Mathematical Writing

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Abstract

This article is written for a course on mathematical writing that lasts about an hour. This lecture is mainly based on Prof. Knuth's extraordinary article *Mathematical Writings*.

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1 Introduction: Why Should We Learn It

In this lecture, we will mainly discuss topics in mathematical writing, accompanied by examples that will hopefully help you better understand the concepts.

Before we move on to serious and overwhelming details about mathematical writing, I would like to spend some time discussing motivations for learning this course.

Everyone who hopes to learn new things about Mathematics should find their own goals before diving into the details, or they would lose themselves otherwise. Questions like "Why should we be here and spend your prestigious time learning how to write mathematical essays?" and "Are there any differences from those we are familiar with?" are of great significance.

One can easily discover that formal mathematical articles tend to contain more formulas, definitions, and theorems than those in other areas. Therefore, it is crucial to learn the following in mathematical writing courses.

1. The art of writing formulas and other mathematical objects in an elegant and amusing way.

2. Methods for organizing your materials well and displaying your ideas fluently and logically.

Inspired by the above analysis, I am convinced that this lecture should revolve around topics in mathematical writing styles and commonly-accepted regulations about mathematical writing.

2 General Principles

First, we will go through some general regulations of mathematical writing.

2.1 Some Basic Principles

Here are some general principles in mathematical writing that you should beware of, which are extremely important from your instructor's perspective.

1. Symbols in different formulas must be separated by words.
   Bad: Consider S_q, $q \leq p$.
   Good: Consider S_q, where $q \leq p$.

2. Don't start a sentence with a symbol.
   Bad: $x^n - a$ has n distinct zeros.
   Good: The polynomial $x^n - a$ has n distinct zeros.

3. Don't use the symbols $\therefore, \Rightarrow, \forall, \exists, \ni$; replace them by the corresponding words.

4. The statement just preceding a theorem, algorithm, etc. should be a complete sentence or should end with a colon.
   Bad: We now have the following
   Theorem. $H(x)$ is continuous.
   Good: We can now prove the following result
   Theorem. The function $H(x)$ defined in (5) is continuous.

5. The statement of a theorem should usually be self-contained, not depending on the assumptions in the preceding text.

6. The word "we" is often useful to avoid passive voice; in most technical writing, "I" should be avoided, unless the author's persona is relevant.

7. There is a definite rhythm in sentences. Read what you have written and change the wording if it does not flow smoothly.

8. Try to state things twice, in complementary ways, especially when giving a definition. This reinforces the reader's understanding.

9. Motivate the reader for what follows.

10. Don't use the same notation for two different things.

11. Display important formulas on a line by themselves.

12. Sentences should be readable from left to right without ambiguity.

13. Don't say "which" when "that" sounds better.
Bad: Don't use commas which aren't necessary.
Good: Don't use commas that aren't necessary.

14. Avoid using long strings of nouns as adjectives.
Bad: If $L^+(P, N_0)$ is the set of functions $f : P \to N_0$ with the property that

$$\underset{n_0 \in N_0}{\exists} \quad \underset{p \in P}{\forall} \quad p \geq n_0 \Rightarrow f(p) = 0 \tag{1}$$

then there exists a bijection $N_1 \to L^+(P, N_0)$ such that if $n \to f$ then

$$n = \prod_{p \in P} p^{f(p)}. \tag{2}$$

Here P is the prime numbers and $N_1 = N_0 \sim \{0\}$.
Good: According to the 'fundamental theorem of arithmetic' (proved in ex. 1.2.4–21), each positive integer u can be expressed in the form

$$u = 2^{u_2} 3^{u_3} 5^{u_5} 7^{u_7} 11^{u_{11}} \cdots = \prod_{p\ prime} p^{u_p}, \tag{3}$$

where the exponents u_2, u_3, $\cdots$ are uniquely determined nonnegative integers, and where all but a finite number of the exponents are zero.

15. Small numbers should be spelled out when used as adjectives, but not when used as names.
Bad: The method requires 2 passes.
Good: Method 2 is illustrated in Fig. 1; it requires 17 passes. The count was increased by 2. The leftmost 2 in the sequence was changed to a 1.

2.2 Examples

Let's apply the principles we have learned to improve to the following solution.

Problem : $x_1 > 0$, $x_{n+1} = x_n + \frac{1}{\sqrt{n}x_n}$. Calculate $\lim_{n\to\infty} \frac{x_n^2}{\sqrt{n}}$.

Original version :

By induction, we have

$$x_n > x_{n-1} > 0.$$

Since

$$x_{n+1} = x_n + \frac{1}{\sqrt{n}x_n} \Rightarrow x_{n+1}^2 - x_n^2 = \frac{1}{nx_n^2} + \frac{2}{\sqrt{n}}.$$

Hence,

$$\begin{aligned}
&\lim_{n\to\infty} \frac{x_n^2}{\sqrt{n}} \\
&= \lim_{n\to\infty} \frac{x_{n+1}^2 - x_n^2}{\sqrt{n+1} - \sqrt{n}} \\
&= \lim_{n\to\infty} \frac{\frac{1}{nx_n^2} + \frac{2}{\sqrt{n}}}{\sqrt{n+1} - \sqrt{n}} \\
&= \lim_{n\to\infty} (\frac{1}{nx_n^2} + \frac{2}{\sqrt{n}})(\sqrt{n+1} + \sqrt{n}) \\
&= \lim_{n\to\infty} (\frac{1}{\sqrt{n}x_n^2} + 2)(1 + \sqrt{1 + \frac{1}{n}}) \\
&= 4.
\end{aligned}$$

Revised version :

When $n = 1$, we have

$$x_2 = x_1 + \frac{1}{x_1} > x_1 > 0.$$

By induction, we have

$$x_n > x_{n-1} > \cdots > x_1 > 0.$$

From

$$x_{n+1} = x_n + \frac{1}{\sqrt{n}x_n},$$

we have

$$x_{n+1}^2 - x_n^2 = \frac{1}{nx_n^2} + \frac{2}{\sqrt{n}}.$$

By Stolz's Theorem,

$$
\begin{aligned}
&\lim_{n\to\infty}\frac{x_n^2}{\sqrt{n}}\\
&=\lim_{n\to\infty}\frac{x_{n+1}^2-x_n^2}{\sqrt{n+1}-\sqrt{n}}\\
&=\lim_{n\to\infty}\frac{\frac{1}{nx_n^2}+\frac{2}{\sqrt{n}}}{\sqrt{n+1}-\sqrt{n}}\\
&=\lim_{n\to\infty}(\frac{1}{nx_n^2}+\frac{2}{\sqrt{n}})(\sqrt{n+1}+\sqrt{n})\\
&=\lim_{n\to\infty}(\frac{1}{\sqrt{n}x_n^2}+2)(1+\sqrt{1+\frac{1}{n}})\\
&=4.
\end{aligned}
$$

3 Common mistakes

Here are some common mistakes that often appear in many naive mathematicians' articles.

3.1 Articles

1. "Derivatives"
   Wrong: The function $-e^{-x}$ is derivative of e^{-x}. The function $-e^{-x}$ is a derivative of e^{-x}.
   Right: The function $-e^{-x}$ is the derivative of e^{-x}.

2. "Such"
   Wrong: Such operator is defined by $\cdots$
   Right: Such an operator is defined by $\cdots$

3. "Section"
   Wrong: In the Section 2.
   Right: In Section 2.

4. Singular or plural forms
   Wrong: There is a finite number of elements such that $\cdots$
   Right: There are a finite number of elements such that $\cdots$

5. Syntax of verbs
   Wrong: Let F denotes a function such that $\cdots$
   Right: Let F denote a function such that $\cdots$

3.2 Pronunciations

1. impliment (Wrong) vs implement (Right)
2. compliment (Wrong) vs complement (Right)
3. occurence (Wrong) vs occurrence (Right)
4. auxilary (Wrong) vs auxiliary (Right)
5. preceeding (Wrong) vs preceding (Right)

3.3 Grammar

1. "join $\cdots$ to"
   Wrong: We can join a with b by a path π.
   Right: We can join a to b by a path π.
2. "contradicts"
   Wrong: $\cdots$, which contradicts to Theorem 2.
   Right: $\cdots$, which contradicts Theorem 2.
3. "Disjoint"
   Wrong: Disjoint with X
   Right: Disjoint from X
4. "coefficient"
   Wrong: The coefficient by x^3 in the expansion
   Right: The coefficient of x^3 in the expansion
5. "greater than or equal to"
   Wrong: Then F is greater or equal to 3.
   Right: Then F is greater than or equal to 3.

4 Writing Styles

Now we turn to learning mathematical writing styles.

4.1 Why should we improve our writing styles

Here are some reasons why you should improve your writing style.

1. Grab your readers' attention immediately.
2. Convey your idea fluently.
3. Help your readers grasp the main idea.
4. Tell your readers about the motivations.
5. Foster further learning and research.

4.2 Some Advice

Here are some recommendations to improve your writing style.

1. Read masterpieces and learn how famous mathematicians develop their articles.
2. Plan your article. Make a map of how your article develops. Figure out the connections among the proofs and organize all your theorems logically.
3. Modify your languages to make them fluent.
4. Add more examples and paraphrase your definitions. Write about your own understanding when necessary.
5. Keep your article audience-oriented.

5 References

For those who are seeking more detailed and advanced materials, here are some notes that Prof. Knuth recommended in his lecture notes *Mathematical Writing*:

1. *The Elements of Style* by Shrunk & White
2. *Writing Mathematics Well* by Leonard Gillman
3. *How to Write Mathematics*, American Mathematical Society, 1973