabilizing controllers $K = \mathcal{F}_\ell(J, Q)$. Now verify that $K_0 = -4$ is a stabilizing controller and find a $Q_0 \in \mathcal{RH}_\infty$ such that $K_0 = \mathcal{F}_\ell(J, Q_0)$.

Problem 11.2 Suppose that $\{P_i : \ i = 1, \ldots, n\}$ is a set of MIMO plants and that there is a single controller K that internally stabilizes each P_i in the set. Show that there exists a single transfer function P such that the set

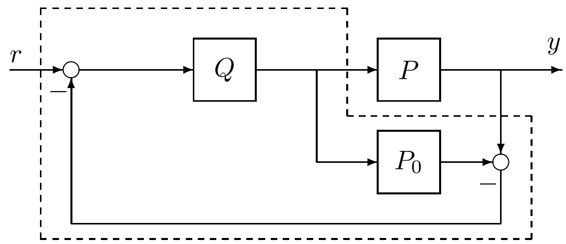
$$\mathcal{P} = \{\mathcal{F}_u(P, \Delta) \mid \Delta \in \mathcal{H}_\infty, \ \|\Delta\|_\infty \leq 1\}$$

is also robustly stabilized by K and that $\{P_i\} \subset \mathcal{P}$.

Problem 11.3 Internal Model Control (IMC): Suppose a plant P is stable. Then it is known that all stabilizing controllers can be parameterized as $K(s) = Q(I - PQ)^{-1}$ for all stable Q. In practice, the exact plant model is not known, only a nominal model P_0 is available. Hence the controller can be implemented as in the following diagram:



The control diagram can be redrawn as follows:



This control implementation is known as internal model control (IMC). Note that no signal is fed back if the model is exact. Discuss the advantage of this implementation and possible generalizations.

Problem 11.4 Use the Youla parameterization (the coprime factor form) to show that a SISO plant cannot be stabilized by a stable controller if the plant does not satisfy the parity interlacing properties. [A SISO plant is said to satisfy the parity interlacing property if the number of unstable real poles between any two unstable real zeros is even; $+\infty$ counts as a unstable zero if the plant is strictly proper. See Youla, Jabr, and Lu [1974] and Vidyasagar [1985].]