UNSTRUCTURED ANALYSIS THEOREM

Given NS & Perturbed Model Sets
Then Closed-Loop Robust Stability
if and only if Robust Stability Tests

# Chapter 8

## Uncertainty and Robustness

In this chapter we briefly describe various types of uncertainties that can arise in physical systems, and we single out ”unstructured uncertainties” as generic errors that are associated with all design models. We obtain robust stability tests for systems under various model uncertainty assumptions through the use of the small gain theorem. We also obtain some sufficient conditions for robust performance under unstructured uncertainties. The difficulty associated with MIMO robust performance design and the role of plant condition numbers for systems with skewed performance and uncertainty specifications are revealed. A simple example is also used to indicate the fundamental difference between the robustness of an SISO system and that of a MIMO system. In particular, we show that applying the SISO analysis/design method to a MIMO system may lead to erroneous results.

**笔记 1**

**包含关系:**

$$Q_d \subset \{\tilde{\Delta}_d(j\omega) : \|\tilde{\Delta}_d(s)\|_\infty \leq 1\}$$

但关键是：在计算最坏情况增益时，这个包含关系实际上是”充分”的。

**等价性定理:**

对于最坏情况增益计算，有：

$$\sup_{\tilde{\Delta}_d \in \Delta_d} \sigma(\mathcal{F}_u(D(j\omega), \tilde{\Delta}_d(j\omega))) = \sup_{Q_d \in Q_d} \sigma(\mathcal{F}_u(D(j\omega), Q_d))$$

这意味着：

- 虽然 $Q_d$ 是 $\tilde{\Delta}_d(j\omega)$ 的一个子集
- 但在寻找最大增益时，我们不会丢失任何信息
- 因为本质上这个 $Q_d$ 就是所有最坏增益对应的不确定性的集合

作者: JC　日期: 2025-10-16

**疑问 1**

**包含关系:**

$$Q_d \subset \{\tilde{\Delta}_d(j\omega) : \|\tilde{\Delta}_d(s)\|_\infty \leq 1\}$$

但关键是：在计算最坏情况增益时，这个包含关系实际上是”充分”的。

**等价性定理:**

对于最坏情况增益计算，有：

$$\sup_{\tilde{\Delta}_d \in \Delta_d} \sigma(\mathcal{F}_u(D(j\omega), \tilde{\Delta}_d(j\omega))) = \sup_{Q_d \in Q_d} \sigma(\mathcal{F}_u(D(j\omega), Q_d))$$

这意味着：

- 虽然 $Q_d$ 是 $\tilde{\Delta}_d(j\omega)$ 的一个子集
- 但在寻找最大增益时，我们不会丢失任何信息
- 因为本质上这个 $Q_d$ 就是所有最坏增益对应的不确定性的集合

作者: JC　日期: 2025-10-16

## 8.1 Model Uncertainty

Most control designs are based on the use of a design model. The relationship between models and the reality they represent is subtle and complex. A mathematical model provides a map from inputs to responses. The quality of a model depends on how closely its responses match those of the true plant. Since no single fixed model can respond exactly like the true plant, we need, at the very least, a set of maps. However, the modeling problem is much deeper - the universe of mathematical models from which a **model set** is chosen is distinct from the universe of physical systems. Therefore, a model set that includes the true physical plant can never be constructed. It is necessary for the engineer to make a leap of faith regarding the applicability of a particular design based on a mathematical model. To be practical, a design technique must help make this leap small by accounting for the inevitable inadequacy of models. A good model should be simple enough to facilitate design, yet complex enough to give the engineer confidence that designs based on the model will work on the true plant.

The term uncertainty refers to the differences or errors between models and reality,
and whatever mechanism is used to express these errors will be called a representation of uncertainty. Representations of uncertainty vary primarily in terms of the amount of structure they contain. This reflects both our knowledge of the physical mechanisms that cause differences between the model and the plant and our ability to represent these mechanisms in a way that facilitates convenient manipulation. For example, consider the problem of bounding the magnitude of the effect of some uncertainty on the output of a nominally fixed linear system. A useful measure of uncertainty in this context is to provide a bound on the power spectrum of the output's deviation from its nominal response. In the simplest case, this power spectrum is assumed to be independent of the input. This is equivalent to assuming that the uncertainty is generated by an additive noise signal with a bounded power spectrum; the uncertainty is represented as additive noise. Of course, no physical system is linear with additive noise, but some aspects of physical behavior are approximated quite well using this model. This type of uncertainty received a great deal of attention in the literature during the 1960s and 1970s, and elegant solutions are obtained for many interesting problems (e.g., white noise propagation in linear systems, Wiener and Kalman filtering, and LQG optimal control). Unfortunately, LQG optimal control did not address uncertainty adequately and hence had less practical impact than might have been hoped.

Generally, the deviation's power spectrum of the true output from the nominal will depend significantly on the input. For example, an additive noise model is entirely inappropriate for capturing uncertainty arising from variations in the material properties of physical plants. The actual construction of model sets for more general uncertainty can be quite difficult. For example, a set membership statement for the parameters of an otherwise known FDLTI model is a highly structured representation of uncertainty. It typically arises from the use of linear incremental models at various operating points (e.g., aerodynamic coefficients in flight control vary with flight environment and aircraft configurations, and equation coefficients in power plant control vary with aging, slag buildup, coal composition, etc.). In each case, the amounts of variation and any known relationships between parameters can be expressed by confining the parameters to appropriately defined subsets of parameter space. However, for certain classes of signals (e.g., high-frequency), the parameterized FDLTI model fails to describe the plant because the plant will always have dynamics that are not represented in the fixed order model.

In general, we are forced to use not just a single parameterized model but model sets that allow for plant dynamics that are not explicitly represented in the model structure. A simple example of this involves using frequency domain bounds on transfer functions to describe a model set. To use such sets to describe physical systems, the bounds must roughly grow with frequency. In particular, at sufficiently high frequencies, phase is completely unknown (i.e., $\pm 180°$ uncertainties). This is a consequence of dynamic properties that inevitably occur in physical systems. This gives a less structured representation of uncertainty.

**笔记 2**

不确定性的非结构化表示如下：下面一个加性不确定性一个乘性不确定性假设 P 是标称模型，K 是控制器标称稳定性（NS）：如果 K 能使得标称模型 P 稳定鲁棒稳定性（RS）：如果 K 能使得一族不确定性系统模型 Π 中的每个系统都稳定标称性能（NP）：如果标称系统 P 的性能目标得到满足鲁棒性能（RP）：如果 Π 中的每个系统都满足性能指标

作者: JC　日期: 2025-10-18

Examples of less structured representations of uncertainty are direct set membership statements for the transfer function matrix of the model. For instance, the statement

$$P_\Delta(s) = P(s) + W_1(s)\Delta(s)W_2(s), \quad \bar{\sigma}[\Delta(j\omega)] < 1, \forall\omega \geq 0, \tag{8.1}$$

where $W_1$ and $W_2$ are stable transfer matrices that characterize the spatial and frequency structure of the uncertainty, confines the matrix $P_\Delta$ to a neighborhood of the nominal model $P$. In particular, if $W_1 = I$ and $W_2 = w(s)I$, where $w(s)$ is a scalar function, then $P_\Delta$ describes a disk centered at $P$ with radius $w(j\omega)$ at each frequency, as shown in Figure 8.1. The statement does not imply a mechanism or structure that gives rise to $\Delta$. The uncertainty may be caused by parameter changes, as mentioned previously or by neglected dynamics, or by a host of other unspecified effects. An alternative statement to equation (8.1) is the so-called multiplicative form:

$$P_\Delta(s) = (I + W_1(s)\Delta(s)W_2(s))\,P(s). \tag{8.2}$$

This statement confines $P_\Delta$ to a normalized neighborhood of the nominal model $P$. An advantage of equation (8.2) over (8.1) is that in equation (8.2) compensated transfer functions have the same uncertainty representation as the raw model (i.e., the weighting functions apply to $PK$ as well as $P$ ). Some other alternative set membership statements will be discussed later.

**笔记 3**

nyqusit 图表示加性不确定性：标称模型 $P(j\omega)$ 是图 8.1 里的弧线，不确定性大小 $W(j\omega)$ 用频率点的圆的半径表示（周老师说加性不确定性是这样的，为什么是这样的，乘性不确定性如果要在这个图上表示会有什么区别）

作者: JC　日期: 2025-10-18

The best choice of uncertainty representation for a specific FDLTI model depends, of course, on the errors the model makes. In practice, it is generally possible to represent some of these errors in a highly structured parameterized form. These are usually the low-frequency error components. There are always remaining higher-frequency errors, however, which cannot be covered this way. These are caused by such effects as infinite-dimensional electromechanical resonance, time delays, diffusion processes, etc. Fortunately, the less structured representations, such as equations (8.1) and (8.2), are well suited to represent this latter class of errors. Consequently, equations (8.1) and
(8.2) have become widely used ”generic” uncertainty representations for FDLTI models. An important point is that the construction of the weighting matrices $W_1$ and $W_2$ for multivariable systems is not trivial.

**笔记 4**

作者: JC　日期: 2025-10-20

Motivated from these observations, we will focus for the moment on the multiplicative description of uncertainty. We will assume that $P_\Delta$ in equation (8.2) remains a strictly proper FDLTI system for all $\Delta$. More general pertur-